

Capital Values and Job Values

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Abstract

This paper explores how the joint behavior of hiring and investment is governed by the expected present values of capital and of jobs. It uses a model of frictions, which is a combination of a search model of the labor market and a q-type model of the capital market, emphasizing the interaction of capital and labor frictions. Relying on structural estimation of private sector U.S. data, it studies the future determinants of capital and job values and the implications for U.S. labor market developments.

Key findings include: (i) complementarity between the hiring and investment processes; (ii) important cross effects, of the value of capital on the mean and the volatility of the hiring rate, and vice versa; (iii) future returns are shown to play a dominant role in determining capital and job values; and (iv) U.S. labor market developments, including the outward shift of the Beveridge curve in the Great Recession and its aftermath 2007-2013, can be accounted for by changes in job and capital values.

Key words: investment, hiring, present values, frictions, returns, Great Recession.

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1 Introduction

This paper explores how the joint behavior of hiring and investment is governed by the expected present value of capital and labor. It uses a model of frictions, a combination of a search model of the labor market and a q-type model of the capital market, emphasizing the interaction of capital and labor frictions. Hiring and investment are modelled as the outcomes of a dynamic, intertemporal optimization problem of the representative firm. The paper uses structural estimation of private sector U.S. data to answer the following four specific questions: (i) how do capital and labor expected present values determine hiring and investment? (ii) how big are these values, i.e., how big are the relevant frictions? (iii) what determinants drive expected present values? (iv) how can recent U.S. labor market developments – including the Great Recession period – be understood in terms of capital and job (labor) values?

The answers to these questions are important for a number of key issues. The evolution of employment and of the capital stock are essential for the understanding of macroeconomic fluctuations. It has been shown that gross hiring is a major factor for understanding employment and unemployment dynamics.² Hiring frictions were shown to play a key role in determining the business cycle properties of labor productivity (including its declining

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²See, for example, Hall (2007) and Rogerson and Shimer (2011).

pro-cyclicality) and of the job finding rate (including its high volatility).³ Investment is key for the understanding of the evolution of the capital stock and consequently of firm market value.⁴

The results of this inquiry can explain the outward shift of the Beveridge curve and the big rise in unemployment in the Great Recession using changes in capital and job values. These findings have implications for business cycle modelling, such as the importance of incorporating joint investment and hiring costs, complete with the cited interaction, into DSGE models. The analysis takes into account the distinct, specific roles played by vacancy creation, gross hiring from non-employment, and job to job movements (as well as the separation flows involved).

A major implication of the findings is that hiring and investment can be treated as forward-looking variables, reflecting the expectations of future discounted profits from employing labor and capital. Using the results of estimation, I employ a restricted VAR analysis, such as the one used in the asset pricing literature, to study this forward-looking aspect. The analysis shows how investment and hiring are related to their expected, future determinants, with future returns turning out to play the dominant role.

This approach naturally links up with stock prices that are also forward-looking and relate to the same expected discounted future profits. Indeed, in previous work, joint with Monika Merz (Merz and Yashiv (2007)), we have shown that this set-up allows one to define asset values for hiring and for investment and that these values can be used to explain the time variation of equity values of firms in the U.S. economy. The current paper retains the focus on forward-looking behavior but does not make use of stock market data or tries to explain them. It updates the previous estimates, using a longer sample period, one that includes the Great Recession and its

³Gali and van Rens (2014) show that a lower degree of hiring frictions may lower the cyclicality of labor productivity in ways which are consistent with actual U.S. aggregate data dynamics. Coles and Mortensen (2013a,b) study the role of hiring costs in dynamic environments which generate a result whereby there is no Shimer “puzzle” and the job finding rate volatility matches the data.

⁴See, for example, Cochrane (2011).

aftermath, and then proceeds to examine a totally different set of empirical implications.

The paper is structured as follows: Section 2 briefly discusses the relevant strands of literature. Section 3 presents the firm's optimization problem and the resulting optimality conditions to be estimated. Section 4 discusses estimation issues and presents the results. It uses the results to look at the implied magnitude of frictions and to gauge the plausibility of the estimates. Section 5 discusses hiring and investment as driven by their present values and compares the derived job values to those implied by the standard matching model. Section 6 undertakes the VAR analysis and decomposes the present value relationships embodied in the model. Section 7 looks at the ability of the results to provide a stylized account of U.S. labor market developments, including the shift of the Beveridge curve and the high unemployment rate of the Great Recession. Section 8 concludes. Technical matters and data issues are treated in appendices.

2 Background Literature

The literature on hiring and on investment is very large. In what follows I allude to those papers that relate directly to the focus of this paper.

First is the literature on search and matching models, which feature dynamic, optimal vacancy decisions by firms in the face of frictions; see Pissarides (2000), Yashiv (2007), and Rogerson and Shimer (2011) for overviews and surveys. Recruiting costs and time lags are the expression of frictions in these models. The first order condition for optimal vacancy creation is a key ingredient and this is one of the two estimating equations examined here. The finding in this literature, as indicated above, is that gross hiring, subject to these frictions, is key in accounting for employment and unemployment dynamics. The model here features a generalization of the hiring problem and a wider concept of costs relative to what has been considered by these models.

The second strand of relevant literature includes investment models,

mostly following the seminal contributions of Lucas and Prescott (1971) and of Tobin (1969) and Hayashi (1982). These models have been studied extensively for over four decades. The idea in these models is that costs are key to the understanding of investment behavior. These models have encountered a lot of empirical difficulties and have engendered much debate (see, for example, the discussion in Chirinko (1993) and Smith (2008)). Like search and matching models, much of this literature does not feature the other factor of production, namely labor. In the current paper I present results both from the “traditional” formulation of the investment costs model and from a formulation which allows for the interaction of investment costs and hiring costs.

It should also be noted that models of the business cycle (evidently) feature optimal hiring and investment decisions. Many of them do not feature frictions, though a large part of the RBC literature assumes lags in the installation of capital. The latest vintage of business cycle models, surveyed by Christiano, Trabandt and Walentin (2010), posits costs for investment but no frictions in hiring. Note, too, that in business cycle models there is no explicit interaction between hiring costs and investment costs.

A key issue in the current paper is the mutual dependence of hiring and investment and the interaction of their costs. This is not a new issue. Mortensen (1973) has examined the interrelation of costs in a theoretical model and over the years some empirical work was attempted; prominent examples include Nadiri and Rosen (1969), Shapiro (1986), and Hall (2004). These studies point to the potential importance of including costs on both capital and labor. However key differences with the current study are that these papers do not model at least one of three elements, which the empirical work below finds to be of crucial importance: (i) an interaction term between the two costs; (ii) gross, as opposed to net, hiring flows; and (iii) aggregate, as opposed to micro-level, hiring and investment. It should also be emphasized, that the current paper stays within the representative firm framework of the cited literature and does not at all attempt to go into a firm-level or sector-level analysis. Hence most of the findings of the latter

type of studies may be different from what is reported here.

This paper stresses the forward-looking aspect of hiring and investment. Consequently an important issue is the future determinants of current behavior. This issue is studied, for the case of stock prices, by a sizeable strand of literature in Finance, launched by the work of Campbell and Shiller (1988). A key concern in this literature has been the question of what is the relative importance of dividend growth and of future returns for stock price volatility. I make use of the methodology developed in this literature, examined by Cochrane (2011), to determine the relative importance of the future determinants of current hiring and current investment. Recently, Hall (2014) has taken up this issue, albeit making use of a different empirical methodology.

3 The Model

I delineate a partial equilibrium model which serves as the basis for estimation. There are identical workers and identical firms, who live forever and have rational expectations. All variables are expressed in terms of output.

Firms. Firms make gross investment (i_t) and vacancy (v_t) decisions. Once a new worker is hired, the firm pays him or her a per-period wage w_t . Firms use physical capital (k_t) and labor (n_t) as inputs in order to produce output goods y_t according to a constant-returns-to-scale production function f with productivity shock z_t :

$$y_t = f(z_t, n_t, k_t), \tag{1}$$

Gross hiring and gross investment are subject to frictions, spelled out below, and hence are costly activities. I represent these costs by a function $g[i_t, k_t, v_t, h_t, n_t]$ which is convex in the firm's decision variables and exhibits constant returns-to-scale, allowing hiring costs and investment costs to interact.

In every period t , the capital stock depreciates at the rate δ_t and is augmented by new investment i_t . Similarly, workers separate at the rate ψ_t and the employment stock is augmented by new hires $q_t v_t = h_t$. The laws

of motion are:

$$k_{t+1} = (1 - \delta_t)k_t + i_t, \quad 0 \leq \delta_t \leq 1. \quad (2)$$

$$n_{t+1} = (1 - \psi_t)n_t + q_t v_t, \quad 0 \leq \psi_t \leq 1 \quad (3)$$

$$q_t v_t = h_t$$

The representative firm chooses sequences of i_t and v_t in order to maximize its profits as follows:

$$\max_{\{i_{t+j}, v_{t+j}\}} E_t \sum_{j=0}^{\infty} \left(\prod_{i=0}^j \rho_{t+i} \right) (1 - \tau_{t+j}) \begin{pmatrix} f(z_{t+j}, n_{t+j}, k_{t+j}) - g(i_{t+j}, k_{t+j}, v_{t+j}, h_{t+j}, n_{t+j}) \\ -w_{t+j}n_{t+j} - (1 - \chi_{t+j} - \tau_{t+j}D_{t+j}) \tilde{p}_{t+j}^I i_{t+j} \end{pmatrix} \quad (4)$$

subject to the constraints (2) and (3), and where τ_t is the corporate income tax rate, w_t is the wage, χ_t the investment tax credit, D_t the present discounted value of capital depreciation allowances, \tilde{p}_t^I the real pre-tax price of investment goods, and ρ_{t+j} is a time-varying discount factor. The firm takes the paths of the variables $q_t, w_t, p_t^I, \delta_t, \psi_t, \tau_t$ and ρ_t as given. This is consistent with the standard models in the search and matching and Tobin's q literatures. The Lagrange multipliers associated with these two constraints are Q_{t+j}^K and Q_{t+j}^N , respectively. These Lagrange multipliers can be interpreted as marginal q for physical capital, and marginal q for employment, respectively. I shall use the term capital value or present value of investment for the former and job value or present value of hiring for the latter.

The first-order conditions for dynamic optimality are:⁵

$$Q_t^K = E_t [\rho_{t+1} [(1 - \tau_{t+1}) (f_{k_{t+1}} - g_{k_{t+1}}) + (1 - \delta_{t+1}) Q_{t+1}^K]] \quad (5)$$

$$Q_t^K = (1 - \tau_t) (g_{i_t} + p_t^I) \quad (6)$$

$$Q_t^N = E_t [\rho_{t+1} [(1 - \tau_{t+1}) (f_{n_{t+1}} - g_{n_{t+1}} - w_{t+1}) + (1 - \psi_{t+1}) Q_{t+1}^N]] \quad (7)$$

$$Q_t^N = (1 - \tau_t) \frac{g_{v_t}}{q_t} \quad (8)$$

⁵where I use the real after-tax price of investment goods, given by:

$$p_{t+j}^I = \frac{1 - \chi_{t+j} - \tau_{t+j}D_{t+j}}{1 - \tau_{t+j}} \tilde{p}_{t+j}^I.$$

I can summarize the firm's first-order necessary conditions from equations (5)-(8) by the following two expressions:

$$(1 - \tau_t)(g_{i_t} + p_t^I) = E_t \left[\rho_{t,t+1} (1 - \tau_{t+1}) \left[\begin{array}{c} f_{k_{t+1}} - g_{k_{t+1}} \\ +(1 - \delta_{t+1})(g_{i_{t+1}} + p_{t+1}^I) \end{array} \right] \right] \quad (9)$$

$$(1 - \tau_t) \frac{g_{v_t}}{q_t} = E_t \left[\rho_{t,t+1} (1 - \tau_{t+1}) \left[\begin{array}{c} f_{n_{t+1}} - g_{n_{t+1}} - w_{t+1} \\ +(1 - \psi_{t+1}) \frac{g_{v_{t+1}}}{q_{t+1}} \end{array} \right] \right] \quad (10)$$

Worker Flows. Consider worker flows. The flow from non-employment – unemployment (U) and out of the labor force (O) – to employment is to be denoted $OE + UE$ and the separation rate ψ_t is rate of the flow in the opposite direction, $EU + EO$. Worker flows within employment – i.e., job to job flows – are to be denoted EE .

I shall denote:

$$\begin{aligned} \frac{h}{n} &= \left(\frac{h^1}{n} \right) + \left(\frac{h^2}{n} \right) \\ \frac{h^1}{n} &= \frac{OE + UE}{E} \\ \frac{h^2}{n} &= \frac{EE}{E} \end{aligned}$$

Hence h^1 and h^2 denote flows from non-employment and from other employment, respectively.

Separation rates are given by:

$$\begin{aligned} \psi &= \psi^1 + \psi^2 \\ \psi^1 &= \frac{EO + EU}{E} \\ \psi^2 &= \frac{EE}{E} = \frac{h^2}{n} \end{aligned}$$

Equation (3) now satisfies:

$$\begin{aligned}
n_{t+1} &= (1 - \psi_t^1 - \psi_t^2)n_t + h_t^1 + h_t^2 & (11) \\
&= (1 - \psi_t)n_t + h_t, \quad 0 \leq \psi_t \leq 1 \\
h_t^2 &= \psi_t^2
\end{aligned}$$

*Matching and Separations.*⁶ Firms hire from non-employment (h_t^1) and from other firms (h_t^2). Each period, the worker's effective units of labor (normally 1 per person) depreciate to 0, in the current firm, with some exogenous probability ψ_t . Thus, the match suffers an irreversible idiosyncratic shock that makes it no longer viable. The worker may be reallocated to a new firm where his/her productivity is (temporarily) restored to 1. This happens with a probability of ψ_t^2 . Those who are not reallocated join unemployment, with probability $\psi_t^1 = \psi_t - \psi_t^2$. So the fraction ψ_t^2 that enters job to job flows depends on the endogenous hiring flow h_t^2 . The firm decides how many vacancies to open and, given job filling rates (q_t^1, q_t^2), will get to hire from the pre-existing non-employed and from the pool of matches just gone sour. The matching rates satisfy:

$$\begin{aligned}
q_t^1 &= \frac{h_t^1}{v_t} & (12) \\
q_t^2 &= \frac{h_t^2}{v_t} \\
q_t &= q_t^1 + q_t^2
\end{aligned}$$

4 Estimation

I estimate equations (9) and (10), using structural estimation and alternative versions of the model. In what follows I present the parameterization of the production and costs functions, the econometric methodology, the data and the estimation results.

⁶I am indebted to Giuseppe Moscarini for very useful suggestions to this section.

4.1 Methodology

To estimate the model I need to parameterize the relevant functions. For the *production function* I use a standard Cobb-Douglas formulation, with productivity shock z_t :

$$f(z_t, n_t, k_t) = e^{z_t} n_t^\alpha k_t^{1-\alpha}, \quad 0 < \alpha < 1. \quad (13)$$

The *costs function* g , capturing the different frictions in the hiring and investment processes, is at the focus of the estimation work and merits discussion. Specifically, hiring costs include costs of advertising, screening and testing, matching frictions, training costs and more. Investment involves implementation costs, financial premia on certain projects, capital installation costs, learning the use of new equipment, etc. Both activities may involve, in addition to production disruption, the implementation of new organizational structures within the firm and new production techniques.⁷ In sum g is meant to capture all the frictions involved in getting workers to work and capital to operate in production, and not, say, just capital adjustment costs or vacancy costs. One should keep in mind that this is formulated as the costs function of the representative firm within a macroeconomic model, and not one of a single firm in a heterogeneous firms micro set-up.

Functional Form. The parametric form I use is the following, generalized convex function.

$$g(\cdot) = \left[\begin{array}{c} \frac{e_1}{\eta_1} \left(\frac{i_t}{k_t} \right)^{\eta_1} \\ + \frac{e_2}{\eta_2} \left[\frac{(1-\lambda_1-\lambda_2)v_t + \lambda_1 q_t^1 v_t + \lambda_2 q_t^2 v_t}{n_t} \right]^{\eta_2} \\ + \frac{e_{31}}{\eta_{31}} \left(\frac{i_t}{k_t} \frac{q_t^1 v_t}{n_t} \right)^{\eta_{31}} + \frac{e_{32}}{\eta_{32}} \left(\frac{i_t}{k_t} \frac{q_t^2 v_t}{n_t} \right)^{\eta_{32}} \end{array} \right] f(z_t, n_t, k_t). \quad (14)$$

This function is linearly homogenous in its arguments i, k, v, h, n . The parameters $e_l, l = 1, 2, 31, 32$ express scale, and the parameters $\eta_1, \eta_2, \eta_{31}, \eta_{32}$ express the convexity of the costs function with respect to its different arguments. λ_1 is the weight in the cost function assigned to hiring from non-employment ($\frac{h_t^1}{n_t}$), λ_2 is the weight assigned to hiring from other firms

⁷See Alexopoulos (2011) and Alexopoulos and Tombe (2012).

$(\frac{h_t^2}{n_t})$, and $(1 - \lambda_1 - \lambda_2)$ is the weight assigned to vacancy $(\frac{v_t}{n_t})$ costs. The weights λ_1 and λ_2 are thus related to the training and production disruption aspects, while the complementary weight is related to the vacancy creation and recruiting aspects. The last two terms in square brackets capture interactions between investment and hiring. I rationalize the use of this form in what follows.

Arguments of the function. This specification captures the idea that frictions or costs increase with the extent of the activity in question – vacancy creation, hiring and investment. This needs to be modelled relative to the size of the firm. The intuition is that hiring 10 workers, for example, means different levels of hiring activity for firms with 100 workers or for firms with 10,000 workers. Hence firm size, as measured by its physical capital stock or its level of employment, is taken into account and the costs function is increasing in the vacancy, hiring and investment rates, $\frac{v}{n}$, $\frac{h}{n}$ and $\frac{i}{k}$. The function used postulates that costs are proportional to output, i.e., the results can be stated in terms of lost output.

More specifically, the terms in the function presented above may be justified as follows (drawing on Garibaldi and Moen (2009)): suppose each worker i makes a recruiting and training effort h_i ; as this is to be modelled as a convex function, it is optimal to spread out the efforts equally across workers so $h_i = \frac{h}{n}$; formulating the costs as a function of these efforts and putting them in terms of output per worker one gets $c(\frac{h}{n}) \frac{f}{n}$; as n workers do it then the aggregate cost function is given by $c(\frac{h}{n}) f$.

Convexity. I use a convex function, allowing for alternative specifications of the degree of convexity (quadratic, cubic) and looking also at a linear specification. The use of such a function may be questioned at the micro-level, as non-convexities were found to be significant at that level (plant, establishment, or firm). But a number of recent papers have given empirical support to the use of a convex function in the aggregate, showing that such a formulation is appropriate at the macroeconomic level.⁸

⁸Thus, Thomas (2002) and Kahn and Thomas (2008, see in particular their discussion on pages 417-421) study a dynamic, stochastic, general equilibrium model with nonconvex

Interaction.. The terms $\frac{e_{31}}{\eta_{31}} \left(\frac{i_t}{k_t} \frac{q_t^1 v_t}{n_t} \right)^{\eta_{31}}$ and $\frac{e_{32}}{\eta_{32}} \left(\frac{i_t}{k_t} \frac{q_t^2 v_t}{n_t} \right)^{\eta_{32}}$ express the interaction of investment and hiring costs. They allow for a different interaction for hires from non-employment (h_t^1) and from other firms (h_t^2). These terms, absent in many studies, have important implications for the complementarity of investment and hiring.

Relation to Known Cases. The function above encompasses widely-used cases as special cases:

- a. The standard Tobin's q model of investment which has $e_2 = e_{31} = e_{32} = 0$ and $\eta_1 = 2$.
- b. An analog Tobin's q model for hiring which has $e_1 = e_{31} = e_{32} = 0$ and $\eta_2 = 2$.
- c. The standard (Pissarides-type) matching model which has $e_1 = e_{31} = e_{32} = 0, \lambda_1 = \lambda_2 = 0$ and $\eta_2 = 1$.
- d. The case that abstracts from job to job flows and considers only flows into (and out of) employment would have $\lambda_2 = e_{32} = 0$ and sets $h_t^2 = \psi_t^2 = 0$. This case enables the use of a much larger data sample, 1976-2013, with 152 quarterly observations.

In estimation, I explore these alternative specifications.

Estimation of the parameters in these functions allows for the quantification of the derivatives g_{i_t} and g_{v_t} that appear in the firms' optimality equations (9) and (10). I structurally estimate the firms' first-order conditions (9) and (10), using Hansen's (1982) generalized method of moments (GMM). The moment conditions estimated are those obtained under rational expectations. I formulate the equations in stationary terms by dividing (9) by $\frac{f_t}{k_t}$ and (10) by $\frac{f_t}{n_t}$. Appendix A spells out the first derivatives included in these equations. Importantly, I check whether the estimated g function capital adjustment costs. One key idea which emerges from their analysis is that there are smoothing effects that result from equilibrium price changes. Favilukis and Lin (2011) use data on asset prices as additional restrictions when examining firm investment behavior and find that "...within such a model, non-convex frictions are unnecessary to match important features of aggregate investment...a model with convex costs alone does nearly as good of a job at matching firm level micro data as our preferred model with both convex and non-convex costs" (page 26).

fulfills the convexity requirement.

4.2 The Data

The data are quarterly, pertain to the private sector of the U.S. economy. For a large part of the empirical work reported below the sample period is 1994-2013. The start date of 1994 is due to the lack of availability of job to job worker flows (h_t^2) data prior to that. For another part of the empirical work, the sample covers 1976-2013 and the 1976 start is due to the availability of credible monthly CPS data from which the gross hiring flows (h_t^1) series is derived. This longer sample period covers five NBER-dated recessions, including the Great Recession of 2007-2009 and its aftermath (2009-2013). The data include NIPA data on GDP and its deflator, capital, investment, the price of investment goods and depreciation, BLS CPS data on employment and on worker flows, and Fed data computations on tax and depreciation allowances. Appendix B elaborates on the sources and on data construction. These data have the following distinctive features: (i) they pertain to the U.S. private sector; (ii) both hiring h and investment i refer to gross flows; likewise, separation of workers ψ and depreciation of capital δ are gross flows; (iii) the estimating equations take into account taxes and depreciation allowances. Table 1 presents key sample statistics.

Table 1

4.3 Estimation Results

Table 2 reports in two panels the results of estimation. The table reports the estimates and their standard errors, Hansen's (1982) J-statistic and its p-value.

Table 2 a,b.

Panel (a) looks at the specifications of the model discussed above. With regard to the convexity of the costs function (14), row 1 examines a quadratic function ($\eta_1 = \eta_2 = 2$) with linear interactions ($\eta_{31} = \eta_{32} = 1$) and row 2

reports the cubic function ($\eta_1 = \eta_2 = 3$) with linear interactions. The weights on the different elements of the hiring process – vacancies, hiring from non-employment, and hiring from other employment – are expressed by the fixed parameters $\lambda_1 = 0.6, \lambda_2 = 0.2$, obtained after some experimentation. The parameters estimated are the scale parameters of the frictions function (e_1, e_2, e_{31} and e_{32}) and the labor share (α) of the production function (13). The results for both these rows have J-statistics with high p-values, are for the most part precisely estimated, and the resulting g function fulfills all convexity requirements. In what follows I prefer to focus on row 1, the quadratic case, for a number of reasons: all its parameters are precisely estimated while e_{31} is not precisely estimated in row 2 (the cubic case); the estimate of α is around the conventional estimate of 0.66, while that of row 2 is lower; and the quadratic is more tractable for the computations which follow. Row 3 shows the results of estimation when ignoring job to job flows and assigning all costs to hiring from non-employment ($\frac{h_t^1}{n_t}$), i.e., setting $\lambda_1 = 1, \lambda_2 = e_{32} = 0$ and $h_t^2 = \psi_t^2 = 0$. This allows for the use of a much longer data sample – 1976:1-2013:4, with 152 quarterly observations. It too yields a J-statistic with a high p-value, is precisely estimated, and the resulting g function fulfills all convexity requirements.

Panel (b) of Table 2 looks at standard specifications in the literature. Row 1 sets $\eta_1 = 2, e_2 = e_{31} = e_{32} = 0$, i.e., quadratic investment costs, with no role for hiring, as is typical of the Tobin’s q investment literature. The results do not reject the model. But some more experimentation, for example using different instrument sets, yields big variations in the estimates, including very high positive e_1 estimates. The production parameter α is estimated to be relatively low at 0.63. Below it will be shown that the implied costs are very high. Row 2 takes the analog formulation for labor, i.e., $\eta_2 = 2, e_1 = e_{31} = e_{32} = 0$, i.e., quadratic vacancy and hiring costs, with no role for capital. Here the results appear reasonable and there is no rejection of the model, but, as shown below this specification too implies very high costs. Row 3 looks at the standard (Pissarides-type) search and matching model with linear vacancy costs, such

that $\eta_2 = 1, e_1 = e_{31} = e_{32} = \lambda_1 = \lambda_2 = 0$. The emerging estimates yield a marginal cost series which is highly correlated with the one that emerges from Row 2, but as will be shown below, implies even higher costs. Moreover, the parameter α is estimated at a high value (0.77).

The conclusions, thus far, are as follows: quadratic costs and linear interaction of investment and hiring costs generate a good fit of the data; the bigger weight (0.6) is placed on the costs of hiring from non-employment, with the remaining weight given equally to vacancy creation and to hiring from other employment; the interactions between both types of hiring and investment are significant and negatively signed, implying complementarity between investment and hiring (to be discussed below). In what follows I shall refer to the results of row 1 in panel (a) as the preferred specification.

4.4 Implied Costs

The estimated costs are interesting and important by and of themselves, as many models rely on their existence. Hence, the results of Table 2 merit inspection for plausibility and the derivation of the time series for the frictions they imply. This is done by constructing the time series for total and marginal costs implied by the point estimates of the parameters of the g function and relating them to what is known on these issues. Total costs are presented in terms of percentage points out of GDP, $\frac{g}{f}$. The marginal costs of investment are compared to the price of investment, $\frac{\frac{g_i}{f}}{\frac{pI}{f}} = \frac{g_i}{pI}$, so they indicate how much the firm has to add in frictions costs to every dollar paid for the investment good. The marginal costs of vacancies are compared to the wage, $\frac{\frac{g_v}{f}}{\frac{q}{w}} = \frac{g_v}{w}$, so they indicate how much the firm pays for vacancy creation and hiring in wage terms. Key moments are presented in Tables 3a and 3b.

Table 3 a,b.

For the preferred estimates, total costs are about 2% of GDP on average, with a standard deviation of 0.4%. Marginal investment costs add about 5%

on average to the price p_t^I of a unit of capital. Marginal hiring costs are on average 30% of quarterly wages, the equivalent of almost 4 weeks of wages. Note that row 3, with the longer data sample, produces almost the same numbers. These numbers constitute moderate or low costs estimates; Appendix C provides a comparison to the literature. The cubic specification of row 2 in panel (a) has slightly higher total costs, lower marginal investment costs, and higher vacancy creation marginal costs.

The implied costs of the standard specifications, reported in panel (b), are all unreasonable: for Tobin's q for capital, for Tobin's q for labor and for the standard search and matching model they are all excessively high.

Overall, then, this section has presented an estimate of a quadratic costs function of the frictions, with linear interaction between hiring and investment, which fits the data. The estimates imply complementarity between hiring and investment and low costs. Standard specifications, which pertain to one factor only – capital or labor – do not produce such reasonable results.

5 Hiring, Investment and Their Present Values

This section examines the implications of the estimates for the relations of hiring and investment with their values – capital and job values. In what follows, I will use both the terms vacancies (v_t) and hiring (h_t) and it should be kept in mind that $h_t = q_t v_t$, with the firm taking q_t as given.

5.1 Vacancy and Investment Rates as Functions of the Present Values

Taking equations (6)-(8), using the definitions of the derivatives of the g function spelled out in Appendix A, and the results of row 1 in Table 2a,⁹ the following relations are derived:

⁹Whereby

$$\eta_1 = \eta_2 = 2, \eta_{31} = \eta_{32} = 1$$

$$\frac{v_t}{n_t} = \frac{e_1 q_t \left(\frac{1}{(1-\tau_t)} \frac{Q_t^N}{n_t} \right) - (e_{31} q_t^1 + e_{32} q_t^2) \left(\frac{1}{(1-\tau_t)} \frac{Q_t^K}{k_t} - \frac{p_t^I}{k_t} \right)}{e_1 e_2 \Lambda_t^2 - (e_{31} q_t^1 + e_{32} q_t^2)^2} \quad (15)$$

$$\frac{i_t}{k_t} = \frac{e_2 \Lambda_t^2 \left(\frac{1}{(1-\tau_t)} \frac{Q_t^K}{k_t} - \frac{p_t^I}{k_t} \right) - q_t (e_{31} q_t^1 + e_{32} q_t^2) \left(\frac{1}{(1-\tau_t)} \frac{Q_t^N}{n_t} \right)}{e_1 e_2 \Lambda_t^2 - (e_{31} q_t^1 + e_{32} q_t^2)^2} \quad (16)$$

where:

$$\Lambda_t = [(1 - \lambda_1 - \lambda_2) + \lambda_1 q_t^1 + \lambda_2 q_t^2]$$

The estimates of Table 2 indicate that $e_1, e_2 > 0, e_{31}, e_{32} < 0$ and $e_1 e_2 \Lambda_t^2 - (e_{31} q_t^1 + e_{32} q_t^2)^2 > 0$.

The implications of these relations are that the vacancy and investment rates, $\frac{v_t}{n_t}$ and $\frac{i_t}{k_t}$, are positive functions of both their present values, Q_t^N and Q_t^K (net of p_t^I), taking into account taxes. It is therefore apparent that models which ignore the present value of the other factor are mis-specified, as $e_{31} \neq 0$ and $e_{32} \neq 0$.

Table 4 shows the first and second moments of the decomposition of the RHS of the equations in (15)-(16).

Table 4

In both cases, the cross effects are substantial. Of the mean quarterly vacancy rate of 2.8%, a fraction of 74% is due to the present value of hiring term $\left[e_1 q_t \left(\frac{1}{(1-\tau_t)} \frac{Q_t^N}{n_t} \right) \right]$ and the remaining 26% are due to the investment term $\left[- (e_{31} q_t^1 + e_{32} q_t^2) \left(\frac{1}{(1-\tau_t)} \frac{Q_t^K}{k_t} - \frac{p_t^I}{k_t} \right) \right]$. The variance of the vacancy rate (std of 0.5%) is decomposed in rows 2 and 3, which sum up to 1. The investment term plays a substantial role – its variance is bigger than that of the hiring term and the co-variance of the two terms is substantial. Hence

and

$$e_1 e_2 - (e_{31} q_t^1 + e_{32} q_t^2) [(1 - \lambda_1 - \lambda_2) + \lambda_1 q_t^1 + \lambda_2 q_t^2]^2 > 0$$

the results imply that the present value of investment plays a substantial role in the determination of the volatility of hiring rates.

The mean quarterly investment rate of 2.6% is due to the present value of hiring term (53%) and the present value of investment term (47%). The variance of the investment rate (std of 0.2%) is decomposed into a smaller part due to the hiring term and a bigger part played by the variance of the investment term and a large (negative) co-variation with hiring. Hence investment is heavily influenced by hiring value both in terms of its mean value and in terms of its volatility.

5.2 Negative Interaction Engenders Simultaneity

In Table 2a, the estimates of the coefficients of the interaction terms, e_{31}, e_{32} are negative. These negative point estimates imply a negative value for g_{vi} and, therefore, as can be seen in equations (15)-(16), a positive sign for $\partial(\frac{v_t}{n_t})/\partial\left(\frac{Q_t^K}{\frac{J_t}{k_t}}\right)$ and for $\partial(\frac{i_t}{k_t})/\partial\left(\frac{Q_t^N}{\frac{J_t}{n_t}}\right)$. Note that $\partial(\frac{i_t}{k_t})/\partial\left(\frac{Q_t^K}{\frac{J_t}{k_t}}\right)$ and $\partial(\frac{v_t}{n_t})/\partial\left(\frac{Q_t^N}{\frac{J_t}{n_t}}\right)$ are positive due to convexity. Hence, when the marginal value of investment $\frac{Q_t^K}{\frac{J_t}{k_t}}$ rises, both investment and vacancies/hiring rise. A similar argument shows that they both rise when the marginal value of vacancies $\frac{Q_t^N}{\frac{J_t}{n_t}}$ rises.

The signs of these derivatives imply that for given levels of investment, total and marginal costs of investment decline as vacancies increase. Similarly, for given levels of vacancies, total and marginal costs of vacancies decline as investment increases. This finding of complementarity between investment and vacancies/hiring is to be expected as it implies that they should be simultaneous. One interpretation of this result is that simultaneous hiring and investment is less costly than sequential hiring and investment of the same magnitude. This may be due to the fact that simultaneous action by the firm is less disruptive to production than sequential action.

5.3 Capital and Job Values Across Models

Figure 1a presents a plot of the capital value. It equals estimated marginal investment costs, defined as $\frac{g_{i_t}}{\frac{f_t}{k_t}}$. Figure 1b plots the value of a job, which equals estimated marginal vacancy costs, defined as $\frac{g_{v_t}}{q_t \frac{f_t}{n_t}}$. Both use the preferred specification (row 1 of Table 2a). Figure 1a also shows the same value according to the Tobin's q model (row 1 in Table 2b) and Figure 1b shows the same value according to the standard, search and matching model (row 3 in Table 2b). NBER-dated recessions are shaded.

Figures 1a and b

In terms of the first moment, as discussed in the preceding section and shown in Table 3b, both the Tobin's q model and the standard search and matching model estimates imply very high marginal costs (read on the right scale of the figures). For the former, these are on average 10% of the purchase price of capital (p^I). For the latter they are equivalent to about 31 weeks of wages on average.¹⁰ In the current model, as shown in Table 3a, they are 4% of the purchase price and equivalent to 4 weeks of wages, respectively. In terms of second moments, the Tobin's q estimates are correlated 0.37 with the preferred estimates here; the standard search and matching model costs are negatively correlated, at -0.57 , with the values implied by the preferred specification. The negative co-movement is highly apparent in the shaded recession periods.

The reasons for these substantial differences are as follows. The Tobin's q model and the standard matching model ignore the interaction with the other factor (as they set $e_{31} = e_{32} = 0$). Moreover, the standard matching model postulates linear costs that pertain to vacancies only. Essentially it is $\frac{(1-\tau_t)e_2}{q_t}$ and its main variation comes from q_t , the firm matching rate, that appears in its denominator. The preferred specification in the current model features convex costs that pertain to both investment ($\frac{i_t}{k_t}$) and hiring ($\frac{h_t}{n_t}$) rates (both from non-employment and job to job flows) as well as to

¹⁰See Table 3b.

vacancy rates.¹¹ Hence the different models take very different stands on the arguments of the investment and hiring frictions function and on its shape. The cyclical implications of these differences are further explored in Yashiv (2014). Implications for recent U.S. experience are discussed in Section 7 below.

6 The Determinants of Capital and Job Values

I have derived, through structural estimation, the costs function (g), from which one can derive the value of the job (i.e., the expected present value of hiring ($\frac{Q^N}{f}$)) and the value of capital (i.e., the expected present value investment ($\frac{Q^K}{k}$)). How are these values related to their expected future determinants, given that both hiring and investment are forward-looking variables? In other words, what in the future drives hiring and investment today? In this section, I follow the empirical methodology of the asset pricing literature in Finance and examine the present value relationships governing hiring and investment. This involves the use of a forecasting VAR. The analysis is based on the framework proposed by Campbell and Shiller (1988) and its more recent elaboration by Cochrane (2011), whose notation I follow.¹² Note that I do not consider stock prices or any financial data here; rather, I apply the empirical framework developed in the cited Finance literature to the current context. The results in the Finance literature do, however, provide a natural benchmark against which to compare the current results.¹³

¹¹It is given by

$$\frac{g v_t}{q_t \frac{f_t}{n_t}} = \frac{1}{q_t} \left[e_2 \left[\frac{(1-\lambda_1-\lambda_2)v_t + \lambda_1 q_t^1 v_t + \lambda_2 q_t^2 v_t}{n_t} \right]^{\eta_{2-1}} \left[(1-\lambda_1-\lambda_2) + \lambda_1 q_t^1 + \lambda_2 q_t^2 \right] \right. \\ \left. + e_{31} q_t^1 \left(\frac{i_t}{k_t} \right)^{\eta_{31}} \left(\frac{q_t^1 v_t}{n_t} \right)^{\eta_{31}-1} \right. \\ \left. + e_{32} q_t^2 \left(\frac{i_t}{k_t} \right)^{\eta_{32}} \left(\frac{q_t^2 v_t}{n_t} \right)^{\eta_{32}-1} \right]$$

¹²The importance of this approach and its wider significance was noted in the Nobel Economics Prize for 2013 (see in particular pp.17-20 in Nobel Prize (2013)). This model is often referred to as the dynamic, dividend-growth model. Cochrane (2011) provides a discussion of empirical findings and their implications for asset pricing.

¹³See Jermann (1998, 2010).

6.1 An Asset Pricing Model

The model begins with the following two-period representation for the stock price (P) and dividends (D):

$$P_t = E_t (R_{t+1}^{-1} [D_{t+1} + P_{t+1}])$$

where R is the gross return. Iterated forward this yields:

$$P_t = E_t \sum_{j=1}^{\infty} \left(\left(\prod_{k=1}^j R_{t+k}^{-1} \right) D_{t+j} \right) \quad (17)$$

These relationships hold true also ex-post if one defines the gross return as:

$$R_{t+1} \equiv \frac{D_{t+1} + P_{t+1}}{P_t} \quad (18)$$

Using logs, this asset pricing relationship can be approximated as:¹⁴

$$p_t \simeq k + E_t (\rho p_{t+1} + (1 - \rho) d_{t+1} - r_{t+1}) \quad (19)$$

Equation (19) is an ex-ante formulation using conditional expectations. The ex-post equation, omitting the expectations operator E_t in the above, holds true as well, when using (18).

The current price (p_t) is related to the future dividend (d_{t+1}) and to the future return (r_{t+1}). The price will be higher when the future dividend is higher and/or when the future return is lower.

6.2 Implementing the Forecasting Model for Hiring and Investment

I cast the estimated model of hiring and investment into this asset pricing framework by defining P and D for the optimal investment equation and for the optimal hiring equation. The “price” P is the value of capital or the

¹⁴where:

$$p_t \equiv \ln P_t, \quad d_t = \ln D_t, \quad r_t = \ln R_t \quad \rho = \frac{\frac{P}{D}}{1 + \frac{P}{D}}$$

and where P, D are steady state or long-term average values.

value of jobs; this is essentially marginal q for capital investment (Q^K) and marginal q for labor hiring (Q^N), each divided by the relevant productivity ($\frac{f}{k}$ or $\frac{f}{n}$); the “dividend” D is the flow of net income from capital or from labor. As shown below, additional terms come into play here. These prices and “dividends” are not observed on the market, as in the Finance literature. Rather, they represent what the firm actually gets from its use of capital and labor in production. Thus, the “dividend” in the investment case is the net marginal productivity of capital; in the hiring case it is net labor profitability, i.e., the net marginal product of labor less the wage. These “dividends” do not depend on institutional or financial considerations of firms as dividends do in the Finance context.

Define:

$$P_t^1 \equiv (1 - \tau_t) \left(\frac{g_{it} + p_t^I}{\frac{f_t}{k_t}} \right) = \frac{Q_t^K}{\frac{f_t}{k_t}}; \quad D_t^1 = (1 - \tau_t) \frac{(f_{k_t} - g_{k_t})}{\frac{f_t}{k_t}}; \quad R_t^1 = \frac{G_{t+1}^{f/k} [(1 - \delta_t)P_t^1 + D_t^1]}{P_{t-1}^1} \quad (20)$$

using $G_{t+1}^{f/k} = \frac{\frac{f_{t+1}}{k_{t+1}}}{\frac{f_t}{k_t}}$.

Comparing equation (20) to (18), one can see that two additional terms in the current context are the one involving capital depreciation (δ_t) and one involving capital productivity growth ($G_{t+1}^{f/k}$). Note, too, that D_t^1 expresses the share in capital productivity received by the firm, which without taxes and investment costs would be $\frac{f_{k_t}}{f_t} = 1 - \alpha$. The term $G_{t+1}^{f/k}$ captures the gross rate of growth of this productivity.

Appendix D shows that this formulation yields the following log-linear approximation for log capital values:

$$p_{t-1}^1 \cong c_2 + \ln G_t^{f/k} + \rho^k \ln(1 - \delta_t) + \rho^k p_t^1 + (1 - \rho^k) d_t^1 - r_t^1 \quad (21)$$

where small letters denote variables in logs and where $\rho^k = \frac{\frac{(1-\delta)P^1}{D^1}}{1 + \frac{(1-\delta)P^1}{D^1}}$.

For labor, define:

$$P_t^2 \equiv \frac{(1 - \tau_t) g_{v_t}}{\frac{f_t}{n_t} q_t} \equiv \frac{Q_t^N}{\frac{f_t}{n_t}}; \quad D_t^2 = (1 - \tau_t) \left(\alpha - \frac{g_{n_t}}{\frac{f_t}{n_t}} - \frac{w_t}{\frac{f_t}{n_t}} \right); \quad R_t^2 = \frac{G_{t+1}^{f/n} [(1 - \psi_t) P_t^2 + D_t^2]}{P_{t-1}^2} \quad (22)$$

$$\text{using } G_{t+1}^{f/n} = \frac{\frac{f_{t+1}}{n_{t+1}}}{\frac{f_t}{n_t}}$$

Comparing equation (22) to (18), one can see that two additional terms in the current context are the one involving worker separations (ψ_t) and one involving labor productivity growth ($G_{t+1}^{f/n}$). Note that D_t^2 are the actual profits from labor, once taxes, costs and wages have been deducted. The term $G_{t+1}^{f/n}$ captures the gross rate of growth of labor productivity. Appendix D shows that this yields the following log-linear approximation of job values:

$$p_{t-1}^2 = c_5 + \ln G_t^{f/n} + \rho^n \ln(1 - \psi_t) + \rho^n p_t^2 + (1 - \rho^n) d_t^2 - r_t^2 \quad (23)$$

$$\text{where } \rho^n = \frac{\frac{(1-\psi)P^2}{D^2}}{1 + \frac{(1-\psi)P^2}{D^2}}.$$

6.3 Empirical Methodology

I use a restricted VAR to examine these relationships. Consider, first, the log-linear pricing equations in the non-stochastic steady state. These are given by:

$$p^1 \cong \frac{c_2}{1 - \rho^k} + \frac{\ln G^{f/k}}{1 - \rho^k} + \frac{\rho^k}{1 - \rho^k} \ln(1 - \delta) + d^1 - \frac{r^1}{1 - \rho^k} \quad (24)$$

$$p^2 \simeq \frac{c_5}{1 - \rho^n} + \frac{\ln G^{f/n}}{1 - \rho^n} + \frac{\rho^n}{1 - \rho^n} \ln(1 - \psi) + d^2 - \frac{r^2}{1 - \rho^n} \quad (25)$$

These equations state that, in the non-stochastic steady state, the value of capital (p^1) and of jobs (p^2) can each be decomposed (using log-linear approximation) into parts due to dividends (d) or shares in net productivity, returns (r), productivity growth ($\ln G^{f/k}$ or $\ln G^{f/n}$) and depreciation (δ) or separation (ψ).

Thus I estimate the following restricted structural VAR:

$$\mathbf{x}_{t+1} = A + B\mathbf{x}_t + \varepsilon_t \quad (26)$$

where $\mathbf{x}_{t+1} = (p_{t+1}^1, d_{t+1}^1, r_{t+1}^1, \ln(G_{t+1}^{f/k}), \ln(1 - \delta_{t+1}))$ for capital, $\mathbf{x}_{t+1} = (p_{t+1}^2, d_{t+1}^2, r_{t+1}^2, \ln(G_{t+1}^{f/n}), \ln(1 - \psi_{t+1}))$ for labor, under the restrictions implied by the steady state equations (24) and (25). Following estimation I compute the relevant long run coefficients (see Appendix D for a full derivation).

6.4 VAR Results

Table 5 reports the results of the VAR for selected coefficients in the B matrix and the implied long run coefficients.

Table 5

For investment, a substantial role is played by returns (a long run coefficient of -1.15), while the other determinants have negligible effects. Productivity growth seems to have some effect but it is imprecisely estimated. The adjusted R^2 of the return regression (that of r^1 on the lagged values of all the other variables) is not high, though at 0.23 it is higher than the value reported in the Finance literature for return regressions using stock prices.

For hiring, the most substantial role is again played by returns (a long run coefficient of -0.73), and to some extent by labor profitability, i.e., productivity less the wage (a long run coefficient of 0.24). The other determinants have much smaller effects.

What, then, do we learn about the various future determinants of investment and hiring values?

First, *returns* play the dominant role, as also found in the empirical Finance literature. Their VAR coefficients (b_{r_p1} and b_{r_p2}) are precisely estimated and the implied long run coefficients are sizeable. The adjusted R^2 in the investment case of the return regression (0.23) is higher than that of regressions in Finance while for hiring it is even much higher (0.39). Note that these coefficients are negative, implying that a rise in log prices is

associated with future declines in returns (r), for both investment and hiring, i.e., high prices predict low subsequent returns, as found in the Finance literature. A similar result is obtained when computing the relation between the log price-dividend ratio ($p - d$) of investment and of hiring with their subsequent returns. This result has also been observed for stock prices and dividends and for house prices and rents (see Cochrane (2011, pp. 1051-1052)).

Second, “*dividends*” in the way defined here – labor profitability – play a role in the hiring case, although a smaller one than returns. In this case, higher prices are associated with subsequent higher labor profitability and the adjusted R^2 is very high (0.95). Productivity does not play a significant role in the capital case.

Third, *productivity growth*, does not appear to play a role in both cases: the VAR coefficients (b_{gk_p1} and b_{gn_p2}) are not significantly different from zero. This is akin to the finding in Finance that dividend growth does not matter much.

Fourth, *investment values* are highly persistent (as measured by $\phi_1 = 0.97$), which is consistent with the findings of the Finance literature. *Job values* are somewhat less persistent ($\phi_2 = 0.73$).

Fifth, *the rates of separation and depreciation* do not play a meaningful role; the coefficients are not significantly different from zero. This means that the variable that determines the length of the hire (ψ determines job duration) does not have much effect on the value of the hire, relative to the other determinants. It is the discounting of future streams which plays the overwhelming role.

7 U.S. Labor Market Experience

In this section I use the model of vacancy creation and hiring studied here to examine broader labor market phenomena. I embed the afore-going set-up in a matching framework which facilitates the analysis of unemployment, including the recent Great Recession experience. The essential idea is to

incorporate the firms' F.O.C – complete with the investment interaction – into a standard search and matching model of vacancies and unemployment. Using a conventional matching function and the afore-going estimates, I relate the model's steady state formulations to U.S. data and then analyze recent U.S. experience.

This exercise uses the estimates of Table 2 to embed the vacancy and investment F.O.C. in a wider framework, albeit still a partial equilibrium one. By calibrating the parameters using the GMM estimates and employing data averages, the steady state of this framework is derived and compared to actual data using graphical analysis. This allows one to see how movements in the data over the sample period may be approximated by movements in the model's steady state curves over sub-periods. The changes in unemployment and vacancies over time can be understood in terms of changes in variables that are at the focus of the analysis – job and capital values. I then compare the results of the current model to those implied by the standard search and matching model.

7.1 Incorporating the Analysis in a Matching Framework

Following Pissarides (2000), a matching function defines the hiring rate. There are two CRS functions for each of the hiring flows:

$$\frac{h_t^1}{n_t} = \mu^1 \left(\frac{u_t}{n_t} \right)^\sigma \left(\frac{v_t}{n_t} \right)^{1-\sigma} \quad (27)$$

$$\frac{h_t^2}{n_t} = \mu^2 \left(\frac{\Phi_t n_t}{n_t} \right)^\sigma \left(\frac{v_t}{n_t} \right)^{1-\sigma} \quad (28)$$

where Φ_t is the fraction of employed workers that are searching for work in another firm. As noted above, $h_t^2 = \psi_t^2$.

The firm matching rates are given by:

$$q_t^1 = \frac{h_t^1}{v_t} = \mu^1 \left(\frac{v_t}{\frac{u_t}{n_t}} \right)^{-\sigma} \quad (29)$$

$$q_t^2 = \frac{h_t^2}{v_t} = \mu^2 (\Phi_t)^\sigma \left(\frac{v_t}{n_t} \right)^{-\sigma} \quad (30)$$

$$q_t = \frac{h_t}{v_t} = \frac{h_t^1 + h_t^2}{v_t} = \mu^1 \left(\frac{v_t}{\frac{u_t}{n_t}} \right)^{-\sigma} + \mu^2 (\Phi_t)^\sigma \left(\frac{v_t}{n_t} \right)^{-\sigma} \quad (31)$$

In the steady state the two FOC are given by:

$$(1 - \tau) \left(\frac{p^I}{\frac{f}{k}} + \frac{g_i \left(\frac{v}{n}, \frac{q^1 v}{n}, \frac{q^2 v}{n}, \frac{i}{k} \right)}{\frac{f}{k}} \right) = \frac{Q^K}{\frac{f}{k}} \quad (32)$$

$$(1 - \tau) \frac{g_v \left(\frac{v}{n}, \frac{q^1 v}{n}, \frac{q^2 v}{n}, \frac{i}{k} \right)}{q \frac{f}{n}} = \frac{Q^N}{\frac{f}{n}} \quad (33)$$

In steady state equilibrium, the flows from and to non-employment are equal so:

$$\mu^1 \left(\frac{v}{n} \right)^{1-\sigma} \left(\frac{u}{n} \right)^\sigma = \psi^1 + g \quad (34)$$

Within employment flows satisfy:

$$\mu^2 \left(\frac{v}{n} \right)^{1-\sigma} (\Phi)^\sigma = \psi^2 \quad (35)$$

where g is the rate of growth of the labor force ($n + u$).

The solution to (32), (33), (34) and (35) determines $\frac{i}{k}$, $\frac{v}{n}$, $\frac{u}{n}$ and Φ .

Making use of the formulation of a quadratic-linear costs function ($\eta_1 = \eta_2 = 2$ and $\eta_{31} = \eta_{32} = 1$) and using (32), I get

$$\frac{i}{k} = \frac{1}{e_1} \left[\frac{1}{(1 - \tau)} \frac{Q^K}{\frac{f}{k}} - \frac{p^I}{\frac{f}{k}} - (e_{31} q^1 + e_{32} q^2) \frac{v}{n} \right] \quad (36)$$

which can be substituted into (33). The steady state equilibrium can thus be presented as follows:

$$\frac{1}{q} \left[\begin{array}{c} e_2 [(1 - \lambda_1 - \lambda_2) + \lambda_1 q^1 + \lambda_2 q^2]^2 \frac{v}{n} \\ + (e_{31} q^1 + e_{32} q^2) \frac{1}{e_1} \left[\frac{1}{(1-\tau)} \frac{Q^K}{\frac{f}{k}} - \frac{p^I}{\frac{f}{k}} - (e_{31} q^1 + e_{32} q^2) \frac{v}{n} \right] \end{array} \right] = \frac{1}{(1-\tau)} \frac{Q^N}{\frac{f}{n}} \quad (37)$$

$$\mu^1 \left(\frac{v}{n} \right)^{1-\sigma} \left(\frac{u}{n} \right)^\sigma = \psi^1 + g \quad (38)$$

$$\mu^2 \left(\frac{v}{n} \right)^{1-\sigma} (\Phi)^\sigma = \psi^2 \quad (39)$$

Substituting (39) into (37), this yields the following two equations, to be plotted in $\frac{u}{n}$ and $\frac{v}{n}$ space:

$$\frac{1}{\left[\mu^1 \left(\frac{u}{n} \right)^\sigma + \frac{\psi^2}{\frac{v}{n}} \right]} \left[\begin{array}{c} e_2 \left[\begin{array}{c} (1 - \lambda_1 - \lambda_2) + \lambda_1 \mu^1 \left(\frac{u}{n} \right)^\sigma \\ + \lambda_2 \left(\frac{\psi^2}{\frac{v}{n}} \right) \end{array} \right]^2 \frac{v}{n} \\ + \left[\begin{array}{c} e_{31} \mu^1 \left(\frac{u}{n} \right)^\sigma \\ + e_{32} \left(\frac{\psi^2}{\frac{v}{n}} \right) \end{array} \right] \frac{1}{e_1} \left[\begin{array}{c} \frac{1}{(1-\tau)} \frac{Q^K}{\frac{f}{k}} - \frac{p^I}{\frac{f}{k}} \\ - \left[\begin{array}{c} e_{31} \mu^1 \left(\frac{u}{n} \right)^\sigma \\ + e_{32} \left(\frac{\psi^2}{\frac{v}{n}} \right) \end{array} \right] \frac{v}{n} \end{array} \right] \end{array} \right] = \frac{1}{(1-\tau)} \frac{Q^N}{\frac{f}{n}} \quad (40)$$

$$\mu^1 \left(\frac{v}{n} \right)^{1-\sigma} \left(\frac{u}{n} \right)^\sigma = \psi^1 + g \quad (41)$$

Using (40), the vacancy creation curve, and (41), the steady state flows curve, one solves for $\frac{u}{n}$ and $\frac{v}{n}$ given the steady state values of the variables $\frac{1}{(1-\tau)} \frac{Q^N}{\frac{f}{n}}, \frac{1}{(1-\tau)} \frac{Q^K}{\frac{f}{k}} - \frac{p^I}{\frac{f}{k}}, \psi^1, \psi^2, g$ and the parameter values $e_1, e_2, e_{31}, e_{32}, \lambda_1, \lambda_2, \mu^1, \mu^2$ and σ .

Note that term ‘‘Beveridge curve’’ is often used to denote the empirical relationship between v and u (see Yashiv (2008)). In the search and matching literature, this term is typically used to designate the steady state flows equation, given here by (41).

7.2 Graphical Analysis

Figure 2 presents a plot of these curves in $\frac{u}{n} - \frac{v}{n}$ space, together with data points that will be described shortly.

Figure 2

The figure depicts two downward-sloping curves:

a. The steady state flows curve (41) is downward-sloping because as $\frac{u}{n}$ rises $\frac{v}{n}$ needs to fall to keep the same hiring flow out of unemployment to match a given flow into unemployment (determined by $\psi^1 + g$).

b. The vacancy creation curve (40) is also downward-sloping for the following reason: when the unemployment rate $\frac{u}{n}$ rises, the matching rates q and q^1 rise as firms face a bigger pool of searching workers. This has two contradictory effects on the firm's behavior: it lowers vacancy duration $\frac{1}{q}$, thereby reducing the cost of vacancies, which operates to increase the vacancy rate. At the same time, for any given vacancy rate $\frac{v}{n}$, more hires are made (with $\frac{h}{n} = q\frac{v}{n}$). With convex costs of training, the firm gets lower profits from the marginal hire, which operates to lower the optimal vacancy rate. It turns out that, for the estimated parameter values, the latter effect is dominant and vacancy rates $\frac{v}{n}$ go down. When job values $(\frac{1}{(1-\tau)}\frac{Q^N}{\frac{f}{n}})$ or capital values $(\frac{1}{(1-\tau)}\frac{Q^K}{\frac{f}{k}} - \frac{p^I}{\frac{f}{k}})$ go up, the vacancy creation curve moves up, towards a higher rate of vacancy creation.

To offer some comparison, I repeat this exercise for the standard search and matching model. Figure 3 shows the same curves in a prototypical Pissarides (2000) model, which may be compared to Figure 2 of the current model.

Figure 3

Essentially the curve of the steady state flows equation (41) remains the same across models. But the equation for vacancy creation in the steady state, using the same matching function, is given by:

$$\frac{c}{q} = \frac{1}{(1-\tau)} \frac{Q_{search}^N}{\frac{f}{n}} \quad (42)$$

$$\frac{c}{\left[\mu^1 \left(\frac{v}{n} \right)^{-\sigma} + \mu^2 (\Phi)^\sigma \left(\frac{v}{n} \right)^{-\sigma} \right]} = \frac{1}{(1-\tau)} \frac{Q_{search}^N}{\frac{f}{n}}$$

This is an upward sloping curve in $\frac{u}{n} - \frac{v}{n}$ space.¹⁵ A rise in $\frac{u}{n}$ lowers vacancy duration, decreases costs and thus increases vacancy rates. There are no offsetting convex training costs. Here, too, the curve moves up towards higher vacancy creation when job values $(\frac{1}{(1-\tau)} \frac{Q_{search}^N}{\frac{f}{n}})$ rise.

7.3 Relating the Models to U.S. Data

I now relate the steady state relationships (40) and (41) in the current model (Figure 2), and equations (42) and (41) in the standard model (Figure 3), to the actual data. The idea is to find a region in $\frac{u}{n} - \frac{v}{n}$ space where these equations are a reasonable approximation of the steady state around which the data points are scattered. This is a “stylized exercise” which needs to be understood as such.

In order to do so one needs to use the relevant unemployment pool u_t . The hiring series (h_t^1) used here includes worker flows to employment from both the out of the labor force pool and the official unemployment pool. I examine three alternative formulations for u_t : in one it is the official unemployment pool; in a second, it is the official unemployment pool plus marginally attached workers; and in a third it is the official unemployment pool plus workers who “want a job.” Using these variables, and a vacancy series, Figure 2 plots the data points (together with the steady state equations) in $\frac{u}{n} - \frac{v}{n}$ space for official unemployment. Appendix E, which elaborates on the data and the procedure, does the same for the other two formulations of unemployment. Figure 2 shows actual U.S. data points of $\frac{u}{n}$ and $\frac{v}{n}$ over the sample period divided into two sub-periods: 1994 - 2006 and 2007-2013. Table 6 presents average values of all relevant variables in these sub-periods using the official unemployment pool. Appendix E does the same for the other two formulations of unemployment.

Table 6

The data points are fairly well distributed around the steady state curves.

¹⁵Note, that it does not start at the origin because of the job to job flow term $\mu^2 (\Phi)^\sigma (\frac{v}{n})^{-\sigma}$ in (42).

By construction, the intersection of the curves lies at the relevant sub-sample average. It turns out that the analysis of the other two non-employment pools yields the same qualitative conclusions. Figure 2 and Table 6, as well as the figures and tables in Appendix E, suggest the following interpretation of U.S. labor market developments: in the Great Recession, both curves shifted up. The outcome was that the unemployment rate increased considerably while the vacancy rate fell somewhat.

The emerging partial equilibrium “story” is as follows. Going from 1994-2006 to 2007-2013 the vacancy creation curve (40) moved up due to a rise in job values $\frac{1}{1-\tau} \frac{Q^N}{\frac{l}{n}}$ and in capital values $\frac{1}{(1-\tau)} \frac{Q^K}{\frac{l}{k}} - \frac{p^l}{\frac{l}{k}}$. The intuition is clear: the higher the job value, the higher is vacancy creation, and the curve for the latter moves up. The less intuitive aspect is the rise in job values in a period marked by a recession. In Yashiv (2014), I show that job values behave counter-cyclically, reflecting expected future job profitability (as opposed to current profitability). The steady state flows curve (41) went up too, due to the rise in the separation rate ψ^1 , and despite a decline in the labor force growth rate g . Overall the following took place: the unemployment rate ($\frac{u}{n}$) rose, as did the rate of hiring from non-employment ($\frac{h^1}{n}$) and the separation rate ψ^1 ; vacancy rates ($\frac{v}{n}$) fell and so did job to job movements (seen by the decline in ψ^2 in Table 6).

How do these same $\frac{u}{n} - \frac{v}{n}$ developments look in the standard search and matching model? Figure 3 features the data points in the same way, as does Appendix E for the other non-employment pools. In this standard model, job values Q_{search}^N go down, which can be seen in Table 6. Therefore the vacancy creation curve underlying (42) moves down, implying lower vacancy creation for a given rate of unemployment. With the further move in the flows curve (41), for the same reason as above, equilibrium moves to a higher rate of unemployment. The increase in the flow into unemployment (higher ψ^1) needs to be balanced by the outflow from unemployment and vacancy rates rise too, though not as far as the initial rate.

Note, then, the difference between the current model and the standard model in accounting for the same developments in the data: in the current

model the job value has gone up (as well as the capital value) while in the standard model it has gone down. Both of these movements in job values may be seen in Figure 1 of Section 5.3 above. Hence, while both models can account for the developments in $\frac{u}{n}$ and $\frac{v}{n}$ space, they attribute different reasons to the changes that took place. The reason for the differences lies in the formulation of hiring frictions. The standard model has linear ($\eta_2 = 1$) costs, which depend only on vacancy rates (no capital interaction), and the ensuing marginal costs function depends mainly on $\frac{v}{u}$ (see the LHS of (42)). In the current model, costs are convex ($\eta_1 = \eta_2 = 2$) and are a function of the three elements of the recruiting process ($\frac{v}{n}, \frac{q^1 v}{n}, \frac{q^2 v}{n}$), as well as the interaction with investment rates ($\frac{i}{k}$).¹⁶ So while the standard model has vacancy duration ($\frac{1}{q}$) as the only element driving fluctuations in costs, the current model adds to this element also hiring and investment rates. Note that the standard model is a special case of the current model (with $e_1 = e_{31} = e_{32} = 0, \lambda_1 = \lambda_2 = 0$ and $\eta_2 = 1$) and in estimation has yielded parameter estimates indicating excessively high vacancy costs.

8 Conclusions

The key notions in this paper are the forward-looking aspect of investment and hiring and their joint determination. More specifically, the results indicate three sets of key implications:

One is the complementarity between hiring and investment and substantial cross-effects – the first and second moments of the hiring rate are heavily influenced by the present value of investment and vice versa. Estimated job values here were shown to differ from those implied by the standard search and matching model. A second, is that the main determinants of these capital values and job values are future returns, in line with what has been found in the Finance literature for asset prices. The third is that U.S. labor market experience, including the rise in unemployment in the Great Recession, can be depicted in a stylized way using the estimated model.

¹⁶As can be seen on the LHS of (40).

This paper, intentionally, did not specify a full DSGE model. This was done in order to focus on firms' investment and hiring decisions and not let the analysis be affected by possible mis-specifications or problematics in other parts of the macroeconomy. To account for firm investment and hiring behavior, one does not need to get into issues such as optimal intertemporal consumption and labor choices of the individual, with all the associated empirical difficulties. However this precludes the analysis of structural shocks. In current research, Faccini and Yashiv (2015) take up such a model in an attempt to map the linkages between the structural shocks to the economy and the differential evolution of the relevant present values.

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Table 1

Descriptive Sample Statistics

Quarterly, U.S. data

a. 1976:1-2013:4 ($n = 152$)

Variable	$\frac{f}{k}$	τ	$\frac{i}{k}$	δ	$\frac{wn}{f}$	$\frac{h^1}{n}$	$\frac{v}{n}$	ψ^1	β
Mean	0.14	0.38	0.024	0.02	0.62	0.126	0.031	0.125	0.99
Standard Deviation	0.01	0.05	0.003	0.003	0.02	0.010	0.007	0.010	0.005

b. 1994:1-2013:4 ($n = 80$)

Variable	$\frac{f}{k}$	τ	$\frac{i}{k}$	δ	$\frac{wn}{f}$	$\frac{h}{n} = \frac{h^1+h^2}{n}$	$\frac{v}{n}$	$\psi = \psi^1 + \psi^2$
Mean	0.15	0.34	0.026	0.02	0.61	0.178	0.028	0.178
Standard Deviation	0.01	0.005	0.002	0.002	0.03	0.012	0.005	0.012

	β
Mean	0.99
Standard Deviation	0.005

Table 2a
GMM estimates
Current Model

	specification	e_1	e_2	e_{31}	e_{32}	α	J-Statistic
1	benchmark	77.3 (6.29)	9.1 (0.98)	-2.8 (1.2)	-19.6 (0.9)	0.66 (0.003)	51.6 (0.74)
2	cubic $\eta_1 = \eta_2 = 3$	124.7 (69.5)	101.5 (10.2)	2.6 (1.8)	-3.6 (0.8)	0.62 (0.005)	57.7 (0.52)
3	constrained case $\lambda_2 = e_{32} = 0; \lambda_1 = 1$ 1976 – 2013	30.8 (6.3)	1.96 (0.29)	-1.3 (0.9)	0 –	0.65 –	83.6 (0.31)

Table 2b
GMM estimates, Standard Specifications

	specification	e_1	e_2	e_{31}	e_{32}	α	J-Statistic
1	Tobin's q for K	49.4 (2.6)	0 –	0 –	0 –	0.63 (0.0009)	60.0 (0.51)
2	Tobin's q for N	0 –	30.8 (0.9)	0 –	0 –	0.70 (0.003)	61.9 (0.48)
3	Standard matching model $\eta_2 = 1, \lambda_1 = \lambda_2 = 0$	0 –	9.3 (0.1)	0 –	0 –	0.77 (0.002)	62.5 (0.46)

Notes:

1. The tables report point estimates with standard errors in parentheses. The J-statistic is reported with p value in parentheses.
2. The following parameter values are set unless indicated otherwise:
 $\lambda_1 = 0.6; \lambda_2 = 0.2; \eta_1 = \eta_2 = 2, \eta_{31} = \eta_{32} = 1.$

3. The instrument set for both equations is $\left\{ \frac{w_t}{f_t}, \frac{i_t}{k_t}, p_t^I \right\}$; for the investment equation also $\frac{f_{t+1}}{k_{t+1}}$ is used; and for the vacancies equation also $\frac{h_t}{n_t}$ is used; all with lags 1 to 6, 8 and 10.

4. The sample period is 1994:1 – 2013:4, except for Row 3 in Table 2a where it is 1976:1-2013:4.

Table 3a
Costs Implied by the GMM Estimation Results
Current Model

	specification	$\frac{g}{f}$		$\frac{g_i}{p^I}$		$\frac{g_v}{qw}$	
		mean	std.	mean	std.	mean	std.
1	benchmark	0.02	0.004	0.05	0.01	0.30	0.08
2	cubic	0.03	0.002	0.02	0.003	0.68	0.05
	$\eta_1 = \eta_2 = 3$						
3	constrained case	0.02	0.002	0.04	0.01	0.35	0.03
	$\lambda_2 = e_{32} = 0; \lambda_1 = 1$						
	1976 – 2013						

Table 3b
Costs Implied by the GMM Estimation Results
Standard Specifications

	specification	$\frac{g}{f}$		$\frac{g_i}{p^I}$		$\frac{g_v}{qw}$	
		mean	std.	mean	std.	mean	std.
1	Tobin's q for K	0.02	0.003	0.10	0.01	–	–
2	Tobin's q for N	0.12	0.01	–	–	2.24	0.14
3	Standard matching model	0.26	0.01	–	–	2.44	0.37
	$\eta_2 = 1, \lambda_1 = \lambda_2 = 0$						

Notes:

1. Mean and std. refer to sample statistics.
2. The functions were computed using the point estimates in Tables 2 a,b.

Table 4
Decomposition of the Vacancy Rate and Investment Rate
Equations

First Two Moments

$$\frac{v_t}{n_t} = \frac{e_1 q_t \left(\frac{1}{(1-\tau_t)} \frac{Q_t^N}{\frac{f_t}{n_t}} \right) - (e_{31} q_t^1 + e_{32} q_t^2) \left(\frac{1}{(1-\tau_t)} \frac{Q_t^K}{\frac{f_t}{k_t}} - \frac{p_t^I}{\frac{f_t}{k_t}} \right)}{e_1 e_2 \Lambda_t^2 - (e_{31} q_t^1 + e_{32} q_t^2)^2}$$

$$\frac{i_t}{k_t} = \frac{e_2 \Lambda_t^2 \left(\frac{1}{(1-\tau_t)} \frac{Q_t^K}{\frac{f_t}{k_t}} - \frac{p_t^I}{\frac{f_t}{k_t}} \right) - q_t (e_{31} q_t^1 + e_{32} q_t^2) \left(\frac{1}{(1-\tau_t)} \frac{Q_t^N}{\frac{f_t}{n_t}} \right)}{e_1 e_2 \Lambda_t^2 - (e_{31} q_t^1 + e_{32} q_t^2)^2}$$

where:

$$\Lambda_t = [(1 - \lambda_1 - \lambda_2) + \lambda_1 q_t^1 + \lambda_2 q_t^2]$$

a. Vacancy (hiring) Equation

			1		2	
			$\frac{e_1 q_t \left(\frac{1}{(1-\tau_t)} \frac{Q_t^N}{\frac{f_t}{n_t}} \right)}{e_1 e_2 \Lambda_t^2 - (e_{31} q_t^1 + e_{32} q_t^2)^2}$		$\frac{-(e_{31} q_t^1 + e_{32} q_t^2) \left(\frac{1}{(1-\tau_t)} \frac{Q_t^K}{\frac{f_t}{k_t}} - \frac{p_t^I}{\frac{f_t}{k_t}} \right)}{e_1 e_2 \Lambda_t^2 - (e_{31} q_t^1 + e_{32} q_t^2)^2}$	
1	mean	0.028	relative mean	0.74		0.26
2	std	0.005	relative var	0.28		0.38
3			relative cova	0.17		

b. Investment Equation

				1	2	
		$\frac{i_t}{k_t}$				
					$\frac{e_2 \Lambda_t^2 \left(\frac{1}{(1-\tau_t)} \frac{Q_t^K}{\frac{f_t}{k_t}} - \frac{p_t^I}{\frac{f_t}{k_t}} \right)}{e_1 e_2 \Lambda_t^2 - (e_{31} q_t^1 + e_{32} q_t^2)^2}$	$\frac{-q_t (e_{31} q_t^1 + e_{32} q_t^2) \left(\frac{1}{(1-\tau_t)} \frac{Q_t^N}{\frac{f_t}{n_t}} \right)}{e_1 e_2 \Lambda_t^2 - (e_{31} q_t^1 + e_{32} q_t^2)^2}$
1	mean	0.026	relative mean	0.47	0.53	
2	std	0.002	relative var	1.84	0.90	
3			relative cova	-0.87		

Notes:

1. Row 1 reports the mean vacancy or investment rate and the relative means of the two decomposition terms indicated in columns 1 and 2 (which sum to 1).

2. Row 2 reports the std. of the vacancy or investment rate and the relative variances of the two decomposition terms indicated in columns 1 and 2 (i.e., $\frac{\text{var term1}}{\text{total var}}, \frac{\text{var term2}}{\text{total var}}$).

3. Row 3 reports the relative co-variance of the two decomposition terms indicated in columns 1 and 2 (i.e., $\frac{\text{co-var term1,term2}}{\text{total var}}$).

4. All results are based on the point estimates of row 1 in Table 2a.

Table 5
VAR Results

Investment				
	coef.	std.	\bar{R}^2	LR coef.
ϕ_1	0.965	0.025	0.95	
b_{d_p1}	-0.002	0.012	0.93	-0.001
b_{r_p1}	-0.070	0.019	0.23	-1.151
b_{gk_p1}	-0.009	0.023	0.23	-0.154
b_{δ_p1}	0.0002	0.0003	0.999	0.004

Vacancies/Hiring				
	coef.	std.	\bar{R}^2	LR coef.
ϕ_2	0.725	0.102	0.82	
b_{d_p2}	0.409	0.147	0.95	0.244
b_{r_p2}	-0.351	0.089	0.39	-0.726
b_{gn_p2}	0.014	0.013	-0.05	0.029
b_{ψ_p2}	0.001	0.007	0.07	0.001

Notes:

1. The VAR formulation is given in Section 6.3, with full derivation provided in Appendix D.
2. The relevant long run coefficients, for capital are:

$$\begin{aligned}
 b_{gk_p1}^{lr} &= \frac{b_{gk_p1}}{1 - \rho^k \phi_1}; & b_{\delta_p1}^{lr} &= \frac{\rho^k b_{\delta_p1}}{1 - \rho^k \phi_1} \\
 b_{d_p1}^{lr} &= \frac{(1 - \rho^k) b_{d_p1}}{1 - \rho^k \phi_1}; & b_{r_p1}^{lr} &= \frac{b_{r_p1}}{1 - \rho^k \phi_1}
 \end{aligned}$$

For labor:

$$\begin{aligned}
b_{gn_p2}^{lr} &= \frac{b_{gn_p2}}{1 - \rho^n \phi_2}; & b_{\psi_p2}^{lr} &= \frac{\rho^n b_{\psi_p2}}{1 - \rho^n \phi_2} \\
b_{d_p2}^{lr} &= \frac{(1 - \rho^n) b_{d_p2}}{1 - \rho^n \phi_2}; & b_{r_p2}^{lr} &= \frac{b_{r_p2}}{1 - \rho^n \phi_2}
\end{aligned}$$

where ϕ_1 is the AR coefficient on p^1 , ϕ_2 is the AR coefficient on p^2 the $b_{_p1,2}$ are the coefficients w.r.t $p^{1,2}$ and lr denotes the long-run.

Table 6
Variables in the $\frac{u}{n} - \frac{v}{n}$ Analysis of Figures 2 and 3
Sample Averages

	1994 – 2006	2007 – 2013
$\frac{v}{n}$	0.0299	0.0255
$\frac{u}{n}$	0.054	0.083
$\frac{1}{(1-\tau)} \frac{Q^K}{\frac{f}{k}} - \frac{p^I}{\frac{f}{k}}$	0.36	0.72
$\frac{1}{1-\tau} \frac{Q^N}{\frac{f}{n}}$	0.16	0.22
$\frac{1}{(1-\tau)} \frac{Q_{search}^N}{\frac{f}{n}}$	1.485	1.447
ψ^1	0.117	0.120
ψ^2	0.068	0.044
g	0.003	0.0006

Notes:

1. For the full computation methodology of the solution depicted in Figures 2 and 3 see Appendix E.

2. The rate of unemployment $\frac{u}{n}$ here pertains to the official unemployment pool. For the other non-employment pools see Appendix E.

3. The numbers are data averages except for the values of $\frac{1}{(1-\tau)} \frac{Q^K}{\frac{f}{k}} - \frac{p^I}{\frac{f}{k}}$ in the current model and of $\frac{1}{(1-\tau)} \frac{Q_{search}^N}{\frac{f}{n}}$ in the standard model, which are solved out of the steady state equations.

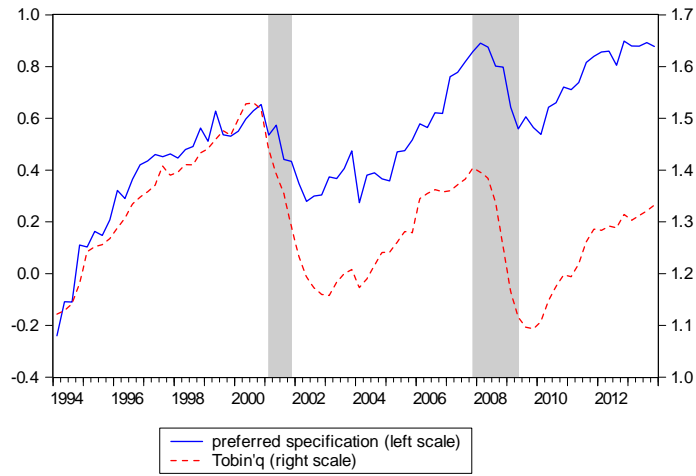


Figure 1a: Marginal Investment Costs/Capital Values ($\frac{g_{i,t}}{\frac{f_t}{k_t}}$) across models
 $\rho = 0.36$

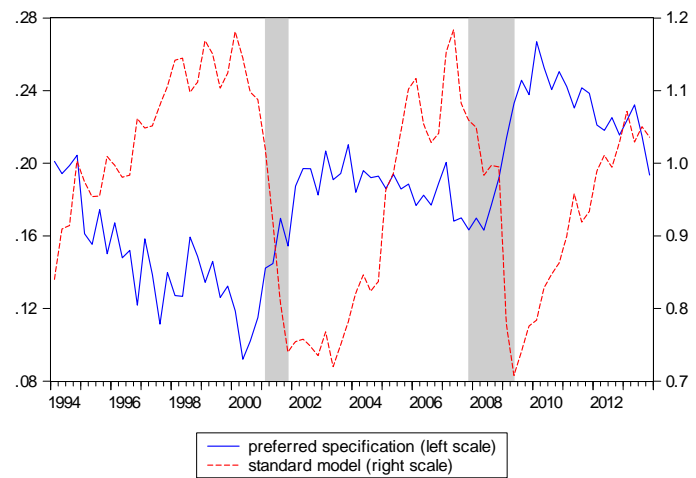
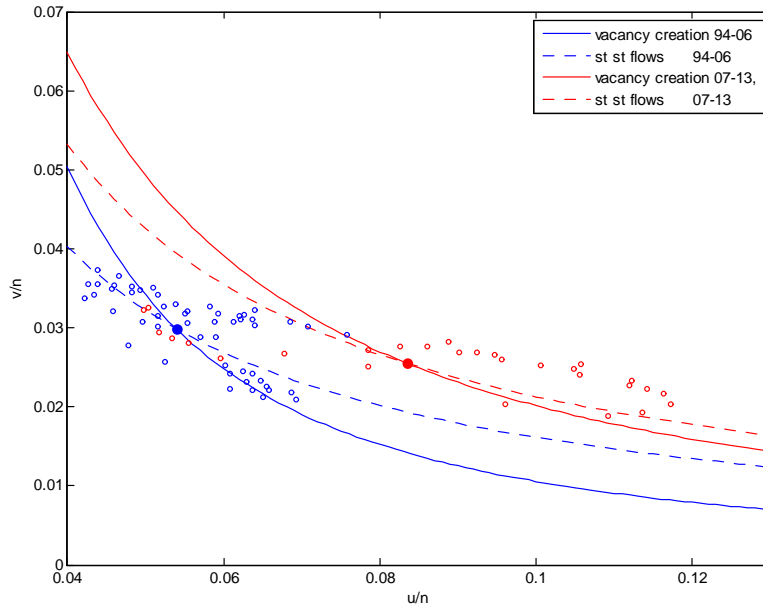


Figure 1b: Marginal Vacancy Costs/Job Values ($\frac{g_{v,t}}{\frac{q_t}{n_t}}$) across models
 $\rho = -0.57$

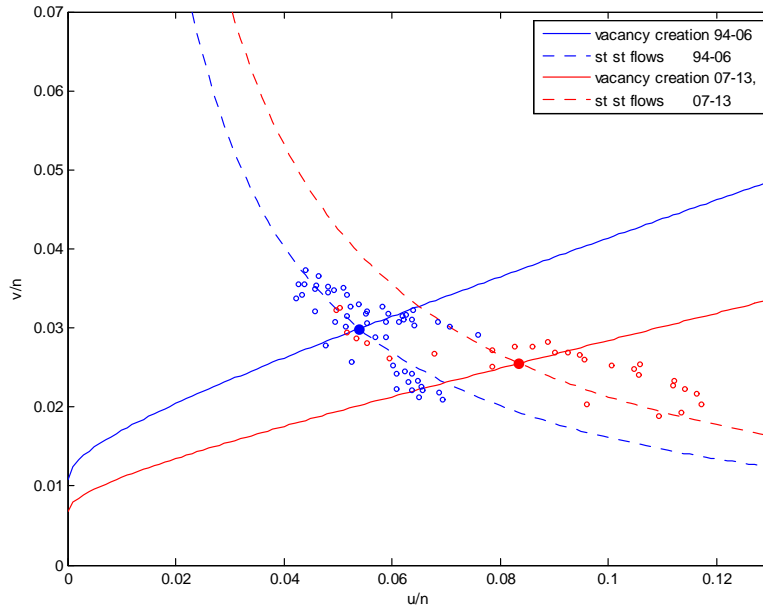
Figure 2
Unemployment-Vacancies Analysis 1994-2013



Notes:

1. The solid lines are the vacancy creation curves as in equation (40) and the dashed lines are the steady state flows curve as in equation (41).
2. The blue lines pertain to the period 1994-2006. The red lines pertain to the period 2007-2013.
3. The circles are the actual data points with the same colors indicating the periods.

Figure 3
Unemployment-Vacancies Analysis of the Standard Search and
Matching Model
1994-2013



Notes: As in Figure 3, except that the vacancy creation curve is given by equation (42).

January 6, 2015

Capital Values and Job Values
by Eran Yashiv
Appendices

1 Appendix A

The Cost Function and its Derivatives

$$g(\cdot) = \left[\begin{array}{c} \frac{e_1}{\eta_1} \left(\frac{i_t}{k_t} \right)^{\eta_1} \\ + \frac{e_2}{\eta_2} \left[\frac{(1-\lambda_1-\lambda_2)v_t + \lambda_1 q_t^1 v_t + \lambda_2 q_t^2 v_t}{n_t} \right]^{\eta_2} \\ + \frac{e_{31}}{\eta_{31}} \left(\frac{i_t}{k_t} \frac{q_t^1 v_t}{n_t} \right)^{\eta_{31}} + \frac{e_{32}}{\eta_{32}} \left(\frac{i_t}{k_t} \frac{q_t^2 v_t}{n_t} \right)^{\eta_{32}} \end{array} \right] f(z_t, n_t, k_t). \quad (1)$$

$$\frac{g_{i_t}}{\frac{f_t}{k_t}} = \left[\begin{array}{c} e_1 \left(\frac{i_t}{k_t} \right)^{\eta_1 - 1} \\ + e_{31} \left(\frac{q_t^1 v_t}{n_t} \right)^{\eta_{31}} \left(\frac{i_t}{k_t} \right)^{\eta_{31} - 1} + e_{32} \left(\frac{q_t^2 v_t}{n_t} \right)^{\eta_{32}} \left(\frac{i_t}{k_t} \right)^{\eta_{32} - 1} \end{array} \right] \quad (2)$$

$$\frac{g_{v_t}}{\frac{f_t}{n_t}} = \left[\begin{array}{c} e_2 \left[\frac{(1-\lambda_1-\lambda_2)v_t + \lambda_1 q_t^1 v_t + \lambda_2 q_t^2 v_t}{n_t} \right]^{\eta_2 - 1} \left[(1 - \lambda_1 - \lambda_2) + \lambda_1 q_t^1 + \lambda_2 q_t^2 \right] \\ + e_{31} q_t^1 \left(\frac{i_t}{k_t} \right)^{\eta_{31}} \left(\frac{q_t^1 v_t}{n_t} \right)^{\eta_{31} - 1} \\ + e_{32} q_t^2 \left(\frac{i_t}{k_t} \right)^{\eta_{32}} \left(\frac{q_t^2 v_t}{n_t} \right)^{\eta_{32} - 1} \end{array} \right] \quad (3)$$

$$\frac{g_{k_t}}{\frac{f_t}{k_t}} = - \left[e_1 \left(\frac{i_t}{k_t} \right)^{\eta_1} + e_{31} \left(\frac{q_t^1 v_t}{n_t} \frac{i_t}{k_t} \right)^{\eta_{31}} + e_{32} \left(\frac{q_t^2 v_t}{n_t} \frac{i_t}{k_t} \right)^{\eta_{32}} \right] \quad (4)$$

$$+ (1 - \alpha) \left[\begin{array}{c} \frac{e_1}{\eta_1} \left(\frac{i_t}{k_t} \right)^{\eta_1} \\ + \frac{e_2}{\eta_2} \left[\frac{(1-\lambda_1-\lambda_2)v_t + \lambda_1 q_t^1 v_t + \lambda_2 q_t^2 v_t}{n_t} \right]^{\eta_2} \\ + \frac{e_{31}}{\eta_{31}} \left(\frac{i_t}{k_t} \frac{q_t^1 v_t}{n_t} \right)^{\eta_{31}} + \frac{e_{32}}{\eta_{32}} \left(\frac{i_t}{k_t} \frac{q_t^2 v_t}{n_t} \right)^{\eta_{32}} \end{array} \right]$$

$$\frac{g_{n_t}}{\frac{f_t}{n_t}} = - \left[\begin{array}{c} e_2 \left[\frac{(1-\lambda_1-\lambda_2)v_t + \lambda_1 q_t^1 v_t + \lambda_2 q_t^2 v_t}{n_t} \right]^{\eta_2} \\ + e_{31} \left(\frac{q_t^1 v_t}{n_t} \frac{i_t}{k_t} \right)^{\eta_{31}} + e_{32} \left(\frac{q_t^2 v_t}{n_t} \frac{i_t}{k_t} \right)^{\eta_{32}} \end{array} \right] \quad (5)$$

$$+ \alpha \left[\begin{array}{c} \frac{e_1}{\eta_1} \left(\frac{i_t}{k_t} \right)^{\eta_1} \\ + \frac{e_2}{\eta_2} \left[\frac{(1-\lambda_1-\lambda_2)v_t + \lambda_1 q_t^1 v_t + \lambda_2 q_t^2 v_t}{n_t} \right]^{\eta_2} \\ + \frac{e_{31}}{\eta_{31}} \left(\frac{i_t}{k_t} \frac{q_t^1 v_t}{n_t} \right)^{\eta_{31}} + \frac{e_{32}}{\eta_{32}} \left(\frac{i_t}{k_t} \frac{q_t^2 v_t}{n_t} \right)^{\eta_{32}} \end{array} \right]$$

2 Appendix B

The Data

variable	symbol	definition
GDP	f	gross value added of NFCB
GDP deflator	p^f	price per unit of gross value added of NFCB
wage share	$\frac{wn}{f}$	numerator: compensation of employees in NFCB
discount rate	r	the rate of non-durable consumption growth minus 1
employment	n	employment in nonfinancial corporate business sector
hiring	h	gross hires
separation rate	ψ	gross separations divided by employment
vacancies	v	adjusted Help Wanted Index
investment	i	gross investment in NFCB sector
capital stock	k	stock of private nonresidential fixed assets in NFCB sector
depreciation	δ	depreciation of the capital stock
price of capital goods	p^I	real price of new capital goods

variable	symbol	source
GDP	f	NIPA accounts, table 1.14, line 41
GDP deflator	p^f	NIPA table 1.15, line 1
wage share	$\frac{wn}{f}$	NIPA; see note 7
discount rate	r	NIPA Table 2.3.3, lines 3, 8, and 13; see note 1
employment	n	CPS; see note 2
hiring	h	CPS; see note 3
separation rate	ψ	CPS; see note 3
vacancies	v	Conference Board; see note 4
investment	i	BEA and Fed Flow of Funds; see note 5
capital stock	k	BEA and Fed Flow of Funds; see note 5
depreciation	δ	BEA and Fed Flow of Funds; see note 5
price of capital goods	p^I	NIPA and U.S. tax foundation; see note 6

The sample period is 1976:2-2013:4 unless noted otherwise; all data are quarterly.

Notes:

1. *The discount rate and the discount factor*

The discount rate is based on a DSGE-type model with logarithmic utility $U(c_t) = \ln c_t$. Define the discount factor as $\rho_t \equiv \frac{1}{1+r_t}$

In this model:

$$U'(c_t) = U'(c_{t+1}) \cdot (1 + r_t) \quad (6)$$

Hence:

$$\rho_t = \frac{c_t}{c_{t+1}} \quad (7)$$

where c is non-durable consumption (goods and services) and 5% of durable consumption.

2. Employment

As a measure of employment in the nonfinancial corporate business sector (n) I take wage and salary workers in non-agricultural industries (series ID LNS12032187) less government workers (series ID LNS12032188), less self-employed workers (series ID LNS12032192). All series originate from CPS databases. I do not subtract workers in private households (the unadjusted series ID LNU02032190) from the above due to lack of sufficient data on this variable.

3. Hiring and Separation Rates

The aggregate flow from non-employment – unemployment (U) and out of the labor force (O) – to employment is to be denoted $OE + UE$ and the separation rate ψ_t is rate of the flow in the opposite direction, $EU + EO$. Worker flows within employment – i.e., job to job flows – are to be denoted EE .

I denote:

$$\begin{aligned} \frac{h}{n} &= \left(\frac{h^1}{n}\right) + \left(\frac{h^2}{n}\right) \\ \frac{h^1}{n} &= \frac{OE + UE}{E} \\ \frac{h^2}{n} &= \frac{EE}{E} \end{aligned} \tag{8}$$

Hence h^1 and h^2 denote flows from non-employment and from other employment, respectively.

Separation rates are given by:

$$\begin{aligned} \psi &= \psi^1 + \psi^2 \\ \psi^1 &= \frac{EO + EU}{E} \\ \psi^2 &= \frac{EE}{E} = \frac{h^2}{n} \end{aligned} \tag{9}$$

Employment dynamics now satisfies:

$$\begin{aligned} n_{t+1} &= (1 - \psi_t^1 - \psi_t^2)n_t + h_t^1 + h_t^2 \\ &= (1 - \psi_t)n_t + h_t, \quad 0 \leq \psi_t \leq 1 \\ h_t^2 &= \psi_t^2 n_t \end{aligned} \tag{10}$$

To calculate hiring and separation rates for the whole economy I use the following:

a. *The h_t^1 and ψ_t^1 flows.* I compute the flows between E (employment), U (unemployment) and O (not-in-the-labor-force) that correspond to the E,U,O

stocks published by the CPS. The methodology of adjusting flows to stocks is taken from BLS, and is presented in Frazis et al (2005).¹The data till 1990:Q1 were kindly provided by Ofer Cornfeld. The data from 1990:Q2 onwards were taken from the CPS (http://www.bls.gov/cps/cps_flows.htm). Employment is the quarterly average of the original seasonally adjusted total employment series from BLS (LNS12000000).

b. *The h_t^2 and ψ_t^2 flows.* The data on EE, available only from 1994:Q1 onward, were computed by multiplying the percentage of people moving from one employer to another using Fallick and Fleischman (2004)'s² data by the NSA population series LNU00000000, taken from the CPS, completing several missing observations and performing seasonal adjustment.

4. *Vacancies*

I use the vacancies series based on the Conference Board Composite Help-Wanted Index that takes into account both printed and web job advertisements, as computed by Barnichon. The updated series is available at

<https://sites.google.com/site/regisbarnichon/research/publications>.

This index was multiplied by a constant to adjust its mean to the mean of the JOLTS vacancies series over the overlapping sample period (2001:Q1–2013:Q4)

5. *Investment, capital and depreciation*

The goal here is to construct the quarterly series for real investment flow i_t , real capital stock k_t , and depreciation rates δ_t . I proceed as follows:

- Construct end-of-year fixed-cost net stock of private nonresidential fixed assets in NFCB sector, K_t . In order to do this I use the quantity index for net stock of fixed assets in NFCB (FAA table 4.2, line 37, BEA) as well as the 2009 current-cost net stock of fixed assets (FAA table 4.1, line 37, BEA).
- Construct annual fixed-cost depreciation of private nonresidential fixed assets in NFCB sector, D_t . The chain-type quantity index for depreciation originates from FAA table 4.5, line 37. The current-cost depreciation estimates (and specifically the 2009 estimate) are given in FAA table 4.4, line 37.
- Calculate the annual fixed-cost investment flow, I_t :

$$I_t = K_t - K_{t-1} + D_t$$

- Calculate implied annual depreciation rate, δ_a :

¹Frazis, Harley J., Edwin L. Robison, Thomas D. Evans and Martha A. Duff, 2005. Estimating Gross Flows Consistent with Stocks in the CPS, **Monthly Labor Review**, September, 3-9.

²Fallick and Fleischman, 2004. "Employer-to-Employer Flows in the U.S. Labor Market: The Complete Picture of Gross Worker Flows," FEDS #2004-34.

$$\delta_a = \frac{I_t - (K_t - K_{t-1})}{K_{t-1} + I_t/2}$$

- Calculate implied quarterly depreciation rate for each year, δ_{qt} :

$$\delta_q + (1 - \delta_q)\delta_q + (1 - \delta_q)^2\delta_q + (1 - \delta_q)^3\delta_q = \delta_a$$

- Take historic-cost quarterly investment in private non-residential fixed assets by NFCB sector from the Flow of Funds accounts, atabs files, series FA105013005).
- Deflate it using the investment price index (the latter is calculated as consumption of fixed capital in domestic NFCB in current dollars (NIPA table 1.14, line 18) divided by consumption of fixed capital in domestic NFCB in chained 2009 dollars (NIPA table 1.14, line 42). This procedure yields the implicit price deflator for depreciation in NFCB. The resulting quarterly series, i_t_unadj , is thus in real terms.
- Perform Denton's procedure to adjust the quarterly series i_t_unadj from the Federal Flow of Funds accounts to the implied annual series from BEA I_t , using the depreciation rate δ_{qt} from above. I use the simplest version of the adjustment procedure, when the discrepancies between the two series are equally spread over the quarters of each year. As a result of adjustment I get the fixed-cost quarterly series i_t .
- Simulate the quarterly real capital stock series k_t starting from k_0 (k_0 is actually the fixed-cost net stock of fixed assets in the end of 1975, this value is taken from the series K_t), using the quarterly depreciation series δ_{qt} and investment series i_t from above:

$$k_{t+1} = k_t \cdot (1 - \delta_{qt}) + i_t$$

6. Real price of new capital goods

In order to compute the real price of new capital goods, p^I , I use the price indices for output and for investment goods.

Investment in NFCB Inv consists of equipment Eq and structures St as well as intellectual property, which I do not include. I define the time- t price-indices for good $j = Eq, St$ as \tilde{p}_t^j . The data are taken from NIPA table 1.1.4, lines 10, 11.

I take from <http://www.federalreserve.gov/econresdata/frbus/us-models-package.htm> the following tax-related rates:

- The parameter τ – the statutory corporate income tax rate as reported by the U.S. Tax Foundation.
- The investment tax credit on equipment and public utility structures, to be denoted ITC .

c. The percentage of the cost of equipment that cannot be depreciated if the firm takes the investment tax credit, denoted χ .

d. The present discounted value of capital depreciation allowances, denoted $ZPDE^{St}$ and $ZPDE^{Eq}$.

I then apply the following equations:

$$\begin{aligned} p^{Eq} &= \tilde{p}^{Eq} (1 - \tau_{Eq}) \\ p^{St} &= \tilde{p}^{St} (1 - \tau_{St}), \end{aligned}$$

$$\begin{aligned} 1 - \tau^{St} &= \frac{(1 - \tau ZPDE^{St})}{1 - \tau} \\ 1 - \tau^{Eq} &= \frac{1 - ITC - \tau ZPDE^{Eq} (1 - \chi ITC)}{1 - \tau} \end{aligned}$$

Subsequently I compute their change between $t-1$ and t (denoted by Δp_t^j):

$$\frac{\Delta p_t^{Inv}}{p_{t-1}^{Inv}} = \omega_t \frac{\Delta p_t^{Eq}}{p_{t-1}^{Eq}} + (1 - \omega_t) \frac{\Delta p_t^{St}}{p_{t-1}^{St}}$$

where

$$\omega_t = \frac{(\text{nominal expenditure share of } Eq \text{ in } Inv)_{t-1} + (\text{nominal expenditure share of } Eq \text{ in } Inv)_t}{2}.$$

The weights ω_t are calculated from the NIPA table 1.1.5, lines 9,11.

I divide the series by the price index for output, p_t^f , to obtain the real price of new capital goods, p^I .

As all of these prices are indices, in estimation I estimate a scaling parameter e^a .

7. Labor share

NIPA table 1.14, line 20 (compensation of employees in NFCB) divided by line 17 in the same table (gross value added in NFCB).

3 Appendix C

Comparison of the Costs Estimates to the Literature

3.1 Vacancy and Hiring Costs

Mortensen and Nagypal (2006, page 30)³ note that “Although there is a consensus that hiring costs are important, there is no authoritative estimate of their magnitude. Still, it is reasonable to assume that in order to recoup hiring costs, the firm needs to employ a worker for at least two to three quarters. When wages are equal to their median level in the standard model ($w = 0.983$), hiring costs of this magnitude correspond to less than a week of wages.” The widely-cited Shimer (2005) paper⁴ calibrates these costs at $\frac{c}{q} = 0.16$ using a linear cost function, which is equivalent to 3.4 weeks of wages. Hagedorn and Manovskii (2008)⁵ decompose this cost into two components: (i) the capital flow cost of posting a vacancy; they compute it to be – in steady state – 47.4 percent of the average weekly labor productivity; (ii) the labor cost of hiring one worker, which, relying on micro-evidence, they compute to be 3 percent to 4.5 percent of quarterly wages of a new hire. The first component would correspond to a figure of 0.037 here; the second component would correspond to a range of 0.02 to 0.03 in the terms used here; together this implies 0.057 to 0.067 in current terms or around 1.1 to 1.3 weeks of wages. Note that the estimates for the preferred specification, i.e., the GMM results reported in row 1 of Tables 2a and 3a, pertain to marginal costs with a convex costs function, while most of the above pertain to average costs, usually with a linear function. The preferred specification here has an estimate of $\frac{q_v}{q}$ which is 0.19 at its sample average and this is the equivalent of almost 4 weeks of wages.

3.2 Investment Costs

The q literature exhibits huge variation across studies over four decades. One finds estimates of marginal costs varying from as low as 0.04 to as high as 60 (in terms of $\frac{f}{k}$). These differences in marginal cost estimates are usually due to differences in the parameter estimates, and not just due to the diversity in the rate of investment used. One can divide the results into three sets: (i) the earlier studies, from the 1980s, suggested high costs, whereby marginal costs range between 3 to 60 in terms of average output per unit of capital and the implied total costs range between 15% to 100% of output; (ii) more recent studies report moderate costs, whereby marginal costs are around 1 in terms of average output per unit of capital and total costs range between 0.5% to 6% of output; (iii) micro-based studies, using cross-sectional or panel data, report

³Mortensen, Dale T. and Eva Nagypal, 2006. “More on Vacancy and Unemployment Fluctuations,” working paper.

⁴Shimer, Robert, 2005. “The Cyclical Behavior of Equilibrium Unemployment and Vacancies,” *American Economic Review*, 95,1, 25-49.

⁵Hagedorn, Marcus, and Iourii Manovskii, 2008. “The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited,” *American Economic Review*, 98, 4, 1692–1706.

low costs, whereby marginal costs are 0.04 to 0.50 of average output per unit of capital and total costs range between 0.1% to 0.2% of output.

The results for the preferred specification, i.e., the GMM results reported in row 1 of Tables 2a and 3a, have marginal costs as a fraction of output per unit of capital ($\frac{g_i}{k}$) estimated at a mean of 0.58. This corresponds to the high part of the third set, of low costs.

4 Appendix D

Derivation and VAR Estimation of the Asset Pricing Model

4.1 Investment in Capital

Define:

$$P_t^1 \equiv (1 - \tau_t) \left(\frac{g_{i_t} + P_t^I}{\frac{f_t}{k_t}} \right) = \frac{Q_t^K}{\frac{f_t}{k_t}} \quad (11)$$

$$D_t^1 = (1 - \tau_t) \frac{(f_{k_t} - g_{k_t})}{\frac{f_t}{k_t}} \quad (12)$$

$$R_t^1 = \frac{\left(1 + g_t^{f/k}\right) [(1 - \delta_t)P_t^1 + D_t^1]}{P_{t-1}^1} \quad (13)$$

Using:

$$G_{t+1}^{f/k} = \frac{\frac{f_{t+1}}{k_{t+1}}}{\frac{f_t}{k_t}}$$

Hence:

$$\begin{aligned} R_t^1 &= \frac{G_t^{f/k} [(1 - \delta_t)P_t^1 + D_t^1]}{P_{t-1}^1} \\ &= G_t^{f/k} \frac{D_t^1 \left(1 + \frac{(1 - \delta_t)P_t^1}{D_t^1}\right)}{P_{t-1}^1} \\ \ln R_t^1 &= \ln \left(G_t^{f/k} \right) \\ &\quad + \ln \left(D_t^1 \left(1 + \frac{(1 - \delta_t)P_t^1}{D_t^1}\right) \right) \\ &\quad - \ln P_{t-1}^1 \end{aligned}$$

Looking into the second term:

$$\begin{aligned} \ln \left(D_t^1 \left(1 + \frac{(1 - \delta_t)P_t^1}{D_t^1}\right) \right) &= \ln D_t^1 + \ln \left(1 + \frac{(1 - \delta_t)P_t^1}{D_t^1}\right) \\ &= \ln D_t^1 + \ln \left(1 + e^{\ln(1 - \delta_t) + p_t^1 - d_t^1}\right) \\ &\cong d_t^1 + c_0 + \rho^k (\ln(1 - \delta_t) + p_t^1 - d_t^1) \end{aligned}$$

where:

$$\rho^k = \frac{\frac{(1-\delta)P^1}{D^1}}{1 + \frac{(1-\delta)P^1}{D^1}}$$

Hence:

$$\ln R_t^1 \cong c_1 + \ln \left(G_t^{f/k} \right) + d_t^1 + c_0 + \rho^k (\ln(1 - \delta_t) + p_t^1 - d_t^1) - p_{t-1}^1$$

So:

$$p_{t-1}^1 \cong c_2 + \ln G_t^{f/k} + \rho^k \ln(1 - \delta_t) + \rho^k p_t^1 + (1 - \rho^k) d_t^1 - r_t^1 \quad (14)$$

4.2 Hiring of Labor

Define:

$$P_t^2 \equiv \frac{(1 - \tau_t) g_{v_t}}{\frac{f_t}{n_t} q_t} \equiv \frac{Q_t^N}{\frac{f_t}{n_t}} \quad (15)$$

$$D_t^2 = (1 - \tau_t) \left(\alpha - \frac{g_{n_t}}{\frac{f_t}{n_t}} - \frac{w_t}{\frac{f_t}{n_t}} \right) \quad (16)$$

$$R_t^2 = \frac{\left(1 + g_t^{f/n} \right) [(1 - \psi_t) P_t^2 + D_t^2]}{P_{t-1}^2} \quad (17)$$

where:

$$G_{t+1}^{f/n} = \frac{\frac{f_{t+1}}{n_{t+1}}}{\frac{f_t}{n_t}}$$

Hence:

$$\begin{aligned} R_t^2 &= \frac{G_t^{f/n} [(1 - \psi_t) P_t^2 + D_t^2]}{P_{t-1}^2} \\ &= \frac{G_t^{f/n} D_t^2 \left[1 + \frac{(1 - \psi_t) P_t^2}{D_t^2} \right]}{P_{t-1}^2} \\ \ln R_t^2 &= \ln G_t^{f/n} \\ &\quad + \ln D_t^2 \\ &\quad + \ln \left[1 + \frac{(1 - \psi_t) P_t^2}{D_t^2} \right] \\ &\quad - \ln P_{t-1}^2 \end{aligned}$$

Looking into the fourth term on the RHS:

$$\begin{aligned} \ln\left(1 + \frac{(1-\psi_t)P_t^2}{D_t^2}\right) &= \ln(1 - e^{\ln((1-\psi_t)+p_t^2-d_t^2)}) \\ &\cong c_3 + \rho^n(\ln(1-\psi_t) + p_t^2 - d_t^2) \end{aligned}$$

where

$$\rho^n = \frac{\frac{(1-\psi)P^2}{D^2}}{1 + \frac{(1-\psi)P^2}{D^2}}$$

Collecting all terms:

$$\ln R_t^2 \cong c_4 + \ln G_t^{f/n} + d_t^2 + c_3 + \rho^n(\ln(1-\psi_t) + p_t^2 - d_t^2) - p_{t-1}^2$$

So:

$$p_{t-1}^2 = c_5 + \ln G_t^{f/n} + \rho^n \ln(1-\psi_t) + \rho^n p_t^2 + (1-\rho^n)d_t^2 - r_t^2 \quad (18)$$

4.3 The VAR

I estimate the following structural VAR:

$$(\mathbf{x}_{t+1}) = A + B\mathbf{x}_t + \varepsilon_t$$

For capital

$$\mathbf{x}_{t+1} = \begin{pmatrix} p_{t+1}^1 \\ d_{t+1}^1 \\ r_{t+1}^1 \\ \ln(G_{t+1}^{f/k}) \\ \ln(1-\delta_{t+1}) \end{pmatrix}$$

The structural restrictions implied by (14):⁶

$$e_1(I - \rho^k B) = ((1-\rho^k)e_2 - e_3 + e_4 + \rho^k e_5) B \quad (19)$$

For labor:

⁶ where
 $e_1 = (1, 0, 0, 0, 0)$
 $e_2 = (0, 1, 0, 0, 0)$
 $e_3 = (0, 0, 1, 0, 0)$
 $e_4 = (0, 0, 0, 1, 0)$
 $e_5 = (0, 0, 0, 0, 1)$

$$\mathbf{x}_{t+1} = \begin{pmatrix} p_{t+1}^2 \\ d_{t+1}^2 \\ r_{t+1}^2 \\ \ln(G_{t+1}^{f/n}) \\ \ln(1 - \psi_{t+1}) \end{pmatrix}$$

The structural restrictions implied by (18) are:⁷

$$e_1(I - \rho^n B) = ((1 - \rho^n)e_2 - e_3 + e_4 + \rho^n e_5) B \quad (20)$$

with similar definitions and where ϕ_2 is the AR coefficient on p^2 .

Following estimation I compute the relevant long run coefficients. For capital:

$$\begin{aligned} b_{gk_p1}^{lr} &= \frac{b_{gk_p1}}{1 - \rho^k \phi_1}; & b_{\delta_p1}^{lr} &= \frac{\rho^k b_{\delta_p1}}{1 - \rho^k \phi_1} \\ b_{d_p1}^{lr} &= \frac{(1 - \rho^k) b_{d_p1}}{1 - \rho^k \phi_1}; & b_{r_p1}^{lr} &= \frac{b_{r_p1}}{1 - \rho^k \phi_1} \end{aligned}$$

where ϕ_1 is the AR coefficient on p^1 , the $b_{_p1}$ are the coefficients w.r.t p^1 and lr denotes the long-run.

For labor:

$$\begin{aligned} b_{gn_p2}^{lr} &= \frac{b_{gn_p2}}{1 - \rho^{n2} \phi_2}; & b_{\psi_p2}^{lr} &= \frac{\rho^{n2} b_{\psi_p2}}{1 - \rho^{n2} \phi_2} \\ b_{d_p2}^{lr} &= \frac{(1 - \rho^n) b_{d_p2}}{1 - \rho^n \phi_2}; & b_{r_p2}^{lr} &= \frac{b_{r_p2}}{1 - \rho^{n2} \phi_2} \end{aligned}$$

with similar definitions and where ϕ_2 is the AR coefficient on p^2 .

⁷where

$$\begin{aligned} e_1 &= (1, 0, 0, 0, 0) \\ e_2 &= (0, 1, 0, 0, 0) \\ e_3 &= (0, 0, 1, 0, 0) \\ e_4 &= (0, 0, 0, 1, 0) \\ e_5 &= (0, 0, 0, 0, 1) \end{aligned}$$

5 Appendix E

Relating the Model to the Data in $\frac{u}{n} - \frac{v}{n}$ Space

5.1 The Data

The unemployment data include the following three alternatives: In one it is the official unemployment pool.⁸ In a second, it is the official unemployment pool plus marginally attached workers; these are defined as persons who want a job, have searched for work during the prior 12 months, and were available to take a job during the reference week, but had not looked for work in the past 4 weeks.⁹ In a third it is the official unemployment pool plus workers who “want a job;” these are workers who are out of the labor force but replied (in the CPS) in the affirmative to the question if they want a job now.¹⁰ Using these variables, and a vacancy series,¹¹ Figures 2 and 3 in the main text plot the data and the model steady state equations in $\frac{u}{n} - \frac{v}{n}$ space for official unemployment. The other two pools are plotted below.

5.2 Construction of Figures 2 and 3

To see how Figure 2 is constructed start off from the equations:

$$\frac{1}{\mu^1 \left(\frac{u}{n}\right)^\sigma + \frac{\psi^2}{n}} \left[\begin{array}{c} e_2 \left[\begin{array}{c} (1 - \lambda_1 - \lambda_2) + \lambda_1 \mu^1 \left(\frac{u}{n}\right)^\sigma \\ + \lambda_2 \left(\frac{\psi^2}{n}\right) \end{array} \right] \frac{v}{n} \\ + \left[\begin{array}{c} e_{31} \mu^1 \left(\frac{u}{n}\right)^\sigma \\ + e_{32} \left(\frac{\psi^2}{n}\right) \end{array} \right] \frac{1}{e_1} \left[\begin{array}{c} \frac{1}{(1-\tau)} \frac{Q^K}{k} - \frac{p^I}{k} \\ - \left[\begin{array}{c} e_{31} \mu^1 \left(\frac{u}{n}\right)^\sigma \\ + e_{32} \left(\frac{\psi^2}{n}\right) \end{array} \right] \frac{v}{n} \end{array} \right] \end{array} \right] = \frac{1}{(1-\tau)} \frac{Q^N}{\frac{f}{n}} \quad (21)$$

$$\mu^1 \left(\frac{v}{n}\right)^{1-\sigma} \left(\frac{u}{n}\right)^\sigma = \psi^1 + g \quad (22)$$

Using (21), the vacancy creation curve, and (22), the steady state flows curve, one solves for $\frac{u}{n}$ and $\frac{v}{n}$ given the steady state values of the variables $\frac{1}{(1-\tau)} \frac{Q^N}{\frac{f}{n}}$, $\frac{1}{(1-\tau)} \frac{Q^K}{k} - \frac{p^I}{k}$, ψ^1 , ψ^2 , g and the parameter values e_1 , e_2 , e_{31} , e_{32} , μ^1 , μ^2 , λ_1 , λ_2 and σ .

⁸BLS code LNS13000000.

⁹This is computed as follows: the rate of official unemployment plus marginally attached (MA) workers is given in BLS series LNS13327708. The MA pool is extracted from the definition of this rate which is

U+MA rate = (U pool + MA pool)/(LF pool + MA pool).

This is done using the pools for U and LF.

¹⁰BLS code LNS15026639.

¹¹See Appendix B for the computation of the vacancy series.

The plots are constructed so that they will pass through the sample means of $\frac{u}{n}$ and $\frac{v}{n}$. In order to do so these sample means are posited in (21) and (22), which are then solved for $\frac{1}{(1-\tau)} \frac{Q^K}{\frac{f}{k}} - \frac{p^I}{\frac{f}{k}}$ and μ^1 , determining the latter values in the analysis.

Parameters and Steady State Values

Based on GMM from Table 2a, row 1

- $\eta_1 = 2$
- $\eta_2 = 2$
- $\eta_{31} = 1$
- $\eta_{32} = 1$
- $e_1 = 77.31$
- $e_2 = 9.07$
- $e_{31} = -2.79$
- $e_{32} = -19.60$
- $\lambda_1 = 0.6$
- $\lambda_2 = 0.2$
- $\alpha = 0.66$

Based on other studies

$\sigma = 0.5$

Sample Average Values

	1994 – 2006	2007 – 2013
$\frac{v}{n}$	0.0299	0.0255
$\frac{u}{n}$	0.054	0.083
$\frac{1}{(1-\tau)} \frac{Q^K}{\frac{f}{k}} - \frac{p^I}{\frac{f}{k}}$	0.36	0.72
$\frac{1}{1-\tau} \frac{Q^N}{\frac{f}{n}}$	0.16	0.22
$\frac{1}{(1-\tau)} \frac{Q_{search}^N}{\frac{f}{n}}$	1.485	1.447
ψ^1	0.117	0.120
ψ^2	0.068	0.044
g	0.003	0.0006

Likewise for the standard model, Figure 3:

$$\frac{c}{\mu^1 \left(\frac{v}{n}\right)^{-\sigma} + \mu^2 (\Phi)^\sigma \left(\frac{v}{n}\right)^{-\sigma}} = \frac{1}{(1-\tau)} \frac{Q_{search}^N}{\frac{f}{n}} \tag{23}$$

$$\mu \left(\frac{v}{n}\right)^{1-\sigma} \left(\frac{u}{n}\right)^\sigma = \psi^1 + g \tag{24}$$

Parameters and Steady State Values for the Standard Model

Based on GMM from Table 2b, row 3

$c = 9.3$

Based on other studies
 $\sigma = 0.5$
Steady State Values
as above

5.3 Additional Tables

Table F-1
Variables in the $\frac{u}{n} - \frac{v}{n}$ Analysis of Figures F-1 and F-2
Sample Averages

$u =$ official unemployment + marginally attached

	1994 – 2006	2007 – 2011
$\frac{v}{n}$	0.0299	0.0255
$\frac{u}{n}$	0.068	0.105
$\frac{1}{(1-\tau)} \frac{Q^K}{\frac{I}{k}} - \frac{p^I}{\frac{I}{k}}$	0.36	0.72
$\frac{1}{1-\tau} \frac{Q^N}{\frac{I}{n}}$	0.16	0.22
$\frac{1}{(1-\tau)} \frac{Q_{search}^N}{\frac{I}{n}}$	1.485	1.447
ψ	0.117	0.120
g	0.003	0.0006

$u =$ official unemployment + “want a job”

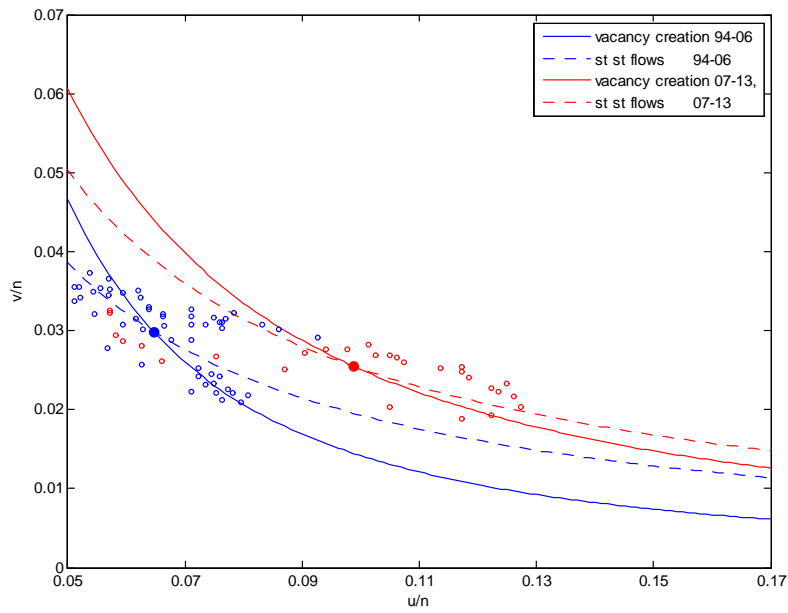
	1994 – 2006	2007 – 2011
$\frac{v}{n}$	0.0299	0.0255
$\frac{u}{n}$	0.096	0.132
$\frac{1}{(1-\tau)} \frac{Q^K}{\frac{I}{k}} - \frac{p^I}{\frac{I}{k}}$	0.36	0.72
$\frac{1}{1-\tau} \frac{Q^N}{\frac{I}{n}}$	0.16	0.22
$\frac{1}{(1-\tau)} \frac{Q_{search}^N}{\frac{I}{n}}$	1.485	1.447
ψ	0.117	0.120
g	0.003	0.0006

Notes:

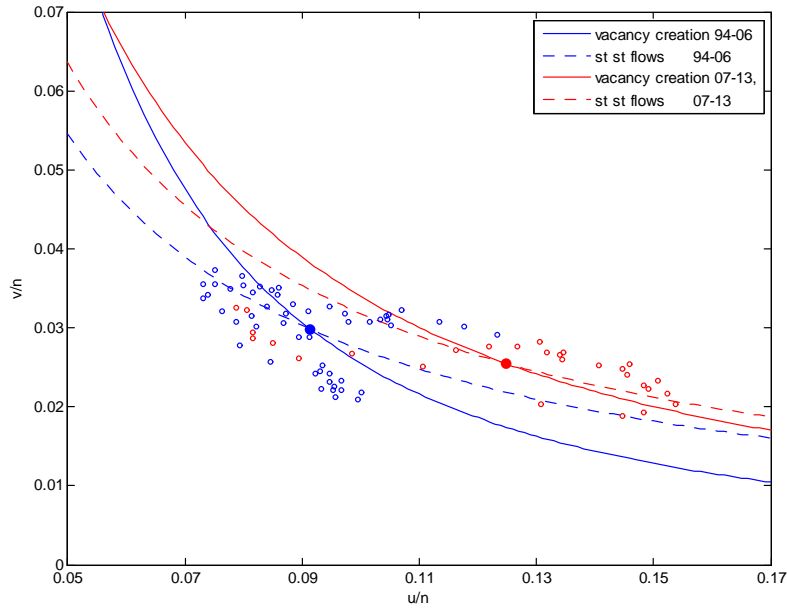
1. The rate of unemployment $\frac{u}{n}$ here pertains to the pools indicated in the titles.
2. The numbers are data averages except for the values of $\frac{1}{(1-\tau)} \frac{Q^K}{\frac{I}{k}} - \frac{p^I}{\frac{I}{k}}$ in the current model and of $\frac{1}{(1-\tau)} \frac{Q_{search}^N}{\frac{I}{n}}$ in the standard model, which are solved out of the steady state equations.

5.4 Additional Figures

Figures F-1
Unemployment-Vacancies Analysis
 $u = \text{official unemployment} + \text{marginally attached}$



$u = \text{official unemployment} + \text{“want a job”}$

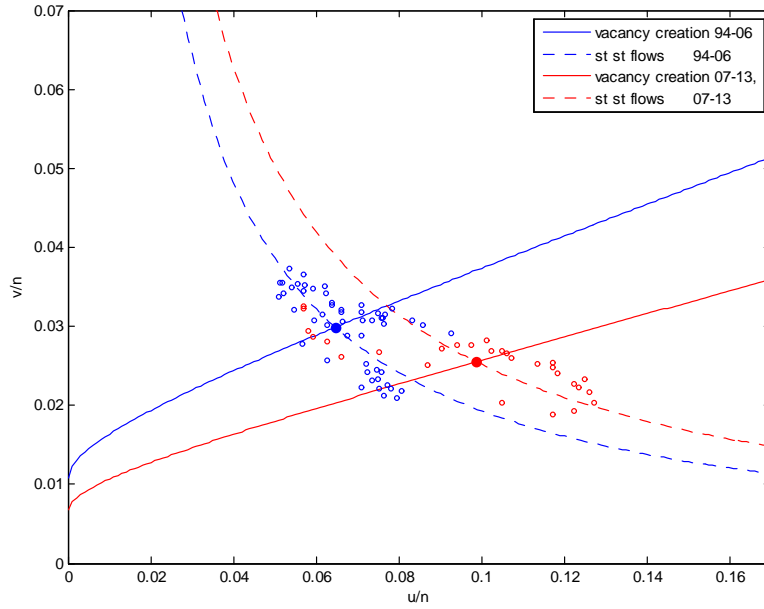


Notes:

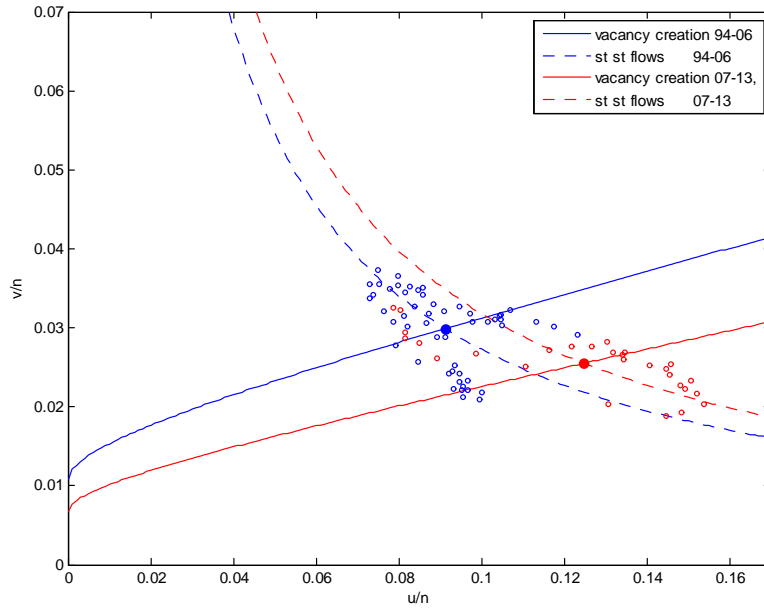
1. The solid lines are the vacancy creation curves as in equation (21) and the dashed lines are the steady state flows curve as in equation (22).
2. The blue lines pertain to the period 1994-2006. The red lines pertain to the period 2007-2013.
3. The circles are the actual data points with the same colors indicating the periods.

Figures F-2
Unemployment-Vacancies Analysis of the standard (Pissarides)
Model

$u = \text{official unemployment} + \text{marginally attached}$



$u = \text{official unemployment} + \text{“want a job”}$



Notes:

1. As in Figures F-2, except that the vacancy creation curve is equation 23) and the dashed line is the steady state flows curve, equation 24).