The "Mystery of the Printing Press" Monetary Policy and Self-fulfilling Debt Crises*

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Abstract

We study the mechanism by which unconventional (balance-sheet) monetary policy can rule out self-fulfilling sovereign default in a model with optimizing but discretionary fiscal and monetary policymakers. By purchasing sovereign debt, the central bank effectively swaps risky government paper for monetary liabilities only exposed to inflation risk, thus yielding a lower interest rate. We characterize a critical threshold for central bank purchases beyond which, absent fundamental fiscal stress, the government strictly prefers primary surplus adjustment to default. Since default may still occur for fundamental reasons, however, the central bank faces the risk of losses on sovereign debt holdings, which may generate inefficient inflation. This risk does not undermine the credibility of a backstop, nor the ability of a central bank to pursue its inflation objectives when the latter enjoys fiscal backing or fiscal authorities are sufficiently averse to inflation.

JEL classification: E58, E63, H63

Key words: Sovereign risk and default, Lender of last resort, Seigniorage, inflationary financing

> "[T]he proposition [is] that countries without a printing press are subject to self-fulfilling crises in a way that nations that still have a currency of their own are not."

Paul Krugman, "The Printing Press Mystery", The conscience of a liberal, August 17, 2011.

> "Public debt is in aggregate not higher in the euro area than in the US or Japan. [T]he central bank in those countries could act and has acted as a backstop for government funding. This is an important reason why markets spared their fiscal authorities the loss of confidence that constrained many euro area governments' market access."

Mario Draghi, Luncheon Address: Unemployment in the Euro Area, Jackson Hole Symposium, August 22, 2014.

1 Introduction

The recent sovereign debt crisis in the euro area and the launch of the Outright Monetary Transactions (OMTs) program by the European Central Bank (ECB) in September 2012 has revived the academic and policy debate on the role of monetary policy in shielding a country from belief-driven speculation in the sovereign debt market (see Section 2 below). In the quote above, the ECB president Mario Draghi argues that providing a backstop for government debt is among the functions normally performed by a central bank—whether operating in a monetary union or in a country with its own currency. This argument raises two crucial questions, namely: what are the mechanisms that allow a central bank to perform such a function successfully? Under what conditions can a backstop be effective without compromising its ability to pursue its primary objectives of inflation and macroeconomic stability?

The contribution of this paper consists of analyzing in detail the core mechanisms by which monetary authorities can rule out self-fulfilling sovereign crises, relying on either conventional or unconventional monetary policies. In our model, welfare-maximizing fiscal and monetary authorities optimally choose their policy under discretion. Ex post, the fiscal authorities set taxes and may choose outright repudiation by imposing haircuts on debt holders, either in response to weak fiscal fundamentals or because of self-fulfilling default expectations. Monetary authorities set inflation generating seigniorage and reducing the real value of debt. Hence default can occur via haircuts on bond holder and/or inflationary debt debasement. In addition to pursuing (conventional) inflation policy, however, monetary authorities can engage in (unconventional) balance sheet policy, through outright purchases of government debt. Whether via debt debasement or interventions in the debt market, a monetary backstop for public debt that rules out self-fulfilling sovereign default can be effective only if credible, i.e., feasible and welfare-improving from the vantage point of monetary policymakers.

We show that the ability to generate (seigniorage) revenue and especially debase nominal debt via inflation may enable a central bank to affect the range of debt over which the economy is vulnerable to belief-driven default, but is generally insufficient to eliminate welfare-decreasing equilibria driven by self-fulfilling expectations (a point also stressed by Aguiar et al. 2013 and Cooper and Camous 2014). This goal may require the use of unconventional balance sheet policy.

In our analysis, this is where the "printing press" argument in the quote by Krugman comes into play. Monetary authorities stand ready to honour their own liabilities—not necessarily government debt—by redeeming them for cash (fiat money) at their nominal value. Hence, by purchasing government paper while simultaneously issuing monetary liabilities, the central bank effectively swaps default-risky public debt for its own liabilities with a guaranteed face value, subject only to the risk of inflation.[1] The implied positive interest differential between government and monetary liabilities is a precondition for a successful backstop. Because of this differential, central bank interventions can reduce uncertainty and the overall cost of debt service, altering the trade-offs faced by a discretionary fiscal authority.

To characterize a credible backstop strategy, we show that there is a critical threshold for central bank purchases of sovereign debt beyond which, absent fundamental fiscal stress, default is never the preferred policy option. This is because purchases of the appropriate size keep the cost of debt low enough that the fiscal authorities will choose to adjust the primary surplus rather than defaulting. Yet, default may still occur if fundamentals turns out to be weak—raising the risk of losses on the central bank balance sheet.

Balance sheet losses need not impinge on the ability of the monetary authorities to pursue optimal inflation if the government accepts negative transfers from the central bank (i.e., it provides fiscal backing, see Del Negro

[1] See Gertler and Karadi (2012) for a similar notion of unconventional monetary policy applied to ouright purchases of private assets. Together with us, the same point has been recently emphasized by Hall and Reis (2015) and Reis (2013).

and Sims 2014). Barring fiscal transfers under any circumstances, however, the central bank's commitment to honour its own liabilities in full becomes inconsistent with pursuing optimal inflation policies in case of fundamental fiscal stress. Budget separation —requiring the central bank to be the sole responsible for its own balance sheet—creates inflation risk, which may in turn cause an equilibrium with central bank interventions to be welfare-dominated by the equilibrium with belief-driven speculation. Budget separation may thus constrain credible backstops to be carried out only on a scale that does not foreshadow strongly adverse consequences on the future inflation choices by the central bank. A key result from our analysis is that these restrictions are not relevant if fiscal and monetary authorities share the same objectives (or more in general if the fiscal authority is sufficiently averse to inflation), and internalize the effects of own policy choices on overall distortions.

These results are important in light of concerns that, on the one hand, large-scale purchases of government debt will eventually cause monetary authorities to run high inflation, even when the backstop is successful in ruling out belief-driven crises. On the other hand, the central bank may not have the ability to expand its balance sheet on a sufficient scale to effectively backstop government debt. Our analysis suggests that an effective backstop neither has to guarantee the government in all circumstances at the cost of high inflation, nor has to match the full scale of the government financing.

While our framework builds upon Calvo (1988), our model and results are related to a vast and growing literature on self-fulfilling debt crises, most notably Cole and Kehoe (2002) and more recently Lorenzoni and Werning (2014) and Nicolini et al. (2014), as well as sovereign default and sovereign risk, see e.g. Arellano (2008) and Uribe (2006) among others. Jeanne (2012) and Roch and Uhlig (2011) analyze the role of an external lender of last resort. Cooper (2012) and Tirole (2012) study debt guarantees and international bailouts in a currency union.

A few recent papers and ours complement each other in the analysis of sovereign default and monetary policy. Discussing self-fulfilling debt crises, Bacchetta et al. (2014) study the role of conventional monetary policy in a new-Keynesian model, while Reis (2013) models the central bank balance sheet in a similar way as ours. Both papers assume fiscal and monetary policy follow exogenous rules, however. In a dynamic framework, Aguiar, Amador, Farhi and Gopinath (2013) analyze a similar problem as in our paper with optimizing fiscal and monetary authorities, focusing on inflation policies rather than balance sheet policies.

The text is organized as follows. The next section gives an intuitive account of the gist of our argument and results, also putting our contribution in the context of the literature. Section 3 presents our model economy, and Section 4 derives the main results on equilibrium multiplicity under conventional monetary policy. Section 5 contains the main results on the central bank backstop, and Section 6 offers some conclusions.

2 What allows a central bank to backstop government debt?

The main goal of our analysis is to understand which policies, if any, a central bank can rely upon to shield a country from the disruptive effects of self-fulfilling crises in the sovereign debt market. When policymakers act under discretion, multiple equilibria are possible because, by determining the equilibrium costs of issuing public debt, agents expectations impact on the ex-post choices by the fiscal and monetary authorities: if agents arbitrarily coordinate their expectations on anticipation of default on public debt, they will require a high interest rate to finance the government; facing a high interest bill, the government is more likely to choose to default (partially or fully) on its liabilities, over the alternative of adjusting the primary surplus, thus validating agents' expectations.

Equilibrium multiplicity is illustrated by Figure 1, which summarizes the main properties of our model economy in the absence of a successful backstop. The figure plots the interest costs of issuing public debt, $R_B B$, measured on the y-axis, against the initial financing need of the government, denoted by B, on the x-axis. As explained below in detail, the market interest rate R_B in the figure is set by risk-neutral rational investors, forming expectations of taxation, default and inflation one-period ahead, knowing that policymaking is discretionary and (exogenous) macroeconomic conditions vary randomly. The states of the economy are parameterized as a weak, average or high output.

In the figure, the interest costs faced by the government are overall increasing in B but not continuously so. Because default has fixed costs, there are threshold values of B, at which the interest rate R_B jumps up, marking a sharp increase in $R_B B$. The three segments in the figure have a steeper slope as we move to the right of the figure and, most crucially for our purposes, overlap over two ranges of B (marked by a shaded area). In other words, as the initial financing need of the government grows larger, higher interest costs imply that the fiscal authority may find it optimal to default more, and in more states of the world, rather than facing the economic distortions associated with adjusting the primary surplus to service debt in all circumstances. Over some ranges of debt, however, under rational expectations the interest rate is not uniquely determined.

In the shaded region of B to the left of the figure, an equilibrium in which the government does not default (labelled ND) coexists with another equilibrium in which, if agents anticipate default to occur under weak macroeconomic conditions, the government validates their expectations (labelled D). Here, the D equilibrium is "non-fundamental", in the sense that fiscal distress is determined not by the exogenous state of the economy, but by market expectations. Between the two shaded areas, the financing need of the government are so high that the government would default under macroeconomic stress even if investors bought government debt at the risk-free rate R. Yet, there is no other equilibrium in which the government would find it optimal to repudiate debt in better states of the economy than the weak one: over the region of debt in between the two shaded area, the D equilibrium is unique and fundamental.

Default in more states of the world becomes again a possibility for higher values of B, under the second shaded area to the right of the figure. Here, an equilibrium with fundamental default only when fundamentals turns out to be weak (D), coexists with an equilibrium with self-fulfilling expectations of (non-fundamental) default *also* under average macroeconomic conditions (DD). In the economic environment illustrated by Figure 1, our question is whether and how a central bank can prevent a rise in interest rates driven by arbitrary anticipations of outright default, *de facto* eliminating the overlap between segments.

Somewhat paradoxically, early seminal work on self-fulfilling debt crises in a monetary economy, by Calvo (1988), envisions monetary policy as part of the problem, rather than as part of the solution. Calvo assumes bounded costs of inflation. In this case (or, as we will see below, when these costs are not bounded but do not grow too fast), as long as the central bank acts under discretion, non-fundamental hikes in interest rates can be driven by self-fulfilling expectations of debt debasement through bouts of inflation—in addition or in alternative to self-fulfilling expectations of outright default).

More recent contributions take quite a different perspective on the role of monetary policy. In a number of papers (see e.g. Aguiar et al. 2013 and Cooper and Camus 2014), the main focus is on the option to inflate away debt, as an off-equilibrium threat the central bank can use to coordinate market expectations on the fundamental equilibrium. The threat works as follows. In response to a hike in interest rates due to belief-driven anticipations of outright default, the central bank stands ready to engineer inflation and reduce ex-post the real value of the interest bill. A fall in the real interest costs in turn eliminates the need for large and costly adjustment in primary surpluses that would otherwise make outright default attractive. The key issue is whether (under what conditions) can such a threat be credible, that is, feasible and welfare-enhancing from the perspective of monetary authorities. Essentially because of the ensuing costs of high inflation, the literature concludes that, in general, this type of policy provides an effective backstop to government debt only under strict conditions.[2] In the model underlying Figure 1, indeed, the option to inflate away debt is available to the central bank, but is used only at the margin. At best, it improves the resilience of the economy to debt crises, but does not rule out multiplicity. Similar conclusions are reached in models allowing for nominal rigidities, whereas inflationary monetary expansions have the additional benefit of lowering the interest rate in real terms and, to the extent that they raise current output and thus current tax revenue, they also reduce the initial financing need of the government (see e.g. Bacchetta et al. 2015).

The threat of inflationary debt debasement, however, is not the only strategy available to monetary authorities. As a matter of fact, the policy debate on monetary backstops typically revolves around another option, that is, purchases of debt by the central bank. Relative to the threat of inflationary debasement of public nominal liabilities, the key difference is that this second option aims at lowering the government's borrowing costs ex-ante, at the time of debt issuance, rather then ex post, after debt has been issued at high nominal costs.

The central question is then how can debt purchases by the monetary authority lower the overall costs of issuing public debt. From an aggregate perspective, the liabilities of the public sector are ultimately backed by current and future primary surpluses cum seigniorage, that is, by resources contributed by domestic taxpayers: the central bank is not an external lender of last resort that can throw in extra resources, in addition to the overall fiscal capacity of a country.

The answer to the above question is arguably easy to grasp when the interest rate is at its zero lower bound and the economy is in a liquidity trap (similar to the circumstances following the global crisis of 2008 in advanced countries). In a liquidity trap, it is well understood that central banks are able to issue fiat money at will and buy government paper, without any

[2] In Aguiar et al. (2013), for instance, these conditions include a contingent lengthening of the maturity of public debt, so that debt debasement can be accomplished via sustained but moderate inflation over time—essentially, smoothing the costs of inflation debasement across periods.

impact on current prices, as long as these purchases are not permanent. In economies vulnerable to self-fulfilling debt crises, however, even temporary purchases have at least one consequential effect: they reduce the amount of default-risky debt that the government needs to sell to the market, substituting it with fiat money—effectively, a safe short-term asset on which agents expect no haircut.[3]

In the theory and practice of monetary policy, imperfect substitutability between monetary liabilities and other assets has long provided the foundations of open market operations and the effectiveness of conventional monetary policy, see Wallace (1981). Central bank liabilities, in the form of cash and bank reserves, do differ from the government's in that the former are a claim to cash—by its nature, fiat money is a claim on itself—and central banks can always make good on their debt by running the printing press. The monetary literature takes for granted that central banks will make no discretionary attempt to tamper with the face value of the monetary base. The point we stress in this paper is that imperfect substitutability between monetary liabilities and other assets is also an integral component of monetary backstops. When a central bank buys debt issuing monetary base and reserves, it effectively swaps default-risky debt, with default-free liabilities—government debt exposed to both inflation and default risk, with core central bank liabilities exposed to inflation risk, but not to default risk. Because of the price difference, such a swap lowers the overall costs of borrowing for the public sector.

While the working of a monetary backstop via non-inflationary debt purchases is easy to grasp when rates are at the zero lower bound, by no means its logic applies only in a liquidity trap. As systematically analyzed by ongoing work on the "new style central banking" exploring the theoretical foundations and practical arrangement of unconventional monetary policy, central banks can and do operate via balance sheet expansions in a variety of circumstances—see Bassetto and Messer (2013), Del Negro and Sims (2014)

[3] The former ECB president, Jean-Claude Trichet writes: "I think we have to reflect more on the reason why the purchases of Treasuries appeared appropriate in the aftermath of the crisis despite the paradox that they seem to have a modest effect on the economy as a whole [...]. Such purchases might have played the role of an insurance policy against any start of materialization of the ultimate tail risk: the challenge to sovereign signatures (not only the weakest European ones) [...]. The counterfactual is naturally impossible to figure out. But it is illegitimate to wonder what could have happened, in the past three years, if a number of central banks had not purchased any Treasuries, at a moment when investors and savers, losing confidence, were starting to put into question all signatures, including the traditionally unchallengeable risk-free?" Jean-Claude Trichet, 2013.

and Hall and Reis (2015) among others. A key institutional development is that policymakers entertain the option to pay interest rates on reserves. By virtue of this option, central banks can engage in non-inflationary expansions of their balance sheet even when policy rates are not at zero, as reserves are issued at the equilibrium interest rate, consistent with expectations of future inflation.[4] As long as it is understood that monetary liabilities will always be convertible into cash/fiat money at face value, reserves will earn a lower interest than government debt.[5]

In the rest of paper, we emphasize a central bank's ability to purchase default-risky government debt financed by issuing default-free monetary liabilities as a precondition for implementing a successful monetary backstop, and analyze in details the conditions under which monetary authorities can rely on it to rule out equilibria with self-fulfilling sovereign crises. As in the case of inflationary debt debasement, the central bank may provide a successful backstop by threatening to intervene in the debt market, rather than carrying out actual debt purchases. Its analysis will thus require a thorough discussion of the credibility of such threat.

In this respect, the second multiplicity region in Figure 1 should be emphasized as a qualifying feature for a model of backstops—that distinguishes our contribution to the literature. Only in this regions is a discussion of central bank's balance sheet losses meaningful, as default may occur (for fundamental reasons) even after monetary interventions in the debt market are successful in eliminating self-fulfilling default. We carry out such a discussion in Section 5. We should also stress that, by virtue of its tractability, our model sheds light on a number of key analytical features of the literature on non-fundamental debt crisis. By way of example, in section 4, we will show that the size of the multiplicity regions is generally proportional to the spread between the interest costs of government debt across the non-fundamental and the non-fundamental equilibrium; and that well-behaved equilibria in which the cost of debt issuance is increasing in B (as shown in the figure), requires that the interest bill is honoured in full in some state of the world with strictly positive probability.

[4]Whether or not the economy is in a liquidity trap, of course, a large change in the size of a central bank balance sheet may create inflation risk in the future. To avoid this risk, enough fiscal and monetary adjustment is required in the future, to manage and possibly accompany a reduction of the size of the central bank balance sheet over time. We will address this issue formally in our analysis below.

[5]It is worth noting that in the literature on the Fiscal Theory of the Price Level, the central bank is committed to ensure that all public liabilities, fiscal and monetary, are always honored at face value—see Leeper (1991) Sims (1994) and Woodford (1995). Here, the central bank only honors monetary liabilities at face value.

3 A model of self-fulfilling sovereign crises and monetary policy

In this section we describe the model, discuss policy instruments and distortions, and characterize the optimal fiscal and monetary strategies. Since we are interested in the mechanism by which, given the government financing needs, outright default is precipitated by agents expectations (rather than, say, in the determinants of public debt accumulation), as in Calvo (1988) we develop our analysis in a two-period economy framework.

3.1 The model setup

Consider a two-period endowment economy, populated by a continuum of identical risk-neutral agents who derive utility from consuming in period 2 only. Initially (in period 1), agents are endowed with a stock of financial wealth W, which they can invest in public debt, B, central bank liabilities $\mathcal{H}$ (if any are issued), as well as in a safe asset K supplied with infinite elasticity and paying a constant real rate, denoted by ρ. In period 2, they receive a random output realization, and the payoffs from their assets; they pay taxes and consume. The economy can be in one of three states: High, Average, or Low (H, A, L) state with (strictly positive) probability $1-\gamma$, $\gamma\mu$ and $\gamma(1-\mu)$, respectively.

Fiscal and a monetary authorities are both benevolent—they maximize the same objective function given by the utility of the presentative agent—but act under discretion and independently of each other. In the first period, the fiscal authority (the government) faces an exogenously given financing need equal to B, and issue bonds at the market-determined nominal rate R_B.[6] The monetary authority may decide to purchase a share $\omega \in [0,1]$ of the outstanding debt at some policy rate $\overline{R}_B$ which may differ from the market. To finance its debt purchases, the central bank issues interest bearing liabilities $\mathcal{H} = \omega B$, at the risk-free nominal rate R. So, out of total debt, $(1-\omega)B$ is held by private investors, ωB is on the central bank balance sheet. Consumers' wealth in the first period is thus equal to $W = (1-\omega)B + K + \mathcal{H}$.

In the second period, taking interest rates and central bank policy as given, the fiscal authority sets taxes T and may choose to impose a haircut

[6] As long as the initial financial need of the government is given, it is immaterial whether we follow Calvo's specification or we model discount bonds—see Lorenzoni and Werning 2014. The set of equilibria would instead be different in a model after Cole and Kehoe 2000, where multiplicity arises via discretionary default on the initial stock of liabilities.

$\theta_i \in [0,1]$ on bond holders, including the central bank. By the same token, taking interest rates and fiscal policy as given, the monetary authority sets inflation π_i and makes good on any liability it may have, paying $R\mathcal{H}$ to private investors.

3.1.1 Policy instruments and distortions

The instruments of fiscal policy, taxation (primary surplus) and default, induce distortions that affect net output and may aggravate the budget. Taxation results in a dead-weight loss of output indexed by $z(T_i, Y_i)$, where from now on a subscript i will refer to the output state $i = H, A, L$.[7] The function $z(.)$ is convex in T_i, satisfying standard regularity conditions. We realistically assume that, to raise a given level of tax revenue, dead-weight losses are larger, and grow faster in T_i, the lower the realization of output, that is:

$$\begin{aligned} z(T;Y_L) &> z(T;Y_A) > z(T;Y_H), \\ z'(T;Y_L) &> z'(T;Y_A) > z'(T;Y_H). \end{aligned} \tag{1}$$

Since what matters in our analysis is the size of the primary surplus, rather than the individual components of the budget, for simplicity we denote with G a constant, nondefaultable level of government spending, and use taxation or primary surplus interchangeably. For notational simplicity, when unambiguous, we will write the function omitting the output argument, i.e., $z(T_i;Y_i) = z(T_i)$.

Sovereign default may entail different types of costs, associated with a contraction of economic activity and transaction costs in the repudiation of government liabilities. In the theoretical literature, some contributions (see e.g. Arellano 2008 and Cole and Kehoe 2000) posit that a default causes output to contract by a fixed amount. In other contributions (see e.g. Calvo 1988) the cost of default falls on the budget and is commensurate to the extent of the loss imposed on investors. While the relative weight

[7] It can be easily shown that the function $z()$ corresponds to the distortions cause by income taxes on the allocation in an economy with an endogenous labor supply. In general, while we encompass trade-offs across different distortions in a reduced-form fashion, in doing so we draw on a vast literature, ranging from the analysis of the macroeconomic costs of inflation, in the Kydland-Prescott but especially in the new-Keynesian tradition (see e.g. Woodford 2003), to the analysis of the trade-offs inherent in inflationary financing (e.g. Barro 1983), or the role of debt in shaping discretionary monetary and fiscal policy (e.g. Diaz et al. 2008 and Martin 2009), and, last but not least, the commitment versus discretion debate in public policy (e.g. Persson and Tabellini 1993).

of different default costs is ultimately an empirical matter (see e.g. Cruces and Trebesch 2012), alternative assumptions are consequential for policy trade-offs and the properties of equilibria. As explained below, multiplicity of well-behaved equilibria can only arise with fixed costs; with variable costs, the equilibrium rate of default responds to central bank interventions. For these reasons, we prefer not to restrict our model to one type of costs only. Rather, we posit that outright default in period 2 entails a loss of Φ units of output regardless of the size of default and the state of the economy,[8] and aggravate the budget in proportion to the size of default. Namely, upon defaulting, the government incurs a financial outlay equal to a fraction $\alpha \in (0, 1)$ of the total size of default on private agents $\theta_i (1 - \omega) BR_B$—the costs of defaulting on the central bank are discussed below.[9]

Like taxation, also inflation has distortionary effects on economic activity. We posit that the costs of inflation are isomorphic to those of taxation: output is lost according to a convex function $C(\pi_i)$, normalized such that $C(0) = C'(0) = 0$ — a standard instance being $C(\pi) = \frac{\lambda}{2}\pi^2$. For simplicity, as in Calvo (1988), we assume that inflation in period 2 generates seigniorage revenue according to the function

$$\text{Seigniorage} = \frac{\pi_i}{1 + \pi_i}\kappa. \tag{2}$$

where a constant κ implies that there is no Laffer curve.[10]

As explained above, in addition to setting inflation in period 2, monetary authorities have the option to purchase government debt, issuing liabilities at the default-free nominal rate in period 1. From a policy perspective, this distinction between conventional and unconventional monetary policy

[8] The fixed component of the cost squares well with the presumption that the decision to breach government contracts, even with a small haircut, marks a discontinuity in the effects of such policy on economic activity. As we show below, effectively this assumption entails that there is a minimum threshold for the haircut θ applied by the government under default.

[9] Calvo (1988) motivates variable costs of default stressing legal and transaction fees associated to debt repudiation. In a broader sense, one could include disruption of financial intermediaries (banks and pension funds) that may require government support. Note that our results would go through if the variable costs of default were in output units, rather than affecting the budget. The main difference would be that the perceived marginal benefit of default for the fiscal authority would be 1 instead of $1 - \alpha$; the marginal cost would remain equal to α.

[10] We refer to the specification in Calvo (1988), who restricts the demand for (non-interest bearing) fiat money to the case of a constant velocity, and abstracts from specifying a terminal condition. Note that our setup can be easily generalized to encompass an inflation Laffer curve, making κ a decreasing function of inflation.

reflects key institutional developments of modern central banking. From a modelling perspective, assuming that monetary reserves H are interest bearing allows us to introduce a demand for central bank liabilities (for a given price level) in the first period, consistent with the discretionary choice of inflation (the conventional monetary instrument) in the second period.[11]

3.1.2 Budget constraints of the fiscal and the monetary authority

In order to write the budget constraint of the government and the central bank, there are at least three interrelated issues that need to be addressed. The first two concern whether a government that opts to default is able/willing (i) to discriminate between private investors and monetary authorities, applying different haircuts; (ii) to transfer resources to the central bank. The third one concerns the budget costs of defaulting, if any, on the central bank.

For clarity of exposition and analytical convenience, we proceed under the following assumptions: first, the government applies the same haircut θ_i rate to all debt holders, corresponding to a *pari passu* clause in government paper; second, the budget cost of defaulting on the central bank is isomorphic to the costs of defaulting on the private investors, i.e. it is a budget cost proportional to the haircut rate, but not necessarily identical—we stipulate $1 > \alpha > \alpha_{CB} \geq 0$.

Under a *pari passu* rule, and allowing for $0 \leq \alpha_{CB} \leq \alpha$, the budget constraint of the fiscal authority reads:

$$T_i - G = \left[1 - \theta_i \left(1 - \alpha\right)\right] \frac{R_B}{1 + \pi_i} \left(1 - \omega\right) B + \left[1 - \theta_i \left(1 - \alpha_{CB}\right)\right] \frac{\overline{R}_B}{1 + \pi_i} \omega B - \mathcal{T}_i \tag{3}$$

where R_B is the market interest rate at which agents buy the share of government debt $(1 - \omega) B$ not purchased by the central bank, $\overline{R}_B$ is the intervention rate at which the central bank purchases bonds, and $\mathcal{T}_i$ denotes transfers from the central bank to the fiscal authority in state i. The budget

[11] In dynamic monetary models, buying government debt by increasing the money stock does not necessarily result in higher current inflation, as the latter mainly reflects future money growth (see e.g. Diaz et al. (2008) and Martin (2009), placing this consideration at the heart of their analysis of time inconsistency in monetary policy).

constraint of the central bank in the second period is:

$$\begin{aligned}\mathcal{T}_i &= \frac{\pi_i}{1+\pi_i}\kappa + \frac{(1-\theta_i)\,\overline{R}_B}{1+\pi_i}\omega B - \frac{R}{1+\pi_i}\mathcal{H} = \\ &= \frac{\pi_i}{1+\pi_i}\kappa + \left(\frac{(1-\theta_i)\,\overline{R}_B}{1+\pi_i} - \frac{R}{1+\pi_i}\right)\omega B.\end{aligned} \tag{4}$$

Budget interactions between the fiscal and monetary authorities are regulated by institutional rules that typically hold central banks responsible for backing their own liabilities — constraining modalities and size of fiscal transfers to the central bank. If the central bank intervenes in the sovereign debt market and exposes its balance sheet to default risk, then, the need to make up for ex-post losses may weigh on monetary policy decisions. Institutional constraints on transfers, however, are not always binding, and may be relaxed in exceptional circumstances, whereas the fiscal authorities stand ready to provide contingent transfers (i.e. "fiscal backing") to the monetary authorities, to prevent balance sheet losses from conditioning the conduct of monetary policy (see e.g. Del Negro and Sims 2014).

In light of these considerations, it is important to analyze monetary backstops under either case. Namely, we will first derive analytical results under the assumption that the constraint on fiscal transfers to the central bank (if any) is either relaxed or not binding. In this case, we can consolidate the two budget constraints above as follows:

$$\begin{gathered} T_i - G + \frac{\pi_i}{1+\pi_i}\kappa = \\ \frac{[1-\theta_i(1-\alpha)]\,R_B}{1+\pi_i}(1-\omega)\,B + \left[\frac{R}{1+\pi_i} + \frac{\alpha_{CB}\theta_i\overline{R}_B}{1+\pi_i}\right]\omega B. \end{gathered} \tag{5}$$

In a final section, we revisit our main results accounting for a binding central bank constraint—which, without loss of generality, we model as the requirement $T_i \geq 0$.

Under full fiscal backing, the expression (5) clarifies that, no matter how large the increase in the central bank balance sheet (ωB) in period 1 is, a large enough primary surplus (net of the ex post interest bill of the government) allows the central bank to redeem its nominal liabilities from the market in period 2, without impinging on the desired level of inflation. Conversely, if transfer to the central bank are ruled out by (an unbreakable) law (i.e., $T_i \geq 0$ always), by (4) it is apparent that, in case of large losses, honoring the outstanding stock nominal liabilities $\mathcal{H}$ at face value requires a rate of inflation large enough to satisfy:

$$\pi_i\kappa + \left((1-\theta_i)\,\overline{R}_B - R\right)\omega B = 0. \tag{6}$$

Monetary authorities must ensure that the (present discounted value of) seigniorage equals repayment of their liabilities.

3.2 Risk neutral agents and debt pricing

Under risk neutrality, the utility of the representative agent coincides with consumption in period 2:

$$\begin{aligned} U_i &= Y_i - z\left(T_i; Y_i\right) - T_i - \frac{\pi_i}{1+\pi_i}\kappa + \\ & \quad KR + \frac{\left(1-\theta_i\right) R_B}{1+\pi_i}\left(1-\omega\right) B + \frac{R}{1+\pi_i}\mathcal{H} - \mathcal{C}\left(\pi_i\right), \end{aligned} \tag{7}$$

where, ex post, the net real asset payoffs are determined by the realization of default and inflation. Ex ante, the expected real returns on government bonds are equalized to the constant, safe return on the real asset:

$$R_B\left\{\left(1-\gamma\right)\frac{1-\theta_H}{1+\pi_H} + \gamma\left[\mu\frac{1-\theta_A}{1+\pi_A} + \left(1-\mu\right)\frac{1-\theta_L}{1+\pi_L}\right]\right\} = \rho. \tag{8}$$

The interest parity condition pins down the price of government debt as a function of both expected default and expected inflation rates.

When both B and $\mathcal{H}$ are traded, there is a second equilibrium interest parity condition: the interest rate on reserves R, free from outright default risk, must equal the real rate ρ, adjusted by expected inflation:

$$R\left[\frac{1-\gamma}{1+\pi_H} + \gamma\left(\frac{\mu}{1+\pi_A} + \frac{\left(1-\mu\right)}{1+\pi_L}\right)\right] = \rho. \tag{9}$$

Comparing this expression with (8), it is apparent that, in equilibrium, the interest rate on government debt must exceed the interest paid on central bank's liabilities by the expected rate of default.

3.3 Optimal discretionary plans for inflation, taxation and default

In this subsection, we characterize the policy plan set by the fiscal and monetary authorities under discretion in period 2. The two authorities independently maximize the same objective function, given by (7), subject to the consolidated budget constraint (5) (with multiplier λ), and the constraint $\mathcal{T}_i \geq 0$ on the central bank budget (with multiplier $\lambda^{CB} \geq 0$). In doing so, they take as given (i) the rates of return on all assets set in period 1, (ii) the scale of interventions by the Central Bank in period 1, and

(iii) each other instruments. These optimal discretionary inflation, taxation and default plans can thus be written in terms of the following functions: $\pi_i^\sigma = \pi_i\left(R_B, B, R, \omega, \overline{R}_B, T_i, \theta_i\right)$, $T_i^\sigma = T_i\left(R_B, B, R, \omega, \overline{R}_B, \pi_i\right)$ and $\theta_i^\sigma = \theta_i\left(R_B, B, R, \omega, \overline{R}_B, \pi_i\right)$. Here a superscript σ indicates that the first order conditions of the policy problems identify "best responses" by the respective policy authorities (to be dropped from equilibrium values). Note that these best responses are each a function of the initial financing need of the government B and central bank's interventions ω.

3.3.1 The authorities' best responses

The fiscal authority will choose to default when the welfare(=consumption) gains from reducing distortionary taxation after the implementation of an optimal haircut exceeds the fixed and variable costs of default, net of output losses due to inflation, so that:

$$U_i\left(\theta_i^\sigma > 0\right) \geq U_i\left(\theta_i^\sigma = 0\right) \tag{10}$$

To study the conditions for optimal default, thus, we need to characterize first the optimal plan for taxation and inflation given the government decision to either default or service its debt in full.

The first order condition of the fiscal authority problem yields:

$$z'\left(T_i^\sigma; Y_i\right) = \frac{\alpha\left(1-\omega\right) R_B + \left(\alpha_{CB} + \lambda_i^{CB}\right) \cdot \omega \overline{R}_B}{\left(1-\alpha\right)\left(1-\omega\right) R_B - \alpha_{CB} \omega \overline{R}_B}, \tag{11}$$

If the constraint on the admissible haircut rate $\theta_i \in (0, 1]$ is not binding, the above condition determines the optimal taxation. Once this is set, the optimal haircut rate θ_i^σ is obtained from the budget constraint of the government (3). In an interior solution, the fiscal authorities set taxes trading off the output costs of distortionary taxation, with the benefits of reducing the haircut rates so to contain the budget (variable) costs of default. Note that, when the central bank budget constraint is binding (the multiplier λ_i^{CB} is strictly positive), a government who cares about inflation will tend to set higher taxes and reduce the optimal haircut rate, so to contain the inflationary consequences of losses on the central bank balance sheet.

If there is no interior solution to the problem, the government either services its liabilities in full, or impose a 100 percent haircut on bond holders. In either case, taxation T_i^σ is no longer chosen optimally, but adjusts as to satisfy the budget constraint (3) (or (5)), evaluated at either $\theta_i^\sigma = 0$ or $\theta_i^\sigma = 1$.

The first order condition of the monetary authority problem is:

$$\begin{aligned}(1+\pi_i^\sigma)^2 \mathcal{C}'(\pi_i^\sigma) &= z'(T_i;Y_i)\left[\kappa+(1-\omega)BR_B+R\omega B\right] \\ &+ \theta_i^\sigma \left\{ \begin{array}{c} z'(T_i;Y_i)\left[-(1-\alpha)(1-\omega)BR_B+\alpha_{CB}\overline{R}_B\omega B\right] \\ +\alpha(1-\omega)BR_B+\left[\alpha_{CB}\overline{R}_B\right]\omega B \end{array} \right\} \\ &+ \lambda_i^{CB}\cdot\left[\kappa-\left((1-\theta_i)\overline{R}_B-R\right)\omega B\right],\end{aligned} \tag{12}$$

The central bank set inflation by trading off the output cost of inflation with the output benefits from reducing distortionary income taxation net of the costs of default (if any). Under discretion, the monetary authorities will always choose a non-negative rate of inflation. The optimal inflation rate would positive even if printing money generates no seigniorage revenue ($\kappa = 0$). This is because a discretionary monetary authority will not resist the temptation to inflate nominal debt, if only moderately so (according to the condition above). Positive and rapidly rising costs of inflation nonetheless prevent policymakers from attempting to wipe away the debt with a bout of very high inflation. It follows that, in state of the world in which there is no default (for $\theta_i^\sigma = 0$), the constraint on the central bank budget never binds ($\lambda^{CB} = 0$), and the above expression simplifies to:

$$(1+\pi_i^\sigma)^2 \mathcal{C}'(\pi_i^\sigma) = z'(T_i^\sigma;Y_i)\left[\kappa+(1-\omega)BR_B+R\omega B\right].$$

Observe that the optimal plan described above minimizes the joint distortions induced by taxation and default, on the one hand, and inflation, on the other hand. In general, since policymakers are benevolent and share the same objective function, they will want to rely on the simultaneous use of all available instruments—ruling out an uneven resort to extreme inflation as a substitute for outright default.

3.3.2 The plan under a consolidated budget constraint

To make progress on the analytical characterization of our results, it is useful to focus on the case in which the constraint on the central bank is not binding, $\lambda_i^{CB} = 0$—an endogenous outcome in Section 4, an assumption in section 5.1 below. Furthermore, throughout our text we will disregard the budget costs of defaulting on the central bank holdings of debt, i.e., we set $\alpha_{CB} = 0$. Notation-wise, we will distinguish policy variables in the case of interior default using a 'hat' ˆ, as opposed to the case of complete default, using a 'tilde'˜.

Under these assumptions and notational convention, the optimal level of taxes, $\widehat{T}_i^\sigma$ in the case of an interior default simplifies to:

$$z'\left(\widehat{T}_i^\sigma; Y_i\right) = z'\left(\widehat{T}_i; Y_i\right) = \frac{\alpha}{1-\alpha}. \tag{13}$$

Taxation is constant across interior default rates—so that we can drop the superscript σ. The optimal inflation plan is set according to:

$$(1+\widehat{\pi}_i^\sigma)^2 \mathcal{C}'\left(\widehat{\pi}_i^\sigma\right) = (1+\widehat{\pi}^\sigma)^2 \mathcal{C}'\left(\widehat{\pi}^\sigma\right) = \frac{\alpha}{1-\alpha}\left[\kappa + (1-\omega)\, BR_B + \omega BR\right] \tag{14}$$

The optimal inflation rate will therefore be independent of the state of the world i. Given $\widehat{T}_i$, then, the optimal default rate (derived from the consolidated budget constraint) is:

$$\widehat{\theta}_i^\sigma = \frac{(1-\omega)\, BR_B + \omega RB - (1+\widehat{\pi}^\sigma)\left(\widehat{T}_i - G\right) - \widehat{\pi}^\sigma \kappa}{(1-\alpha)(1-\omega)\, BR_B}.$$

In the case of complete default, instead, taxation $\widetilde{T}_i^\sigma$ (set to satisfy the consolidated budget constraint) is always state contingent:

$$\widetilde{T}_i^\sigma = G - \frac{\widetilde{\pi}_i^\sigma}{1+\widetilde{\pi}_i^\sigma}\kappa + \frac{\alpha R_B}{1+\widetilde{\pi}_i^\sigma}(1-\omega)\, B + \frac{R}{1+\widetilde{\pi}_i^\sigma}\omega B.$$

So is the inflation plan, determined by:

$$\begin{aligned}(1+\widetilde{\pi}_i^\sigma)^2 \mathcal{C}'\left(\widetilde{\pi}_i^\sigma\right) = z'\left(\widetilde{T}_i^\sigma\right)&\left[\kappa + \alpha(1-\omega)\, BR_B + \omega BR\right] \\ &+ \alpha(1-\omega)\, BR_B,\end{aligned}$$

For the case of an interior solution to the haircut rate, the optimal outright default condition (10) can then be written as

$$\begin{aligned}\Phi + z\left(\widehat{T}_i\right) + \mathcal{C}\left(\widehat{\pi}^\sigma\right) + \alpha\widehat{\theta}_i^\sigma \frac{R_B}{1+\widehat{\pi}^\sigma}(1-\omega)\, B \\ \leq z\left(T_i^\sigma\right) + \mathcal{C}\left(\pi_i^\sigma\right)\end{aligned} \tag{15}$$

where $\underline{\theta}_i$ is the minimum rate at which the government finds it optimal to default (derived solving the expression above with an equality sign). Because of the fixed output costs Φ, optimal default only occurs at strictly positive rates, hence $\underline{\theta}_i > 0$. If an interior solution does not exist, $\widehat{\theta}_i^\sigma$, $\hat{T}_i$, and $\widehat{\pi}^\sigma$ in the condition above are replaced with 1, $\widetilde{T}_i^\sigma, \widetilde{\pi}_i^\sigma$ and the default condition reads:

$$\textbf{If } \widehat{\theta}_i^\sigma > 1 : \Phi + z\left(\widetilde{T}_i^\sigma\right) + \mathcal{C}\left(\widetilde{\pi}_i^\sigma\right) + \alpha \frac{R_B}{1+\widetilde{\pi}_i^\sigma}\left(1-\omega\right)B \\ \leq z\left(T_i^\sigma\right) + \mathcal{C}\left(\pi_i^\sigma\right). \tag{16}$$

4 Inflation and macroeconomic resilience to self-fulfilling crises

We start our study of monetary policy in economies vulnerable to self-fulfilling debt crises restricting the central bank to rely exclusively on conventional policies, i.e. the central bank only sets inflation. As stressed by Calvo (1988), some degree of "stealth repudiation" is a natural outcome in a monetary economy, because unexpected changes in inflation rates affect the ex-post real returns on assets which are not indexed to the price level. In our monetary model, indeed, repudiation in period 2 can take the form of either outright default on the nominal value of debt, or inflation surprises reducing the real value of debt, or both.[12]

In what follows, we will characterize the set of equilibria conditional on no debt purchases by the central bank, i.e. $\omega = 0$. We will first discuss the properties of inflation implied by the optimal discretionary policy plan, state the definition of equilibrium and establish that, under a standard specification of its distortionary costs, inflation is uniquely determined in equilibrium. We will then identify the conditions for equilibrium multiplicity to occur, and discuss the extent to which inflation policy can enhance the resilience of a country to sovereign debt crises.

4.1 Properties of the optimal inflation plans

We have seen that, under discretion, inflation rates will always be positive in equilibrium—in our nominal economy, government spending will be financed at least in part through seigniorage and some debasement of outstanding (ex-default) public liabilities. Since $\mathcal{T}_i > 0$ in all states of nature, and we can replace the budget constraints of the two authorities with the consolidated one, the optimal inflation plan will always be set according to the first order

[12] This is different from Calvo (1988), where default is implemented alternatively through outright repudiation (in the real version of the model), or inflation only (in the monetary version).

condition:

$$(1+\pi_i^\sigma)^2\,\mathcal{C}'\left(\pi_i^\sigma\right) = z'(T_i^\sigma)\left(\kappa + BR_B\right) + \theta_i^\sigma BR_B\left[\alpha - z'(T_i^\sigma)\left(1-\alpha\right)\right] \quad (17)$$

The optimal policy plan is synthesized in Table 1 below, where the minimum (interior) default rate (27) is obtained from the default condition (15) setting $\widehat{\theta}_i^\sigma = \underline{\theta}_i$ and using the fact that, for $\omega = 0$, the cost of debt issuance can be written as

$$BR_B = \frac{(1+\hat{\pi})\left(\hat{T}_i - G\right) + \hat{\pi}\kappa}{1 - \widehat{\theta}_i^\sigma\left(1-\alpha\right)}. \quad (18)$$

When (27) is satisfied for a $\underline{\theta}_i$ exceeding 100 percent (as is the case if the primary surplus under interior default $\left(\hat{T}_i - G\right) + \hat{\pi}\kappa/\left(1+\hat{\pi}\right)$ is non-positive) the government opts for complete default.

In words: under an optimal default plan $\underline{\theta}_i \leq \widehat{\theta}_i^\sigma \leq 1$, inflation is identical across states of the world, i.e., $\widehat{\pi}_A^\sigma = \widehat{\pi}_L^\sigma = \widehat{\pi}^\sigma$—see the conditions (21) and (22).[13] If the constraint $1 \geq \widehat{\theta}_i^\sigma$ is binding and default is complete, inflation plans are instead state dependent, as taxes and seigniorage will have to adjust to cover current non-interest expenditure according to (23) and (24). The same applies under no outright default ($\theta_i^\sigma = 0$), whereas the revenue from taxation and seigniorage needs to adjust according to (25) and (26) to finance the government real expenditure and interest bill in full. Furthermore, in either corner solution for default, π_i^σ and $\widetilde{\pi}_i^\sigma$ always comoves positively with T_i^σ and $\widetilde{T}_i^\sigma$, hence inflation inherits the same properties as taxation. For instance, it must be that taxes under full default are not lower than taxes under partial default, namely $\widetilde{T}_i^\sigma \geq \hat{T}_i$. It follows that output distortions due to both taxation *and* inflation are higher under full default, including at the margin, since

$$z'(\widetilde{T}_i^\sigma) \geq \alpha/\left(1-\alpha\right). \quad (19)$$

[13]This property of the optimal inflation rate depends on the simplifying assumption that the cost of inflation does not vary with the state of the world. It would be easy to relax this assumption, at the cost of cluttering the notation without much gain in terms of economic intuition.

Table 1

$$\textbf{If } 1 \geq \widehat{\theta}_i^{\sigma} = \frac{1}{1-\alpha}\left[1 - \frac{(1+\widehat{\pi}^{\sigma})\left(\hat{T}_i - G\right) + \widehat{\pi}^{\sigma}\kappa}{BR_B}\right] \geq \underline{\theta}_i > 0: \tag{20}$$

$$\theta_i^{\sigma} = \widehat{\theta}_i^{\sigma} \qquad T_i^{\sigma} = \widehat{T}_i = z'^{-1}\left(\tfrac{\alpha}{1-\alpha}; Y_i\right) \tag{21}$$

$$\text{and} \qquad (1+\widehat{\pi}^{\sigma})^2 \mathcal{C}'(\widehat{\pi}^{\sigma}) = \tfrac{\alpha}{1-\alpha}(\kappa + BR_B) \tag{22}$$

$$\textbf{If } \widehat{\theta}_i^{\sigma} > 1: \qquad \theta_i^{\sigma} = 1 \qquad \widetilde{T}_i^{\sigma} = G + \alpha \tfrac{R_B}{1+\widetilde{\pi}_i^{\sigma}} B - \tfrac{\widetilde{\pi}_i^{\sigma}}{1+\widetilde{\pi}_i^{\sigma}}\kappa \tag{23}$$

$$\text{and} \qquad (1+\widetilde{\pi}_i^{\sigma})^2 \mathcal{C}'(\widetilde{\pi}_i^{\sigma}) = z'(\widetilde{T}_i^{\sigma})(\kappa + \alpha BR_B) + \alpha BR_B \tag{24}$$

$$\textbf{If } \widehat{\theta}_i^{\sigma} < \underline{\theta}_i: \qquad \theta_i^{\sigma} = 0 \qquad T_i^{\sigma} = \tfrac{R_B}{1+\pi_i^{\sigma}} B + G - \frac{\pi_i^{\sigma}}{1+\pi_i^{\sigma}}\kappa \tag{25}$$

$$\text{and} \qquad (1+\pi_i^{\sigma})^2 \mathcal{C}'(\pi_i^{\sigma}) = z'(T_i^{\sigma})(\kappa + BR_B) \tag{26}$$

$$\begin{aligned} \underline{\theta}_i \text{ solves } \Phi = {} & z\left(G - \frac{\pi_i^{\sigma}}{1+\pi_i^{\sigma}}\kappa + \frac{1+\hat{\pi}}{1+\pi_i^{\sigma}} \frac{\left(\hat{T}_i - G\right) + \frac{\hat{\pi}}{1+\hat{\pi}}\kappa}{1-\underline{\theta}_i(1-\alpha)}\right) - z\left(\hat{T}_i\right) \\ & + \mathcal{C}(\pi_i^{\sigma}) - \mathcal{C}(\hat{\pi}) - \alpha\underline{\theta}_i \frac{\left(\hat{T}_i - G\right) + \frac{\hat{\pi}}{1+\hat{\pi}}\kappa}{1-\underline{\theta}_i(1-\alpha)}. \end{aligned} \tag{27}$$

The following lemma summarizes key properties of inflation that will play an important role in our results. Namely, the best response of inflation (and thus its equilibrium value) is increasing in the ex-ante interest rate R_B and stock of debt B. The best responses are such that inflation rates under full default cannot be lower than inflation rates under partial default. Finally, at the corner solutions for the default rate, $\theta_i^{\sigma} = \{0, 1\}$, inflation rates π_i and $\widetilde{\pi}_i$ are both increasing across states H, A, L, that is, inflation is higher when the state of the economy is worse and tax distortions are higher.

Lemma 1 *Inflation best responses* $(\pi_i^\sigma, \hat{\pi}, \widetilde{\pi}_i^\sigma)$ *are i) increasing in the expected sovereign rate* R_B *and stock of debt* B*, where* $\hat{\pi} \leq \widetilde{\pi}_i^\sigma$*; ii) such that* $(\pi_i^\sigma, \widetilde{\pi}_i^\sigma) > \left(\pi_j^\sigma, \widetilde{\pi}_j^\sigma\right)$ *if* $z(\cdot; Y_i) > z(\cdot; Y_j)$*, while* $\hat{\pi}$ *is constant across states.*

These properties are intuitive in light of our assumption of convex costs of inflation, $\mathcal{C}(\pi_i^\sigma)$, which translates into decreasing marginal benefits from its use. Property *i)* follows from inspection of the inflation reaction function (17). In this expression, the right-hand-side is increasing in $R_B B$, since taxes are weakly increasing in the interest rate bill. Moreover, by (19), taxes under full default are at least as high as taxes under partial default $(\widetilde{T}_i^\sigma \geq \hat{T}_i)$. Property ii) descends directly from our ordering of tax distortions $z(\cdot; Y_i)$ across states, stipulating that distortions are worse, the weaker the fundamentals.

4.2 Equilibrium definition

A rational-expectation equilibrium is defined by the pricing conditions (8), together with the consolidated budget constraint (5) with $\omega = 0$, the optimal tax rates, either (21) or (23), or (25), given the default option (15) or (16) with $\omega = 0$, and the optimal inflation, either (22) or (24) or (26).

For the purpose of our analysis, we are interested in studying economies where, in equilibrium, fundamental default may or may not occur in state L under fiscal stress, but there is no fundamental reason for defaulting in states A, H. Consistent with this goal, we find it convenient to impose mild conditions on our specification such that, for increasing initial financing needs of the government B, we obtain the set of equilibria represented in Figure 1:

1. an equilibrium with no default, which we denote with a superscript ND;
2. an equilibrium with full default in state L and no default in the other states, which we denote with a superscript D;
3. an equilibrium with full default in L and partial default in A which we denote with DD.

The following three assumptions detail sufficient conditions for these equilibria to be admissible.

Assumption 1: The primary surplus across states of the world satisfies the following restrictions:

$$\overline{B}_L < (1-\gamma)\left(\widehat{T}_A - G\right)/\rho < \left(\widehat{T}_A - G\right)/\rho < \left(\widehat{T}_H - G\right)/\rho \qquad (28)$$

The condition (28) establishes a reasonable ordering between the primary surplus under interior default in the high (H) and the average (A) state, and stipulates that both must be larger than required to service the maximum level of debt $\left(\overline{B}_L\right)$ sustainable in a state of fiscal stress (L), at the real risk free rate ρ.

A second assumption is motivated by our interest in studying equilibria which are well-behaved, i.e., "stable" by the Walrasian criterion discussed, e.g., by Lorenzoni and Werning (2014). By this criterion, a small increase in the supply of government bonds should not lower the notional interest rate (i.e., it should not raise the price of the bond).

Assumption 2: The probability of state H, $1-\gamma$, and of state A, $\gamma\mu$, are such that:

$$\begin{aligned} 1-\gamma &> \alpha, \\ \frac{1+\hat{\pi}^{DD}}{1+\pi_H^{DD}} &> \mu > 0. \end{aligned} \qquad (29)$$

These conditions ensure that there is a range of debt B for which our model features well-behaved, stable multiple equilibria, in which sovereign rates have the intuitive, desirable property of being increasing in the initial level of public debt, B. Note that the second condition, stipulating that the probability of the intermediate state should not be too high, is always satisfied if $\pi_i = 0$ for any i. We should nonetheless stress that (29) does not rule out the existence of other equilibria which are not well-behaved, i.e., "unstable" by the Walrasian criterion, which may coexist with the stable ones. By the same argument set forth by Lorenzoni and Werning (2014), in what follows we will abstract from these equilibria, on the ground that they have pathological, unpalatable implications for policy.[14]

The last condition—that the primary surplus in state L is at most zero when seigniorage revenue is at its maximum κ—is imposed for the sake of analytical tractability.

[14] In (Walrasian-)unstable equilibria, such as the one discussed by Calvo (1988), the economy is vulnerable to self-fulfilling crisis for small levels (but not for high levels) of debt, and sovereign rates are decreasing in the stock of debt. In an analysis of backstops, interventions by the central bank should be negative, i.e., the central bank should actually sell government debt in response to the threat of a run on debt.

Assumption 3:

$$\left(\widehat{T}_L - G\right) + \kappa \to 0. \tag{30}$$

As shown above, see (27), this implies that a rational expectation equilibrium will have either no default or complete default in state L. We will nonetheless study partial default in the A state in the DD equilibrium—whereas the interior optimal rate $\widehat{\theta}_A^{DD}$ is given by:

$$\widehat{\theta}_A^{DD} = \frac{B\rho - \left[(1-\gamma)\frac{1+\widehat{\pi}^{DD}}{1+\pi_H^{DD}} + \gamma\mu\right]\left[\left(\hat{T}_A - G\right) + \frac{\widehat{\pi}^{DD}}{1+\widehat{\pi}^{DD}}\kappa\right]}{(1-\alpha)\, B\rho - \gamma\mu\left[\left(\hat{T}_A - G\right) + \frac{\widehat{\pi}^{DD}}{1+\widehat{\pi}^{DD}}\kappa\right]} \geq \underline{\theta}_A^{DD} > 0. \tag{31}$$

Under the assumptions just spelled out, the sovereign interest rates across equilibria are, respectively,

$$R_B^{ND} = R = \left\{\frac{(1-\gamma)}{1+\pi_H^{ND}} + \gamma\left[\frac{\mu}{1+\pi_A^{ND}} + \frac{(1-\mu)}{1+\pi_L^{ND}}\right]\right\}^{-1}\rho \tag{32}$$

$$R_B^D = \left\{\frac{(1-\gamma)}{1+\pi_H^D} + \frac{\gamma\mu}{1+\pi_A^D}\right\}^{-1}\rho \tag{33}$$

$$R_B^{DD} = \left\{\frac{(1-\gamma)}{1+\pi_H^{DD}} + \gamma\mu\frac{1-\widehat{\theta}_A^{DD}}{1+\widehat{\pi}^{DD}}\right\}^{-1}\rho. \tag{34}$$

where inflation rates are determined according to (24) in states with full default (i.e. state L in the D and DD equilibria); according to (26) in states with partial default (namely state A in the DD equilibrium), and according to (22) in (all other) states with no default.

For expositional convenience, we will present and discuss our main results in three steps, by stating one lemma and two propositions.

4.3 Uniqueness of inflation

The following lemma establishes that, under our assumptions, the equilibrium inflation rate is uniquely determined, for any equilibrium level of the nominal interest rate R_B^j.

Lemma 2 *With convex costs of inflation and the normalization $C'(0) = 0$, the optimal reaction function for inflation under default, (22) or (24), and under no default (26), yields unique equilibrium rates π_i,$\hat{\pi}$, $\widetilde{\pi}_i$, for given agents expectations and haircuts θ_i^j, embedded in the equilibrium market interest rate R_B^j, $j = ND, D, DD$.*

Consider the reaction function under no default, (26), rewritten here for convenience:

$$\mathcal{C}'(\pi_i) = z'(\frac{R_B^j}{1+\pi_i}B + G - \frac{\pi_i}{1+\pi_i}\kappa)\frac{BR_B^j+\kappa}{(1+\pi_i)^2},$$

where the expressions for R_B^j are given by (32) through (34). Under convexity of inflation costs $C'(\cdot)$, the left-hand-side is increasing in inflation. Therefore, inflation is uniquely determined if the right-hand-side—the marginal benefit of inflation evaluated in equilibrium— is decreasing in π_i. It is easy to see that this is always the case in an equilibrium with no default where $R_B^{ND} = R$, since $R/(1+\pi_i)$ is decreasing in π_i. In turn, this establishes that the inflation level is unique in states of the world in which there is no default across all equilibria (including D and DD). A similar argument applies to the cases of full default,[15] and to the case of interior default. In the latter case (relevant under our assumptions only in the DD equilibrium), we can rewrite the expression (22) as follows:

$$\mathcal{C}'\left(\widehat{\pi}^{DD}\right) = \frac{\alpha}{1-\alpha}\left(\frac{\kappa}{\left(1+\widehat{\pi}^{DD}\right)^2} + \frac{R_B^{DD}B}{\left(1+\widehat{\pi}^{DD}\right)^2}\right).$$

where the first and the second term in brackets on the right hand side are decreasing in $\widehat{\pi}^{DD}$.[16]

A unique inflation rate conditional on a realized haircut rate rules out the possibility of self-fulfilling hikes in interest rates driven by anticipation of debt debasement via inflation—the policy scenario analyzed by Calvo 1988 under the restriction that the costs of inflation are bounded. When these costs are convex, as we assume in our model, multiplicity (if any) only obtains in outright repudiation rates. Convexity of inflation costs

[15] In the case of full default, the term on the right-hand side of (23), capturing the effects of changes in inflation rates on tax distortions, will be again decreasing in π_i. Moreover, it is clear from R_B^D and R_B^{DD} that the level of inflation in the state of full default, L, does not affect the nominal sovereign rate. Therefore, in state L inflation is uniquely determined under both the D and DD equilibrium.

[16] It is clear that since $0 < \widehat{\theta}_A^{DD} \leq 1$,

$$\frac{\rho}{\frac{(1-\gamma)}{1+\pi_H^{DD}}\left(1+\widehat{\pi}^{DD}\right)^2} \geq R_B^{DD}/\left(1+\widehat{\pi}^{DD}\right)^2,$$

but the term on the left hand side is obviously decreasing in $\widehat{\pi}^{DD}$.

could actually be relaxed somewhat, without affecting uniqueness of inflation rates. It would be enough to assume that, for some sufficiently large $\bar{\pi}\left(e.g. \bar{\pi} > \widehat{\pi}^{DD}\right)$, $C'(\cdot)$ falls at a lower rate than the left-hand side (e.g., as approximated by $(1+\pi_i)^{-1}$).[17]

This result offers a new perspective on the key lesson from Calvo (1988). Namely, if, in the policymakers' preferences, the cost of inflation does not rise sufficiently fast, belief-driven sovereign debt crises may be rooted in the behavior of the monetary authorities, rather than the fiscal authorities. In other words, they may occur independently of expectations of outright default. It follows that a sufficiently strong aversion to inflation is a key precondition for the central bank to be effective in ruling out non-fundamental equilibria. Contrary to popular arguments in the press, a low social costs of inflation provides no firm foundations for the central bank to act as a "lender of last resort" to the government.

4.4 Multiple equilibria for intermediate levels of sovereign debt

We are now ready to characterize the multiplicity of equilibria in the sovereign rate and haircuts over some ranges for the government financing needs, as depicted in Figure 1. For clarity of exposition, we articulate our main result in two propositions. The first establishes that there exists a range of B for which both the ND and the D equilibrium are possible. The second shows that there is another, non overlapping, debt range for which the ND equilibrium no longer exists, but both the D and the DD equilibrium are possible. Under the maintained Assumption 2, these equilibria are all well-behaved (Walrasian-stable) in the sense defined above. We will show that inflation and seigniorage do affect the equilibrium policy trade-offs and the debt range over which multiplicity obtains—but the option to inflate debt away does not rule out the possibility of self-fulfilling non-fundamental sovereign crises.

4.4.1 Multiplicity between the ND and the D equilibrium

To state our first proposition, we define two thresholds for B. The first threshold, denoted by $\underline{B}_L$, is defined as the *minimum* level of B at which, if

[17] This condition may not be satisfied by some specifications of $\mathcal{C}(\cdot)$, for instance, if $\mathcal{C}(\cdot)$ is bounded for large but finite values of the inflation rate (see e.g. Calvo 1988). Even in this case, however, it would be possible to obtain a unique equilibrium by assuming that seigniorage is not increasing in inflation, but instead obeys a Laffer curve.

markets coordinate their expectations on anticipating a 100% haircut in the low output state (and thus charge a destabilizing high market rate $R_B^D > R$), ex post the government will validate their expectations and default in the low output state. This threshold is obtained from the counterpart of (16), written as an equality and evaluated at the sovereign rate R_B^D:

$$\Phi + \alpha \frac{R_B^D \underline{B}_L}{1+\widetilde{\pi}_L^D} + z\left(G + \alpha \frac{R_B^D \underline{B}_L}{1+\widetilde{\pi}_L^D} - \frac{\widetilde{\pi}_L^D}{1+\widetilde{\pi}_L^D}\kappa\right) + \mathcal{C}\left(\widetilde{\pi}_L^D\right) \tag{35}$$
$$= z\left(G + \frac{R_B^D \underline{B}_L}{1+\pi_L^D} - \frac{\pi_L^D}{1+\pi_L^D}\kappa\right) + \mathcal{C}\left(\pi_L^D\right).$$

The second threshold, denoted by $\overline{B}_L$, is defined as the *maximum* level of B at which, if markets expect no default and thus charge the risk free rate R, the government will be indifferent between default and no default in any state of the world. The threshold $\overline{B}_L$ is also obtained from (16), again written as an equality but now evaluated at the sovereign rate $R_B^{ND} = R$:

$$\Phi + \alpha \frac{R\overline{B}_L}{1+\widetilde{\pi}_L^{ND}} + z\left(G + \alpha \frac{R\overline{B}_L}{1+\widetilde{\pi}_L^{ND}} - \frac{\widetilde{\pi}_L^{ND}}{1+\widetilde{\pi}_L^{ND}}\kappa\right) + \mathcal{C}\left(\widetilde{\pi}_L^{ND}\right) \tag{36}$$
$$= z\left(G + \frac{R\overline{B}_L}{1+\pi_L^{ND}} - \frac{\pi_L^{ND}}{1+\pi_L^{ND}}\kappa\right) + \mathcal{C}\left(\pi_L^{ND}\right).$$

Note that, when debt is above the threshold $\overline{B}_L$, the government would default in the weak fundamental state even if markets charged the risk free rate R. For $B \geq \overline{B}_L$, then, ND cannot be an equilibrium.

Multiplicity arises if $\overline{B}_L > \underline{B}_L$. For B comprised between the two thresholds, a fundamental equilibrium with no default coexists with another, non-fundamental equilibrium with sovereign risk.

Proposition 3 *Holding assumptions (28) through (29), for* $0 < B < \overline{B}_L$, *where* $\overline{B}_L$ *is defined by (36), there is one well-behaved rational expectations equilibrium with no default which satisfies the budget constraint (5), the optimal tax plan (25), the optimal inflation plan (26) and the pricing condition (8) with* $R_B^{ND} = R$. *For* $B \geq \underline{B}_L$ *where* $\underline{B}_L$ *is defined by (35), there is a second well-behaved rational expectations equilibrium where the government borrows at the rate* R_B^D *and defaults in the L state only, in which it follows optimal plans (23) and (24). These two equilibria will coexist if* $\underline{B}_L < \overline{B}_L$, *for B in the range* $\underline{B}_L \leq B < \overline{B}_L$. *A sufficient condition for multiplicity is that* $\kappa \to 0$.

Under the sufficient condition $\kappa \to 0$ (i.e., seigniorage revenue is negligible), the proof of the proposition is straightforward and particularly instructive in the special case in which all government debt is real (inflation-indexed), so that inflation is always zero in equilibrium and by (30) $\widehat{T}_L - G = 0$. This is the case of a non-monetary economy (or a monetary economy where the costs of inflation are arbitrarily high). To show that $\underline{B}_L < \overline{B}_L$, we combine the equations determining $\underline{B}_L$ and $\overline{B}_L$ imposing $\pi_i = 0$, so to obtain the following expression:

$$\begin{aligned}\Phi &= z\left(G + \rho\overline{B}_L\right) - z\left(G + \alpha\rho\overline{B}_L\right) - \alpha\rho\overline{B}_L \qquad (37)\\ &= z\left(G + \frac{\rho}{(1-\gamma)+\gamma\mu}\underline{B}_L\right) - z\left(G + \frac{\alpha\rho}{(1-\gamma)+\gamma\mu}\underline{B}_L\right) - \frac{\alpha\rho}{(1-\gamma)+\gamma\mu}\underline{B}_L,\end{aligned}$$

where $R_B^D = \frac{\rho}{(1-\gamma)+\gamma\mu} > R = \rho$. The expressions on the first and the second line are both increasing in debt B: this follows from the fact that when $\widehat{T}_L - G = 0$, taxes and distortions under no default and full default are always larger than under partial default — see (19). Since the sovereign (real) rate is higher when agents anticipate default, then, if evaluated at the same level of B, the expression on the second line would be bigger than the expression one on the first line: it must be that $\overline{B}_L > \underline{B}_L$. As (32) and (33) evaluated at zero inflation are constant, it is apparent that the D and the ND equilibria (both admissible for B within the two thresholds) are well-behaved: the ex-ante interest rate payments $R_B^j B$ are always increasing in B for $j = ND, D$.

Similarly to Lorenzoni and Werning (2014), this argument establishes that, in an economy with real (indexed) debt and no seigniorage revenues, there is always the possibility of multiple well-behaved equilibria in default rates. For $\alpha \to 0$ (as, e.g., in Lorenzoni and Werning (2014)), one can easily show that the range of multiplicity $\left[\underline{B}_L, \overline{B}_L\right]$ has size $\gamma(1-\mu)\overline{B}_L$, where $\gamma(1-\mu)$ is (approximately) the spread between R_B^D and R. This suggests that the range $\left[\underline{B}_L, \overline{B}_L\right]$ will generally be larger, the larger the spread between R_B^D and R, and the higher $\overline{B}_L$.

A similar argument applies to the case of nominal debt, whereas we know that a discretionary government will choose positive inflation even if $\kappa = 0$. The expression combining the two thresholds $\underline{B}_L$ and $\overline{B}_L$ becomes:

$$\begin{aligned}\Phi &= z\left(G + \frac{R}{1+\pi_L^{ND}}\overline{B}_L\right) - z\left(G + \alpha\frac{R}{1+\widetilde{\pi}_L^{ND}}\right) - \alpha\frac{R}{1+\widetilde{\pi}_L^{ND}}\overline{B}_L + \mathcal{C}\left(\pi_L^{ND}\right) - \mathcal{C}\left(\widetilde{\pi}_L^{ND}\right)\\ &= z\left(G + \frac{R_B^D}{1+\pi_L^D}\underline{B}_L\right) - z\left(G + \alpha\frac{R_B^D}{1+\widetilde{\pi}_L^D}\underline{B}_L\right) - \alpha\frac{R_B^D}{1+\widetilde{\pi}_L^D}\underline{B}_L + \mathcal{C}\left(\pi_L^D\right) - \mathcal{C}\left(\widetilde{\pi}_L^D\right)\end{aligned}$$

Also in this case, it is easy to see that the expressions on the first and second line are increasing in the level of debt (since (19) holds and costs of inflation are convex) and, in state L, the ex-post sovereign interest rate is higher under expected default than under no default, namely:

$$(1-\gamma)\frac{1+\pi_L^{ND}}{1+\pi_H^{ND}}+\gamma\left(\mu\frac{1+\pi_L^{ND}}{1+\pi_A^{ND}}+1-\mu\right)>(1-\gamma)\frac{1+\pi_L^{D}}{1+\pi_H^{D}}+\gamma\mu\frac{1+\pi_L^{D}}{1+\pi_H^{D}}.$$

Hence, $\overline{B}_L > \underline{B}_L$: multiple equilibria exist also when debt is not indexed.[18] Again, these equilibria are well behaved, since inflation is increasing in B, implying that both R and R_B^D are also increasing in B.

Positive seigniorage $\kappa > 0$ will make the proof analytically cumbersome, but will not overturn its logic. Multiplicity will obtain as long as the elasticity of seigniorage to inflation is not exceedingly large.

4.4.2 Multiplicity between the D and the DD equilibrium

In the range of B with multiplicity identified in the first proposition, sovereign debt is risk free in the fundamental equilibrium, but may become risky in a non-fundamental equilibrium when agents expect the government to default conditional on macroeconomic distress, in the weak state of the world. We now characterize a second range of debt over which default occurs for fundamental reasons. Namely, debt is never risk-free, since the government will always repudiate debt in the weak state of the world, which materializes with a positive probability. Driven by self-fulfilling expectations, however, there is a second equilibrium in which the government defaults, either partially or fully, also under stronger economic conditions.

Analogously to $\underline{B}_L$ and $\overline{B}_L$, we define two further thresholds for B ($\underline{B}_A$ and $\overline{B}_A$), relevant for levels of debt above $\overline{B}_L$. For values of debt $\overline{B}_L \leq B < \overline{B}_A$, our model economy admits one (fundamental) equilibrium with full default in state L, whereas markets charge the interest rate R_B^D. For debt $B \geq \underline{B}_A > \overline{B}_L$ a second (non-fundamental) equilibrium emerges, DD, in which markets coordinate their expectations on anticipating a 100

[18] Recall that inflation rates will be determined as follows

$$z'\Big(\frac{\rho}{(1-\gamma)\frac{1+\pi_L^{ND}}{1+\pi_H^{ND}}+\gamma\left(\mu\frac{1+\pi_L^{ND}}{1+\pi_A^{ND}}+1-\mu\right)}B+G\Big)\frac{\rho}{(1-\gamma)\frac{1+\pi_L^{ND}}{1+\pi_H^{ND}}+\gamma\left(\mu\frac{1+\pi_L^{ND}}{1+\pi_A^{ND}}+1-\mu\right)}B=\mathcal{C}'\left(_L^{ND}\right)\left(1+_L^{ND}\right)$$

$$z'\Big(\frac{\rho}{(1-\gamma)\frac{1+\pi_L^{D}}{1+\pi_H^{D}}+\gamma\mu\frac{1+\pi_L^{D}}{1+\pi_A^{D}}}B+G\Big)\frac{\rho}{(1-\gamma)\frac{1+\pi_L^{D}}{1+\pi_H^{D}}+\gamma\mu\frac{1+\pi_L^{D}}{1+\pi_A^{D}}}B=\mathcal{C}'\left(\pi_L^{D}\right)\left(1+_L^{D}\right)$$

percent haircut in the low output state and partial default in state A, and thus charge a destabilizing high market rate $R_B^{DD} > R_B^D$. Ex post, the government defaults in both states, validating these expectations. When debt is above the threshold $\overline{B}_A$, however, default in the intermediate state becomes inevitable, and the D equilibrium no longer exists. Therefore, multiplicity arises when $\overline{B}_A > \underline{B}_A$.

To save on space, we omit a formal statement of a proposition, but directly write out the condition for multiplicity, implying $\overline{B}_A > \underline{B}_A$:

$$\left[(1-\gamma)\frac{1+\widehat{\pi}^D}{1+\pi_H^D} + \gamma\mu\frac{1+\widehat{\pi}^D}{1+\pi_A^D}\right]\left[\left(\hat{T}_A - G\right) + \frac{\widehat{\pi}^D}{1+\widehat{\pi}^D}\kappa\right] > \tag{38}$$
$$\left[(1-\gamma)\frac{1+\widehat{\pi}^{DD}}{1+\pi_H^{DD}} + \left(1-\underline{\theta}_A^{DD}\right)\gamma\mu\right]\left[\left(\hat{T}_A - G\right) + \frac{\widehat{\pi}^{DD}}{1+\widehat{\pi}^{DD}}\kappa\right]$$

This condition is easy to verify in the same special cases analyzed in relation to the first multiplicity range. When seigniorage revenue is nil ($\kappa = 0$) and debt is inflation-indexed (or inflation costs are arbitrarily high) so that inflation is zero, the above condition becomes

$$[(1-\gamma)+\gamma\mu]\left(\hat{T}_A - G\right) > \left[(1-\gamma) + \left(1-\underline{\theta}_A^{DD}\right)\gamma\mu\right]\left(\hat{T}_A - G\right) > \rho\overline{B}_L. \tag{39}$$

This first inequality is always satisfied because $\underline{\theta}_A^{DD}$ (solving (27) in state A) is strictly positive, i.e., $\underline{\theta}_A^{DD} > 0$. The second inequality follows from Assumption 1. Observe that, with zero inflation, the size of the debt range $\left[\underline{B}_A, \overline{B}_A\right]$ over which multiplicity occurs is $\gamma\mu\underline{\theta}_A^{DD}\left(\hat{T}_A - G\right)$. As above, this range is proportional to the (minimum) spread between the non-fundamental and the fundamental value of the sovereign rate, R_B^{DD} and R_B^D, approximately equal to the term $\gamma\mu\underline{\theta}_A^{DD}$. With an endogenous haircut rate, the second term is (the present discounted value of) the post-default primary surplus in state A, $\left(\hat{T}_A - G\right)$.

When debt is nominal and thus inflation is positive, the multiplicity condition (with $\kappa \to 0$) boils down to:

$$(1-\gamma)\left(\frac{1+\widehat{\pi}^D}{1+\pi_H^D} - \frac{1+\widehat{\pi}^{DD}}{1+\pi_H^{DD}}\right) + \gamma\mu\left(\frac{1+\widehat{\pi}^D}{1+\pi_A^D} - \left(1-\underline{\theta}_A^{DD}\right)\right) > 0, \tag{40}$$

where inflation rates are determined by (22) and (26) evaluated at R_B^D and R_B^{DD}. The above expression is approximately equal to the (ex-post) real spread between the non-fundamental and the fundamental sovereign rate,

$\frac{R_B^{DD}}{1+\widehat{\pi}^{DD}} - \frac{R_B^D}{1+\widehat{\pi}^D}$. In other words, a positive spread implies multiplicity—a sufficient conditions being $\frac{1+\widehat{\pi}^D}{1+\pi_H^D} \geq \frac{1+\widehat{\pi}^{DD}}{1+\pi_H^{DD}}$ and $\frac{1+\widehat{\pi}^D}{1+\pi_A^D} > \left(1 - \underline{\theta}_A^{DD}\right)$, for $\underline{\theta}_A^{DD} > 0$. By way of example, (40) is satisfied when $\underline{\theta}_A^{DD} \to 1$ and inflation rates across equilibria in state H are such that $\frac{1+\pi_H^{DD}}{1+\pi_H^D} \geq \frac{1+\widehat{\pi}^{DD}}{1+\widehat{\pi}^D}$ (e.g., because the function $z(\cdot, Y_H)$ would be relatively steep).

Relative to an economy with indexed bonds, the option to reduce the real value of debt via inflation may/may not increase the resilience of the economy to multiplicity. Holding seigniorage constant at zero (or more generally, holding it constant independently of inflation), the economy may be more or less resilient depending on whether

$$(1-\gamma)\left(\frac{1+\widehat{\pi}^D}{1+\pi_H^D} - \frac{1+\widehat{\pi}^{DD}}{1+\pi_H^{DD}}\right) + \gamma\mu\left(\frac{1+\widehat{\pi}^D}{1+\pi_A^D} - 1\right) \lesseqgtr 0.$$

Observe that resilience is helped by a uniformly low level of inflation in state H—which would tend to turn the term $\left(\frac{1+\widehat{\pi}^D}{1+\pi_H^D} - \frac{1+\widehat{\pi}^{DD}}{1+\pi_H^{DD}}\right)$ negative—together with a high probability of the no-default state H, $1-\gamma$. Intuitively, the lower the anticipated inflation in the non-default state H, and the higher the probability attached to such a state, the lower the interest rate charged by investors in period 1. A contained debt service in turn enhances the effects of discretionary inflation in the L and A states. Relative to this case, when seigniorage is increasing in inflation (as we assume in our model), inflation revenue will be higher in the DD equilibrium (since inflation increases with the nominal interest rate). Seigniorage will tend to raise $\underline{B}_A$ and thus, other things equal (see (38)), reduce the multiplicity range.

Finally, it is easy to verify that, under Assumption 2, the ex-ante interest rate bill $R_B^{DD}B$ in nominal terms will be rising in the initial level of nominal liabilities B, ensuring that the also the DD equilibrium is well-behaved. Namely, in the non-fundamental DD equilibrium:

$$BR_B^{DD} = \frac{(1-\alpha)\,B\rho - \gamma\mu\left[\left(\hat{T}_A - G\right) + \frac{\hat{\pi}^{DD}}{1+\hat{\pi}^{DD}}\kappa\right]}{\frac{(1-\gamma)(1-\alpha)}{1+\pi_H^{DD}} - \frac{\alpha\gamma\mu}{1+\hat{\pi}^{DD}}}.$$

Provided κ is not exceedingly large, $1-\gamma > \alpha$ and $\frac{1+\hat{\pi}^{DD}}{1+\pi_H^{DD}} > \mu > 0$ will ensure that $R_B^{DD}B$ be increasing in B.

To conclude this section, we illustrate the numerical example underlying Figure 1. To draw this figure, we impose Assumptions 1 and 2, but relax

Assumption 3 (motivated only by analytical convenience). To draw the figure, we set the primary surpluses in the L and A states to $\hat{T}_L - G =$ 0.30, $\hat{T}_A - G$ =0.52, (where $G = 0$ without loss of generality), and the maximum value of seigniorage equal to $\kappa = 0.50$. Note that, given the two period structure of our model, these figures are to be interpreted in terms of present discounted values. Normalizing expected output to 1, hence, they can be read as, respectively, 30, 52 and 50 percent of expected GDP—by the same token, $B = 1$ in the figure would correspond to 100 percent of expected GDP. As long as endogenous inflation is always close to zero in the H state, outcomes in this state are practically irrelevant for the equilibria we study. We then set $\hat{T}_H - G$ =2.22 (in line with the last inequality on the right in Assumption 1) and the probabilities such that $\mu = 1/2$ and $\gamma = 0.2$ (in line with Assumption 2). The sovereign rate spread over the risk free rate is then around 10 percent in the D equilibrium, and 20 percent in the DD equilibrium. The fixed and variable costs of default are, respectively, $\Phi = 0.1$ (equal to 10 percent of expected GDP) and $\alpha = 0.1$ (equal to 10 percent of GDP if total debt service is as high as GDP). The cost of inflation is assumed to be quadratic and equal to $C(\pi) = 1.125 \cdot \pi^2$. This implies that an inflation rate of 10 percent ($\pi = 0.1$) causes output costs (in present discounted) equal to 1.1 percent of expected GDP. Similarly, tax distortions are set to $z(\cdot; Y_i) = \psi_i \cdot T_i^2$, where $2 \cdot \psi_i \cdot \hat{T}_i = \alpha / (1 - \alpha)$. By way of example, this implies that in state A $\psi_A = 0.11$ and, in present discounted value, the output cost of a primary surplus of 50 percent of expected GDP is around 2.5 percent of expected GDP. From this parameterization, we obtain minimum haircut rates around 75 percent in the L and the A state (precisely, $\underline{\theta}_L =$ 0.7904 and $\underline{\theta}_A$ =0.7230). As shown in the figure, equilibrium multiplicity is possible for B in the following ranges [1.07,1.15], and [1.39,1.49].

5 Ruling out bad equilibria with a credible monetary backstop

When multiple equilibria are possible, social welfare is lower when markets coordinate on the equilibrium with default in more states of the world. A high interest bill driven by self-fulfilling expectations of default causes unwarranted output and budget costs—associated to non-fundamental default when this occurs, but also with the need to raise distortionary primary surpluses in states of the world where the government opts for servicing the debt in full.

The fact that equilibria with non-fundamental default are detrimental to social welfare motivates the search for effective backstop policies. In this section we analyze the workings of monetary backstops, whereby the central bank announces that it will stand ready to purchase an amount of public debt ωB if agents coordinate on a non-fundamental default. As it is customary in the literature, we posit that market coordination across equilibria is regulated by a device akin to a "traffic signal" that switches between red and green: when red appears, agents coordinate their expectations on the non-fundamental equilibrium, provided this equilibrium exists (see e.g. Evans, Honkapohja and Romer (1998)). If the central bank backstop policy is successful, however, the very announcement of debt purchases rules out the non-fundamental interest rate as an equilibrium outcome, and markets expectations cannot but coordinate on the unique, fundamental, equilibrium.

Debt purchases do not need to be actually carried out but, to have the desired effect, the policy has to be credible. Namely, it has to satisfy the requirement that, if debt purchases were to be effectively carried out, the ensuing outcome would be feasible, unique, and welfare-improving. So, even if balance sheet losses and/or the possibility of elevated inflation as a consequence of central bank purchases of debt are merely off-equilibrium outcomes, their assessment is crucial to the design of a backstop policy. Should central bank interventions result in expected welfare losses relative to a non-fundamental equilibrium, the announcement of a backstop would clearly not be credible, hence would not work.

As discussed in Section 3, backstop can be implemented under different regimes of fiscal and monetary interactions. We find it analytically convenient to study the backstop under the two polar assumptions of either a consolidated or a separate budget constraint, whereby the transfers by the central bank to the fiscal authorities are restricted to be non-negative. In each case, we organize our analysis focusing separately on each of the two multiplicity regions established in the previous section, as these allow us to discuss different issues. In the first region for B, the ND equilibrium is fundamental, the D equilibrium is the welfare-dominated, non-fundamental one, where, by assumption 3, default is complete. In the second region, the D equilibrium is fundamental. In the DD equilibrium (now the welfare-dominated, non-fundamental one), default in the A state is interior, and the optimal haircut rate generally responds to monetary policy.

5.1 Budget consolidation (fiscal backing)

When the constraint on the central bank budget is not binding, default on its holding of debt automatically generates an equivalent increase in tax liabilities—the transfer to the fiscal authority falls and may become negative. Effectively, central bank purchases of debt reduce the 'tax base' (the outstanding stock of debt held by private investors) on which the fiscal authority can impose haircuts and produce net budget saving. As a result, interventions raise taxation conditional on debt repudiation and, taxation and inflation comove together, exacerbate the overall output costs of default.

We have seen that debt purchases reduce the overall debt service, hence the size and costs of primary surpluses the government needs to generate under no default. Hence, by raising tax distortions under default, while facilitating the full service of debt at lower costs, central bank interventions unambiguously make the resort to haircuts less attractive for the fiscal policymaker.

Consider debt levels in the multiplicity range, $\underline{B}_L \leq B < \overline{B}_L$, where the two thresholds satisfy the condition for full default in state L conditional on $\omega = 0$. Holding these thresholds constant, the condition for optimal (non fundamental) default in state L given the amount of purchases ωB is:

$$\Phi \leq z\left(T_L^D\right) - z\left(\widetilde{T}_L^D\right) + \mathcal{C}\left(\pi_L^D\right) - \mathcal{C}\left(\widetilde{\pi}_L^D\right) - \alpha \frac{R_B^D}{1+\widetilde{\pi}_L^D}\left(1-\omega\right)B \qquad (41)$$

where

$$\begin{aligned}
\widetilde{T}_L^D &= G - \frac{\widetilde{\pi}_L^D}{1+\widetilde{\pi}_L^D}\kappa + \alpha\frac{R_B^D}{1+\widetilde{\pi}_L^D}\left(1-\omega\right)B + \frac{\rho}{\left(1-\gamma\right)\frac{1+\widetilde{\pi}_L^D}{1+\pi_H^D} + \gamma\mu\frac{1+\widetilde{\pi}_L^D}{1+\pi_A^D} + \gamma\left(1-\mu\right)}\omega B \\
T_L^D &= G - \frac{\pi_L^D}{1+\pi_L^D}\kappa + \frac{R_B^D}{1+\widetilde{\pi}_L^D}\left(1-\omega\right)B + \frac{\rho}{\left(1-\gamma\right)\frac{1+\pi_L^D}{1+\pi_H^D} + \gamma\mu\frac{1+\pi_L^D}{1+\pi_H^D} + \gamma\left(1-\mu\right)\frac{1+\pi_L^D}{1+\widetilde{\pi}_L^D}}\omega B
\end{aligned}$$

and inflation rates under central bank interventions are determined by the counterparts of (12) in Section 3. Since, as discussed above, $z\left(\widetilde{T}_L^D\right)$ is increasing in ω, it is apparent that a higher amount of debt purchases reduces the expression on the right hand side of the inequality (41). Given Φ, a sufficiently large amount of interventions can overturn the fiscal authority decision to default.

The effect of a backstop is most clearly seen in the limiting case in which government debt and central bank reserves are both indexed to inflation, and there is no seigniorage. The condition (41) simplifies to:

$$\begin{aligned}\Phi &\leq z\left(G+\left(\frac{1}{(1-\gamma)+\gamma\mu}(1-\omega)+\omega\right)\rho B\right)+\\ &\quad -z\left(G+\left(\frac{\alpha}{(1-\gamma)+\gamma\mu}(1-\omega)+\omega\right)\rho B\right)-\frac{\alpha}{(1-\gamma)+\gamma\mu}(1-\omega)\rho B.\end{aligned}$$

The function on the right-hand side of the equation is decreasing in ω, for any $0 \leq \omega < 1$, since under full default the marginal tax distortions must be larger than $\alpha/(1-\alpha)$. Moreover, when $\omega = 1$, the right-hand side is equal to zero. Thus, for any level of debt $\underline{B}_L \leq B < \overline{B}_L$ there is always a level of purchases $0 < \underline{\omega}_L(B) - \varepsilon < 1$ for which the equation is satisfied with equality, with ε arbitrarily small, implying that default in state L cannot be optimal for central bank purchases $\underline{\omega}_L(B)$. At this level of intervention, the equilibrium D with non-fundamental default in state L does not exist anymore.

The following proposition states the result for the case with nominal bonds and positive seigniorage.

Proposition 4 *Assume that there is a range of debt $\underline{B}_L \leq B < \overline{B}_L$, where $\underline{B}_L$ and $\overline{B}_L$ are defined in (35) and (36), respectively, for which both equilibria ND and D exist. Then there exists a minimum level of announced purchases, $1 > \underline{\omega}_L(B) > 0$, for which the only equilibrium is ND. In this equilibrium, conditional on a level of purchases $\underline{\omega}_L(B)$, welfare is higher than in the D equilibrium without purchases.*

In the second multiplicity region $\underline{B}_A \leq B < \overline{B}_A$, matters are complicated by the fact that the haircut rate in state A is endogenous when default is interior. As interventions reduce the stock of liabilities held by private investors, the government may optimally decide to raise the haircut rate. In the case of indexed liabilities and no seigniorage, for instance, the equation determining $\underline{\theta}_A$ when $\omega > 0$ is:

$$\begin{aligned}\Phi = z&\left(G+\frac{\left(\hat{T}_A-G\right)-(1-\alpha)\underline{\theta}_A\rho\omega B}{1-(1-\alpha)\underline{\theta}_A}\right)+\\ &-z\left(\hat{T}_A\right)-\alpha\underline{\theta}_A\frac{\left(\hat{T}_A-G\right)-\rho\omega B}{1-(1-\alpha)\underline{\theta}_A}.\end{aligned}$$

It is apparent that a higher ω decreases the expression on the right hand side of the equality (since $z'(\cdot) > \alpha/(1-\alpha)$). Therefore, $\underline{\theta}_A$ will have to also rise in order for the expression to hold with equality.

Yet, as shown in Section 3, a higher ω will result in an overall lower debt service and thus in lower inflation and taxation across all states. So, while unambiguously raising $\underline{\theta}_A$ up to its upper limit, interventions have opposing effects on the lower default threshold $\underline{B}_A$, which, for $\omega > 0$, is determined by the following expression:

$$\underline{B}_A = \frac{\left[(1-\gamma)\frac{1+\hat{\pi}^{DD}}{1+\pi_H^{DD}} + (1-\underline{\theta}_A)\gamma\mu\right]\left[\left(\hat{T}_A - G\right) + \frac{\hat{\pi}^{DD}}{1+\hat{\pi}^{DD}}\kappa\right]/\rho}{(1-\omega)\left[1-(1-\alpha)\underline{\theta}_A\right] + \omega\frac{(1-\gamma)\frac{1+\hat{\pi}^{DD}}{1+\pi_H^{DD}} + (1-\underline{\theta}_A)\gamma\mu}{(1-\gamma)\frac{1+\hat{\pi}^{DD}}{1+\pi_H^{DD}} + \gamma\mu + \gamma(1-\mu)\frac{1+\hat{\pi}^{DD}}{1+\tilde{\pi}_L^{DD}}}}.$$

Even keeping inflation constant, it is clear that that, for a given $\underline{\theta}_A$, increasing ω may either increase or decrease such threshold. To wit: with no seigniorage and indexed debt, ω in the denominator is multiplied by $[(1-\alpha)\underline{\theta}_A - \gamma(1-(1-\underline{\theta}_A)\mu)]$. This expression is negative for $\underline{\theta}_A \to 0$ and positive for $\underline{\theta}_A \to 1$, hence the debt threshold $\underline{B}_A$ will be initially increasing, then will become decreasing, as the central bank picks higher $\omega's$, until $\underline{\theta}_A = 1$. At this point, however, the same logic we used to study the lower multiplicity region (with full default in state L) will also apply to the higher multiplicity region (when default in state A is complete). Once $\underline{\theta}_A = 1$, higher purchases unambiguously raise the debt threshold for multiple equilibria.

On a sufficient scale, purchases of government debt by the central bank are bound to result in an unique equilibrium also in the second region of multiplicity. Relative to the non-fundamental equilibrium, the economy is better off both in the D-equilibrium, and on the off-equilibrium path, i.e. conditional on debt purchases effectively implemented in the first period. This is because, on the off-equilibrium path, the reduction in the burden of debt service would mean that the economy does not incur the suboptimal costs of default in state A, and inflation and taxes are lower in all states of the world.[19]

Figure 2a,b illustrate these results using our numerical example—hence in reference to the two regions of multiplicity shown Figure 1. The upper panel of the figure plots the minimum level of interventions required to eliminate multiplicity. The lower panel reports (ex ante) welfare conditional on

[19] One slight complication to keep in mind is that large debt purchases may increase the threshold for fundamental default in state L—so that conditional on central bank interventions the equilibrium may feature no default in any state. But to the extent that the two multiplicity regions are further from each other (as they are in our specification), there will be some level of interventions that will rule out non-fundamental default in state A without ruling out fundamental default in state L.

no intervention ($\omega = 0$) and conditional on the minimum-level interventions shown in the panel above. Two features of the numerical example stands out. First, multiplicity over the relevant range disappears for values of ω between 1/4 and 1/2. Second, welfare conditional on actual (minimum) interventions is always higher than welfare in a non-fundamental equilibrium—confirming that the minimum-intervention backstop is feasible and welfare-improving as off-equilibrium outcome.

5.2 Budget separation

Under budget separation, central bank purchases of debt no longer reduce the 'tax base' for a default, but raise the possibility of high inefficient inflation. We have seen above that a monetary backstop rules out non-fundamental equilibria, without necessarily ruling out fundamental default—this is the case when debt falls in the upper multiplicity region. Thus, under budget separation, if only off-equilibrium, debt purchases may result in balance sheet losses—implying that the monetary authorities may have to deviate from the optimal policy and run high inflation. The key issue is whether this deviation impinges on the credibility of backstops.

Relative to the case of budget consolidation, high prospective inflation in case of default reduces welfare conditional on interventions. This is because, as monetary policy cannot pursue efficient inflation plans, distortions are no longer optimally smoothed across policy instruments, and the output costs of inflation are convex. But exactly for this reason, as long as preferences over inflation are sufficiently similar across policy makers, the large output distortions from high prospective inflation also weigh on the decision to default by the fiscal authority. Indeed, a key result of this section is that the consequences of budget separation for the conduct of monetary policy act as a deterrent against the choice of debt repudiation by fiscal policymakers adverse to inflation.

Starting with the low multiplicity region, focus again on debt levels within the two thresholds $\underline{B}_L \leq B < \overline{B}_L$, defined conditional on $\omega = 0$ in (35) and (36). The decision to default as a function of ω is determined again by condition (41). If no default takes place, the central bank budget constraint does not bind. Since taxes adjust to satisfy the consolidated budget constraint:

$$T_i^D = G - \frac{\pi_i^D}{1+\pi_i^D}\kappa + \frac{R_B^D}{1+\pi_i^D}\left(1-\omega\right)B + \left[\frac{R}{1+\pi_i^D}\right]\omega B,$$

they are decreasing in ω. So does inflation, determined by the optimality condition:

$$\left(1+\pi_i^D\right)^2 \mathcal{C}'\left(\pi_i^D\right) = z'\left(T_i^D\right)\left[\kappa + (1-\omega) BR_B^D + \omega BR\right],$$

where the risk free rate is given by $R = \frac{\rho}{(1-\gamma)\frac{1}{1+\pi_H^D} + \gamma\mu\frac{1}{1+\pi_A^D} + \gamma(1-\mu)\frac{1}{1+\widetilde{\pi}_L^D}}$. Clearly, debt purchases lower the cost of servicing the debt, as is the case with budget consolidation.

Different from the previous subsection, however, purchases now result in *lower* taxes but *higher* inflation in case of default. For instance, under full default $\theta_L^D = 1$, inflation has to increase at least to balance the central bank budget constraint:

$$\widetilde{\pi}_L^D \geq \frac{\omega}{\kappa} RB;$$

while, if the central bank budget constraint is binding, taxation will be lower, the larger ω is:

$$\widetilde{T}_L^D = G + \alpha \frac{R_B^D}{1+\widetilde{\pi}_L^D}(1-\omega) B.$$

Because of these opposing movements, the effects of debt purchases by the central bank is in principle ambiguous. However, unless seigniorage revenue is unrealistically high and elastic to inflation, the negative costs of inflation are bound to prevail. Focus on the default condition (41) when the central bank constraint binds (so that $\widetilde{\pi}_L^D = \frac{\omega}{\kappa} RB$),

$$\Phi \leq z\left(T_L^D\right) + \mathcal{C}\left(\pi_L^D\right) +$$
$$-z\left(G + \alpha \frac{R_B^D}{1+\frac{\omega}{\kappa}RB}(1-\omega) B\right) - \alpha \frac{R_B^D}{1+\frac{\omega}{\kappa}RB}(1-\omega) B - \mathcal{C}\left(\frac{\omega}{\kappa} RB\right).$$

From the last line of this expression, it is easy to derive the following sufficient condition for the cost of default to increase in ω:

$$\left(1+\widetilde{\pi}_L^D\right)^2 \mathcal{C}'\left(\widetilde{\pi}_L^D\right) \geq \left[1 + z'\left(\widetilde{T}_L^D\right)\right] \alpha \left(1 + \left(\frac{\kappa}{RB}\right)\right) BR_B^D.$$

Holding interest rates constant, the left-hand side of the expression is increasing in inflation and thus in ω; the right-hand side is decreasing in marginal tax distortions and thus in ω. In light of the optimal choice of inflation under full default when the central bank is unconstrained:

$$\left(1+\widetilde{\pi}_L^D\right)^2 \mathcal{C}'\left(\widetilde{\pi}_L^\sigma\right) = \left[1 + z'\left(\widetilde{T}_L^D\right)\right] \alpha (1-\omega) BR_B^D +$$
$$z'\left(\widetilde{T}_L^D\right)\left[\kappa + \omega BR\right],$$

it is clear that for $\omega \to 1$ the above sufficient condition must hold unless seigniorage revenue is very large. This is quite intuitive, as the lower κ and the largerω, the higher the inflation the central bank will have to generate to meet its nominal obligations at face value. With a convex function $\mathcal{C}(.)$, there will be a level of purchases for which the decision to default is overturned.

A similar argument applies to the higher multiplicity region, $\underline{B}_A \leq B < \overline{B}_A$. In this case, when the central bank is constrained, inflation and taxes under non-fundamental default in state A are given by

$$\pi_A \kappa = \left(R - \left(1 - \underline{\theta}_A\right) \overline{R}_B\right) \omega B,$$

$$\widehat{T}_A - G = \left[1 - \underline{\theta}_A \left(1 - \alpha\right)\right] \frac{R_B}{1 + \pi_A} \left(1 - \omega\right) B + \left[1 - \underline{\theta}_A\right] \frac{\overline{R}_B}{1 + \pi_A} \omega B.$$

So, with a binding budget constraint, central bank purchases reduce overall taxation and increase inflation under default. But now any inflationary consequences of purchases (in case of default, with a binding central bank constraint) also impinge on the minimum threshold $\underline{\theta}_A$:

$$\Phi + \alpha \underline{\theta}_A \frac{R_B B}{1 + \dfrac{\left(R - \left(1 - \underline{\theta}_A\right) \overline{R}_B\right) \omega B}{\kappa}} + z(\widehat{T}_A) +$$
$$\mathcal{C}\left(\frac{R - \left(1 - \underline{\theta}_A\right) \overline{R}_B}{\kappa} \omega B\right) = z(T_A^{\sigma}) + \mathcal{C}\left(\pi_i^{\sigma}\right).$$

making an analytical characterization of the effects of interventions quite cumbersome.

The results for the case of budget separation are nonetheless clearly illustrated by Figure 3a,b, based on the same parameterization and layout of the previous figure. The upper panel shows the minimum level of interventions required to eliminate multiplicity. The lower panel reports ex ante welfare conditional on no intervention ($\omega = 0$) and conditional on the successful (minimum-level) interventions shown in the panel above.

As in Figure 2, welfare conditional on actual (minimum) interventions is always higher than welfare in a non-fundamental equilibrium—confirming that the minimum-intervention backstop is feasible and welfare-improving as off-equilibrium outcome also under budget separation. There is however a notable difference relative to the previous figure: multiplicity over the relevant range disappears for values of ω between 1/20 and 1/10, much lower than in the case of a consolidated budget constraint. As discussed above, provided fiscal authorities are adverse to inflation and budget separation is credibly in place, budget separation does not undermine at all monetary backstops. Credibly committing the central bank to be responsible for its own budget constraint can strengthen its ability to backstop government debt.

5.3 Discussion

There are a number of factors and considerations that may complicate the design of a successful backstop. We have already discussed the possibility of multiplicity in equilibrium inflation rates—in the conclusion below we will briefly consider the possibility of non-market interventions by the central bank, e.g. measures affecting banks' reserves. Here we focus on the issue of "moral hazard", i.e., whether a backstop could feed opportunistic behavior by the fiscal authority *in equilibrium*, exacerbating fiscal fragility and therefore the likelihood of (fundamental) default.[20] This is a widely-debated issue that would require extensive analysis; here we focus on a basic consideration.

Conceptually, backstops are distinct from bailouts, in the form of contingent transfers that occur ex post with positive probability. The literature has long clarified that backstops may actually strengthen the incentives for a government to undertake costly actions to strengthen the economic resilience to fiscal stress—the opposite of the "moral hazard" consequences of a bailout (see Morris and Shin 2006, Corsetti et al. 2005, Corsetti and Dedola 2011 and Nicolini et al. 2014 among others). This is because, without a backstop, the possibility of belief-driven crises tends to reduce the expected future benefits from these actions.

Nonetheless, a backstop does not necessarily eliminate fundamental default. With weak fundamentals creating fiscal stress, a central bank may run the risk of being drawn into quite a different policy, of ex-post debt monetization a la Sargent and Wallace (1981), which may threaten its independence and ability to deliver on its objectives. But it is hardly a reason for a central bank to avoid recognizing its important function in the government debt market.

These considerations also raise a deeper theoretical and practical issue, concerning why central banks do not engage more systematically in the sovereign debt market, to ensure that government debt is non-defaultable

[20] We have seen above that, on the off-equilibrium path, the actual implementation of debt purchases may affect the optimal default rate in the case of an interior solution. The elasticity of θ to ω is however irrelevant in the equilibrium allocation resulting from a successful backstop.

under any circumstances. The public finance literature has shown that even when the government can commit, it is optimal to ex-post affect the value of public debt when the latter is not state-contingent, see, e.g., Adam and Grill (2011). An intriguing direction of research may build on the observation that eliminating default under any circumstances through monetary interventions may not be efficient.

6 Conclusions

This paper has reconsidered the question of whether and how a central bank can backstop debt issued by the fiscal authority, as to rule out self-fulfilling sovereign debt crises. Our main conclusions resonate with the widespread policy view that under appropriate conditions, a central bank has indeed the power to backstop the government debt, although for different reasons that many observers invoke. Our model highlights crucial conditions. Firstly, a monetary backstop rests on the ability of the central bank to issue liabilities at a lower interest rate than a government subject to default risk. In our analysis, successful intervention strategies translate into a swap of (default-) risky government debt with nominal liabilities which can always be redeemed against currency. Secondly, policymakers should be sufficiently adverse to inflation, so that monetary policy is not itself a source of multiple equilibria in inflation and interest rates. Namely, conditional on a realized haircut, inflation rates should be uniquely determined, ruling out the possibility of high interest rates and taxation in the presence of sound fiscal fundamentals and no default.

Our results are at odds with views often voiced in the public debate, claiming that the central bank can freely play the role of lender of last resort to the government' because it is subject to either a soft or to no budget constraint. These views stress, alternatively, that a central bank can always consolidate its liabilities and force private banks to hold them indefinitely, or debase them by a bout of unexpected inflation. In light of our analysis, both views have fundamental weaknesses. The latter view stressing the need for the central bank to impose financial repression over private banks by forcing them to hold reserves, *de facto* introduces the possibility of default on monetary liabilities, without however working out its consequences. If the central bank is expected to tamper with its liabilities, it is easy to see that the arbitrage condition relating the rate on monetary liabilities and the risk free rate would have to include terms in the anticipated central bank's haircut θ_i^{CB}: the optimal monetary policy would have to account for the optimal haircut on the holders of reserves. The logic of self-fulfilling beliefs would then apply to a discretionary central bank as well as to the government.

The alternative, inflationary-debasement view downplays the social costs of running high inflation, historically conducive to financial and macro instability. If anything, in line with Calvo (1988), our analysis suggests that downplaying the costs of inflation may actually raise the prospects of self-fulfilling sovereign debt crises driven by expectations of debt debasement, rather than outright default. Our analysis calls attention on the non-trivial fact that, exactly because high inflation is costly, a monetary backstop is credible even under budget separation. Most importantly, inflation rates are higher in an equilibrium with belief-driven outright defaults: an effective monetary backstop prevents high (let alone runaway) inflation, rather than creating price instability.

An important conclusion from our analysis is that a common objective function among fiscal and monetary authorities (or enough aversion to inflation costs by the fiscal authority) greatly facilitates the implementation of a monetary backstop. As each authority internalizes the effects of own policy choices on overall distortions, a monetary backstop is effective under reasonably mild conditions, even when the central bank is held responsible for its own balance sheet losses, barring contingent fiscal transfers under any circumstance. It follows that the conditions for a monetary backstop to be credible may be stricter when political economy or distributional considerations cause the two authorities to trade-off self-interested objectives with socially efficient policies.

Although we have developed our model from the perspective of a national economy with an independent currency, our analysis bears lessons for a currency union. In a monetary union among essentially independent states, national governments may pursue conflicting, inward-looking objectives and/or be adverse to extending large-scale fiscal backing to the common central bank. In case of budget separation, the inflationary consequences from budget losses due to default by one country may be quite contained and, most importantly, diffuse through the entire currency union. This means that a national fiscal authority choosing to default may not face the full inflationary costs of its decision. Even under these circumstances, however, a common central bank can still engineer a successful backstop to member states, to the extent that, as is the case for the OMTs in the euro area, governments have access to the benefit of a backstop only provided they agree to strict conditionality, ensuring stability of public finances and possibly eliciting stricter cross-border cooperation.

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Figure 1
Interest costs of issuing public debt
as a function of the initial financing need of the government

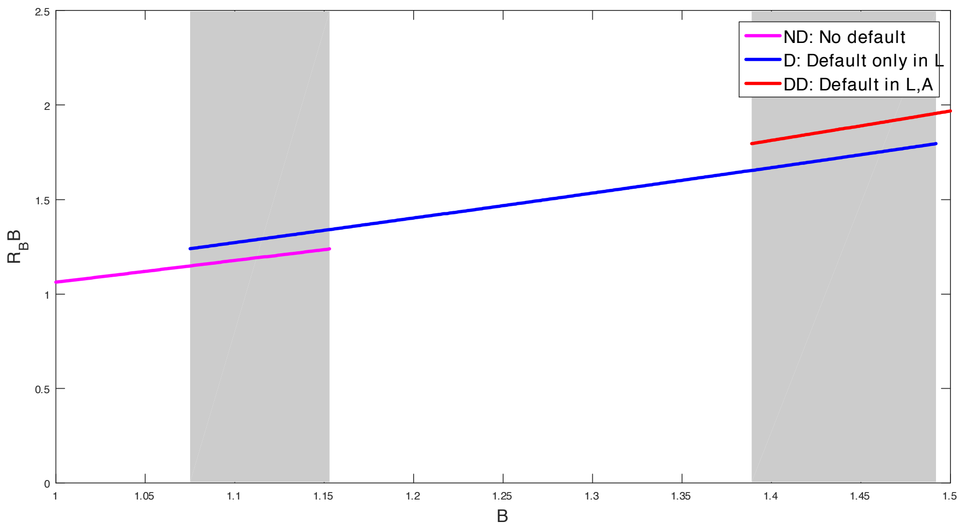

Figure 2a
Minimum Interventions required to eliminate the non fundamental equilibrium
Case of budget consolidation

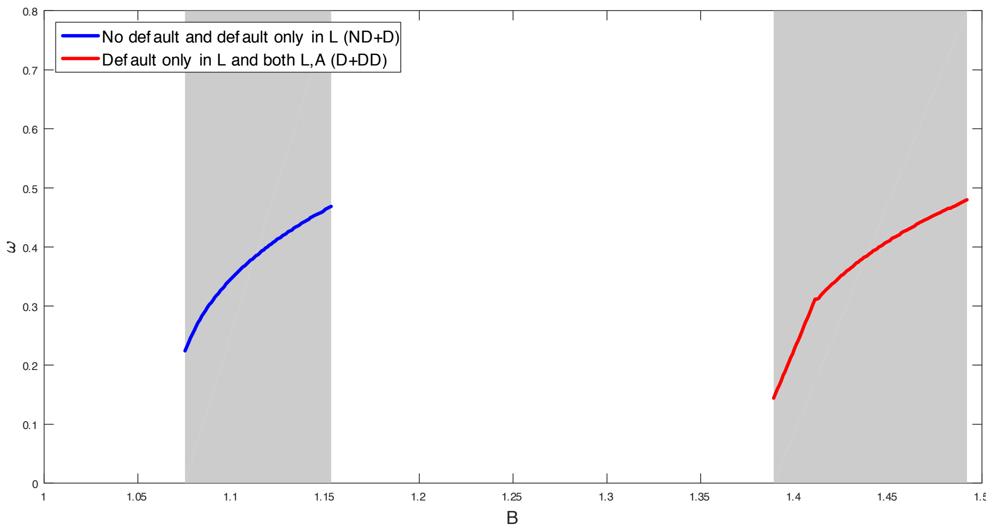

Figure 2b
Welfare with no backstop and with a minimum-intervention backstop
Case of budget consolidation

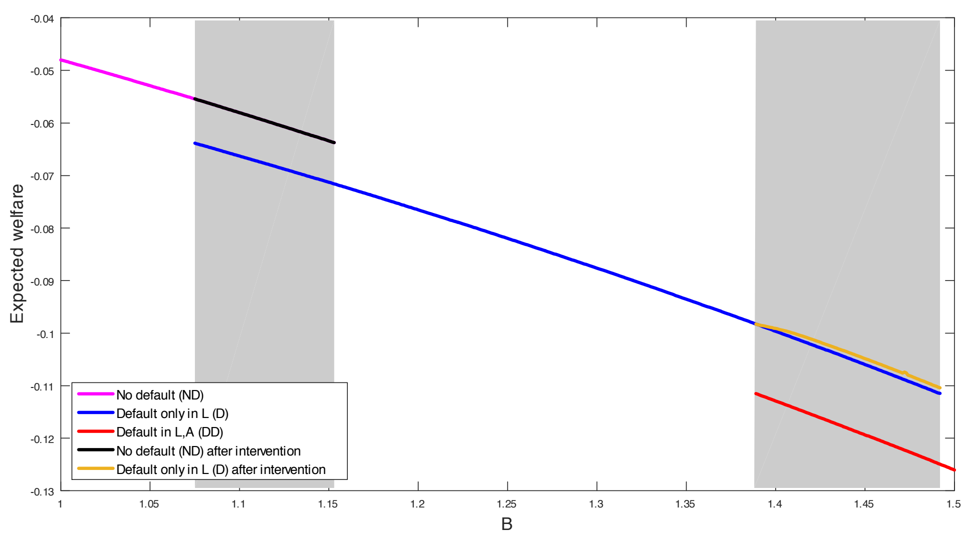

Figure 3a
Minimum Interventions required to eliminate the non fundamental equilibrium
Case of budget separation with a binding central bank constraint

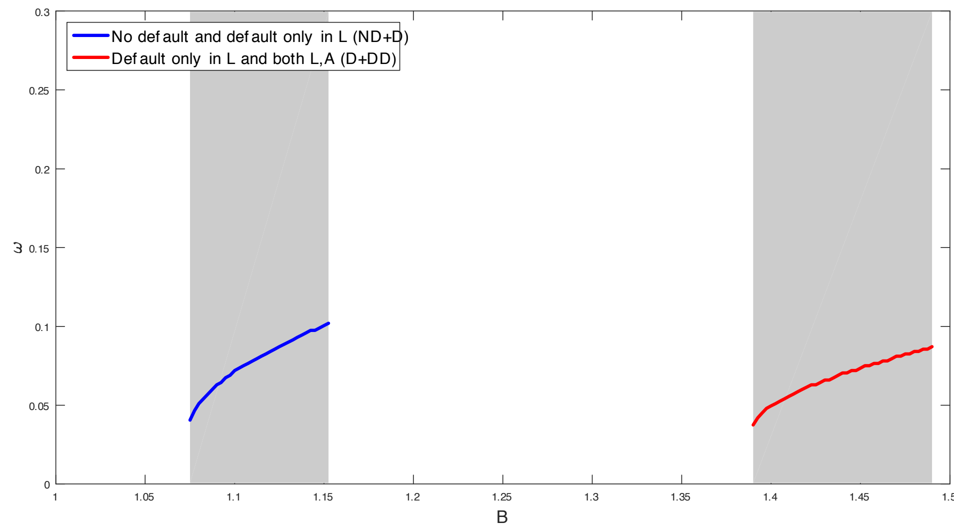

Figure 3b
Welfare with no backstop and with a minimum-intervention backstop
Case of budget separation with a binding central bank constraint