The Transmission of Monetary Policy Operations through Redistributions and Durable Purchases

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Abstract

A large literature has documented statistically significant effects of monetary policy on economic activity. The central explanation for how monetary policy transmits to the real economy relies critically on nominal rigidities, which form the basis of the New Keynesian (NK) framework. This paper studies a different transmission mechanism that operates even in the absence of nominal rigidities. We show that in an OLG setting, standard open market operations (OMO) carried by central banks have important revaluation effects that alter the level and distribution of wealth and the incentives to work and save for retirement. Specifically, expansionary OMO lead households to front-load their purchases of durable goods and work and save more, thus generating a temporary boom in durables, followed by a bust. The mechanism can account for the empirical responses of key macroeconomic variables to monetary policy interventions. Moreover, the model implies that different monetary interventions (e.g., OMO versus helicopter drops) can have different qualitative effects on activity. The mechanism can thus complement the NK paradigm. We study an extension of the model incorporating labor market frictions.

JEL Codes: E1, E52, E58, E32, E31.

1 Introduction

A central question in monetary economics is how monetary policy interventions transmit to the real economy. As emphasized by Woodford (2012) in his influential Jackson Hole symposium paper, in standard modern, general-equilibrium, frictionless asset pricing models, open-market purchases of securities by central banks have no effect on the real economy. This result, which goes back to Wallace (1981)'s seminal article, is at odds with the widely held view that open-market operations (OMO) by central banks affect interest rates—and at odds indeed with the very practice of central banks.

The irrelevance result is easiest to see in the context of a representative agent model, as clearly explained by Woodford (2012);[1] however, Wallace (1981)'s widely cited result applies to a more general setting with heterogeneous agents. A key premise for Wallace's irrelevance result, however, is that OMO by the central bank are accompanied by fiscal transfers that ensure no change in the income distribution following the policy intervention. In other words, by construction, distributional effects of OMO are muted by fiscal transfers that neutralize distributional changes—and hence preclude any change in individuals's decisions following the intervention.[2]

The goal of this paper is to study the effects of monetary policy interventions when, realistically, OMO are not accompanied by neutralizing fiscal transfers—nor is there a complete set of state-contingent securities that would ensure an unchanged income distribution. The motivation is necessarily a practical one. When researchers estimate the effects of (exogenous) monetary policy interventions, they do not (cannot) abstract from or control for the

[1] Suppose the central decides to sell a risky asset (an asset with lower return in a bad state); one would think the private sector would be in principle only willing to buy it at a lower price. However, note that in the settings analyzed by Woodford (2012), even if the central bank keeps the risky asset, the risk does not disappear from the economy. The central bank's earning on its portfolio are lower in the bad state and this means lower earnings distributed to the Treasury (and hence higher taxes to be collected from the private sector). So the representative agent's after-tax income is equally dependent on the real-estate risk as before. Thus asset prices are unaffected by the open-market operation.

[2] Wallace (1981) refers to this condition as "unchanged fiscal policy". An unchanged fiscal policy for him is one in which there is no change in government consumption and no change in distribution. Note that this means that to implement his OMO without the redistributional effects, a Central Bank needs to ask the government to manipulate transfers in a particular way to keep the income distribution unchanged. An alternative way of obtaining this result would be to have a complete set of contingent securities that would undo any change in the income distribution.

distributional effects they cause—and there is no accompanying fiscal policy that undoes them. Hence, to understand the effects of those interventions on activity, researchers need to take into account the potential impact of the redistribution caused by the policy intervention. Indeed, Doepke and Schneider (2006)'s empirical study points to a significant revaluation of assets and redistributional effects from retired, old agent to younger generations following monetary expansions. Relatedly, Coibion et al (2012) find that monetary expansions reduce inequality, as measured by Gini coefficients, suggesting a redistribution away from wealthier individuals.

Under these premises, we analyse the effect of OMO on economic activity in a dynamic, stochastic, general-equilibrium (DSG) model. In the model, overlapping generations (OLG) of households use money, bonds, and durable goods to smooth consumption over their life cycle as well as the business cycle. In the baseline model, we assume that all prices are flexible and there are no labour market frictions. (In extensions of the model, we study the impact of labour frictions due to search and matching and wage rigidities.) The inclusion of durable goods is motivated by the empirical finding that the response of economic activity is primarily driven by the response of the durable-good sector (i.e., durable goods, residential and business investment), whereas the response of non-durable consumption is weak at best. The inclusion of government bonds is aimed at representing the standard OMO, entailing sales and purchases of Treasury bonds, that is, changes in the composition of the central bank's balance sheet. Moreover, we realistically assume that the central bank transfers its interest income to the Treasury. We also study expansions in the size of the balance sheet, akin to "helicopter drops," i.e. tax cuts financed by an increase in the money supply, and analyse differences in the macroeconomic impact of these two policies.

We show that in such setting, a monetary expansion carried out through OMO triggers a durable-driven boom in output, even under fully flexible prices and wages.[3] The operation increases the central bank's bonds holdings and consequently its interest revenues; the higher revenues in turn lead to an increase in transfers to households via the Treasury.[4] Moreover,

[3] In a representative-agent model with durables, this nominal flexibility will immediately lead to money neutrality.

[4] As in reality, the model assumes that the Central Bank's interest revenues are remitted to the Treasure. In addition, the model assumes that interest revenues received by the Treasury are fully rebated to agents.

this leads to an increase in prices and a downward revaluation of nominal wealth. These effects alter the distribution of wealth across generations, benefiting in particular future tax payers. Because of this redistribution, currently young agents who suffer from the negative revaluation of their money holdings, decide to work and save more for retirement, causing a decrease in the nominal and real interest rate. This drives up the demand for durable goods, as durables become a more appealing way of saving (money and bonds suffer the erosion from the inflation tax) and relatively more appealing than non-durables. (Retired old agents who suffer a bigger downward revaluation of their wealth cannot re-enter the labour market and thus become the biggest losers from the intervention.) In the baseline model the output increase is driven by the labour-supply response. In the extension with search and matching frictions in the labour market, the increase in output is instead driven by an increase in firms' labour demand.

To understand the importance of agents' life cycle savings considerations, we also study a limit case of our baseline model with an infinitely-lived representative agent. In this limit case, monetary neutrality is obtained, as in Sidrauski (1967). This is because agents suffering a revaluation effect on their financial assets are compensated in equal amounts by current and future transfers from a fiscal authority rebating lump-sum transfers, thus precluding wealth effects and any change in behaviour. In the absence of nominal rigidities, real wages and relative prices are thus entirely determined by real factors. Nominal wage income and durable good prices therefore increase in tandem in the presence of inflation, and the increase in nominal wage income exactly offsets the desire to bring forward durable good purchases. This is true even though inflation does reduce the real value of financial wealth.[5] Money neutrality in our model obtains under the same conditions in which Ricardian Equivalence holds (Barro 1974). By (realistically) precluding risk sharing of aggregate monetary policy shocks across generations, the model yields money non-neutrality even with flexible prices.[6]

[5]Recall the assumption that the government makes lump-sum transfers from seignioriage revenuesto to agents. Following Weil (1991)'s arguments, based on an endowment economy with helicopter drops, we show that also in an economy with production and durable goods, the reduction in wealth caused by OMO, is exactly offset by future increases in government transfers, which renders money neutral.

[6]Allowing for fiscal transfers to exactly offset the heterogenous effects of monetary policy across different agents would restore the money neutrality in our model. Realistically, however, monetary policy shocks are not accompanied by targeted fiscal transfers aimed at undoing the monetary effects. Hence, to interpret

In sharp contrast, expansionary helicopter drops in the baseline model that generate a comparable fall in the nominal interest rate as in the OMO, cause a bust in durables and a fall in output and hours, though the effects are quantitatively small. In other words, two interventions causing a similar fall in nominal interest rates lead to markedly different effects in economic activity.[7]

Our model thus offer a setting consistent with both i) the way in which Central Banks and in particular the Fed affects its policy rate, i.e., mostly through OMO and ii) empirical estimates on how such changes affects the macroeconomy and more specifically, the durable good sector. As a by-product, our results speak to a criticism fired by Barsky, House and Kimball (2003, 2007) against the standard New Keynesian (NK) representative-agent model. The authors ague that the standard NK model generate counterfactual responses of durable and non-durable consumption to monetary policy interventions—and, moreover, under reasonable parametrizations, counterfactual responses for aggregate output. Specifically, when durable goods' prices are relatively flexible, as appears to be the case in the data, these models predict that following a monetary expansion, non-durable purchases increase, while durable purchases, remarkably, decrease. And, indeed, in the case of fully flexible durable prices, the predicted contraction in the durable goods producing sector is so large that the monetary expansion has almost no effect on total aggregate output.[8,9] By introducing "retire-

the data and in particular the empirical evidence on the effect of monetary policy interventions, one cannot assume away the redistributional effets of monetary policy.

[7]The difference is driven by the distributional effects of the two policies. Since we assume a (delayed) balanced budget, a monetary expansion generated by a one-off helicopter drop leads to transfers that are immediately rebated to working agents. So, the loss generated by the revaluation effect on the working agents is more than compensated by the transfers, thus generating a positive wealth effect and a fall in labour and output. For the old, instead, the revaluation effect, as in the OMO case, leads to a negative wealth effect.

[8]The literature typically focuses on completely flexible durable prices, for which the comovement problem is most severe. Bils and Klenow (2004) report a median price duration of only two months for new cars. Barsky et al. (2007) argue that prices of new homes are highly flexible. A key result in Barsky et al. (2007) is that the rigidity of durable good prices plays a crucial role: if these prices are flexible, monetary policy is nearly neutral, irrespectively of the rigidity of non-durable good prices.

[9]These predictions are in sharp contrast with conventional wisdom and the empirical evidence for the U.S. economy. As we discuss in the next Section, the response of GDP to monetary policy changes is driven almost entirely by the response of durable goods consumption and residential investment. These two items are the most sensitive components of GDP, increasing fast and sharply in response to a monetary expansion. In contrast, non durable goods and services consumption show virtually no response to monetary policy interventions. We shall go back to this in the empirical section, where we carefully document these responses. See for example Bernanke and Gertler (1995) and Barsky et al.(2003),

ment" our model provides a mechanism that counteracts the channel highlighted by Barsky et al (2007) and can thus help the NK model in mimicking the response of the economy. As such, our model can complement standard NK by adding a realistic feature and restoring its ability to match the data.

In the Appendix we further extend the model to allow for search and matching frictions in the labour market and real wage rigidities. In this version of the model, the increase in employment due to the monetary expansion is caused by an increase in the demand for workers, rather than by the increased labour supply, as in our baseline model. The introduction of these frictions—together with other frictions we abstract from— can help fine-tune the model to better match the data. For expositional clarity and to focus on the value-added, however, we keep the baseline model relatively simple.

The paper is organized as follows. Section 2 reviews the main empirical facts that motivate our model. Section 3 introduces the model. Section 4 performs various numerical exercises. and discusses the findings in light of the empirical evidence. Section 5 offers concluding remarks.

2 Empirical Evidence

This Section first revisits the aggregate effects of monetary policy on the macroeconomy, highlighting the role of durables. It then reviews the evidence on redistributional effects.

2.1 Monetary expansions and the response of durables

Policy and academic discussions on the economic effects of monetary policy interventions often rely on the relatively high sensitivity of the durables sector to interest rate changes. We corroborate this premise by studying the U.S. evidence from 1966 until 2007 using a standard recursive VAR approach. In the Supplemental Appendix we study the data using Romer and Romer (2004)'s approach to identify exogenous monetary policy interventions. The two methods show that monetary expansions lead to temporary booms in durables, with little or no response of non-durables. This motivates the introduction of durables in our model, as the key variable driving the response of output. We discuss the data and estimation next.

The empirical analysis for measuring the effect of monetary policy shocks relies on a general linear dynamic model of the macroeconomy whose structure is given by the following system of equations (see for example, Olivei and Tenreyro, 2007):

$$\mathbf{Y}_t = \sum_{s=0}^{S} \mathbf{B}\mathbf{Y}_{t-s} + \sum_{s=1}^{S} \mathbf{C}p_{t-s} + \mathbf{A}^y\mathbf{v}_t^y \tag{1}$$

$$p_t = \sum_{s=0}^{S} \mathbf{D}_s\mathbf{Y}_{t-s} + \sum_{s=1}^{S} \mathbf{g}_s\ p_{t-s} + v_t^p. \tag{2}$$

Boldface letters are used to indicate vectors or matrices of variables or coefficients. In particular, $\mathbf{Y}_t$ is a vector of non-policy macroeconomic variables (e.g., output, durable and non-durable consumption, aggregate and relative sectoral prices), and p_t is the scalar variable that summarizes the policy stance. We take the federal funds rate as our measure of policy, and use innovations in the federal funds rate as a measure of monetary policy shocks. Federal Reserve operating procedures have varied in the past 40 years, but several authors have argued that funds-rate targeting provides a good description of Federal Reserve policy over most of the period (see Bernanke and Blinder, 1992, and Bernanke and Mihov, 1998). Equation (1) allows the non-policy variables $\mathbf{Y}_t$ to depend on both current and lagged values of $\mathbf{Y}$, on lagged values of p, and on a vector of uncorrelated disturbances $\mathbf{v}^y$. Equation (2) states that the policy variable p_t depends on both current and lagged values of $\mathbf{Y}$, on lagged values of p, and on the monetary policy shock v^p.[10] As such, the system embeds the key restriction for identifying the dynamic effects of exogenous policy shocks on the various macro variables $\mathbf{Y}$: policy shocks do not affect macro variables within the current period. Although debatable, this identifying assumption is standard in VAR analyses.[11]

Given the identifying assumption that policy shocks do not affect macro variables within

[10] Policy shocks are assumed to be uncorrelated with the elements of $\mathbf{v}^y$. Independence from contemporaneous economic conditions is considered part of the definition of an exogenous policy shock. The standard interpretation of v^p is a combination of various random factors that might affect policy decisions, including data errors and revisions, preferences of participants at the FOMC meetings, politics, etc. (See Bernanke and Mihov 1998).

[11] See, among others, Bernanke and Blinder (1992), Bernanke and Mihov (1998), Christiano et al. (1999) and (2005), Jean Boivin and Marc Giannoni (2006), and Julio Rotemberg and Michael Woodford (1997).

the current period, we can rewrite the system in a standard VAR reduced-form, with only lagged variables on the right-hand side. The system can then be estimated equation-by-equation using ordinary least squares. The effect of policy innovations on the non-policy variables is identified with the impulse-response function of $\mathbf{Y}$ to past changes in v^p, with the federal funds rate placed last in the ordering.[12] An estimated series for the policy shock can be obtained via a Choleski decomposition of the covariance matrix of the reduced-form residuals.

In the benchmark estimation, we use seasonally adjusted data from 1966:Q1 to 2007:Q4. The beginning of the estimation period is dictated by the behavior of monetary policy. After 1965 the federal funds rate started to exceed the discount rate, becoming the primary instrument of monetary policy. We stop in 2007:Q4 to avoid concerns with the potentially confounding effects from the financial crisis and the zero-lower bound.

The non-policy variables in the system include real GDP, the GDP deflator, durable-sector consumption (including residential investment), non-durable consumption (including both non durable goods and services consumption), the relative price deflator of non-durables vis-a-vis durables, and an index of spot commodity prices.[13] As is now standard in the literature, the inclusion of the commodity price index in the system is aimed at mitigating the "price puzzle," whereby a monetary tightening initially coincides with an increasing rather than decreasing price level.

We estimate each equation in the reduced-form VAR separately by OLS. In our benchmark specification, all the variables in the vector $\mathbf{Y}$ are expressed in log levels. The policy variable, the federal funds rate, is expressed in levels. We formalize trends in the non-policy variables as deterministic, and allow for a linear trend in each of the equations of the VAR.

<<Figure 1 here>>

[12] The ordering of the variables in $\mathbf{Y}_t$ is irrelevant. Since identification of the dynamic effects of exogenous policy shocks on the macro variables $\mathbf{Y}$ only requires that policy shocks do not affect the given macro variables within the current period, it is not necessary to identify the entire structure of the model.

[13] The source for all aggregates and their deflators is the Bureau of Economic Analysis, Quarterly National Income and Product Accounts. The sectoral deflators are chain-weighted indexes of the real deflators for the individual sub-categories, with the weight being the nominal shares of the sub-category on the sector's consumption. The source for the spot commodity price index is the Commodity Research Bureau.

The estimated impulse-responses are depicted in Figure 1, together with 95 percent confidence bands around the estimated responses. We consider a monetary policy shock that corresponds to a 75-basis point decline in the funds rate on impact. For ease of comparison, the response of the three economic aggregates (GDP, durables and non-durables) to the shock are graphed on the same scale across plots. The top right panel shows that the GDP response to the policy shock is persistent, peaking around 7 quarters after the shock and slowly decaying thereafter. The top left plot shows the response of durables, which, as it is apparent, is both fast and sizable. Durables fall on impact, reaching a level close to its maximum response 3 quarters after the shock. Moreover, the peak response for durables is more than three times as large as the GDP response. In contrast, the response of non-durables, depicted in the center left panel, is virtually insignificant, with its peak response being less than a fourth of that for durables. The center right panel shows that, despite controlling for commodity prices, there is still a "price puzzle," although the increase in prices is not statistically significant. It takes about 7 quarters after the shock for prices to start falling. The bottom left panel shows that the relative price of durables over nondurables tends to increase following the monetary shock, thought the increase is not statistically significant. The response of the fed funds rate, shown in the bottom right panel, converges back to 0 around the 7th quarter after the shock.

The differences in the responses of durables and non durables are substantial from an economic standpoint. The policy shock leads to a fall in durables in the following 8 quarters by almost 135 basis points. In contrast, the fall in nondurables is less than 10 basis points.

As said, Appendix we carry out an alternative exercise based on Romer and Romer's narrative approach. The monetary policy shocks ε_t are quarterly averages of the monetary policy shocks identified by Romer and Romer (2004), extended through 2008 by Coibion et al (2012). Reassuringly, the results, illustrated in the Appendix, are qualitatively similar (and quantitatively close) to those obtained using the VAR approach. The Appendix also investigates the response of taxes following a monetary intervention, a response that becomes interesting in light of the model. With a monetary expansion, bond holdings held by the Central Bank increases—and so do its interest revenues. This leads to an increase in remittances from the Central Bank to the Treasury and thus to higher transfers (or lower taxes) to individuals.

2.2 Redistributional Effects of Monetary Policy

A main goal of our paper is to study the redistributional effects of monetary policy and their impact on aggregate variables in a quantitative model. Two empirical papers substantiate our motivation. The first paper is Doepke and Schneider (2006), which documents significant wealth redistribution in the US economy following (unexpected) inflationary episodes. The authors find that the main losers from a monetary expansion are rich, old households holding nominal bonds (as in our model). Their analysis is based on detailed data on assets and liabilities held by different segments of the population, from which they calculate the revaluation effects caused by inflation.

Our model will embed these redistributional revaluation effects and will bring two additional considerations to the analysis. The first consideration is how these redistributional effects alter the various demographic groups' incentives to work, consume, and save in different types of assets and how these changes affect the macroeconomy. The second consideration is how the Treasury redistributes the higher revenues stemming from the monetary policy intervention. These higher revenues consist of i) higher value of remittances received from the Central Bank as a result of the interest on bonds earned by the central bank; and ii) gains from the revaluation of government debt—assuming the government is a net debtor. The revaluation gains by the government can be large, as Doepke and Schneider (2006)'s calculations illustrate. The remittances are also considerable, amounting to an average of two percent of total government revenues during our period of analysis, with significant volatility. In the baseline model, we assume that these remittances are rebated to the young (working agents), as in practice the taxation burden tends to fall on the working population. However, the framework can be easily adjusted to allow for different tax-tranfer configurations.

A second empirical paper motivating our analysis is Coibion et al. (2012), who find that unexpected monetary contractions as well as permanent decreases in the inflation target lead to an increase in inequality in earnings, expenditures, and consumption. Their results rely on the CEX survey, and thus exclude top income earners. The authors however argue that their estimates provide lower bounds for the increase in inequality following monetary policy contractions. This is because individuals in the top one-percent of the income distribution receive a third of their income from financial assets—a much larger share than any other segment of the population; hence, the income of the top one-percent likely rises even more than for most other households following a monetary contraction.

Consistent with these findings, in our model, monetary policy contractions cause a redistribution of income to old agents, who rely more heavily on wealth, from young agents and future tax payers. The consumption of goods by the old agents also increases relative to that of young agents, consistent with Coibion et al. (2012)'s findings.

3 Open market operations in a model without nominal rigidities

We study the dynamic effects of monetary policy shocks in a general equilibrium model which embeds overlapping generations and a parsimonious life cycle structure with two stages: working life and retirement. Transitions from working life to retirement and from retirement to death are stochastic but obey fixed probabilities, following Gertler (1999). Financial markets are incomplete in the sense that there exists no insurance against risks associated with retirement and longevity. As a result, agents accumulate savings during their working lives, which they gradually deplete once retired. These savings can take the form of money, bonds and durable consumption goods.

The money supply is controlled by a central bank, who implements monetary policy using open market operations, that is, by selling or buying bonds. Realistically, we assume that the central bank transfers its profits to the treasury. The treasury in turn balances its budget by setting lump-sum transfers to households. In this environment we study the dynamic effects of persistent monetary policy shocks. We contrast our benchmark model with an alternative economy in which the central bank uses "helicopter drops" of money rather than open market operations to implement monetary policy.

We solve the model using a standard numerical method.[14] This may seem challenging

[14]Specifically, we use first-order perturbation, exploiting its certainty-equivalence property. See the ap-

given the presence of heterogeneous households and incomplete markets. In particular, the presence of aggregate fluctuations implies that a time-varying wealth distribution is part of the state of the macroeconomy. To render the model tractable, we introduce a government transfer towards newborn agents which eliminates inequality among young agents.[15] We show that aggregation then becomes straightforward and only the distribution of wealth between the group of young and old agents is relevant for aggregate outcomes. At the same time, our setup preserves the most basic life-cycle savings pattern: young agents save for old age and retired agents gradually consume their wealth.

Another advantage of our model with limited heterogeneity is that it nests a model with an infinitely-lived representative agent. One can show analytically that monetary policy shocks do not affect real activity under the representative agent assumption, provided that money and consumption enter the utility function separably.[16] This result is closely related to the fact that by construction redistributional effects are absent in an economy without heterogeneity. Also, the operating procedures of monetary policy (OMO versus helicopter drop) have the same effect on prices, a result that is broken down once we move beyond the representative agent assumption.

In the benchmark model discussed in this section we do not incorporate any form of product or labor market friction. Hence, the monetary transmission in the model is very different to the transmission in New Keynesian models, which typically abstract from demographics and household heterogeneity in wealth. In the Appendix we analyze the combined transmission of monetary policy shocks by introducing labor market frictions to the model.

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pendix for details.

[15] Wealth inequality among retired agents is preserved in our framework.

[16] This result by itself is unsuprising, as (super)neutrality results for representative agent models with productive durables, have been known since the seminal work of Sidrauski (1967) and Fischer (1979). Sidrauski (1967) shows that when money enters the utility function separably, the rate of inflation does not affect real outcomes in the steady state. Fischer (1979) shows that under logarithmic utility this is also true along transition paths. Under alternative utility functions this is generally not true, but in quantitative exercises deviations from neutrality are often found to be quantitatively small, see for example Danthine, Donaldson and Smith (1987). In our benchmark model we will assume logarithmic utility and thus focus on a different source of non-neutrality.

3.1 Model

3.1.1 Agents and demographics

We model a closed economy which consists of a continuum of households, a continuum of perfectly competitive firms and a government, which is comprised of a treasury and a central bank. In every period a measure of new young agents is born. Young agents retire and turn into old agents with a time-invariant probability $\rho_o \in [0, 1)$ in each period. Upon retirement, agents face a time invariant death probability $\rho_x \in (0, 1]$ in each period, including the initial period of retirement. The population size and distribution over the age groups remains constant over time and the total population size is normalized to one. The fraction of young agents in the economy, denoted ν, can be solved for by exploiting the implication that the number of agents retiring equals the number of deaths in the population, i.e.

$$\rho_o \nu = \rho_x \left(1 - \nu + \rho_o \nu\right). \tag{3}$$

The age status of an agent is denoted by a superscript $\mathbf{s} \in \{\mathbf{n}, \mathbf{y}, \mathbf{o}\}$, with $\mathbf{n}$ denoting a newborn young agent, $\mathbf{y}$ a pre-existing young agent, and $\mathbf{o}$ an old agent.

Households derive utility from non-durables, denoted $c \in \mathbb{R}^+$, a stock of durables, $d \in \mathbb{R}^+$, and real money balances, denoted $m \in \mathbb{R}^+$. They can also invest in nominal bonds, the real value of which we label $b \in \mathbb{R}$. Bonds pay a net nominal interest rate $r \in \mathbb{R}^+$.

Young agents, including the newborns, supply labor to firms on a competitive labor market whereas old agents are not productive. Durables depreciate at a rate $\delta \in (0, 1)$ per period and are produced using the same technology as non-durables. Because of the latter, durables and non-durables have the same market price. All agents take laws of motion of prices, interest rates, government transfers and idiosyncratic life-cycle shocks as given. We describe the decision problems of the agents in turn.

3.1.2 Old agents

Agents maximize expected lifetime utility subject to their budgets, taking the law of motion of the aggregate state, denoted by Γ, as given. Letting primes denote next period's variables, we can express the decision problem for old agents ($\mathbf{s} = \mathbf{o}$) recursively and in real terms as:

$$\begin{aligned}
& V^{\mathbf{o}}(a, \Gamma) = \max_{c,d,m,b} U(c, d, m) + \beta (1 - \rho_x) \mathbb{E} V^{\mathbf{o}}(a', \Gamma') \\
& s.t. \\
& c + d + m + b = a + \tau^{\mathbf{o}} \\
& a' \equiv (1 - \delta) d + \frac{m}{1 + \pi'} + \frac{(1 + r) b}{1 + \pi'}, \\
& c, d, m \geq 0,
\end{aligned} \tag{4}$$

where $V^{\mathbf{o}}(a, \Gamma)$ is the value function of an old agent which depends on the aggregate state and the real value of wealth, denoted by a, $\mathbb{E}$ is the expectation operator conditional on information available in the current period, $\beta \in (0, 1)$ is the agent's subjective discount factor, and $\pi \in \mathbb{R}$ is the net rate of inflation. $U(c, d, m)$ is a utility function and we assume that $U_j(c, d, m) > 0$, $U_{jj}(c, d, m) < 0$ and $\lim_{j \to 0} U_j(c, d, m) = \infty$ for $j = c, d, m$. Finally, $\tau^{\mathbf{s}} \in \mathbb{R}$ is a transfer from the government to an agent with age status $\mathbf{s}$, so $\tau^{\mathbf{o}}$ is the transfer to any old agent.

The budget constraint implies that old agents have no source of income other than from wealth accumulated previously. Implicit in the recursive formulation of the agent's decision problem is a transversality condition $\lim_{t \to \infty} \mathbb{E}_t \beta^t (1 - \rho_x)^t U_{c,t} x_t = 0$, where $x = d, m, b$ and where $U_{c,t}$ denotes the marginal utility of non-durable consumption. Finally, we assume that agents derive no utility from bequests and that the wealth of the deceased agents is equally distributed among the currently young agents.

3.1.3 Young agents

Young agents supply labor in exchange for a real wage $w \in \mathbb{R}^+$ per hour worked. The optimization problem for newborn agents ($\mathbf{s} = \mathbf{n}$) and pre-existing young agents ($\mathbf{s} = \mathbf{y}$) can be written as:

$$
\begin{aligned}
&V^{\mathbf{s}}(a,\Gamma) = \max_{c,d,m,b,h} U(c,d,m) - \zeta\frac{h^{1+\kappa}}{1+\kappa} + \beta\left(1-\rho_o\right)\mathbb{E}V^{\mathbf{y}}(a',\Gamma') + \beta\rho_o\left(1-\rho_x\right)\mathbb{E}V^{\mathbf{o}}(a',\Gamma') \\
&\mathbf{s} = \mathbf{n},\mathbf{y} \qquad (5) \\
&s.t. \\
&c + d + m + b = a + wh + \tau^{bq} + \tau^{\mathbf{s}}, \\
&a' \equiv \left(1-\delta\right)d + \frac{m}{1+\pi'} + \frac{\left(1+r\right)b}{1+\pi'}, \\
&c,d,m \geq 0,
\end{aligned}
$$

where young agents too obey transversality conditions. The term $\zeta\frac{h^{1+\kappa}}{1+\kappa}$ captures the disutility obtained from hours worked, denoted h, with $\zeta > 0$ being a scaling's parameter and $\kappa > 0$ being the Frisch elasticity of labor supply. Bequests from deceased agents are denoted τ^{bq}. Moreover, $\tau^{\mathbf{s}}$ is a again lump-sum transfer from the government. When making their optimal decisions, young agents take into account that in the next period they may be retired, which occurs with probability $\rho_o\left(1-\rho_x\right)$, or be deceased which happens with probability $\rho_o\rho_x$. We thus assume that upon retirement, young agents may be immediately hit by a death shock.

3.1.4 Firms

Goods are produced by a continuum of perfectly competitive and identical goods firms. These firms operate on a linear production function:

$$
y_t = h_t. \tag{6}
$$

Profit maximization implies that $w_t = 1$, that is, the real wage equals one.

3.1.5 Central bank

Although we do not model any frictions within the government, we make a conceptual distinction between a central bank conducting monetary policy and a treasury conducting fiscal policy. We make this distinction for clarity and in order to relate the model to real-world practice.

The central bank controls the nominal money supply, $M_t \in \mathbb{R}^+$, by conducting open market operations. In particular, the central bank can sell or buy government bonds. We denote the nominal value of the bonds held by the central bank by $B_t^{\mathbf{cb}} \in \mathbb{R}$. The use of these open market operations implies that in every given period the change in bonds held by the central bank equals the change in money in circulation, i.e.

$$B_t^{\mathbf{cb}} - B_{t-1}^{\mathbf{cb}} = M_t - M_{t-1}. \tag{7}$$

By implication, the size of the central bank's balance sheet, i.e. the total amount of its assets/liabilities, is kept constant over time. Correspondingly, the central bank transfers its accounting profit -typically called seigniorage- to the treasury.[17] The real value of the seigniorage transfer, labeled $\tau_t^{\mathbf{cb}} \in \mathbb{R}$, is given by:

$$\tau_t^{\mathbf{cb}} = \frac{r_{t-1} b_{t-1}^{\mathbf{cb}}}{1 + \pi_t}. \tag{8}$$

The above description is in line with how central banks conduct monetary policy, as well as with the typical arrangement between a central bank and the treasury. By contrast, many models of monetary policy assume monetary policy is implemented using "helicopter drops", i.e. expansions of the money supply that are not accompanied by a purchase of assets but instead by a fiscal transfer that is equal to change in the money supply. Modern monetary models are often silent on how monetary policy is implemented and directly specify an interest rate rule. In our framework, it is important to be careful about modeling the precise implementation of monetary policy since the associated monetary-fiscal arrangements pin down redistributional effects and hence the impact of changes in monetary policy on the real economy.

When we implement the model quantitatively, we simulate exogenous shocks to monetary policy, that is, unexpected open market operations. We do so by specifying a stochastic process that affects the growth rate of the money supply M_t.

[17] We abstract from operational costs incurred by the central bank.

3.1.6 Treasury

The treasury conducts fiscal policy. For simplicity, we abstract from government purchases of goods and assume that the treasury follows balanced budget policy, with the exception that we allow for some delay in transferring income to households.[18] The government has an initial level of bonds $B^{\mathbf{g}}_{t-1}$ which gives rise to interest income (or expenditure if the government has debt) on top of the seigniorage transfer from the central bank. To balance its budget, the government makes lump-sum transfers to the households, which can be either positive or negative. Letting $j \geq 0$ denote the transfer delay, the government's budget constraint can be written as:

$$\frac{r_{t-j-1} b^{\mathbf{g}}_{t-j-1}}{1+\pi_{t-j}} + \tau^{\mathbf{cb}}_{t-j} = \nu \rho_o \tau^{\mathbf{n}}_t + \nu \left(1-\rho_o\right) \tau^{\mathbf{y}}_t + \left(1-\nu\right) \tau^{\mathbf{o}}_t \tag{9}$$

Here, $\nu \rho_o \tau^{\mathbf{n}}_t$ is the total transfer to the newborns, $\nu \left(1-\rho_o\right) \tau^{\mathbf{y}}_t$ is the transfer to pre-existing young agents and $b^{\mathbf{g}}_t$ is the real value of government bonds.

For tractability we also assume that the government provides newborn agents with an initial transfer that equalizes the wealth levels with the average after-tax wealth among pre-existing agents, i.e.

$$\tau^{\mathbf{n}}_t = a^{\mathbf{y}}_t + \tau^{\mathbf{y}}_t, \tag{10}$$

where $a^{\mathbf{y}}_t \equiv \int\limits_{i:\mathbf{s}=\mathbf{y}} a_{i,t} di$ is the *average* wealth among pre-existing young agents. Since before-tax wealth is the only source of heterogeneity among young agents, all young agents make the same decisions and what arises is a representative young agent. This implication makes the model tractable. Note that although we eliminate heterogeneity among young agents by assumption, we do preserve heterogeneity between young and old agents, as well as heterogeneity among old agents.

Finally, we assume that only productive agents are affected by transfers/taxes, i.e. we set $\tau^{\mathbf{o}}_t = 0$. This assumption is motivated by the reality that the majority of the tax burden falls on people in their working life, since due to the progressivity of tax systems.[19]

[18] The assumption that the government changes tax policy within the same quarter in response to shocks seems somewhat extreme and hence we allow for a delay.

[19] We have solved a version of our model in which instead taxes are proportional to wealth levels, and

3.1.7 Market clearing and equilibrium

Aggregate non-durables and durables are given by:

$$
\begin{aligned}
c_t &= \nu c_t^{\mathbf{y}} + (1-\nu)\, c_t^{\mathbf{o}} && (11)\\
d_t &= \nu d_t^{\mathbf{y}} + (1-\nu)\, d_t^{\mathbf{o}}, && (12)
\end{aligned}
$$

where superscripts **y** and **o** denote the averages among young and old agents, defined analogously to the definition of $a_t^{\mathbf{y}}$.[20] Clearing in the markets for goods, money and bonds requires:

$$
\begin{aligned}
c_t + d_t &= \nu h_t^{\mathbf{y}} + (1-\delta)\, d_{t-1}, && (13)\\
m_t &= \nu m_t^{\mathbf{y}} + (1-\nu)\, m_t^{\mathbf{o}}, && (14)\\
0 &= b_t^{\mathbf{g}} + b_t^{\mathbf{cb}} + \nu b_t^{\mathbf{y}} + (1-\nu)\, b_t^{\mathbf{o}} && (15)
\end{aligned}
$$

Finally, the size of the bequest received per young agent is given by:

$$
\tau_t^{bq} = \frac{\rho_x a_t^{\mathbf{o}} + \rho_o \rho_x a_t^{\mathbf{y}}}{\nu} \qquad (16)
$$

We are now ready define a recursive competitive equilibrium:

> **Definition.** *A recursive competitive equilibrium is defined by policy rules for non-durable consumption, $c^{\mathbf{s}}(a,\Gamma)$, durable consumption, $d^{\mathbf{s}}(a,\Gamma)$, money holdings, $m^{\mathbf{s}}(a,\Gamma)$, bond holdings, $b^{\mathbf{s}}(a,\Gamma)$, labor supply, $h^{\mathbf{s}}(a,\Gamma)$, with $\mathbf{s} = \mathbf{n}, \mathbf{y}, \mathbf{o}, \mathbf{cb}, \mathbf{g}$, as well as laws of motion for inflation, the nominal interest rate and the real wage, such that households optimize their expected life-time utility subject to their constraints and the law of motion for the aggregate state, the treasury and central banks follow their specified policies, and the markets for bonds, money, goods and labor clear in every period. The aggregate state Γ includes the value of the monetary policy shock, the distribution of wealth among agents, as well as the initial*

obtained results similar to the ones obtained from our benchmark model.

[20] Due to the transfer to newborns $c_t^{\mathbf{y}} = c_t^{\mathbf{n}}$, $d_t^{\mathbf{y}} = d_t^{\mathbf{n}}$, $b_t^{\mathbf{y}} = b_t^{\mathbf{n}}$ and $m_t^{\mathbf{y}} = m_t^{\mathbf{n}}$.

holdings of assets by households, the treasury and the central bank.

3.1.8 Three analytical results in a representative agent version

A special case of our model is obtained when we set the death probability to one, i.e. $\rho_x = 1$. In this case, agents immediately die upon retirement and old agents are effectively removed from the model. Given the absence of heterogeneity among young agents, the model becomes observationally equivalent to one with an infinitely-lived representative household with subjective discount factor $\widetilde{\beta} = \beta(1-\rho_o)$. This special case is useful to understand the role of household heterogeneity in the transmission of monetary policy, as several analytical results can be derived which contrast our numerical results to be presented in the next section. The first result is:

Result 1. *Monetary policy is neutral with respect to real activity in the representative agent model.*

The arguments for the monetary neutrality follow Sidrauski (1967). The representative agents' first-order conditions for durables in and labor supply, and the aggregate resource constraint are, respectively:

$$
\begin{aligned}
U_{c,t} &= U_{d,t} + \widetilde{\beta}(1-\delta)\mathbb{E}_t U_{c,t+1} && (17)\\
U_{c,t} &= h_t^{\kappa} && (18)\\
c_t + d_t &= h_t + (1-\delta) d_{t-1} && (19)
\end{aligned}
$$

for $t = 0, 1, ...$Given an initial level of durables and that the utility function is separable in its arguments, these three equations pin down the equilibrium solution paths for c_t, d_t and h_t in any period t without any reference to variables related to monetary policy. Given this solution it straightforward to pin down output and the real interest rate as well.

Next, we consider how an unexpected monetary policy shock impacts on the price level in the representative agent world. We can derive the following result:

Result 2. *Monetary policy shocks impact on the prices solely through their effect on government transfers to representative agent.*

This result can be seen from the government's consolidated present value budget constraint, which is derived in the appendix. In expectations it can be written as:

$$\mathbb{E}_t \sum_{s=t}^{\infty} D_k \left(\frac{r_s}{1+r_s} m_s - \tau_s^{\mathbf{g}} \right) = \frac{m_{t-1} - (1+r_{t-1}) \left(b_{t-1}^{\mathbf{g}} + b_{t-1}^{\mathbf{cb}} \right)}{1+\pi_t} \tag{20}$$

or, where $D_s \equiv \prod_{k=t}^{s-1} \frac{1+\pi_{k+1}}{1+r_k}$ is the agent's valuation of one unit of nominal wealth received in period $s > t$, and $D_t \equiv 1$, and $\tau_t^{\mathbf{g}} \equiv \nu \rho_o \tau_t^{\mathbf{n}} + \nu (1-\rho_o) \tau_t^{\mathbf{y}}$ is the total transfer to the households. The left-hand side represents the expected present value of the opportunity costs of holding money paid by the household, $\frac{r_s}{1+r_s} m_s$, which are income to the government, minus the transfers to the households, $\tau_s^{\mathbf{g}}$. On the right hand side are the initial liabilities of the government. The appendix also demonstrates that from the neutrality of Result 1 it follows that D_s and $\frac{r_s}{1+r_s} m_s$, with $s \geq 1$, are unaffected by changes in monetary policy. Given that π_t is the only variable on the right-hand side that is not predetermined, it follows that its initial response to a monetary policy shock is fully pinned down by the change in the expected present value of the transfer. Thus, the impact of monetary policy shock on the price level depends crucially on how seigniorage transfers to the households via the treasury respond.

Intuition for Result 1, the irrelevance of monetary policy for real outcomes in the representative agent model, is obtained by considering the net wealth effects of changes in monetary policy, following Weil (1991). From the same equation we can infer our third result:

Result 3. *Changes in monetary policy do not create net wealth effects in the representative agent model.*

The flip-side of the government's budget constraint is the consolidated budget constraint of the households, excluding labor income. In particular, the initial liability of the government are equal to the value of bonds and money held by the public. The present value constraint shows that any such revaluation is *exactly* offset by a decline in the expected present value of transfers. Thus, a negative revaluation of the representative agent's nominal wealth following a surprise monetary expansion is exactly offset by a decline in transfers to be obtained from the government. Hence, there is no net wealth effect, an insight that is closely related to the seminal work of Barro (1978) and that was spelled out by Weil (1991) in the context of monetary model.

3.1.9 The transmission of OMOs with heterogeneous agents

The results derived for the special representative agent case help to understand the real effects of open market operations in the full model with heterogeneous agents. In this full version, agents are affected differently by monetary policy shocks for two reasons. The is that portfolio size and compositions are heterogeneous across agents, affecting the extent to which they are affected by a surprise revaluation of nominal wealth. Second, agents are affected differently by a change in the path of transfers depending on their individual age status. Old agents can be expected to be disadvantaged by an expansionary monetary policy shock, since they suffer the negative revaluation of wealth but do not benefit from an increase in transfers. The same holds for a young agent who retires soon after a persistent and expansionary monetary shock. A monetary expansion also impacts directly on yet unborn generations, as the change in policy affects the transfers they will receive from the government.

The redistributional effects impact on agents' savings decisions. We can make this effect explicit by analyzing the young agents' first-order conditions for durables and bonds, respectively:

$$U_{c,t}^{\mathbf{y}} = U_{d,t}^{\mathbf{y}} + \beta\left(1-\rho_o\right)\mathbb{E}_t\frac{U_{c,t+1}^{\mathbf{y}}}{1+\pi_{t+1}} + \beta\left(1-\rho_x\right)\mathbb{E}_t\frac{U_{c,t+1}^{\mathbf{yo}}}{1+\pi_{t+1}}, \tag{21}$$

$$U_{c,t}^{\mathbf{y}} = \beta\left(1-\rho_o\right)\mathbb{E}_t\frac{\left(1+r_t\right)U_{c,t+1}^{\mathbf{y}}}{1+\pi_{t+1}} + \beta\left(1-\rho_x\right)\mathbb{E}_t\frac{\left(1+r_t\right)U_{c,t+1}^{\mathbf{yo}}}{1+\pi_{t+1}}, \tag{22}$$

where superscript **yo** denotes a newly retired agent. Agents who retire face a reduction in their expected lifetime income and hence it holds in the stationary equilibrium that $U^{\mathbf{y}} < U^{\mathbf{yo}}$, i.e. the marginal utility of wealth is higher when young than once newly retired. The redistributional effect brought about by an expansionary monetary shock further exacerbates the increase in the marginal utility of wealth upon retirement, i.e. $U^{\mathbf{yo}}$ increases further relative to $U^{\mathbf{y}}$. Young agents thus become more strongly incentivised to save for retirement during a monetary expansion. The above two first-order conditions make clear that this additional desire to save pushes down the interest rate the real interest rate and encourages young agents to accumulate more durables. In our numerical simulations these effects result in an increase in aggregate durable expenditures in the equilibrium, pushing up aggregate output.

4 Quantitative simulations

In this section we analyze the effects of open market operations in our model using numerical simulations. Before doing so we specify the details of household preferences and the monetary policy rule. We assume that the utility function is a CES basket of non-durables, durables and money, nested in a CRRA function:

$$\begin{aligned} U(c_{i,t}, d_{i,t}, m_{i,t}) &= \frac{x_{i,t}^{1-\sigma} - 1}{1-\sigma}, \\ x_{i,t} &\equiv \left[c_{i,t}^{\frac{\epsilon-1}{\epsilon}} + \eta d_{i,t}^{\frac{\epsilon-1}{\epsilon}} + \mu m_{i,t}^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}}, \end{aligned} \tag{23}$$

where $\epsilon, \sigma, \eta, \mu > 0$. Here, ϵ is the elasticity of substitution between non-durables, durables and money, σ is the coefficient of relative risk aversion, and η and μ are parameters giving utility weights to durables and money, respectively. Computation of the dynamic equilibrium path seems complicated due to the high dimensionality of the aggregate state Γ_t. In the Appendix we show that the solving the model using a standard first-order perturbation (linearization) method is nonetheless straightforward under the above preference specification.[21]

The central bank is assumed to set the money supply according to the following process:

$$\frac{M_t}{M_{t-1}} = 1 + z_t \tag{24}$$

where z_t is an exogenous shock process to the rate of nominal money growth, assumed to be

[21] In particular, we exploit the properties of first-order perturbation and show that the implied certainty equivalence with respect to the aggregate state allows us to express the decision rules of the old agents as linear functions of their wealth levels. This in turn implies that aggregation is straightforward and that only the distribution of wealth between *between* old agents and young agents is relevant for aggregate outcomes.

of the following form:

$$z_t = \xi \left(\overline{m} - m_{t-1} \right) + \varepsilon_t, \ \xi \in [0, 1], \tag{25}$$

where ε_t is an i.i.d. shock innovation and $\overline{m}$ is the steady-state value of real money balances. A positive shock increases the money supply. When $\xi < 1$ this increase is gradually reversed in subsequent periods due to the above feedback rule.[22]

4.1 Parameter values

Parameter values are chosen corresponding to a model period of one quarter. The subjective discount factor, β, is set to 0.9732 which implies an annual real interest rate of 4 percent in the deterministic steady state. The steady state real interest rate is lower than the subjective discount rate, $1/\beta - 1$, due to the retirement savings motive arising in the presence of incomplete insurance markets. The durable preference parameter η is chosen to target a steady-state spending ratio of 20 percent on durables. To set the money preference parameter, we target a quarterly money velocity, defined as $\frac{y}{m}$, of 1.8. The intratemporal elasticity of substitution between non-durables, durables and money, ϵ, is set equal to one, as is the coefficient of relative risk aversion, σ. These two parameter settings imply that money and consumption enter the utility function additively in logs. Hence, our benchmark results are not driven by non-separability of money and consumption in the utility function. We set the Frisch elasticity of labor supply κ equal to one following many macro studies. The parameter scaling the disutility of labor, ζ, such as to normalize aggregate quarterly output to one.

Life-cycle transition parameters are set to imply a life expectancy of 60 years, with on expectation 40 years in working life and 20 in retirement. Accordingly, we set $\rho_o = 0.0063$ and $\rho_x = 0.0125$ which imply $\nu = 0.6677$. The depreciation rate of durables, δ, is set to 0.04 following Baxter (1996). The initial level of government debt is set to sixty percent of annual output. For simplicity we assume the central bank starts off without any bond holdings or debt. The shock process parameter ξ is set tot 0.2 which implies that the half life of the response for the nominal interest rate is about 2.5 years. Finally, we set the government's

[22] In equilibrium, both real an nominal money balances increase following the shock. Also, the rule implies that the net rate of inflation is zero in the steady state.

delay in transferring income to households to one year, i.e. $j = 4$. Parameter values are presented in Table 1.

<<Table 1 here>>

4.2 The dynamic effects of open market operations

Figure 2 presents the responses to an expansionary monetary policy shock, implemented using open market operations. The magnitude of the shock is scaled to imply a reduction in the nominal interest rate of about 75 basis points. The shock increases the price level as well as aggregate output on impact. The responses of durables and non-durables make clear that this increase in output is entirely driven by an increase in expenditures on durables. Non-durables decline on impact, although the magnitude of the response is much smaller than the response of durables. Finally, the shock leads to a moderate decline in the real interest rate.

<<Figure 2 here>>

In the periods after the initial shock, the nominal interest rate and the price level gradually revert back to their initial levels and the same holds for non-durable purchases. This happens as a result of the reversion in the monetary policy rule. The booms in durables and output are also gradually reversed but the responses overshoot and turn into a busts several quarters after the shock, in line with the empirical impulse responses. The overshooting in durable purchases and output is related to the large degree of endogenous persistence in the model, which is also reflected in the response of the real interest rate which continues to decline in the year following the shock and remains low quite persistently.

Figure 3 plots several variables that provide insight into the impact of monetary policy shocks as well as their endogenous propagation over time. The top left panel plots the response of real money balances which increase on impact and then gradually revert back to the steady state, akin to the responses of the nominal interest rate and the price level. From the positive response of the price level it follows that nominal money balances increase as well. The top right panel plots the transfer to the young households, which increases by about 0.6 percent of output one year following the shock and gradually revert back in the years after.

<<Figure 3 here>>

The bottom four panels shows the aggregated responses of non-durable consumption and (the stock of) durable consumption for the young and the old agents. Following the shock, the old agents reduce both types of consumption. Underlying, old households face a reduction in their real wealth due to the increase in prices, but are not compensated by an increase in transfers. The young, by contrast, increase durable consumption. This response can be understood by noting that a young agent benefits from the increase in transfers, but only up to the period in which she retires. This implies a steepening of the decline in the agent's income profile over the life cycle which increases the desire to save for retirement. In equilibrium, aggregate bond holdings are zero and the supply of money is determined largely by monetary factors and therefore cannot easily accommodate an increased aggregate desire to save. Bringing forward durable purchases, however, is an alternative way of saving that not much restricted by supply factors since production can be shifted from non-durables towards durables. Indeed, non-durable purchases by the young decline on impact, driving the decline in aggregate non-durable purchases.

Several quarters after the shock, the response of non-durable purchases by the young turns from negative to positive. This sign switch is important to understand the bust in output that follows several periods after the shock. To see this, note that under the assumed preferences, the young agents' labor supply equation directly links output and the non-durable consumption of the young, as it can be written as:

$$\frac{1}{c_t^y} = \frac{y_t}{\nu}.$$

Thus, under these preferences output and non-durable purchases necessarily co-move negatively. Intuitively, leisure is a normal good and the urge to buy durables makes young agents willing to forego leisure and non-durable consumption in the initial periods of the expansion. While remaining young, however, the young agents increasingly reap the benefits from the redistributional effects of the monetary policy, which increases their lifetime wealth and thus their demand for leisure and non-durables. The latter "pure wealth effect" starts to dominate several quarters after the initial shock and the non-durables response turns from negative to positive. The above discussion makes clear that this effect is also behind the boom-bust response of aggregate output.

4.3 Helicopter drops

We now contrast the effects of open market operations to the effects of shocks in an alternative economy in which monetary policy is implemented using "helicopter drops" of money. By a helicopter drop we mean an expansion in the money supply that is not accompanied by an increase in central bank bond holdings, but rather an outright transfer to the treasury.[23] It then follows that the total transfer from the treasury to the households is given by its interest earnings on bond holdings (which can be negative) plus the change in the money supply. In real terms, the transfer to the households becomes:

$$\frac{r_{t-1-j}b^{\mathbf{g}}_{t-j-1}}{1+\pi_{t-j}} + m_{t-j} - \frac{m_{t-j-1}}{1+\pi_{t-j}} = \nu\rho_o\tau^{\mathbf{n}}_t + \nu\left(1-\rho_o\right)\tau^{\mathbf{y}}_t + \left(1-\nu\right)\tau^{\mathbf{o}}_t \tag{26}$$

where j is again a delay in transferring government income to households. We assume again that helicopter drops are gradually reversed after the initial shock, following the same feedback rule as used for in the economy with market operations.[24]

Figures 4 and 5 plot the responses for the economy with helicopter drops, together with those for our benchmark economy with open market operations. These figures show that although responses of prices and real money balances to helicopter drops are comparable to those in our benchmark economy with open market operations, the effects on real economic outcomes are drastically different. In particular, with helicopter drops output and durable expenditures *decline* following an expansion of the money supply, whereas the real interest rate *increases* several periods after the shock. Thus, the transmission of monetary policy depends importantly on the operating procedures of the central bank.

<<**Figure 4 here**>>

[23] Consequently, $b^{\mathbf{cb}}_t$ remains zero at all times.

[24] For comparability, we do not re-scale the magnitude of the shock relative to the benchmark model.

<<Figure 5 here>>

The responses plotted in Figure 5 reveal why the effects of a monetary expansion are so different in the two economies. First of all, the response of the government transfer to households is very different when helicopter drops are used. Rather than a persistent increase in these transfers, there is a large increase after one year with a magnitude of about 2 percent of annual GDP. In later periods, there is a persistent decline in government transfers, relative to the steady state. Thus, in the economy with open market operations a monetary expansion is relatively favorable for those households in their working life long after the initial shock, which includes generations yet unborn in the initial period of the shock. The impact of a helicopter drop, by contrast, is more similar to a one-time redistribution between retired agents and agents in their working life. Hence the pure wealth effect dominates immediately following the initial helicopter drop and young households increase both leisure and non-durable consumption, as well as durables. As all three utility components are increased simultaneously, however, the response of durables is weaker than in the economy with OMO. Hence, the increase in durable purchases by the young is insufficient to offset the decline in durable purchases by the old, resulting in a decline in aggregate durable purchases.

4.4 The role of risk aversion

Although the most of the responses implied by our benchmark model are in line with the VAR evidence, the model predicts a decline in non-durables whereas the VAR predicts a very small but nonetheless positive response. Also, the increase in output in our benchmark model is relatively short-lived. Since our model is rather stylized we do not attempt to estimate its parameters. Instead we explore whether a plausible reparameterization of the model can be helpful to bringing the model closer to the VAR.

Figure 6 plots the response to a monetary expansion implemented using OMOs, comparing the benchmark model to a version in which the coefficient of risk aversion, σ, is lowered from 1 to 0.6.[25] The figure shows that under this parameterization, the output increase becomes more persistent. Whereas in the benchmark the output turns negative after about

[25] We also recalibrate to match the steady-state targets described in the previous subsection.

one and a half year, this is increase to 2.5 years under lower risk aversion, close to moment of the sign switch in the empirical response. Also, the model now predicts a joint increase in non-durables, durables and output following a monetary expansion. This is a result that is difficult to obtain in sticky-price models, see Barsky, House and Kimball (2007). The response of durables is markedly smaller under the alternative parameterization, but still much larger than the response of non-durable consumption.

<<**Figure 6 here**>>

5 Concluding remarks

We study the effects of open market operations in a real general equilibrium model with a parsimonious life cycle structure. We show that monetary expansions stemming from OMO generate negative wealth effects in the population, with a more negative impact on old agents whose income stems from financial assets. Working agents respond to higher inflation by working and saving more and by accumulating durable goods. This causes a boom in output driven by the durable good sector, consistent with the empirical evidence.

The distributional effects embedded in our model are consistent with empirical evidence on the effects of monetary interventions in the US economy. They point to a different transmission mechanism of monetary policy that can complement the standard NK channel based on nominal rigidities. In a model extension, we allow for search and matching frictions. In this setting, the increase in employment results from higher labour demand, rather than a voluntary increase in labour supply as in the baseline case.

References

References

[1] Algan, Yann, Allais, Olivier, Challe, Edouard, and Xavier Ragot (2012), "Monetary Shocks under Incomplete Markets," mimeo.

[2] Andolfatto, David (1996), "Business Cycles and Labor-Market Search," *American Economic Review*, 86(1), 112-32.

[3] Ascari, Guido and N. Rankin (2007), "Perpetual Youth and Endogenous Labor Supply: A Problem and a Possible Solution," *Journal of Macroeconomics*, 29, 708-723

[4] Barro, Robert J. (1974), "Are Government Bonds Net Wealth?," *Journal of Political Economy*, 82(6), 1095-1117.

[5] Barsky, R.B., House, C.L., Kimball, M.S. (2003), "Do Flexible Durable Goods Prices Undermine Sticky Price Models?" National Bureau of Economic Research Working Paper 9832.

[6] Barsky, R.B., House, C.L., Kimball, M.S. (2007), "Sticky-price Models and Durable Goods," *American Economic Review*, 97(3), 984–998.

[7] Baxter, Marianne (1996), "Are Consumer Durables Important for Business Cycles?," *The Review of Economics and Statistics*, 78(1), 147-55.

[8] Bernanke, Ben and Alan Blinder (1992), "The Federal Funds Rate and the Channels of Monetary Transmission," *American Economic Review*, 82(4), 901-921.

[9] Bernanke, B.S.,Gertler, M.L. (1995), "Inside the Black Box: the Credit Channel of Monetary Policy Transmission," *Journal of Economic Perspectives*, 9, 27–48.

[10] Bernanke, Ben and Ilian Mihov (1998), "Measuring Monetary Policy," *Quarterly Journal of Economics*, 113, 315-334.

[11] Bils, Mark and Peter Klenow (2004), "Some evidence on the Importance of Sticky Prices," *Journal of Political Economy*, 112, 947–985.

[12] Boivin, Jean and Marc Giannoni (2006), "Has Monetary Policy Become More Effective?" *Review of Economics and Statistics*, 88, 433-444.

[13] Calvo, Guillermo (1983), "Staggered Prices in a Utility-Maximizing Framework," *Journal of Monetary Economics*, 12, 383-398.

[14] Card, D. (1990), "Unexpected Inflation, Real Wages, and Employment Determination in Union Contracts," *American Economic Review*, 80, 669–688.

[15] Cass, David, Masahiro Okuno and Itzhak Zilcha (1979), "The Role of Money in Supporting the Pareto Optimality of Competitive Equilibrium in Consumption-Loan Type Models," *Journal of Economic Theory*, 20, 41-80.

[16] Christiano, Lawrence, Martin Eichenbaum and Charles Evans (1999), "Monetary Policy Shocks: What Have We Learn and to What End?," in John B. Taylor and Michael Woodford, (eds), *Handbook of Macroeconomics*. Volume 1A, 65-148, Elsevier Science, North-Holland.

[17] Christiano, Lawrence, Martin Eichenbaum and Charles Evans (2005), "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Polical Economy,* 113, 1-45.

[18] Coibion, O., Gorodnichenko, Y., Kueng, L., Silvia, J. (2012), "Innocent Bystanders? Monetary Policy and Inequality in the U.S.," Berkeley manuscript.

[19] Danthine, Jean-Pierre, John Donaldson and Lance Smith (1987), "On the Superneutrality of Money in a Stochastic Dynamic Macroeconomic Model," *Journal of Monetary Economics*, 20, 475-499.

[20] Diamond, Peter (1965), "National Debt in a Neoclassical Growth Model," *American Economic Review*, 55, 1126-1150.

[21] Doepke, M., Schneider, M. (2006), "Inflation and the Redistribution of Nominal Wealth," *Journal of Political Economy* 114 (6), 1069-1097.

[22] Dotsey, Michael, Robert King and Alexander Wolman (1999), "State-Dependent Pricing and the General Equilibrium Dynamics of Money and Output," *Quarterly Journal of Economics*, 114, 655-90.

[23] Fischer, Stanley (1979), "Capital Accumulation on the Transition Path in a Monetary Optimizing Model," *Econometrica*, 47, 1433-1439.

[24] Fuhrer, Jeffrey (2000), "Habit Formation in Consumption and Its Implications for Monetary Policy," *American Economic Review*, 90, 367-390.

[25] Gertler, Mark (1999), "Government Debt and Social Security in a Life-Cycle Economy," *Carnegie-Rochester Conference on Public Policy,* 50, 61-110.

[26] Gornemann, Nils, Kuester, Keith, and Makoto Nakajima (2012), "Monetary Policy with Heterogeneous Agents" (2012), Philadelphia Fed Working Paper 12-21.

[27] Iacoviello, Matteo (2005), "House Prices, Borrowing Constraints, and Monetary Policy in the Business Cycle." *American Economic Review* 95, 739–764.

[28] Ireland, Peter M. (2005), "The Liquidity Trap, The Real Balance Effect, and the Friedman Rule", *International Economic Review*, 46(4), 1271-1301.

[29] Rotemberg, J., (1982), Sticky prices in the United States. *Journal of Political Economy* 90, 1187-1211.

[30] Rotemberg, Julio and Michael Woodford (1997), "An Optimization-Based Econometric Framework for the Evaluation of Monetary Policy," *NBER Macroeconomics Annual,* 297-346.

[31] Schabert, Andreas (2004), Interactions of Monetary and Fiscal Policy via Open Market Operations, *The Economic Journal*, 114, 2004, C186-C206

[32] Sidrauski, Miguel (1967), "Rational Choice and Patterns of Growth in a Monetary Economy," *American Economic Review* 57, 534-544.

[33] Sims, Christopher and Tao Zha (1999), "Error Bands for Impulse Responses," *Econometrica*, 67, 1113-55.

[34] Wallace, Neil (1981), "A Modigliani-Miller Theorem for Open Market Operations," *American Economic Review*, 71, 267–274.

[35] Weil, Philippe (1991), "Is Money Net Wealth?," *International Economic Review*, 32, 37-53.

[36] Woodford, Michael (2003), "Interest and Prices: Foundations of a Theory of Monetary Policy," Princeton: Princeton University Press.

[37] Woodford, Michael (2012), "Methods of Policy Accommodation at the Interest-Rate Lower Bound," Jackson Hole symposium paper.

Tables and Figures

Figure 1: Impulse response function of headline variables to monetary policy shocks using VAR

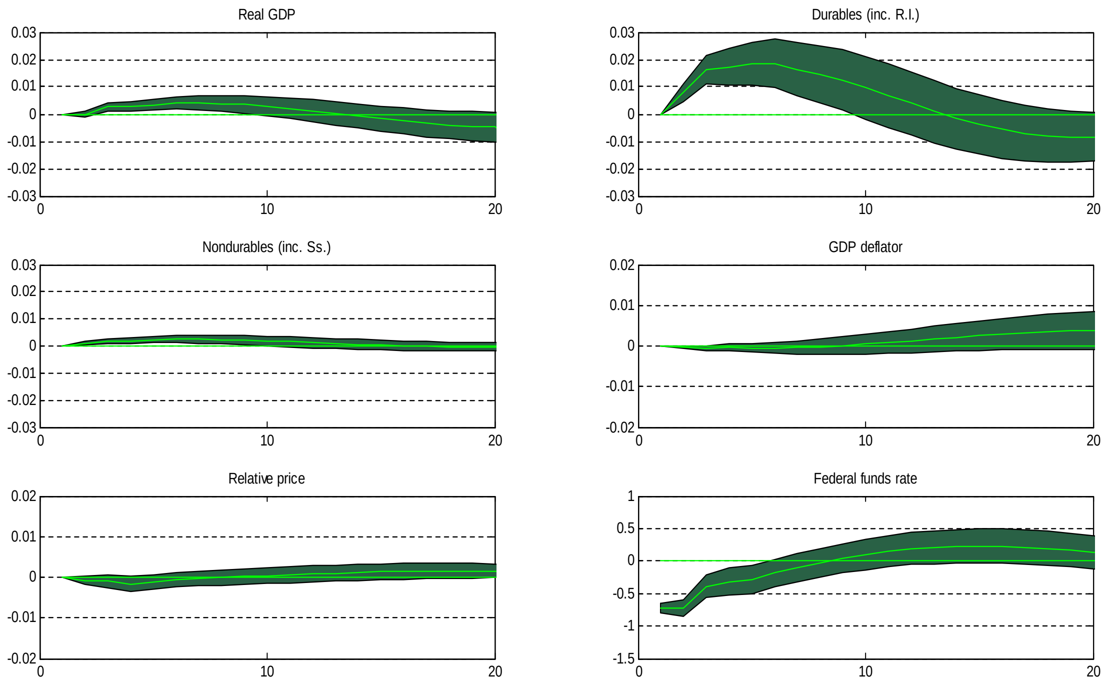


Figure 2: Model responses to expansionary OMO.

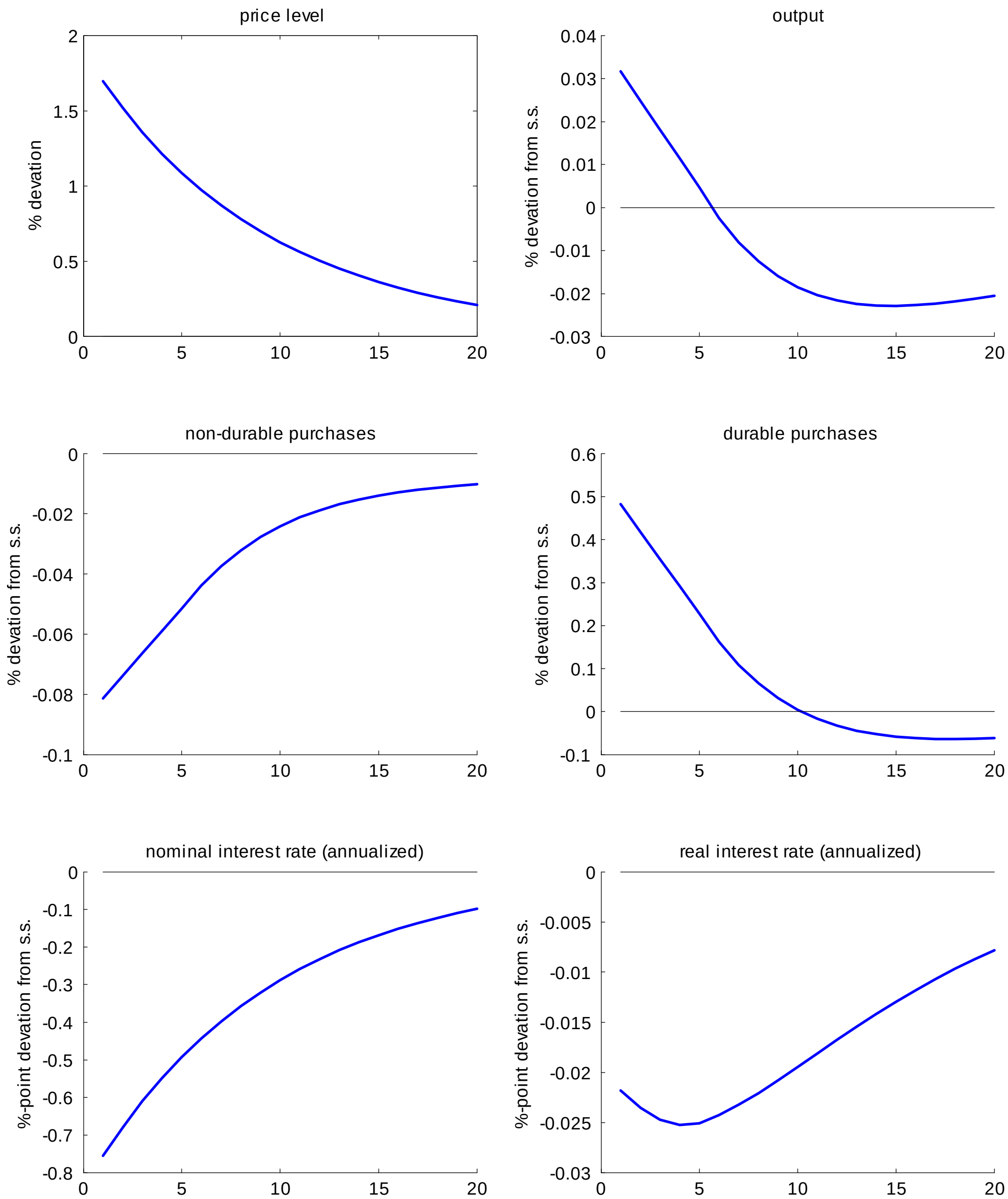


Figure 3: Model responses to expansionary OMO.

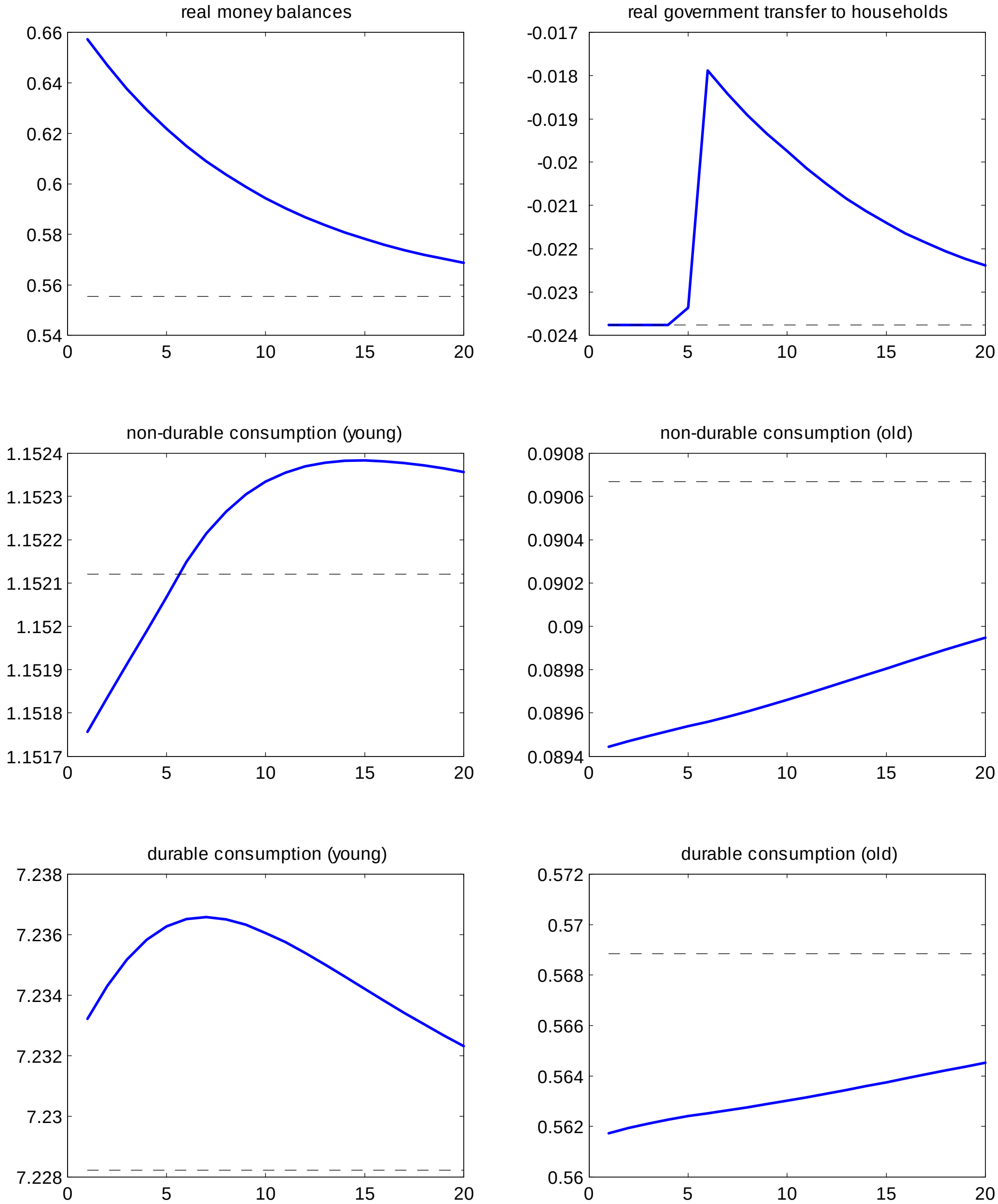


Figure 4: Model responses to expansionary OMO versus helicopter drop.

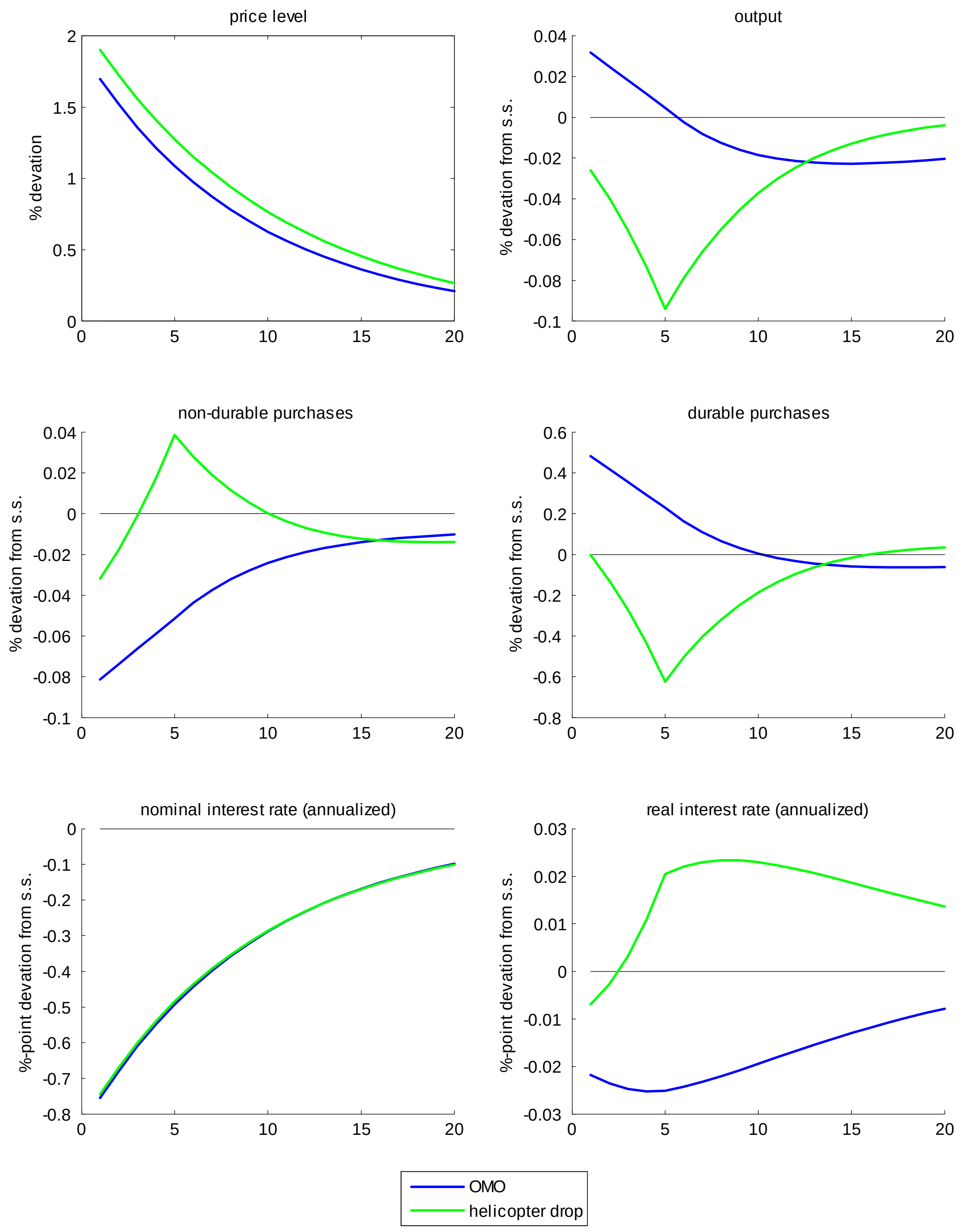


Figure 5: Model responses to expansionary OMO versus helicopter drop.

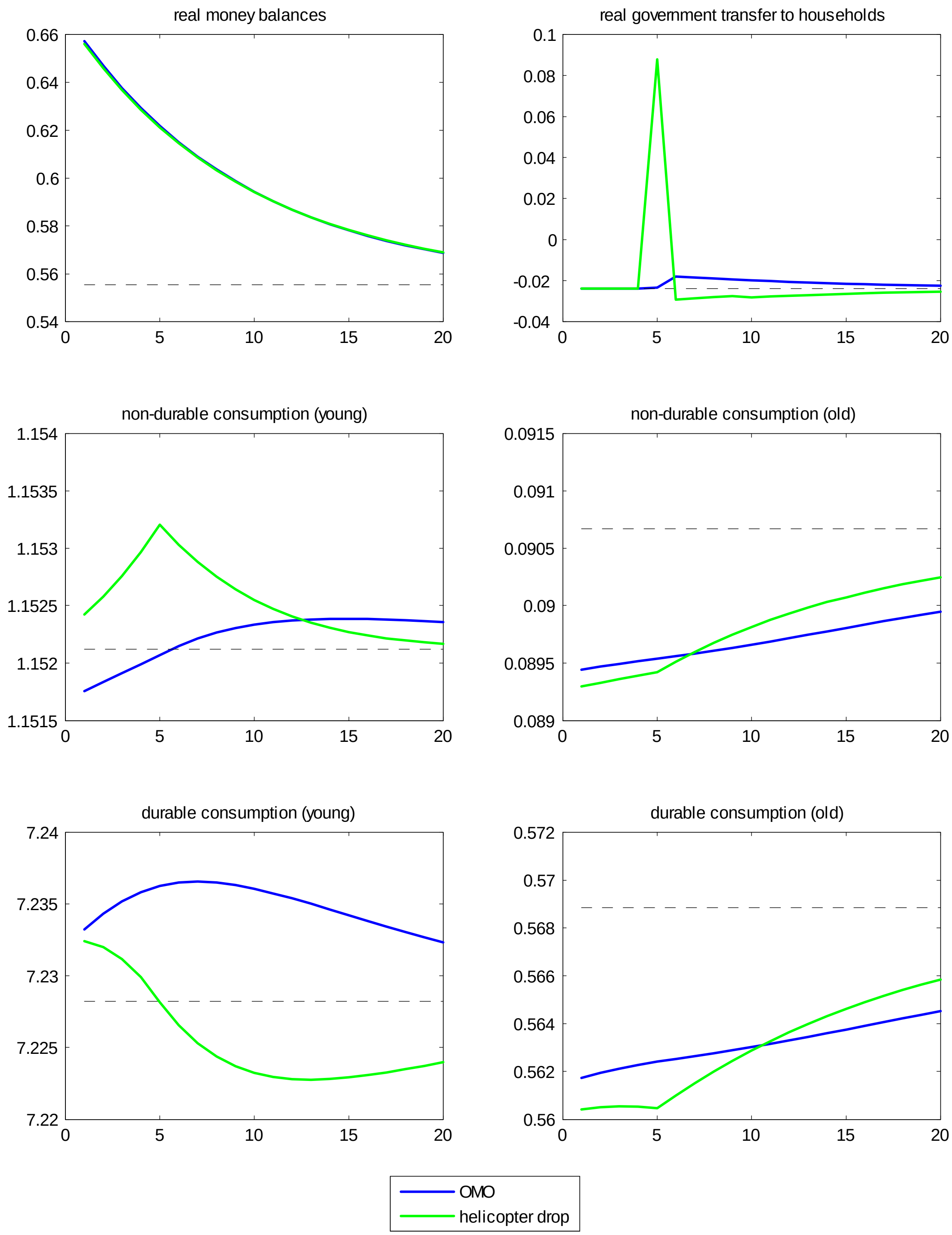


Figure 6: Model responses to expansionary OMO; benchmark versus lower risk aversion

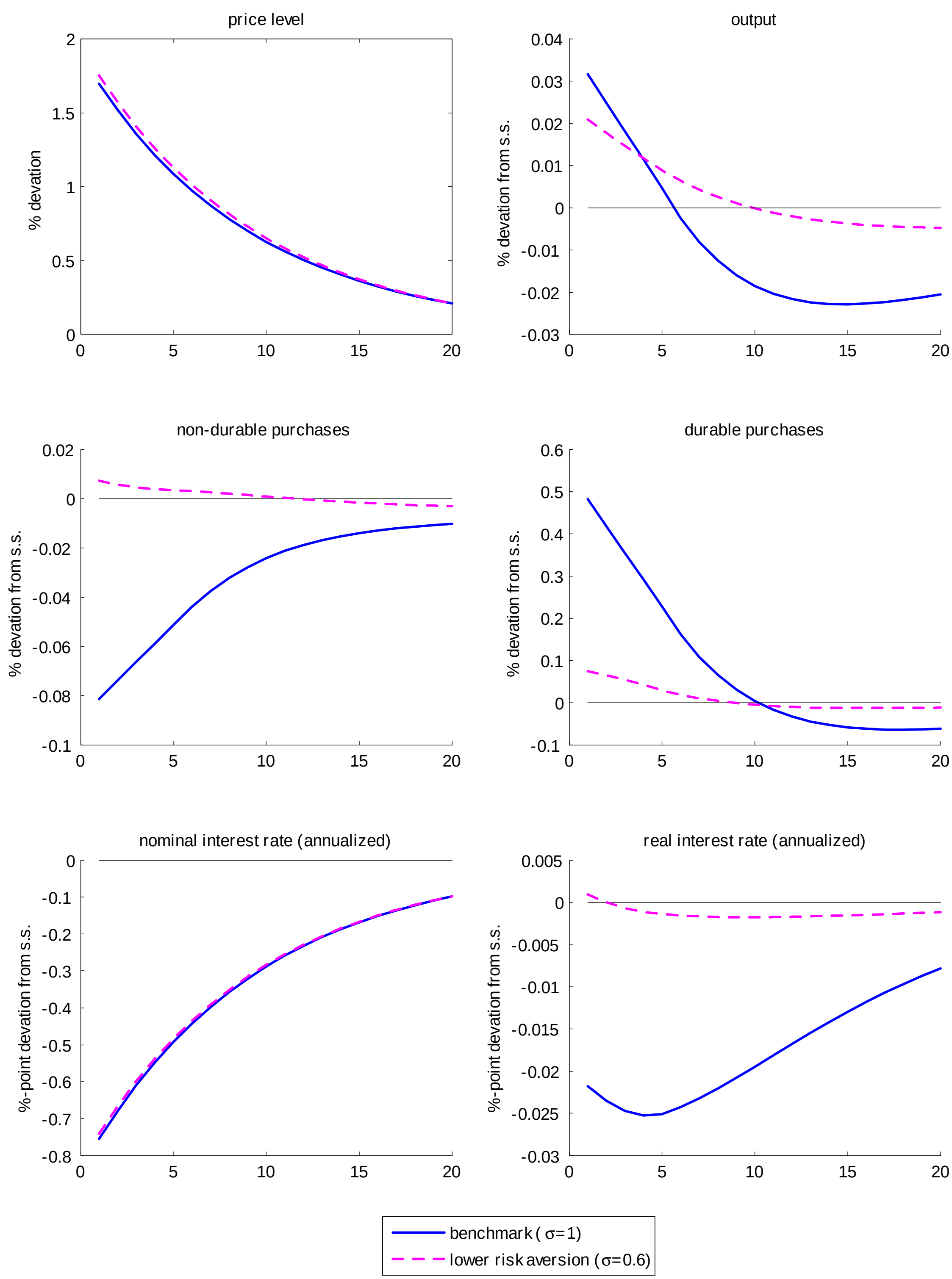


Table 1. Parameter values.

	value	description	motivation
β	0.9732	subjective discount factor	target 4% s.s. annual interest rate
η	0.31	durable preference param.	target 20% s.s. spending on durables (NIPA)
μ	0.0068	money preference param.	target 1.8 s.s. M2 velocity ($\frac{y}{m}$) (FRB/NIPA)
σ	1	coefficient of relative risk aversion	convention macro literature
ϵ	1	intratemporal elast. of substitution	convention macro literature
κ	1	inv. elasticity labor supply	convention macro literature
ζ	0.5795	disutility of labor	normalize aggregate quarterly output to one
ρ_o	0.0063	ageing probability	average duration working life 40 years
ρ_x	0.0125	death probability	average duration retirement 20 years
δ	0.04	depreciation rate durables	Baxter (1996)
b_0^g	-2.4	initial bond holdings treasury	government debt 60% of annual output
b_0^{cb}	0	initial bond holdings central bank	no initial central bank debt
ξ	0.15	coefficient monetary rule	half life response nominal interest rate 2.5 years
j	4	transfer delay treasury	one year delay

Appendix

In this Appendix we present additional evidence supplementing the empirical results, provide full derivation of the results and study extensions of the model that allow for search and matching frictions in the labour market as well as wage rigidity.

A1. Alternative Estimation Approach

Figure A1 shows the empirical response of the same macroeconomic variables as in Figure 1 when the identification of shocks relies on Romer and Romer (2004). As illustrated in the Figure, the results are qualitatively similar to those obtained from a recursive VAR, and remarkably close also from a quantitative point of view.

Figure A2 shows the empirical response of the macroeconomic variables using the recursive VAR identification as in Figure 1, but including personal income and social security taxes in the system. Figure A3 shows the same plots using the Romer and Romer identification. The responses of all variables is similar to those resulting from the baseline estimations without taxes. Interestingly, however taxes decrease following the monetary expansion. This is consistent with our model: with a monetary expansion, the bond holdings by the Central Bank increase, which leads to higher transfers from the Central Bank to the Treasury. This, in turn, leads to lower taxes (higher transfers) to individuals. The effect on taxes is quite persistent.

Figure A1: Impulse response function of headline variables to monetary policy shock Romer&Romer

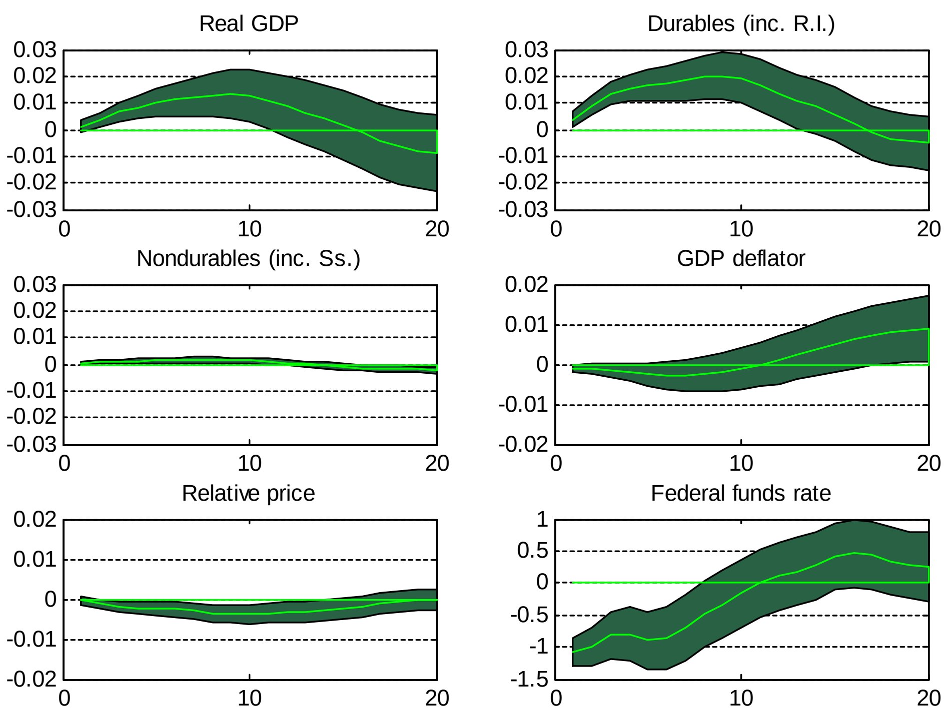


Figure A2: Impulse response function of headline variables to monetary policy shock VAR approach

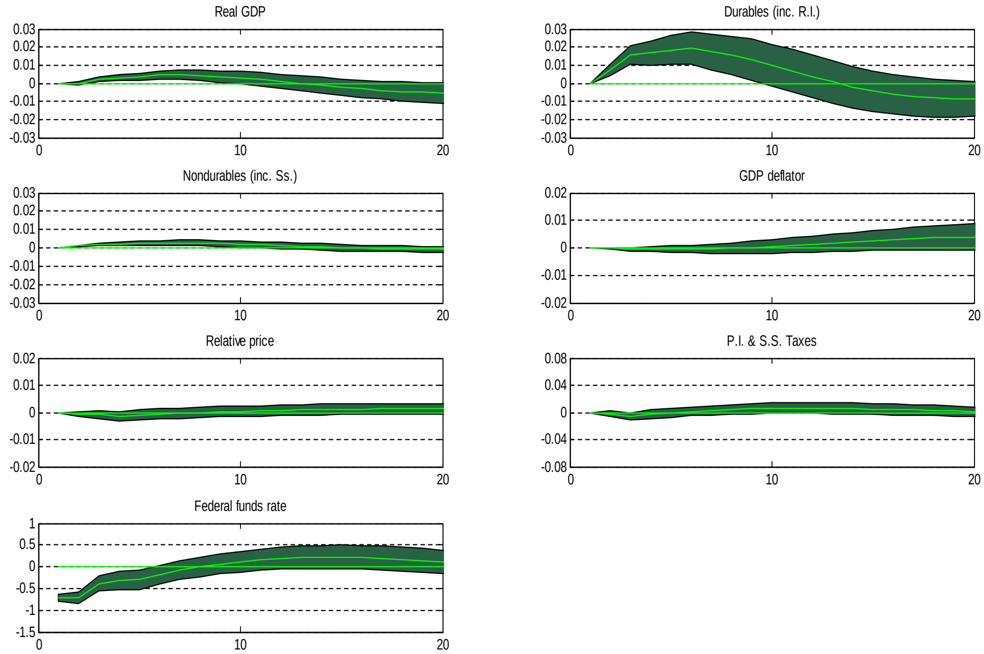
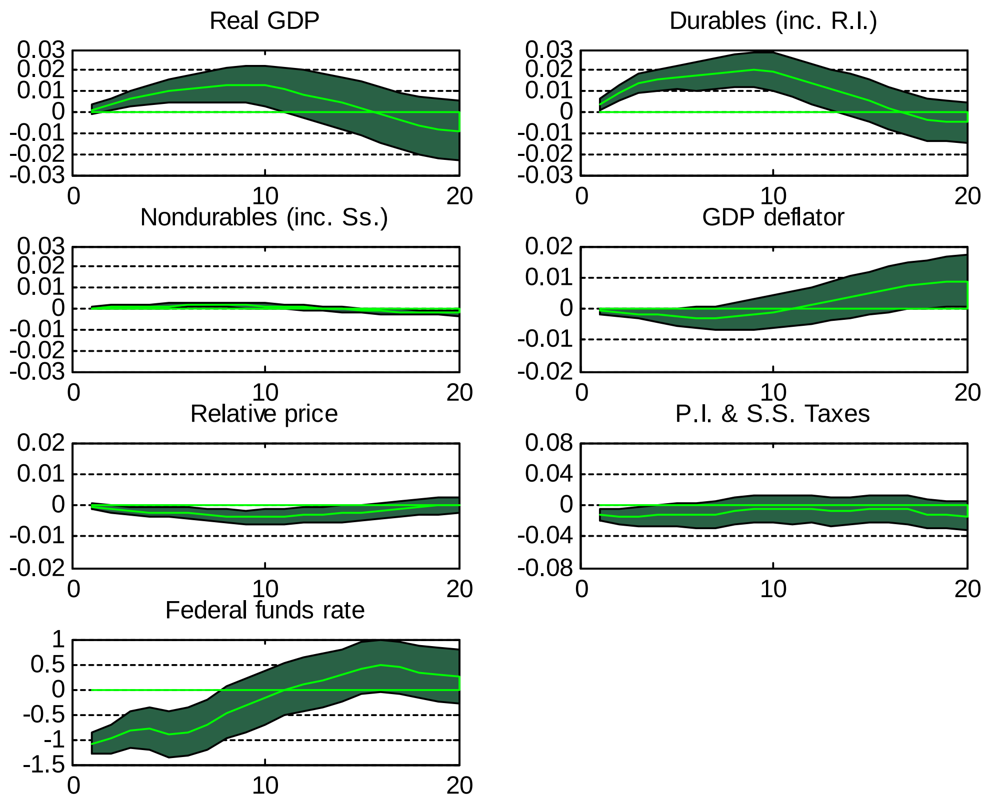


Figure A3: Impulse response function of headline variables to monetary policy shock Romer&Romer adding fiscal variables



A2. Model derivations

A2.1 The government's budget constraint

The consolidated government budget constraint in real terms can be written as:

$$b_t^{\mathbf{g}} + b_t^{\mathbf{cb}} - m_t = \frac{1 + r_{t-1}}{1 + \pi_t} \left(b_{t-1}^{\mathbf{g}} + b_{t-1}^{\mathbf{cb}} \right) - m_{t-1} - \tau_t^{\mathbf{g}}$$

where $\tau^{\mathbf{g}} \equiv \nu \rho_o \tau_t^{\mathbf{n}} + \nu \left(1 - \rho_o\right) \tau_t^{\mathbf{y}} + (1 - \nu) \tau_t^{\mathbf{o}}$ is the total transfer to the households. Define:

$$
\begin{aligned}
\varpi_{t+1} &\equiv \frac{1+r_t}{1+\pi_{t+1}}\left(b_t^{\mathbf{g}}+b_t^{\mathbf{cb}}\right)-m_t, \\
&= \frac{1+r_t}{1+\pi_{t+1}}\left(\frac{1+r_{t-1}}{1+\pi_t}\left(b_{t-1}^{\mathbf{g}}+b_{t-1}^{\mathbf{cb}}\right)-m_{t-1}-\tau_t^{\mathbf{g}}+\frac{r_t}{1+r_t}m_t\right), \\
&= \frac{1+r_t}{1+\pi_{t+1}}\left(\varpi_t-\tau_t^{\mathbf{g}}+\frac{r_t}{1+r_t}m_t\right).
\end{aligned}
$$

Also, define D_s as in the main text note that $\frac{1+r_s}{1+\pi_{s+1}}D_{s+1}=D_s$. Consider budget constraint for period s and multiply both sides by D_{s+1}:

$$
D_{s+1}\varpi_{s+1}=D_s\left(\varpi_s-\tau_s^{\mathbf{g}}+\frac{r_s}{1+r_s}m_s\right).
$$

Sum all constraints from period t to infinity:

$$
\sum_{s=t}^{\infty}D_{s+1}\varpi_{s+1}=\sum_{s=t}^{\infty}D_s\left(\varpi_s-\tau_s^{\mathbf{g}}+\frac{r_s}{1+r_s}m_s\right),
$$

where we impose the limit condition $\sum_{s\to\infty}^{\infty}D_s\varpi_s=0$. Finally, rearrange to obtain:

$$
\sum_{s=t}^{\infty}D_s\left(\frac{r_{s-1}}{1+r_{s-1}}m_s-\tau_s^{\mathbf{g}}\right)=m_t-(1+r_t)\left(b_t^{\mathbf{g}}+b_t^{\mathbf{cb}}\right)
$$

A2.2 Solving the model

The model is solved using first-order perturbation (linearization). This part of the Appendix describes the first-order conditions for the optimization problems of the individuals and discusses aggregation of the individuals' choices.

Old agents and aggregation. Although the model features a representative young agent, there is wealth heterogeneity among the old agents. Typically, dynamic models with a large number of heterogeneous agents are challenging to solve. For our model, however, it turns out that policy functions are linear in wealth, which implies that aggregation is straightforward. Hence we can solve for aggregates without reference to the distribution of wealth among old agents. Wealth heterogeneity between young and old agents, however, is a key factor driving aggregate dynamics.

We exploit that the use of first-order perturbation implies certainty equivalence (see Schmitt-Grohé and Uribe (2004)). As a consequence, first-order approximations to the equilibrium laws of motion of the model coincide with those obtained for a version without aggregate uncertainty.[26] In what follows, we therefore omit expectations operators.[27]

The first-order conditions for the choices of durables, money and bonds by an old household i can be written, respectively, as:

$$
\begin{aligned}
U_{c,i,t} &= U_{d,i,t} + \beta\left(1-\rho_x\right)\left(1-\delta\right)U_{c,i,t+1}, \\
U_{c,i,t} &= U_{m,i,t} + \frac{\beta\left(1-\rho_x\right)}{\left(1+\pi_{t+1}\right)}U_{c,i,t+1}, \\
U_{c,i,t} &= \frac{\beta\left(1-\rho_x\right)\left(1+r_t\right)}{\left(1+\pi_{t+1}\right)}U_{c,i,t+1}.
\end{aligned}
$$

Now introduce four auxiliary variables $\gamma_{c,i,t} \equiv \frac{c_{i,t}}{a_{i,t}}$, $\gamma_{d,i,t} \equiv \frac{d_{i,t}}{a_{i,t}}$, $\gamma_{m,i,t} \equiv \frac{m_{i,t}}{a_{i,t}}$ and $\gamma_{b,i,t} \equiv \frac{b_{i,t}}{a_{i,t}}$.The crucial step is to show that there are four restrictions that pin down $\gamma_{c,i,t}$, $\gamma_{d,i,t}$, $\gamma_{m,i,t}$ and $\gamma_{b,i,t}$ as functions of *only* aggregate variables. To find these coefficients, first combine the first-order conditions to obtain:

$$
\begin{aligned}
U_{c,i,t} &= U_{d,i,t} + \left(1-\delta\right)\left(1+\pi_{t+1}\right)\left(U_{c,i,t} - U_{m,i,t}\right) \\
U_{c,i,t} &= \left(1+r_t\right)\left(U_{c,i,t} - U_{m,i,t}\right)
\end{aligned}
$$

Under the assumed nested CES preferences we obtain:

$$
\begin{aligned}
U_{c,i,t} &= x_{i,t}^{-\sigma}\frac{\epsilon}{\epsilon-1}\left[c_{i,t}^{\frac{\epsilon-1}{\epsilon}} + \eta d_{i,t}^{\frac{\epsilon-1}{\epsilon}} + \mu m_{i,t}^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}-1}\frac{\epsilon-1}{\epsilon}c_{i,t}^{\frac{\epsilon-1}{\epsilon}-1}, \\
&= x_{i,t}^{\frac{-\sigma\epsilon+1}{\epsilon}}c_{i,t}^{\frac{-1}{\epsilon}}, \\
U_{d,i,t} &= x_{i,t}^{\frac{-\sigma\epsilon+1}{\epsilon}}\eta d_{i,t}^{\frac{-1}{\epsilon}}, \\
U_{m,i,t} &= x_{i,t}^{\frac{-\sigma\epsilon+1}{\epsilon}}\mu m_{i,t}^{\frac{-1}{\epsilon}}.
\end{aligned}
$$

[26] Both versions preserve *idiosyncratic* uncertainty.

[27] Alternatively, one could first linearize the model equations and then perform the steps described below.

The combined first-order conditions become:

$$\gamma_{c,i,t}^{\frac{-1}{\epsilon}} = \eta\gamma_{d,i,t}^{\frac{-1}{\epsilon}} + (1-\delta)(1+\pi_{t+1})\left(\gamma_{c,i,t}^{\frac{-1}{\epsilon}} - \mu\gamma_{m,i,t}^{\frac{-1}{\epsilon}}\right) \tag{27}$$

$$\gamma_{c,i,t}^{\frac{-1}{\epsilon}} = (1+r_t)\left(\gamma_{c,i,t}^{\frac{-1}{\epsilon}} - \mu\gamma_{m,i,t}^{\frac{-1}{\epsilon}}\right) \tag{28}$$

To get the third restriction, consider the Euler equation for bonds, which can be written as:

$$\left(\frac{x_{i,t}}{x_{i,t+1}}\right)^{\frac{-\sigma\epsilon+1}{\epsilon}}\left(\frac{c_{i,t}}{c_{i,t+1}}\right)^{\frac{-1}{\epsilon}} = \frac{\beta(1-\rho_x)(1+r_t)}{(1+\pi_{t+1})} \tag{29}$$

and use the fact that $a_{i,t+1} = \left((1-\delta)\gamma_{d,i,t} + \frac{\gamma_{m,i,t}}{1+\pi_{t+1}} + \frac{(1+r_t)\gamma_{b,i,t}}{1+\pi_{t+1}}\right)a_{i,t}$ to write:

$$\frac{c_{i,t}}{c_{i,t+1}} = \frac{\gamma_{c,i,t}}{\gamma_{c,i,t+1}\left((1-\delta)\gamma_{d,i,t} + \frac{\gamma_{m,i,t}}{1+\pi_{t+1}} + \frac{(1+r_t)\gamma_{b,i,t}}{1+\pi_{t+1}}\right)}$$

$$\frac{x_{i,t}}{x_{i,t+1}} = \left[\left(\frac{\gamma_{c,i,t}^{\frac{\epsilon-1}{\epsilon}} + \eta\gamma_{d,i,t}^{\frac{\epsilon-1}{\epsilon}} + \mu\gamma_{m,i,t}^{\frac{\epsilon-1}{\epsilon}}}{\gamma_{c,i,t+1}^{\frac{\epsilon-1}{\epsilon}} + \eta\gamma_{d,i,t+1}^{\frac{\epsilon-1}{\epsilon}} + \mu\gamma_{m,i,t+1}^{\frac{\epsilon-1}{\epsilon}}}\right)\right]^{\frac{\epsilon}{\epsilon-1}} \frac{1}{(1-\delta)\gamma_{d,i,t} + \frac{\gamma_{m,i,t}}{1+\pi_{t+1}} + \frac{(1+r_t)\gamma_{b,i,t}}{1+\pi_{t+1}}}$$

The budget constraint gives the fourth restriction since it can be written as:

$$\gamma_{c,i,t}a_{i,t} + \gamma_{d,i,t}a_{i,t} + \gamma_{m,i,t}a_{i,t} + \gamma_{b,i,t}a_{i,t} = a_{i,t}$$

or:

$$\gamma_{c,i,t} + \gamma_{d,i,t} + \gamma_{m,i,t} + \gamma_{b,i,t} = 1 \tag{30}$$

Equations (1)-(4) pin down $\gamma_{c,i,t}, \gamma_{d,i,t}, \gamma_{m,i,t}$ and $\gamma_{b,i,t}$ as functions of only aggregate variables, as we have substituted out individual wealth from all the equations. Hence we can omit individual i-subscripts for these variables. Given the average wealth level among old agents, $a_t^{\mathbf{o}}$, we can now compute averages for the old agents' decision variables as $c_t^{\mathbf{o}} = \gamma_{c,t}a_t^{\mathbf{o}}$, $d_t^{\mathbf{o}} = \gamma_{d,t}a_t^{\mathbf{o}}$, $m_t^{\mathbf{o}} = \gamma_{m,t}a_t^{\mathbf{o}}$ and $b_t^{\mathbf{o}} = \gamma_{b,t}a_t^{\mathbf{o}}$. Note that these objects do not depend on the distribution of wealth among old agents. Finally, note that $a_t^{\mathbf{o}}$ satisfies:

$$\begin{aligned} a_t^{\mathbf{o}} &= (1-\rho_x)\left((1-\delta)\, d_{t-1}^{\mathbf{o}} + \frac{m_{t-1}^{\mathbf{o}} + (1+r_{t-1})b_{t-1}^{\mathbf{o}}}{1+\pi_t}\right) \\ & +\rho_o\,(1-\rho_x)\,\frac{\nu}{1-\nu}\left[(1-\delta)\, d_{t-1}^{\mathbf{y}} + \frac{m_{t-1}^{\mathbf{y}} + (1+r_{t-1})b_{t-1}^{\mathbf{y}}}{1+\pi_t}\right]. \end{aligned}$$

Young agents. As discussed in the main text there is effectively a representative young agent. Its first-order conditions for the choices of durables, money and bonds can be written as:

$$\begin{aligned} U_{c,t}^{\mathbf{y}} &= \zeta h_t^{\kappa} \\ U_{c,t}^{\mathbf{y}} &= U_{d,t}^{\mathbf{y}} + \beta\,(1-\rho_o)\,(1-\delta)\, U_{c,t+1}^{\mathbf{y}} + \beta\rho_o\,(1-\rho_x)\,(1-\delta)\, U_{c,t+1}^{\mathbf{yo}}, \\ U_{c,t}^{\mathbf{y}} &= U_{m,t}^{\mathbf{y}} + \beta\left(\frac{1-\rho_o}{1+\pi_{t+1}}\right) U_{c,t+1}^{\mathbf{y}} + \beta\frac{\rho_o\,(1-\rho_x)}{1+\pi_{t+1}} U_{c,t+1}^{\mathbf{yo}}, \\ \frac{U_{c,t}^{\mathbf{y}}}{(1+r_t)} &= \beta\frac{1-\rho_o}{1+\pi_{t+1}} U_{c,t+1}^{\mathbf{y}} + \beta\frac{\rho_o\,(1-\rho_x)}{1+\pi_{t+1}} U_{c,t+1}^{\mathbf{yo}}. \end{aligned}$$

Here, $U_{c,t}^{\mathbf{y}}$ and $U_{c,t}^{\mathbf{yo}}$ are the marginal utility of non-durables of the young and newly retired agents, respectively, which satisfy:

$$\begin{aligned} U_{c,t}^{\mathbf{y}} &= \left(x_t^{\mathbf{y}}\right)^{\frac{-\sigma\epsilon+1}{\epsilon}} \left(c_t^{\mathbf{y}}\right)^{\frac{-1}{\epsilon}} \\ U_{d,t}^{\mathbf{y}} &= \left(x_t^{\mathbf{y}}\right)^{\frac{-\sigma\epsilon+1}{\epsilon}} \eta\left(d_t^{\mathbf{y}}\right)^{\frac{-1}{\epsilon}} \\ U_{m,t}^{\mathbf{y}} &= \left(x_t^{\mathbf{y}}\right)^{\frac{-\sigma\epsilon+1}{\epsilon}} \mu\left(m_t^{\mathbf{y}}\right)^{\frac{-1}{\epsilon}} \\ U_{c,t}^{\mathbf{yo}} &= \left(x_t^{\mathbf{yo}}\right)^{\frac{-\sigma\epsilon+1}{\epsilon}} \left(c_t^{\mathbf{yo}}\right)^{\frac{-1}{\epsilon}} \end{aligned}$$

where $x_t^{\mathbf{yo}} = \left[\left(c_t^{\mathbf{yo}}\right)^{\frac{\epsilon-1}{\epsilon}} + \eta\left(d_t^{\mathbf{yo}}\right)^{\frac{\epsilon-1}{\epsilon}} + \mu\left(m_t^{\mathbf{yo}}\right)^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}}$, $c_t^{\mathbf{yo}} = \gamma_{c,t} a_t^{\mathbf{y}}$, $d_t^{\mathbf{yo}} = \gamma_{d,t} a_t^{\mathbf{y}}$ and $m_t^{\mathbf{yo}} = \gamma_{m,t} a_t^{\mathbf{y}}$. Finally, the wealth of a young agent can be expressed as:

$$\begin{aligned} a_t^{\mathbf{y}} &= (1-\rho_o+\rho_o\rho_x)\left((1-\delta)\, d_{t-1}^{\mathbf{y}} + \frac{m_{t-1}^{\mathbf{y}} + (1+r_{t-1})b_{t-1}^{\mathbf{y}}}{1+\pi_t}\right) \\ & +\frac{1-\nu}{\nu}\rho_x\left((1-\delta)\, d_{t-1}^{\mathbf{o}} + \frac{m_{t-1}^{\mathbf{o}} + (1+r_{t-1})b_{t-1}^{\mathbf{o}}}{1+\pi_t}\right). \end{aligned}$$

Below we describe two special cases.

The full system. Old agents:

$$
\begin{aligned}
\gamma_{c,t}^{\frac{-1}{\epsilon}} &= \eta\gamma_{d,t}^{\frac{-1}{\epsilon}} + (1-\delta)(1+\pi_{t+1})\left(\gamma_{c,t}^{\frac{-1}{\epsilon}} - \mu\gamma_{m,t}^{\frac{-1}{\epsilon}}\right) && (31)\\
\gamma_{c,t}^{\frac{-1}{\epsilon}} &= (1+r_t)\left(\gamma_{c,t}^{\frac{-1}{\epsilon}} - \mu\gamma_{m,t}^{\frac{-1}{\epsilon}}\right) && (32)\\
\frac{\beta(1-\rho_x)(1+r_t)}{(1+\pi_{t+1})} &= (\Phi_t)^{\frac{-\sigma\epsilon+1}{\epsilon}}\left(\frac{\gamma_{c,t+1}}{\gamma_{c,t}}\left((1-\delta)\gamma_{d,t} + \frac{\gamma_{m,t}}{1+\pi_{t+1}} + \frac{(1+r_t)\gamma_{b,t}}{1+\pi_{t+1}}\right)\right)^{\frac{1}{\epsilon}} && (33)\\
\Phi_t &= \left[\left(\frac{\gamma_{c,t}^{\frac{\epsilon-1}{\epsilon}} + \eta\gamma_{d,t}^{\frac{\epsilon-1}{\epsilon}} + \mu\gamma_{m,t}^{\frac{\epsilon-1}{\epsilon}}}{\gamma_{c,t+1}^{\frac{\epsilon-1}{\epsilon}} + \eta\gamma_{d,i,t+1}^{\frac{\epsilon-1}{\epsilon}} + \mu\gamma_{m,t+1}^{\frac{\epsilon-1}{\epsilon}}}\right)\right]^{\frac{\epsilon}{\epsilon-1}} \frac{1}{(1-\delta)\gamma_{d,t} + \frac{\gamma_{m,t}}{1+\pi_{t+1}} + \frac{(1+r_t)\gamma_{b,t}}{1+\pi_{t+1}}} && (34)\\
c_t^{\mathbf{o}} &= \gamma_{c,t}a_t^{\mathbf{o}} && (35)\\
d_t^{\mathbf{o}} &= \gamma_{d,t}a_t^{\mathbf{o}} && (36)\\
m_t^{\mathbf{o}} &= \gamma_{m,t}a_t^{\mathbf{o}} && (37)\\
b_t^{\mathbf{o}} &= \gamma_{b,t}a_t^{\mathbf{o}} && (38)\\
a_t^{\mathbf{o}} &= (1-\rho_x)\left((1-\delta)d_{t-1}^{\mathbf{o}} + \frac{m_{t-1}^{\mathbf{o}} + (1+r_{t-1})b_{t-1}^{\mathbf{o}}}{1+\pi_t}\right) && (39)\\
&\quad +\rho_o(1-\rho_x)\frac{\nu}{1-\nu}\left[(1-\delta)d_{t-1}^{\mathbf{y}} + \frac{m_{t-1}^{\mathbf{y}} + (1+r_{t-1})b_{t-1}^{\mathbf{y}}}{1+\pi_t}\right] && (40)\\
a_t^{\mathbf{o}} &= c_t^{\mathbf{o}} + d_t^{\mathbf{o}} + m_t^{\mathbf{o}} + b_t^{\mathbf{o}} && (41)
\end{aligned}
$$

Young / newly retired agents:

$$
\begin{aligned}
\left(x_t^{\mathbf{y}}\right)^{\frac{-\sigma\epsilon+1}{\epsilon}}\left(c_t^{\mathbf{y}}\right)^{\frac{-1}{\epsilon}} &= \zeta h_t^{\kappa} && (42)\\
\left(c_t^{\mathbf{y}}\right)^{\frac{-1}{\epsilon}} &= \eta\left(d_t^{\mathbf{y}}\right)^{\frac{-1}{\epsilon}}+\beta\left(1-\rho_o\right)\left(1-\delta\right)\left(\frac{x_{t+1}^{\mathbf{y}}}{x_t^{\mathbf{y}}}\right)^{\frac{-\sigma\epsilon+1}{\epsilon}}\left(c_{t+1}^{\mathbf{y}}\right)^{\frac{-1}{\epsilon}} && (43)\\
&\quad+\beta\rho_o\left(1-\rho_x\right)\left(1-\delta\right)\left(\frac{x_{t+1}^{\mathbf{yo}}}{x_t^{\mathbf{y}}}\right)^{\frac{-\sigma\epsilon+1}{\epsilon}}\left(c_{t+1}^{\mathbf{yo}}\right)^{\frac{-1}{\epsilon}},\\
\left(c_t^{\mathbf{y}}\right)^{\frac{-1}{\epsilon}} &= \mu\left(m_t^{\mathbf{y}}\right)^{\frac{-1}{\epsilon}}+\beta\left(\frac{1-\rho_o}{1+\pi_{t+1}}\right)\left(\frac{x_{t+1}^{\mathbf{y}}}{x_t^{\mathbf{y}}}\right)^{\frac{-\sigma\epsilon+1}{\epsilon}}\left(c_{t+1}^{\mathbf{y}}\right)^{\frac{-1}{\epsilon}} && (44)\\
&\quad+\beta\frac{\rho_o\left(1-\rho_x\right)}{1+\pi_{t+1}}\left(\frac{x_{t+1}^{\mathbf{yo}}}{x_t^{\mathbf{y}}}\right)^{\frac{-\sigma\epsilon+1}{\epsilon}}\left(c_{t+1}^{\mathbf{yo}}\right)^{\frac{-1}{\epsilon}},\\
\left(c_t^{\mathbf{y}}\right)^{\frac{-1}{\epsilon}} &= \beta\frac{\left(1-\rho_o\right)\left(1+r_t\right)}{1+\pi_{t+1}}\left(\frac{x_{t+1}^{\mathbf{y}}}{x_t^{\mathbf{y}}}\right)^{\frac{-\sigma\epsilon+1}{\epsilon}}\left(c_{t+1}^{\mathbf{y}}\right)^{\frac{-1}{\epsilon}} && (45)\\
&\quad+\beta\frac{\rho_o\left(1-\rho_x\right)\left(1+r_t\right)}{1+\pi_{t+1}}\left(\frac{x_{t+1}^{\mathbf{yo}}}{x_t^{\mathbf{y}}}\right)^{\frac{-\sigma\epsilon+1}{\epsilon}}\left(c_{t+1}^{\mathbf{yo}}\right)^{\frac{-1}{\epsilon}}.\\
a_t^{\mathbf{y}} &= \left(1-\rho_o+\rho_o\rho_x\right)\left(\left(1-\delta\right)d_{t-1}^{\mathbf{y}}+\frac{m_{t-1}^{\mathbf{y}}+(1+r_{t-1})b_{t-1}^{\mathbf{y}}}{1+\pi_t}\right) && (46)\\
&\quad+\frac{1-\nu}{\nu}\rho_x\left(\left(1-\delta\right)d_{t-1}^{\mathbf{o}}+\frac{m_{t-1}^{\mathbf{o}}+(1+r_{t-1})b_{t-1}^{\mathbf{o}}}{1+\pi_t}\right)\\
c_t^{\mathbf{y}}+d_t^{\mathbf{y}}+m_t^{\mathbf{y}}+b_t^{\mathbf{y}} &= a_t^{\mathbf{y}}+h_t^{\mathbf{y}}+\tau_t^{\mathbf{s}} && (47)\\
c_t^{\mathbf{yo}} &= \gamma_{c,t}a_t^{\mathbf{y}} && (48)\\
x_t^{\mathbf{yo}} &= \left[\left(\gamma_{c,t}a_t^{\mathbf{y}}\right)^{\frac{\epsilon-1}{\epsilon}}+\eta\left(\gamma_{d,t}a_t^{\mathbf{y}}\right)^{\frac{\epsilon-1}{\epsilon}}+\mu\left(\gamma_{m,t}a_t^{\mathbf{y}}\right)^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}} && (49)\\
x_t^{\mathbf{y}} &= \left[\left(c_t^{\mathbf{y}}\right)+\eta\left(d_t^{\mathbf{y}}\right)^{\frac{\epsilon-1}{\epsilon}}+\mu\left(m_t^{\mathbf{y}}\right)^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}} && (50)
\end{aligned}
$$

Government policy:

$$
\begin{aligned}
\frac{r_{t-1}\left(b_{t-1}^{\mathbf{g}}+b_{t-1}^{\mathbf{cb}}\right)}{1+\pi_t} &= \nu\left(1-\rho_o\right)\tau_t^{\mathbf{s}} && (51)\\
\frac{m_t}{m_{t-1}}\left(1+\pi_t\right) &= 1+z_t && (52)\\
z_t &= \xi\left(\overline{m}-m_{t-1}\right)+\varepsilon_t && (53)
\end{aligned}
$$

Market clearing:

$$c_t + d_t = \nu h_t^{\mathbf{y}} + (1-\delta)\, d_{t-1} \tag{54}$$
$$c_t = \nu c_t^{\mathbf{y}} + (1-\nu)\, c_t^{\mathbf{o}} \tag{55}$$
$$d_t = \nu d_t^{\mathbf{y}} + (1-\nu)\, d_t^{\mathbf{o}} \tag{56}$$
$$m_t = \nu m_t^{\mathbf{y}} + (1-\nu)\, m_t^{\mathbf{o}} \tag{57}$$
$$0 = b_t^{\mathbf{g}} + b_t^{\mathbf{cb}} + \nu b_t^{\mathbf{y}} + (1-\nu)\, b_t^{\mathbf{o}} \tag{58}$$

These are 28 equations in 28 variables, being c_t, $c_t^{\mathbf{o}}$, $c_t^{\mathbf{yo}}$, $c_t^{\mathbf{y}}$, d_t, $d_t^{\mathbf{o}}$, $d_t^{\mathbf{y}}$, m_t, $m_t^{\mathbf{o}}$, $m_t^{\mathbf{y}}$, $b_t^{\mathbf{o}}$, $b_t^{\mathbf{y}}$, $b_t^{\mathbf{g}}$, $b_t^{\mathbf{cb}}$, $x_t^{\mathbf{y}}$, $x_t^{\mathbf{yo}}$,Φ_t, $\gamma_{c,t}$, $\gamma_{d,t}$, $\gamma_{m,t}$, $\gamma_{b,t}$, $h_t^{\mathbf{y}}$, r_t, π_t, $\tau_t^{\mathbf{s}}$, z_t, $a_t^{\mathbf{o}}$, and $a_t^{\mathbf{yo}}$. We leave out the government's budget constraint, which is redundant by Walras' law.

Special cases **Special case 1** ($\epsilon = 1$). When the utility elasticity ϵ equals one, the utility function becomes a Cobb-Douglas basket nested in a CRRA function:

$$U(c_{i,t}, d_{i,t}, m_{i,t}) = \frac{\left(c_{i,t} d_{i,t}^{\eta} m_{i,t}^{\mu}\right)^{1-\sigma} - 1}{1-\sigma}$$

and the marginal utilities become $U_{c,i,t} = \frac{x_{i,t}^{1-\sigma}}{c_{i,t}}$ $U_{d,i,t} = \eta \frac{x_{i,t}^{1-\sigma}}{d_{i,t}}$ and $U_{m,i,t} = \mu \frac{x_{i,t}^{1-\sigma}}{m_{i,t}}$. In the system to be solved, we correspondingly set:

$$x_t^{\mathbf{y}} = (c_t^{\mathbf{y}})\,(d_t^{\mathbf{y}})^{\eta}\,(m_t^{\mathbf{y}})^{\mu}$$
$$x_t^{\mathbf{yo}} = (c_t^{\mathbf{yo}})\,(d_t^{\mathbf{yo}})^{\eta}\,(m_t^{\mathbf{yo}})^{\mu}$$
$$\Phi_t = \left(\frac{\gamma_{c,t}}{\gamma_{c,t+1}}\right)\left(\frac{\gamma_{d,t}}{\gamma_{d,t+1}}\right)^{\eta}\left(\frac{\gamma_{m,t}}{\gamma_{m,t+1}}\right)^{\mu}\left((1-\delta)\,\gamma_{d,t} + \frac{\gamma_{m,t}}{1+\pi_{t+1}} + \frac{(1+r_t)\,\gamma_{b,t}}{1+\pi_{t+1}}\right)^{-(1+\eta+\mu)}$$

Special case 2 ($\sigma = \epsilon = 1$). When both the risk aversion coefficient σ and the utility elasticity ϵ are unity, the utility function further simplifies to:

$$U(c_{i,t}, d_{i,t}, m_{i,t}) = \ln c_{i,t} + \eta \ln d_{i,t} + \mu \ln m_{i,t}$$

and the marginal utilities become $U_{c,i,t} = \frac{1}{c_{i,t}}$, $U_{d,i,t} = \frac{\eta}{d_{i,t}}$ and $U_{m,i,t} = \frac{\mu}{m_{i,t}}$. In the program we therefore set $(x_t^{\mathbf{y}})^{\frac{-\sigma\epsilon+1}{\epsilon}} = (x_t^{\mathbf{yo}})^{\frac{-\sigma\epsilon+1}{\epsilon}} = (\Phi_t)^{\frac{-\sigma\epsilon+1}{\epsilon}} = 1$.

A2.3 Extensions of the Model: Unemployment and wage rigidities

In this section we introduce frictions in the labor market to the model. In particular, we model a simple search and matching friction between workers and firms following the approach of Diamond, Mortensen and Pissarides.

Young agents can be either unemployed or matched with a firm.[28] A worker-firm pair produces one unit of output. A separation between a worker and a firm takes place if the worker retires. If the worker does not retire, the match dissolves with an exogenous probability ρ_s. The overall separation rate, denoted $\widetilde{\rho}_s$, is therefore given by $\widetilde{\rho}_s = \rho_o + (1-\rho_o)\,\rho_s$. Newborn agents enter the workforce as unemployed. It follows that the number of job searchers in the economy, which we denote s_t, is given by $s_t = \rho_o \nu + (1-\rho_o)\,\rho_s n_{t-1}$.

Following Andolfatto (1996) and many others, we assume that there is full income insurance among workers. Hence, we preserve our setup without heterogeneity among young agents. Matching in the labor market takes place at the beginning of the period, after aggregate and individual shocks have realized, but before production takes place. The evolution of the employment rate among young agents, denoted n_t, is given by:

$$n_t = (1-\widetilde{\rho}_s)\, n_{t-1} + g_t,$$

where g_t denotes the number of new hires in period t.

The asset value of a firm matched with a worker is given by:

$$V_t = 1 - w_t + (1-\widetilde{\rho}_s)\,\Lambda_{t,t+1} V_{t+1},$$

where $\Lambda_{t,t+1}$ is the stochastic discount factor of the owner of the firm. For simplicity we assume that only young agents are able to run firms.[29,30] Unmatched firms are enabled to

[28] We set $\zeta = 0$ in this model version, i.e. there is no disutility from work.

[29] Thus, upon retirement agents are forced to sell off the ownership of the firm to a young agent.

[30] It follows that $\mathbb{E}_t \Lambda_{t,t+1} V_{t+1} = \beta\,(1-\rho_o)\,\mathbb{E}_t \frac{U^{\mathbf{y}}_{c,t+1}}{U^{\mathbf{y}}_{c,t}} V_{t+1} + \beta \rho_o\,(1-\rho_x)\,\mathbb{E}_t \frac{U^{\mathbf{yo}}_{c,t+1}}{U^{\mathbf{y}}_{c,t}} V_{t+1}$

search on the labor market after paying a vacancy cost χ. There is free entry of firms, which implies that

$$\chi = \lambda_t V_t,$$

where $\lambda_t \equiv \frac{g_t}{v_t}$ is the probability of finding a worker, where v_t is the total number of vacancies posted in the economy. Given a number of vacancies and a number of searchers, the total number of new matches follows from an aggregate matching function given by:

$$g_t = s_t^{\alpha} v_t^{1-\alpha}.$$

We assume the real wage is fixed, i.e. $w_t = w$, which is consistent with equilibrium.[31] Hence we treat w as a parameter which we use to target a steady-state unemployment rate of seven percent. The matching function elasticity, α, is set to 0.5. The separation rate ρ_s is chosen to imply $\widetilde{\rho}_s = 0.1$, i.e. an overall separation rate of ten percent per quarter. The vacancy cost, χ, is calibrated to imply that the expected steady-state cost of hiring a worker is five percent of quarterly output. Finally, the monetary feedback parameter ξ is set to 0.35.

The blue line in Figure 7 plots the responses to a monetary expansion implemented using OMOs. Like in the model with a Walrasian market, durables increase on impact, whereas non-durables decline somewhat. Output declines marginally initially, but quickly rises above its steady state level, showing a much more persistent increase than in the model with a Walrasian labor market. Underlying is an effect that is not present in the model with a Walrasian labor market: with matching frictions, firm investment serves as an additional way of saving. Hence, the increased desire to save following the monetary expansion is not only reflected in a surge in durable purchases, but also in an increase in firm investment, leading to a persistent increase in vacancy posting. The latter in turn results in a gradual increase in output.

Figure 7 also plots the responses to a monetary shock implemented using helicopter drops. Output and durables increase, but less than in the economy with OMOs. Also, the decline in the real interest rate is substantially smaller. Thus, the implementation of monetary policy

[31] One can verify the real wage is always inside the bargaining set in our simulations.

continues to affect real outcomes when the labor market is subject to search and matching frictions.

Figure 7: Model responses to expansionary OMO versus helicopter drop.