Abstract

The interaction of incomplete markets and sticky nominal wages is shown to magnify business cycles even though these two features – in isolation – dampen them. During recessions, fears of unemployment stir up precautionary sentiments which induces agents to save more. The additional savings may be used as investments in both a productive asset (equity) and an unproductive asset (money). But even a small rise in money demand has important consequences. The desire to hold money puts deflationary pressure on the economy, which, provided that nominal wages are sticky, increases wage costs and reduces firm profits. Lower profits repress the desire to save in equity, which increases (the fear of) unemployment, and so on. This is a powerful mechanism which causes the model to behave differently from both its complete markets version, and a version with incomplete markets but without aggregate uncertainty. In contrast to previous results in the literature, agents uniformly prefer non-trivial levels of unemployment insurance.

Keywords: Keynesian unemployment, business cycles, search friction, magnification, propagation, heterogeneous agents.

JEL Classification: E12, E24, E32, E41, J64, J65.
1 Introduction

The empirical literature documents that workers suffer substantial losses in both earnings and consumption levels during unemployment. For instance, Kolsrud, Landais, Nilsson, and Spinnewijn (2015) use Swedish data to document that consumption expenditures drop on average by 32% during the first year of an unemployment spell. This observed inability to insure against unemployment spells has motivated several researchers to develop business cycle models with a focus on incomplete markets. The hope (and expectation) has been that such models would not only generate more realistic behavior for individual variables, but also be able to generate volatile and prolonged business cycles without relying on large and persistent exogenous shocks. While in existing models, individual consumption is indeed much more volatile than aggregate consumption, aggregate variables are often not substantially more volatile than their counterparts in the corresponding complete markets (or representative-agent) version. Krusell, Mukoyama, and Sahin (2010), for instance, find that imperfect risk sharing does not help in generating more volatile business cycles. McKay and Reis (2013) document that a decrease in unemployment benefits—which exacerbates market incompleteness—actually decreases the volatility of aggregate consumption. The reason is that an increase in unemployment benefits reduces precautionary savings, investment, the capital stock, and ultimately makes the economy as a whole less well equipped to smooth consumption.

We develop a model in which the inability to insure against unemployment risk generates business cycles which are much more volatile than the corresponding complete markets version. Moreover, although the only aggregate exogenous shock has a small standard deviation, the outcome of key exercises such as changes in unemployment benefits depends crucially on whether there is aggregate uncertainty. This result is obtained by combining incomplete asset markets with incomplete adjustments of the nominal wage rate to changes in the price level. The impact of shocks is prolonged by Diamond-Mortensen-Pissarides search frictions in the labor market.

Before explaining why the combination of incomplete markets and sticky nominal wages amplifies business cycles, we first explain why these features by themselves dampen business cycles in our model in which aggregate fluctuations are caused by productivity shocks. First, consider the model in which there are complete markets, but nominal wages

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1Section 3.2 provides a more detailed discussion of the empirical literature investigating the behavior of individual consumption during unemployment spells.

2As discussed in section 7.1, a decrease in unemployment benefits does increase the volatility of output in the model of McKay and Reis (2013), but the effects are small relative to our results.

3We discuss the empirical motivation for these assumptions in section 3.
do not respond one-for-one to price level changes. A negative productivity shock operates like a negative supply shock which puts upward pressure on the price level. Provided that nominal wages are sticky, the resulting downward pressure on real wages mitigates the reduction in profits caused by the direct negative effect of a decline in productivity. The result is a muted aggregate downturn. Next, consider a model in which nominal wages are flexible, but workers cannot fully insure themselves against unemployment risk. Forward-looking agents understand that a persistent negative productivity shock increases the risk of being unemployed in the near future. If workers are not fully insured against this risk, the desire to save increases for precautionary reasons. However, increased savings leads to an increase in demand for all assets, including productive assets such as firm ownership. This counteracting effect alleviates the initial reduction in demand for productive assets, induced by the direct negative effect of a reduced productivity level and, therefore, dampens the increase in unemployment. In either case, sticky nominal wages or incomplete markets lead – in isolation – to a muted business cycle.

Why does the combination of incomplete markets and sticky nominal wages lead to the opposite results? As before, the increased probability of being unemployed in the near future increases agents’ desire to save more in all assets. However, the increased desire to hold money puts downward pressure on the price level, which in turn increases real wage costs and reduces profits. This latter effect counters any positive effect that increased precautionary savings might have on the demand for productive investments. Once started, this channel will reinforce itself. That is, if precautionary savings lead – through downward pressure on prices – to increased unemployment, then this will in turn lead to a further increase in precautionary savings, and so on. When does this process come to an end? At some point, the nonlinearities in the matching function, combined with an expanding number of workers searching for a new job, makes it attractive to resume job creating investments.

In addition to endogenizing unemployment, the presence of search frictions in the labor market adds further dynamics to this propagation mechanism. First, the value of a firm – i.e. the price of equity – is forward-looking. As a consequence, a prolonged increase in real wage costs leads to a sharp reduction in economic activity already in the present, with an associated higher risk of unemployment. Second, with low job-finding rates unemployment becomes a slow moving variable. Thus, the increase in unemployment is more persistent than the reduction in productivity itself.

We use our framework to study the advantages of alternative unemployment insurance (UI) policies. We first document that the effects of changes in unemployment benefits on the behavior of aggregate variables and on the well-being of workers differ from the
effects in other models. For example, in the model of Krusell, Mukoyama, and Sahin (2010) most agents benefit from reductions in unemployment benefits even when benefits are reduced to very low levels. We consider a permanent increase in the replacement rate from the benchmark value of 50% of the prevailing wage rate to 55% and document that this increase in insurance improves the welfare of all agents, provided that the policy switch occurs in a recession. This is true even if wage rates adjust upwards to take into account the strengthened bargaining position of workers.\footnote{Wage increases reduce job creation which has negative welfare consequences. Whether all agents prefer the switch during an expansion depends crucially to which extent wages adjust.}

There are a number of factors affecting agents’ welfare that are important for this result. As a preview of the analysis, let us mention some that operate in our model, but have not been previously emphasized in the literature. Consider a permanent increase in unemployment benefits at the onset of a recession. This obviously benefits the unemployed directly. But the employed benefit too. Firstly, they benefit because they are better insured against future unemployment risk. Secondly, by reducing the negative downward spiral discussed above, the employed are now less likely to be unemployed in the near future. Thirdly, and perhaps more surprisingly, employed agents also benefit because the dampening of the downward spiral implies that the value of their asset holdings drops less relative to the case in which unemployment benefits are not increased.

These features contrast with those exposed in the existing literature, in which increased unemployment benefits brings forth adverse aggregate consequences that eclipse the gains of reduced income volatility (e.g. Young (2004) and Krusell, Mukoyama, and Sahin (2010)). In particular, with lower fluctuations in individual income the precautionary motive weakens, and aggregate investment falls. The result is a decline in average employment and output, with adverse effects on welfare. This channel is important in our model as well. In the version of our model with aggregate uncertainty, however, there are two quantitatively important factors that push average employment in the opposite direction, and can overturn the negative effect associated with the reduction in precautionary savings. The first is that the demand for the productive asset can increase, because an increase in the level of unemployment benefits stabilizes business cycles and asset prices. The second is that the nonlinearity in the matching process is such that increases in employment during expansions are smaller than reductions during recessions. Consequently, a reduction in volatility can lead to an increase in average employment (cf. Jung and Kuester (2011)).

An important aspect of our model is that precautionary savings can be used for investments in both the productive asset (firm ownership) and the unproductive asset (money). This complicates the analysis, because the numerical procedure requires a simultaneous
solution for a portfolio choice problem for each agent, and for equilibrium prices. Our numerical analysis ensures that the market for firm ownership (equity) is in equilibrium and all agents owning equity discount future equity returns with the correct, that is, their own individual-specific, intertemporal marginal rate of substitution (MRS). By contrast, a typical set of assumptions in the literature is that workers jointly own the productive asset at equal shares, that these shares cannot be sold, and that discounting of the returns of this asset occurs with some average MRS or an MRS based on aggregate consumption.

One exceptions is Krusell, Mukoyama, and Sahin (2010) who – like us – allow trade in the productive asset and discount agents’ returns on this asset with the correct marginal rate of substitution.

In section 2, we describe the model. In section 3, we provide empirical motivation for the key assumptions underlying our model: sticky nominal wages and workers’ inability to insure against unemployment risk. We also discuss the relationship between savings and idiosyncratic uncertainty. In section 4, we discuss the calibration of our model. In sections 5 and 6, we describe the behavior of individual and aggregate variables, respectively. In section 7, we discuss how business cycle behavior is affected by alternative UI policies.

2 Model

The economy consists of a unit mass of households, a large mass of potential firms, and one government. The mass of active firms is denoted $q_t$, and all firms are identical. Households are ex-ante homogenous, but differ ex-post in terms of their employment status (employed or unemployed) and their asset holdings.

**Notation.** Upper (lower) case variables denote nominal (real) variables. Variables with subscript $i$ are household specific. Variables without a subscript $i$ are either aggregate variables or variables that are identical across agents, such as prices.

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5See section 2.7 for a detailed discussion.


7An alternative simplifying assumption is that the only agents who are allowed to invest in the productive asset are agents that are not affected by idiosyncratic risk (of any kind). Examples are Rudanko (2009), Bils, Chang, and Kim (2011), Challe, Matheron, Ragot, and Rubio-Ramirez (2014), and Challe and Ragot (2014). Bayer, Luettticke, Pham-Dao, and Tjaden (2014) analyze a more challenging problem than ours, in which firms are engaged in intertemporal decision making. However, in contrast to our model, these firms are assumed to be risk neutral, consume their own profits, and discount the future at a constant geometric rate.

8The procedure in Krusell, Mukoyama, and Sahin (2010) is only exact if the aggregate shock can take on as many realizations as there are assets and no agents are at the short-selling constraint. Our procedure does not require such restrictions, which is important, because the fraction of agents at the constraint is nontrivial in our model.
2.1 Households

Each household consists of one worker who is either employed, \( e_{i,t} = 1 \), or unemployed, \( e_{i,t} = 0 \). The period-\( t \) budget constraint of household \( i \) is given by

\[
P_t c_{i,t} + J_t (q_{i,t+1} - (1 - \delta) q_{i,t}) + M_{i,t+1} = (1 - \tau_t) W_t e_{i,t} + \mu (1 - \tau_t) W_t (1 - e_{i,t}) + D_t q_{i,t} + M_{i,t},
\]

where \( c_{i,t} \) denotes consumption of household \( i \), \( P_t \) the price of the consumption good, \( M_{i,t} \) the amount of the liquid asset held at the beginning of period \( t \) (chosen in period \( t - 1 \)), \( W_t \) the nominal wage rate, \( \tau_t \) the tax rate on nominal income, and \( \mu \) the replacement rate. The variable \( q_{i,t} \) denotes the amount of equity held at the beginning of period \( t \). Equity pay out nominal dividends \( D_t \). In each period, a fraction \( \delta \) of all firms go out of business which leads to a corresponding loss in equity.\(^9\) When the term \( q_{i,t+1} - (1 - \delta) q_{i,t} \) is positive, the worker is buying equity, and vice versa. The nominal value of this transaction is equal to \( J_t (q_{i,t+1} - (1 - \delta) q_{i,t}) \), where \( J_t \) denotes the nominal price of equity ex dividend.

Households are not allowed to take short positions in equity, that is

\[
q_{i,t+1} \geq 0.
\]

The household maximizes the objective function\(^10\)

\[
\mathbb{E}_t \left[ \sum_{j=0}^{\infty} \beta^j \left( \frac{c_{i,t+j}^{1-\gamma} - 1}{1 - \gamma} + \chi \left( \frac{M_{i,t+j}}{P_{t+j}} \right)^{1-\zeta} - 1 \right) \right],
\]

subject to constraints (1) and (2).

The first-order conditions are given as

\[
c_{i,t}^{-\gamma} = \beta \mathbb{E}_t \left[ c_{i,t+1}^{-\gamma} \frac{P_t}{P_{t+1}} \right] + \chi \left( \frac{M_{i,t+1}}{P_t} \right)^{-\zeta},
\]

\[
c_{i,t}^{-\gamma} \geq \beta \mathbb{E}_t \left[ c_{i,t+1}^{-\gamma} \left( \frac{D_{t+1} + (1 - \delta) J_{t+1}}{J_t} \right) \frac{P_t}{P_{t+1}} \right],
\]

\[
0 = q_{i,t+1} \left( c_{i,t}^{-\gamma} - \beta \mathbb{E}_t \left[ c_{i,t+1}^{-\gamma} \left( \frac{D_{t+1} + (1 - \delta) J_{t+1}}{J_t} \right) \frac{P_t}{P_{t+1}} \right] \right).
\]

\(^9\)We assume that households hold a diversified portfolio of equity, which means that the portfolio depreciates at rate \( \delta \).

\(^{10}\)If money and consumption enter the utility function additively, then money does not enter the Euler equation of other assets directly, which is consistent with the empirical results in Ireland (2004).
Equation (3) represents the Euler equation with respect to real money balances; equation (4) the Euler equation with respect to equity; and equation (5) captures the complementary slackness condition associated with the short-selling constraint in equation (2).

Telyukova (2013) documents that households hold more liquid assets than they need for buying goods. This is consistent with our model, in which agents do not only hold money to facilitate transactions, but also to insure themselves against unemployment risk. The utility specification implies that agents will always choose a positive value of real money balances. Short positions in the liquid asset would become possible if the argument of the utility function is equal to \( \frac{M_{i,t} + \Phi}{P_t} \) with \( \Phi > 0 \) instead of \( \frac{M_{i,t}}{P_t} \). At higher values of \( \Phi \), agents can take larger short positions in money and are, thus, better insured against unemployment risk. Increases in \( \chi \) – while keeping \( \Phi \) equal to zero – have similar implications, since higher values of \( \chi \) imply higher average levels of financial assets.

### 2.2 Active firms

An active firm produces \( z_t \) units of the output good in each period, where \( z_t \) is an exogenous stochastic variable that is identical across firms. The value of \( z_t \) follows a first-order Markov process with a low (recession) and a high (expansion) value. The partition into a recession and an expansion regime simplifies the characterization of the model’s properties.\(^{11}\)

There is one worker attached to each active firm. Thus, the number of active firms, \( q_t \), is equal to the economy-wide employment rate. The nominal wage rate, \( W_t \), is the only cost to the firm. Consequently, nominal firm profits, \( D_t \), are given by

\[
D_t = P_t z_t - W_t. \tag{6}
\]

The nominal wage rate is set according to the rule

\[
W_t = \omega_0 \left( \frac{z_t}{\bar{z}} \right)^{\omega_z} \bar{z} \left( \frac{P_t}{\bar{P}} \right)^{\omega_p} \bar{P}, \tag{7}
\]

where \( \bar{z} \) is the average productivity level, \( P_t \) is the price level, and \( \bar{P} \) is the average price level.\(^{12}\) A key aspect of this paper is on the responsiveness of nominal wages, \( W_t \), to nominal prices, \( P_t \). Therefore, we need a wage setting rule which allows us to vary this

\(^{11}\)Although the model is solvable for richer processes, this simple specification for \( z_t \) helps in keeping the computational burden manageable.

\(^{12}\)The specified wage is always in the worker’s bargaining set, since the wage rate exceeds unemployment benefits, there is no home production nor any disutility from working, and the probability of remaining employed exceeds the probability of finding a job. The parameters are chosen such that the wage rate is never so high that the firm would prefer to fire the worker.
responsiveness. The parameter $\omega_P$ controls how responsive wages are to changes in the price level. If $\omega_P$ is equal to one, for instance, then nominal wages adjust one-for-one to changes in $P_t$. If $\omega_P$ instead is equal to zero, by contrast, nominal wages are entirely unresponsive to changes in $P_t$. The coefficient $\omega_0$ indicates the fraction of output that goes to the worker when $z_t$ and $P_t$ take on their average values, and pins down the steady state value of firm profits in real terms. The coefficient $\omega_z$ indicates the sensitivity of the wage rate to changes in productivity and, thus, controls how wages vary with business cycle conditions. This sensitivity is a key question in the labor search literature. In particular, Hall and Milgrom (2008) argue that the popular Nash bargaining framework renders wages too procyclical by making the relevant reference point the value of being unemployed.  

2.3 Government

The government taxes wages to finance unemployment benefits. Since the level of unemployment benefits is equal to a fixed fraction of the wage rate – and since taxes are proportional to wage income – the government’s budget constraint can be written as

$$\tau_t q_t W_t = (1 - q_t) \mu (1 - \tau_t) W_t.$$  

From this equation, we get an expression for the tax rate, $\tau_t$, which only depends on the employment rate. That is,

$$\tau_t = \mu \frac{1 - q_t}{q_t + \mu (1 - q_t)}.$$  

An increase in $q_t$ means that there is an increase in the tax base and a reduction in the number of unemployed. Both lead to a reduction in the tax rate.

2.4 Firm creation and equity market

Agents that would like to increase their equity position in firm ownership, i.e., agents for whom $q_{i,t+1} - (1 - \delta) q_{i,t} > 0$, can do so by buying equity at the price $J_t$ from agents that would like to sell equity, i.e., from agents for whom $q_{i,t+1} - (1 - \delta) q_{i,t} < 0$. Alternatively, agents who would like to obtain additional equity can also acquire new firms by creating them. How many new firms are created by investing $v_{i,t}$ real units depends on the number of unemployed workers, $u_t$, and the aggregate amount invested, $v_t$. In particular, the

\[13\] Under Nash bargaining, workers’ wages vary with their individual wealth level, which would increase the computational burden. One could question whether this is an empirically relevant feature. Moreover, the results in Krusell, Mukoyama, and Sahin (2010) indicate that this complication has a negligible effect on agents’ wages apart from the very poorest.
aggregate number of new firms created is equal to
\[ h_t \equiv q_t - (1 - \delta) q_{t-1} = \psi v_t^{\eta} u_t^{1-\eta} \] (10)

and an individual investment of \( v_{i,t} \) results in \( h_t / v_t \) new firms. In equilibrium, the cost of creating one new firm, \( v_t / h_t \), has to be equal to the real market price, \( J_t / P_t \), since new firms are identical to existing firms. Setting \( v_t / h_t \) equal to \( J_t / P_t \) and using equation (10) gives
\[ v_t = \left( \frac{\psi J_t}{P_t} \right)^{1/(1-\eta)} u_t. \] (11)

Thus, investment in new firms/jobs is increasing in \( J_t / P_t \) and increasing in the mass of workers looking for a job, \( u_t \).

Equilibrium in the equity market requires that the supply of equity is equal to the demand of equity. That is,
\[
  h_t + \int_{i \in A_-} ((1 - \delta) q_i - q(e_i, q_i, M_i; s_t)) \, dF_t(e_i, q_i, M_i)
  - \int_{i \in A_+} (q(e_i, q_i, M_i; s_t) - (1 - \delta) q_i) \, dF_t(e_i, q_i, M_i),
\] (12)

with
\[
  A_- = \{ i : q(e_i, q_i, M_i; s_t) - (1 - \delta) q_i \leq 0 \},
  A_+ = \{ i : q(e_i, q_i, M_i; s_t) - (1 - \delta) q_i \geq 0 \},
\]

and where \( F_t(e_i, q_i, M_i) \) denotes the cross-sectional cumulative distribution function in period \( t \) of the three individual state variables: the employment state, \( e_i \), money holdings, \( M_i \), and equity holdings, \( q_i \). The variable \( s_t \) denotes the set of aggregate state variables and its elements are discussed in Section 2.6.

Combining the last three equations gives
\[
  \psi^{1/(1-\eta)} \left( \frac{J_t}{P_t} \right)^{\eta/(1-\eta)} u_t = \int_{i \in A} (q(e_i, q_i, M_i; s_t) - (1 - \delta) q_i) \, dF_t(e_i, q_i, M_i),
\] (13)

with \( A = \{ A_+ \cup A_- \} \). In appendix B.2, we discuss how our algorithm ensures that this equilibrium condition always holds.

Our representation of the “matching market” looks somewhat different than usual. As documented in appendix C, however, it is equivalent to the standard search-and-matching
setup. Our way of “telling the story” has two advantages. First, there is only one type of investor, namely the household. That is, we do not have entrepreneurs who fulfill a crucial arbitrage role in the standard setup, but attach no value to their existence or activities pursued. Second, all agents in our economy have access to the same two assets; firm ownership and money. By contrast, households and entrepreneurs have different investment opportunities in the standard setup.\textsuperscript{14}

2.5 Money market

Equilibrium in the market for money holdings requires that the net demand of households wanting to increase their money holdings is equal to the net supply of households wanting to decrease their money holdings. That is,

\[
\int_{i \in B_-} (M_i - M(e_i, q_i, M_i; s_i)) \, dF_i (e_i, q_i, M_i) = \int_{i \in B_+} (M(e_i, q_i, M_i; s) - M_i) \, dF_i (e_i, q_i, M_i), \tag{14}
\]

with

\[
B_- = \{ i : M(e_i, q_i, M_i; s_i) - M_i \leq 0 \},
\]

\[
B_+ = \{ i : M(e_i, q_i, M_i; s) - M_i \geq 0 \}.
\]

Money supply, $M$, is constant in the benchmark economy. In section 7.2, we describe how liquidity injections would affect model outcomes and whether central banks are likely to pursue such policies.

2.6 Equilibrium and model solution

In equilibrium, the following conditions hold: (i) asset demand is determined by the households’ optimality conditions, (ii) the cost of creating a new firm equals the market price of an existing firm, (iii) the demand for equity from households that want to buy equity equals the creation of new firms plus the supply of equity from households that

\textsuperscript{14}There is one other minor difference. In our formulation, there is no parameter for the cost of posting a vacancy and there is no variable representing the number of vacancies. Our version only contains the product, i.e., the total amount invested in creating new firms. In the standard setup, the vacancy posting cost parameter is not identified unless one has data on vacancies. The reason is that different combinations of this parameter and the scalings coefficient of the matching function can generate the exact same model outcomes as long as vacancies are not taken into consideration.
want to sell equity, (iv) the demand for the liquid assets from households that want to increase their holdings is equal to the supply from households that want to reduce their holdings, and (v) the government’s budget constraint is satisfied.

The state variables for agent $i$ are individual asset holdings, employment status, and the aggregate state variables. The latter consist of the aggregate productivity level, $z_t$, and the cross-sectional joint distribution of employment status and asset holdings, $F_t$. We use an algorithm similar to the one used in Krusell and Smith (1998) to solve for the laws of motion of aggregate variables. Details on the numerical procedure are given in appendix B.2.

### 2.7 Discounting firm profits correctly with heterogeneous ownership

With incomplete markets and heterogeneous firm ownership, the question arises how to discount future firm profits. In our model, each and every firm owner discounts firm profits as indicated by the agent’s individual optimality condition. The reason is that agents can buy and sell equity. This means that the Euler equation for equity is satisfied with equality for all investors holding equity, which implies that all firm owners discount the proceeds of the equity investment with the correct, i.e., their own, individual-specific, MRS.\(^{15}\) Our numerical algorithm ensures that market prices and quantities are such that the equilibrium conditions as well as each agent’s Euler equations are satisfied.

In our model, all agents can choose to invest in the risky productive asset and in the less risky and unproductive asset. Previous research studying precautionary savings and idiosyncratic risk often assumes that agents can only trade in the unproductive asset. The productive asset is then subject to some form of communal ownership, with fixed ownership shares that can never be sold no matter how keen an agent would be to do so.\(^{16}\) Aggregate investment decisions in the productive asset are then determined by an “Euler equation” using an MRS based either on aggregate consumption; on an average of the marginal rate of substitution of all agents; or on risk neutral geometric discounting. Another approach is to assume that there exist two distinct types of agents: One type of agent faces idiosyncratic risk but cannot invest in the productive asset; the other who

\(^{15}\)Krusell, Mukoyama, and Sahin (2010) also describe a procedure to discount firm profits (almost) correctly. They assume that the number of assets is equal to the number of realizations of the aggregate shock. Firm profits can then be discounted with the prices of the two corresponding contingent claims and this would be exactly correct if borrowing or short-sell constraints are not binding for any investor. Our procedure allows investors to be constrained and the number of realizations of the aggregate shock can exceed the number of assets.

can invest in the productive asset, but is not affected by idiosyncratic risk. Since there is no ex-post heterogeneity within the group of the latter type, their analysis lends itself to a representative agent, which then dictates the aggregate investment decisions in the economy.\(^{17}\) Both approaches simplify the analysis considerably, but both direct any possible consequences of precautionary savings induced by idiosyncratic risk towards the unproductive asset only, which limits our understanding of the effect of idiosyncratic risk on business cycles.

A long outstanding and unresolved debate in corporate finance deals with firm decision making when owners are heterogeneous and markets are incomplete. This is not an issue here since active firms do not take any decisions beyond that of having entered the market.\(^{18}\) If firms had to make such decisions, we would have to deal with this challenging issue and specify how firm decisions are made.\(^{19}\) Conditional on this specification, however, our approach can still be used and firm owners would still discount firm profits correctly.

### 3 Empirical motivation for key model components

In this section, we discuss some key empirical observations that motivate our analysis. First, we discuss the evidence in favor of sticky nominal wages and whether that has or has not affected wage costs during the recent economic downturn. Second, we discuss the inability of individuals to insure themselves against unemployment spells. Lastly, we discuss whether savings respond to an increase in idiosyncratic uncertainty. The discussion mainly highlights the behavior of key Eurozone variables during the recent financial crisis, although we will also discuss evidence from other periods and countries outside the Eurozone. Details on the data sources are given in appendix A.

\(^{17}\)Examples are Rudanko (2009), Bils, Chang, and Kim (2011), McKay and Reis (2013), Challe, Matheron, Ragot, and Rubio-Ramirez (2014), and Challe and Ragot (2014).

\(^{18}\)Note that firm creation is a static decision and all agents in the economy would compare the cost of creating one firm, \(v_t/h_t\), and its market value, \(J_t/P_t\), in the same way.

\(^{19}\)The analysis would be complicated even if the firms’ objective function is given and all firms have the same objective. For example, suppose that all firms maximize their current market value. Identical firms could then very well end up making different decisions. To see why, suppose that all firms make the same intertemporal decision. By deviating and providing different future payoff realizations, a firm can create value by “completing the market”. There are, however, some special cases for which this analysis is tractable. As discussed in Ekern and Wilson (1974), if firms decisions do not alter the set of returns available to the whole economy, then investors can “undo” the effects of firm decisions on the payoffs of their individual portfolio. Consequently, investors would agree on what choices the firm should make. Carceles-Poveda and Coen-Pirani (2009) show that this happens in their model in which firms have constant return to scale technology and there are no binding borrowing constraints.
3.1 Deflationary pressure and sticky nominal wages

In our heterogeneous-agent model, precautionary savings put upward pressure on the demand for money, which in turn puts downward pressure on prices. If nominal wages do not fully respond to changes in prices, then this puts upward pressure on real unit wage costs during recessions.

There are four elements to this story. First, there is downward pressure on prices. Second, nominal wages do not fully adjust to changes in the price level. Third, real unit wage costs increase, that is, upward pressure on real wages is not offset by increases in labor productivity. Fourth, the increase in wage costs is also relevant for new jobs. These elements are discussed next.

Deflationary pressure. Our paper focuses on recessions during which households’ inability to fully insure themselves against increased idiosyncratic risk increases households’ desire to save, which puts downward pressure on prices. The top panel of figure 1 shows the GDP deflator for the Eurozone alongside its pre-crisis trend. The figure shows that the growth in the price level slowed considerably during the crisis relative to the trend.

Nominal wage stickiness and inflation. There are many papers that document that nominal wages are sticky. However, what is important for our paper is the question of to which extent nominal wages adjust to aggregate shocks and, in particular, to changes in the aggregate price level. Druant, Fabiani, Kezdi, Lamo, Martins, and Sabbatini (2009) provide survey evidence for a sample of European firms with a focus on the wages of the firms’ main occupational groups; these would not change for reasons such as promotion. Another attractive feature of this study is that it explicitly investigates whether nominal wages adjust to inflation or not. In their survey, only 29.7% of Eurozone firms indicate that they have an internal policy of taking inflation into account when setting wages, and only

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20 Our story does not require prices to be procyclical. That is, the channel we identify is also present when the precautionary motive only dampens countercyclical behavior.

21 Our model has ambiguous predictions for the cyclicality of real wages. If nominal wages respond little to lower inflation and little to lower productivity, then it is possible that the real wage rate increases in response to a negative shock. In our benchmark model, real wages initially increase following a negative productivity shock, but then start to decrease and fall below previous levels two periods after the shock.

22 The pre-crisis trend is defined as the time path the deflators would have followed if inflation beyond the fourth quarter of 2007 had been equal to the average inflation rate over the five preceding years.

23 Remarkable deflationary pressure is also visible in the US consumer price index (CPI), which dropped by 3.4% during the period from September 2008 to December 2008.

24 See, for example, Dickens, Goette, Groshen, Holden, Messina, Schweitzer, Turunen, and Ward (2007), Druant, Fabiani, Kezdi, Lamo, Martins, and Sabbatini (2009), Barattieri, Basu, and Gottschalk (2010), Daly, Hobijn, and Lucking (2012), and Daly and Hobijn (2013).
half of these firms do so by using automatic indexation. Moreover, most firms that take inflation into account are backward looking. Both findings imply that real wages increase (or decrease by less) when inflation rates fall.

![Graph showing price level and trend from 2000 to 2013](image)

**Figure 1:** Key Eurozone variables before and after the financial crisis.

**Notes.** Panel (a) illustrates the Eurozone GDP deflator together with its pre-crisis trend. Panel (b) illustrates nominal hourly earnings, the GDP deflator, and their associated pre-crisis trends. Panel (c) illustrates nominal unit labor costs together with the GDP deflator. Source: OECD.

Papers that document nominal wage rigidity typically highlight the importance of *downward* nominal wage rigidity. Suppose there is downward, but no upward nominal wage rigidity. Does this imply that all nominal wages respond fully to changes in aggregate prices as long as aggregate prices increase? The answer is no. The reason is that firms are
heterogeneous and a fraction of firms can still be constrained by the inability to adjust nominal wages downward. In fact, downward nominal wage rigidity is supported by the empirical finding that the distribution of firms’ nominal wage changes has a large mass-point at zero. The fraction of firms that is affected by this constraint would increase if the aggregate price level increases by less. In fact, Daly, Hobijn, and Lucking (2012) document that the fraction of US workers with a constant nominal wage increased from 11.2% in 2007 to 16% in 2011, whereas the fraction of workers facing a reduction in nominal wages was roughly unchanged. This indicates that there is upward pressure on real wages when the inflation rate falls, even if it remains positive and nominal wages are only rigid downward.

To investigate whether nominal wages followed the slowdown in inflation, the second panel of figure 1 displays nominal hourly earnings together with the GDP deflator. The panel also shows the realizations of both variables if they would have grown at rates equal to their pre-crisis trends. We find that nominal wages continued to grow at pre-crisis rates or above, despite a substantial reduction in inflation rates. This means that real wages increased relative to trend.

**Real wage costs.** The observed increases in real wages are not necessarily due to a combination of low inflation and downward nominal wage rigidity. It is possible that solid real wage growth reflects an increase in labor productivity, for example, because workers that are laid off are less productive than those that are not. To shed light on this possibility, we compare the nominal unit wage cost with the price level. The results are shown in the bottom panel of figure 1. The panel shows that nominal unit labor costs have grown faster than prices since the onset of the crisis, whereas the opposite was true before the crisis. This indicates that real labor costs increased during the crisis even if one corrects for productivity.

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26 Similarly, at http://nadaesgratis.es/?p=39350, Marcel Jansen documents that from 2008 to 2013 there was a massive increase in the fraction of Spanish workers with no change in the nominal wage. There is some increase in the fraction of workers with a decrease in the nominal wage, but this increase is small relative to the increase in the spike of the histogram at constant nominal wages.

27 Similarly, Daly, Hobijn, and Lucking (2012) and Daly and Hobijn (2013) document that real wages increased during the recent recession in the US.

28 The nominal unit wage cost is defined as the cost of producing one unit of output, i.e., the nominal wage rate divided by labor productivity. The price index used as comparison is the price index used in defining labor productivity.

29 The observation that real unit labor costs are not constant over the business cycle is interesting in itself. If the real wage rate is equal to the marginal product of capital and the marginal product is proportional to average labor productivity – properties that hold in several business cycle models – then real unit labor costs
These observations are consistent with the hypothesis that the combination of deflationary pressure and nominal wage stickiness increased wage costs. In principle, it is still possible that nominal wages in the Eurozone did respond fully to prices. However, in that case, it must be true that the reduction in employment is mainly due to an outflow of workers that earn low wages and could produce at low real unit labor cost, since both real wages and real unit labor costs increased. That is, it must be the case that the workers who left employment were the ones who had a wage that was low relative to their productivity. This does not seem plausible.

Wages of new and existing relationships. What matters in labor market matching models is the flexibility of wages of newly hired workers. Haefke, Sonntag, and van Rens (2013) argue that wages of new hires respond almost one-to-one to changes in labor productivity. Gertler, Huckfeldt, and Trigari (2014), however, argue that this result reflects changes in the composition of new hires and that – after correcting for such composition effects – the wages of new hires are not more cyclical than wages of existing employees. More importantly, however, what matters for our paper is whether nominal wages respond to changes in the price level, and this question is not addressed in either paper. As mentioned above, Druant, Fabiani, Kezdi, Lamo, Martins, and Sabbatini (2009) find that many firms do not adjust wages to inflation. One would think that their results apply to new as well as old matches, since their survey evidence focuses on the firms’ main occupational groups.

3.2 Inability to insure against unemployment risk

An important feature of our model is that workers are poorly insured against unemployment risk. That is, that consumption decreases considerably following a displacement. Using Swedish data, Kolsrud, Landais, Nilsson, and Spinnewijn (2015) document that expenditures on consumption goods drop sharply during the first year of an unemployment spell, after which they settle down at 34% below the pre-displacement level. This sharp fall is remarkable given that Sweden has a quite generous unemployment benefits program. As will be discussed in section 4, one reason is that the amount of assets workers hold at the start of an unemployment spell is low. Another reason is that average borrowing actually decreases during observed unemployment spells.

Using US data Stephens Jr. (2004), Saporta-Eksten (2014), Aguiar and Hurst (2005), Chodorow-Reich and Karabarbounis (2015) provide empirical support for substantial drops in consumption follow job loss, even when expenditures on durables are not in-
Using Canadian survey data, Browning and Crossley (2001) find that workers that have been unemployed for six months report that their total consumption expenditures level during the last month is 14% below consumption in the month before unemployment.

3.3 Savings and idiosyncratic uncertainty

The idea that idiosyncratic uncertainty plays an important role in the savings decisions of individuals has a rich history in the economics literature. From a theoretical point of view Kimball (1992) shows that idiosyncratic uncertainty increases savings when the third-order derivative of the utility function with respect to consumption is positive and/or the agent faces borrowing constraints. Moreover, idiosyncratic uncertainty regarding unemployment is more important in recessions which are characterized by a prolonged downturn and an increase in the average duration of unemployment spells. Krueger, Cramer, and Cho (2014) document that during the recent recession the number of long-term unemployed increased in Canada, France, Italy, Sweden, the UK, and the US. They only case in which they found a decrease is Germany. The results are particularly striking for the US. During the recent recession, the amount of workers who were out of work for more than half a year relative to all unemployed workers reached a peak of 45%, whereas the highest peak observed in previous recessions was about 25%.

Several papers have provided empirical support for the hypothesis that increases in idiosyncratic uncertainty increases savings. Using 1992-98 data from the British Household Panel Survey (BHPS), Benito (2004) finds that an individual whose level of idiosyncratic uncertainty would move from the bottom to the top of the cross-sectional distribution reduces consumption by 11%. An interesting aspect of this study is that the result holds both for a measure of idiosyncratic uncertainty based on an individuals’ own perceptions as well as on an econometric specification. Further empirical evidence for this relationship during the recent downturn can be found in Alan, Crossley, and Low (2012). They

30Using the four 1992-1996 waves of the Health and Retirement Study (HRS), Stephens Jr. (2004) finds that annual food consumption is 16% lower when a worker reports that he is no longer working for the employer of the previous wave either because of a layoff, business closure, or business relocation, that is, the worker was displaced between two waves. Similar results are found using the Panel Study of Income Dynamics (PSID). Using the 1999-2009 biannual waves of the PSID, Saporta-Eksten (2014) finds that job loss leads to a drop in total consumption of 17%. About half of this loss occurs before job loss and the other half around job loss. The drop before job loss suggests that either the worker anticipated the layoff or labor income was already under pressure. Moreover, this drop in consumption is very persistent and is only slightly less than 17% six years after displacement. Using data for food and services, Chodorow-Reich and Karabarounis (2015) find that the consumption level of workers that are unemployed for a full year is 21% below the consumption level of employed workers. Using scanner data for food consumption, Aguiar and Hurst (2005) report a drop of 19%.

31Although the sign is correct, the results based on individuals’ own perceptions are not significant.
argue that the observed sharp rise in the savings ratio of the UK private sector is driven by increases in uncertainty, rather than other explanations such as tightening of credit standards. In line with the mechanism emphasized in this paper, Carroll (1992) argues that employment uncertainty is especially important because unemployment spells are the reason for the most drastic fluctuations in household income. In addition, Carroll (1992) provides empirical evidence to support the view that the fear of unemployment leads to an increased desire to save even when controlling for expected income growth.

4 Calibration

This section starts with a discussion of the parameter values that play a key role in generating the results, followed by a discussion of the remaining and less crucial parameter values. The model period is one quarter. Targets are constructed using Eurozone data from 1980 to 2012.\(^{32}\) We focus on the Eurozone for two reasons. First, our empirical results for the Eurozone indicate that inflation slowed down during the crisis and nominal unit wage costs did not.\(^{33}\) Second, many economists have warned of the risks of a deflationary spiral in the Eurozone.\(^{34}\)

**Key parameter values.** Regarding the choice of key parameter values, our strategy is to show that our main results can be generated with conservative choices. For example, we set the coefficient of relative risk aversion equal to 2. Even though risk aversion is not that high, the differences between our heterogeneous-agent model and the representative-agent version are substantial.

The incidence and duration of unemployment spells are obviously important. The probability of job destruction, \(\delta\), and the parameter characterizing efficiency in the matching market, \(\psi\), are chosen to ensure that the unemployment rate and the expected duration of an unemployment spell in the economy without aggregate risk match their observed counterparts, which are equal to 10.7\% and 3.57 quarters, respectively.\(^{35}\) These numbers

32 Average unemployment duration data are based on all of Europe, since no Eurozone data is available for this time period. Details about data sources are given in appendix A.

33 We found that this is not the case for the US even though – similar to Eurozone developments – real wages did increase sharply during the crisis.

34 According to the May 2014 survey of the Centre for Macroeconomics, half of the macroeconomists in the panel thought that there was a significant risk of sustained negative inflation in the coming two years. Details can be found at http://cfmsurvey.org/surveys/euro-area-deflation-and-risk-uk-economy-may-2014. For our story, inflation does not have to be negative. It is sufficient if deflationary pressure lowers inflation, which increases real wage costs when nominal wages are sticky.

35 Finding the right parameter values requires solving the model numerous times, which is very computer intensive for the model with aggregate uncertainty. For that reason, we calibrate these parameters by
suggest a 3.36% quarterly job separation rate and a value for $\psi$ equal to 0.574, implying a quarterly matching probability of 28%.

Unemployment insurance regimes vary a lot across countries in Europe. Esser, Ferrarini, Nelson, Palme, and Sjöberg (2013) report that net replacement rates for insured workers vary from 20% in Malta to just above 90% in Portugal. Most countries have net replacement rates between 50% and 70% with an average duration of around one year. Coverage ratios vary from about 50% in Italy to 100% in Finland, Ireland, and Greece. Net replacement rates for workers that are not covered are much lower. In most countries, these are less than 40%. In the model, unemployment benefits are set equal to 50% and – for computational convenience – are assumed to last for the entire duration of the unemployment spell. A replacement rate of 50% is possibly a bit less than the average observed, but this is compensated for by the longer duration of unemployment benefits in the model.

The inability to fully insure against unemployment risk plays a key role. It is, therefore, important that the model generates a realistic drop in consumption during an unemployment spell. While data for the Eurozone is unavailable for this purpose, Kolsrud, Landais, Nilsson, and Spinnewijn (2015) provide evidence for Sweden. They report that consumption drops on average by 34% during the first year of an unemployment spell. A key parameter to target this number is the scale parameter, $\chi$, which characterizes the liquidity benefits of money. This parameter affects the average level of financial assets held and, thus, the ability of agents to insure against unemployment spells. The literature also provides some evidence on pre-displacement wealth levels. Gruber (2001) provides evidence for the US. In section 5, we will show that our calibration is conservative. That is, we generate the targeted consumption drop without making agents unrealistically poor.

The main focus of this paper is on the interaction between sticky nominal wages and the deflationary pressure arising from uncertain job prospects. Consequently, a key role is played by $\omega_P$, the parameter that indicates how responsive nominal wages are to changes in the price level. Our benchmark value for $\omega_P$ is equal to 0.7, which means that a 1% increase in the price level leads to an 0.7% increase in nominal wages. As mentioned before, Druant, Fabiani, Kezdi, Lamo, Martins, and Sabbatini (2009) report that only 6% of European firms adjust wages (of their main occupational groups) more than once a year to inflation and only 50% do so once a year.

\[ 36 \]
\[ 37 \]

matching the targets in the model without aggregate uncertainty. The corresponding statistics in the model with aggregate uncertainty are somewhat different; the average unemployment rate is equal to 11.7% and the average duration is equal to 4.03 quarters.

36Its calibrated value is equal to $4.00e - 5$.

37Moreover, even if firms adjust for inflation they typically do so using backward looking measures of inflation, which reduces the responsiveness to changes in inflationary pressure.
Finally, the curvature parameter in the utility component for liquidity services, $\zeta$, plays an important role, because it directly affects the impact of changes in future job security on the demand for the liquid asset. With more curvature, the demand for the liquid asset is less sensitive and increased concerns about future job prospects will generate less deflationary pressure. We follow Lucas (2000), and target a money demand elasticity with respect to the nominal interest rate equal to $-0.5$. The resulting value of $\zeta$ is equal to 2.38

**Other parameter values.** Based on the empirical estimates in Petrongolo and Pissarides (2001), the elasticity of the job finding rate with respect to tightness, $\eta$, is set equal to 0.5. The average share of the surplus received by workers, $\omega_0$, and the elasticity of the wage rate with respect to changes in aggregate productivity, $\omega_z$, are set such that the standard deviation of employment relative to the standard deviation of output are in line with their empirical counterpart.39

In our model, the presence of idiosyncratic risk lowers average real rates of return. This motivates us to set the discount factor, $\beta$, to 0.985, which is slightly below its usual value of 0.99.40 The two values for $z_t$ are 0.978 and 1.023 and the probability of switching is equal to 0.025.41 Finally, money supply, $M$, is chosen such that the average price level, $P$, is equal to 1.

**Parameters values in the representative-agent model.** We will compare the results of our model with those generated by the corresponding representative-agent economy. Parameter values in the representative-agent model are identical to those in the

\[ 1 = \beta (1 + R_t) \beta \mathbb{E}_t \left[ \left( \frac{c_{i,t+1}}{c_{i,t}} \right)^{-\gamma} \frac{P_t}{P_{t+1}} \right] . \]

Using $(1 + R_t)^{-1} \approx 1 - R_t$, we get

\[ \ln \left( \frac{M_{i,t+1}}{P_t} \right) \approx -\zeta^{-1} \ln R_t + \zeta^{-1} (\ln \chi + \gamma \ln c_{i,t}) . \]

The other key parameter in money demand functions is the elasticity with respect to income, which captures the volume of transactions. Our transactions variable is consumption and the elasticity of money demand with respect to consumption is equal to $\gamma/\zeta$, which equals 1 for our choices for the coefficient of relative risk aversion, $\gamma$, and $\zeta$.39

In the benchmark economy, $\omega_0 = 0.97$ and $\omega_z = 0.3$. Without our deflationary mechanism, one would have to choose a higher value for $\omega_0$ and/or a lower value for $\omega_z$ to generate the same amount of volatility in employment as pointed out in Hagedorn and Manovskii (2008).

At this relatively high 6% annual discount rate, the average real rate of return is already quite low, namely 0.72% on an annual basis in the economy with aggregate uncertainty.40

These values are chosen to ensure that $\mathbb{E}[\ln z_t] = 0$, $\mathbb{E}_t[\ln z_{t+1}] = 0.95 \ln z_t$, and $\mathbb{E}_t[(\ln z_{t+1} - \mathbb{E}_t[\ln z_{t+1}])^2] = 0.007^2$.
heterogeneous-agent model, except for $\beta$. Using the model without aggregate shocks, we choose the value of $\beta$ for the representative-agent model such that the MRS in the representative-agent model is equal to the MRS for the agents holding equity in the heterogeneous-agent model.\footnote{Notice that the expected intertemporal marginal rate of substitution, $\beta E_t \left[ (c_{i,t+1}/c_{i,t})^{-\gamma} \right]$, is constant and the same for all agents holding equity in the model without aggregate shocks.} Without this adjustment, the agent in the representative-agent economy would have a more shortsighted investment horizon and average employment would be lower.

5 Agents’ consumption, investment and portfolio decisions

In section 5.1, we describe key aspects of the behavior of individual consumption, and in particular its behavior during an unemployment spell. In section 5.2, we focus on the individuals’ investment portfolio decisions.

5.1 Post-displacement consumption

In this section, we first discuss the reduction of individual consumption following displacement, and then turn to the factors that are behind the substantial drop generated by the model.

Magnitude of the post-displacement drop in consumption. Figure 2 displays the average post-displacement change in consumption. The model’s parameters are calibrated such that the one-year drop equals its empirical equivalent, that is 34%. Although not targeted, the model predicts a proportional decrease over the first year similar to what is observed in the data.\footnote{See Kolsrud, Landais, Nilsson, and Spinnewijn (2015).} However, whereas the data indicate that the fall in consumption settles down after one year, the model suggests that this happens first after two years. Nevertheless, figure 3 documents the distribution of the duration of unemployment and shows that most unemployment spells do not exceed one year.\footnote{81% of all unemployment spells that start in an expansion last at most four quarters. The corresponding number for recessions is 61%.}

There are several reasons why the drop in consumption is of such a nontrivial magnitude. One reason is, of course, that unemployment benefits are only half as big as labor income. But a key factor affecting the magnitude of the drop is the average level of wealth at the beginning of an unemployment spell. Using US data, Gruber (2001) finds that the median agent holds enough gross financial assets to cover 73% of the average net-income
loss during an unemployment spell. Moreover, in terms of net financial assets, the median

Figure 2: Evolution of consumption drop over the unemployment spell.

*Notes.* The black line illustrates the average path of consumption of an individual that becomes unemployed in period 1, conditional on being in an expansion at the time of displacement. The grey line illustrates the equivalent path conditional on being in a recession.

Figure 3: Distribution of the unemployed.

*Notes.* The black bars measure the fraction of unemployed at various durations conditional on being in an expansion at the time of displacement. The grey bars provide the corresponding measure conditional on being in a recession.
agent does not even have enough to cover 10% of the average net-income loss.\footnote{Gross financial assets would be the relevant measure if debt can be rolled over during an unemployment spell. Net financial assets would be the relevant measure if that is not the case. Kolsrud, Landais, Nilsson, and Spinnewijn (2015) find that average debt decreases during unemployment spells, which means that the observed gross measure clearly overestimates the amount of funds agents have to cover income losses.} One would expect that agents accumulate less savings in Europe where unemployment benefits are higher.\footnote{In contrast to Gruber (2001), Kolsrud, Landais, Nilsson, and Spinnewijn (2015) do not provide pre-displacement wealth levels as a function of expected earnings losses. But some information is available. In particular, using an average unemployment spell duration of 4 months, the median Swedish agent’s level of gross financial assets is equal to roughly 13% of average net-income loss. In our model, calibrated to an average level of European unemployment benefits, net income drops by more (by half as opposed to one third in Sweden), but agents that become unemployed are wealthier.} In our model, the median agent’s asset holdings are equal to 54% of the average net-income loss during unemployment spells. This is true for both gross and net asset holdings, since there is no debt in our model. Thus, relative to these observed levels, the median agent is less wealthy in terms of gross financial assets, but a lot wealthier in terms of net financial assets.

Figure 4 displays the complete cumulative distribution function of the value of assets at the beginning of an unemployment spell relative to the average net-income loss. Agents in the bottom of the wealth distribution are substantially richer than their real world counterparts, even if we focus on gross assets. For example, Gruber (2001) documents that 38% of all workers do not have enough assets to cover 25% of the average net-income loss. In our model, the corresponding fraction is only 7%. Thus, it is not the case that we rely on unrealistically low wealth levels to generate the sizeable fall in post-displacement consumption observed in the data.

The question arises why infinitely-lived agents do not build a wealth buffer that insulates them better against this consumption volatility, as is the case in the model of Krusell and Smith (1998).\footnote{For the model of Krusell and Smith (1998), solved in Den Haan and Rendahl (2010) using a 15% unemployment replacement rate, the average post-displacement consumption drop is 5% after one year, whereas we find a 34% drop with a 50% unemployment replacement rate.} The key parameter used to match the observed decline in post-displacement consumption is the scaling coefficient affecting the utility of money, $\chi$. Choosing a low value for $\chi$ to match the observed decline in consumption implies that money holdings – one of the two wealth components – are, on average, lower. The other wealth component is the value of equity holdings, $J_t q_i t$. Elevated uncertainty about future individual consumption increases the expected value of an agent’s MRS, which would increase the price of equity, $J_t$. As a consequence, the number of new firms as well as the total number of shares outstanding would therefore rise. However, there are several reasons why this component of wealth is not very large in our model. First, the equity price, $J_t$, cannot increase by too much, because the presence of a liquid asset with a positive
transactions benefit puts a lower bound on the average real return on equity. Moreover, the nonlinearity of the matching function dampens the impact of an increase in equity prices on the creation of new firms. Lastly, the equity price depends positively on the average share of output going to firm owners, $1 - \omega_0$. To generate sufficient volatility in employment we chose a relatively high value for $\omega_0$, which reduces the value of $J_t$.\textsuperscript{48} For all these reasons, agents in our model do not build up large buffers of real money balances or equity to insure themselves against the large declines in consumption upon and during unemployment.

![Figure 4: Financial assets at the beginning of an unemployment spell.](image)

Another aspect affecting consumption during unemployment is the ability to borrow. In our model, agents cannot go short in any asset, and they would presumably hold less financial assets if they had the option to borrow. Kolsrud, Landais, Nilsson, and Spinnewijn (2015) report, however, that the amount of consumption that is financed out of an increase in debt actually decreases following a displacement. More importantly, we think that the key feature to capture is the level of the drop in consumption upon and during

\textsuperscript{48}A small modification of the model would decrease the value of output going to workers and have only marginal effects on the main mechanism. In particular, suppose that output is increased from $z_t$ to $(1 + \Lambda)z_t$ with $\Lambda > 0$. Also, suppose that operating a firm depends on using an additional factor, say land. This factor is owned by another agent for whom this is the only source of income. If bargaining is such that this factor receives $\Lambda z_t$ each period, then our numerical results are exactly the same except that total GDP is now equal to $(1 + \Lambda)z_t q_t$. This modification increases output by $\Lambda\%$, but has a much bigger impact on the net present value of the additional non-labor income stream. Consequently, with a small value for $\Lambda$ the model would be more realistic in terms of having more wealthy agents while leaving the other properties of the model largely unchanged.
unemployment, and not whether this is accomplished by limited borrowing or by some borrowing combined with a lower level of financial assets.

**State dependence of consumption drop.** Figure 5 presents a scatter plot of the reduction in consumption (y-axis) and beginning-of-period cash on hand (x-axis), where both are measured in the period when the agent becomes unemployed.\(^{49}\) There are two distinct patterns, one for expansions and one for recessions.\(^{50}\)

![Figure 5: Consumption drop upon becoming unemployed.](image)

Consistent with figure 2, figure 5 documents that the drop in consumption is, on average, *much* more severe if the unemployment spell initiates in a recession. The figure also underscores the non-trivial role played by the agents' wealth levels. In particular, during recessions the decline in consumption varies from 18.9% for the richest agent to 35.1% for the poorest. This range increases during an expansion: The richest agent faces a modest drop of 8.8%, whereas the poorest agent can expect to see consumption fall by 33.9%. The latter is only slightly below the equivalent number in a recession.

There are several reasons why consumption falls by more during recessions. First, job finding rates are on average lower in recessions than in expansions. As a consequence,\(^{49}\) cash on hand is equal to the sum of non-asset income (here unemployment benefits), money balances, dividends, and the value of equity holdings.\(^{50}\) The level of employment is also important for the observed decline in consumption, which explains the scatter of observations. In particular, the fall in the level of consumption is smaller at the beginning of an expansion and larger at the beginning of a recession. The reason is that expected investment returns are higher (lower) at the beginning of the expansion (recession), which would put upward (downward) pressure on consumption when the income effect dominates the substitution effect.

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\(^{49}\)Cash on hand is equal to the sum of non-asset income (here unemployment benefits), money balances, dividends, and the value of equity holdings.

\(^{50}\)The level of employment is also important for the observed decline in consumption, which explains the scatter of observations. In particular, the fall in the level of consumption is smaller at the beginning of an expansion and larger at the beginning of a recession. The reason is that expected investment returns are higher (lower) at the beginning of the expansion (recession), which would put upward (downward) pressure on consumption when the income effect dominates the substitution effect.
agents anticipate longer unemployment spells and will, for a given amount of cash on hand, therefore reduce consumption more sharply. A second factor that affects agents’ reduced consumption is the amount of cash on hand they hold. Because the price of equity declines in recessions, so does the agents’ cash on hand. Indeed, the average value of cash on hand held by a newly unemployed agent is equal to 1.26 in a recession compared to 1.68 in an expansion.

In reality, a typical worker may not face such a large decline in the value of their equity position when the economy enters a recession. After all, quite a few workers do not own equity at all. We think, however, that the cyclicality of post-displacement consumption behavior that is driven by the cyclicality of equity prices, capture real world phenomena. First, although not all workers hold equity, many hold assets such as housing that also have volatile and cyclical prices. Second, unemployed workers may receive handouts, and or loans, from affluent family members, friends, or financial intermediaries whose ability and willingness may be affected by the value of their assets, which is likely to be cyclical.

5.2 Investment decisions

A key aspect of our heterogeneous-agent model is that money demand increases during recessions, whereas it decreases in the representative-agent version. In this section, we shed light on this difference. In particular, we first discuss how portfolio shares vary with agents’ wealth levels, employment status, and the business cycle. We then turn to the behavior of money demand along the same dimensions.

Portfolio composition and cash-on-hand levels. Figure 6 presents a scatter plot of the liquid asset’s share in the agents’ investment portfolios (y-axis) and the beginning-of-period cash-on-hand levels (x-axis). Although the pattern is somewhat intricate, the figure can be characterized reasonably well as follows. First, the fraction invested in money is higher at lower cash-on-hand levels. Second, conditional on the cash-on-hand level, this fraction also increases when an agent becomes unemployed. Third, conditional on the cash-on-hand level and employment status, this fraction increases when the economy enters a recession. These three properties imply that the portfolio share invested in money increases during a recession. Without large enough increases in money portfolio shares, aggregate demand for money would decrease during recessions, like it does in the representative-agent model. This is because the total amount of funds carried over into the next period decreases during recessions, which in turn implies that the value of agents’ portfolios is lower.
Which forces explain the observed patterns? The first is that the transaction benefits of money are subject to diminishing returns. As a consequence, agents whose total demand for financial assets is high tend to invest a smaller fraction in money. This explains why the fraction invested in money is generally lower for agents with higher cash-on-hand levels. The second driving force is that money is less risky than equity. Therefore, agents whose total demand for financial assets is high relative to their non-asset income invest a larger fraction in money. For a given cash-on-hand level, this explains why the fraction invested in money increases when a worker becomes unemployed, and why the fraction increases when the economy enters a recession. The observed patterns are intricate because these two features push the money portfolio share in different directions and the outcome depends on the relative strengths of each force.

**Money demand and cash on hand.** Figure 7 presents a scatter plot of the demand for real money balances ($y$-axis) and beginning-of-period cash-on-hand levels ($x$-axis). There are four distinct patterns depending on the stance of the business cycle (expansion or

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51 Notice that for a given cash-on-hand level unemployed agents are more asset rich than employed agents, and therefore demand more money. The Euler equation for money, equation (3), implies a monotone positive relationship between the level of money demand and the level of consumption when there is no aggregate uncertainty, because the expected marginal rate of substitution is constant in that case. This implies that the unemployed hold a lower amount of money balances.
recession) and the agent’s employment status. As discussed above, almost all unemployed workers hold larger shares of their portfolio in the liquid asset than employed workers, and both types of workers typically hold a larger share in the liquid asset during recessions than during expansions. Figure 7 shows that both properties are also true when we consider the amount of real money balances as opposed to its share in the portfolio. The figure also illustrates that – everything else equal – the demand for real money balances increases with beginning-of-period cash on hand.

![Figure 7: Money demand (real).](image)

**Notes.** This figure displays the amount invested in the liquid asset as a function of beginning-of-period cash on hand for workers of the indicated employment status and for both outcomes of aggregate productivity.

A key result of this paper is that the interaction between sticky nominal wages and the inability to insure against unemployment risk deepens recessions. An integral part of the mechanism underlying this result is the upward pressure on money demand that emerges when job prospects worsen. When the economy enters a recession, aggregate money demand is pushed in opposite directions by different factors. In particular, during recessions aggregate cash on hand falls, since equity prices fall. This reduces demand for all assets, including real money balances. Because of incomplete markets, however, there are two further reasons that explain why aggregate demand for money increases in our economy. As documented in figure 7, for given cash-on-hand levels, all agents demand more money during recessions. Lastly, unemployed agents demand more money.

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52 Whereas the observations are *typically* true when the *share* invested in the liquid asset is considered, the observations are *always* true when the *level* of money demand is considered.
than employed agents, and there are more unemployed agents in the economy during recessions. The last two effects dominate the first, and aggregate money demand increases during recessions. This result stands in sharp contrast with the representative-agent version of our economy, in which aggregate money demand unambiguously decreases during recessions.

To see that this is a remarkable result, envisage a partial equilibrium version of our model in which the price level, wages, and the equity price are all constant, and there is no short-sale constraint. Markets are still incomplete because the agents cannot insure fully against unemployment risk. Now consider a temporary decrease in the job finding probability. Could this lead to an increase in the demand for real money balances? The answer is no. In this economy, demand for real money balances, $M_{i,t+1}/P_t$, and consumption, $c_{i,t}$, always move in the same direction. In particular, when agents lower consumption in response to a decrease in the job finding probability, money demand will decrease as well. The reason is the following. The Euler equation for equity in (4) – which now holds with equality – implies that the individual MRS is unaffected by the increase in risk. The Euler equation for money in (3) then directly implies that $c_{i,t}$ and $M_{i,t+1}/P_t$ move in the same direction. By contrast, in our model – in which prices adjust to clear markets and the short-sale constraint is binding for some agents – aggregate money demand and aggregate consumption move in opposite directions.

### Financial assets during unemployment spells.

Consumers dampen the drop in consumption following displacement by selling financial assets. Figures 8 and 9 document what this means for equity and money holdings, respectively. Although the total amount of financial assets, and the amount invested in equity, sharply decrease, the amount held in the liquid asset actually increases during the first two periods of an unemployment spell. The loss of wage income means that workers’ cash-on-hand levels drop when they become unemployed. This reduces the demand for real money balances. However, and as discussed above, the unemployed actually hold more money for a given level of cash on hand. Figure 9 documents that the last effect dominates in the beginning of an unemployment spell.

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53In this case, equation (4) can be rearranged as

$$J_t = \beta \mathbb{E}_t \left[ \left( \frac{c_{i,t+1}}{c_{i,t}} \right)^{-\gamma} \right] (D + (1 - \delta)J_{t+1}),$$

which implies that the individual MRS, $\beta \mathbb{E}_t \left[ (c_{i,t+1}/c_{i,t})^{-\gamma} \right]$, is pinned down by aggregate prices only, and is therefore unaffected by risk.
Figure 8: Post displacement equity holdings.

Notes. The black line illustrates the average path for equity holdings of an individual that becomes unemployed in period 1, conditional on being in an expansion at the time of displacement. The grey line illustrates the equivalent path conditional being in a recession.

Figure 9: Post displacement money holdings.

Notes. The black line illustrates the average path for money holdings of an individual that becomes unemployed in period 1, conditional on being in an expansion at the time of displacement. The grey line illustrates the equivalent path conditional on being in a recession.
6 Economic aggregates over the business cycle

In the previous section, we showed that the inability of agents to insure against unemployment risk means that workers face a sharp drop in consumption when they become unemployed. We also discussed how imperfect insurance affects money demand in ways that are not present in an economy with complete markets. In this section, we discuss what this implies for aggregate activity. In particular, we document and explain why the interactions between sticky nominal wages, gloomy outlooks regarding future employment prospects, and the inability to insure against unemployment risk can deepen recessions. We first compare the business cycle properties of the benchmark economy with imperfect risk sharing and sticky nominal wages, i.e. $\omega_p < 1$, to those of an economy with full risk sharing. Subsequently, we discuss the same comparison when nominal wages are not sticky, i.e., $\omega_p = 1$.

6.1 The role of imperfect insurance when nominal wages are sticky

Figure 10 shows the impulse response functions (IRFs) of key aggregate variables to a negative productivity shock for our benchmark economy and for the corresponding representative-agent economy. The two graphs in the top row of the figure display the responses for output and employment. These two graphs document that the economy with incomplete risk sharing faces a much deeper recession than the economy with complete risk sharing. In particular, output drops by 7.2% in the heterogeneous-agent economy and by only 4.3% in the representative-agent economy.

The key aspect in understanding this large difference is the behavior of the price level. In the representative-agent economy, the reduction in real activity decreases the demand for money and increases the price level. In our benchmark calibration wages are sticky ($\omega_p = 0.7$), and a 1% increase in the price level leads to a 0.7% increase in nominal wages and therefore a 0.3% decrease in real wages. Thus, the direct effect of the reduction of productivity, $z_t$, on profits is counteracted, because nominal wages do not fully respond to the associated increase in the price level. That is, our starting point is an economy in which the sluggish response of nominal wages to changes in prices actually dampens the economic downturn.

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54In our benchmark calibration, productivity takes on only two values. The IRFs are calculated as follows. The starting point is period $s$, when productivity takes on its “expansion” value and employment is equal to its mean value conditional on being in an expansion. We then calculate the following two time paths for each variable. The “no-shock” time path is the expected time path from this point onward. The “shock” time path is the expected time path when the productivity switches to the low value in period $s + 1$. The IRF is the difference between these two time paths.
Figure 10: Impulse responses with sticky nominal wages

Notes. These graphs illustrate the difference between the expected time path when the economy is in an expansion in period 0 and the expected time path conditional on being in a recession in period 1. $\omega_p = 0.7$, i.e., nominal wages increases with 0.7% when prices increase with 1%.

By contrast, the price level falls in the heterogeneous-agent economy. This fall is caused by an increase in the aggregate demand for the safer asset, i.e., money. To understand this different outcome, consider again figure 7, which illustrates the relationship between the demand for money as a function of beginning-of-period cash-on-hand levels during expansions and recessions, for both employed and unemployed agents. The reduction in real activity lowers cash-on-hand levels which reduces the demand for money by both employed and unemployed agents. The drop in cash-on-hand levels is substantial because the value of equity declines sharply. Nevertheless, aggregate money demand increases. As
previously discussed, both employed and unemployed agents hold more money during recessions for the same cash-on-hand level. In addition, there are more unemployed agents during recessions, and unemployed agents have larger money holdings than employed at the same level of cash on hand.

Whereas sticky nominal wages reduce the depth of recessions in the representative-agent economy, they worsen recessions in the heterogeneous-agent economy. This is a quantitatively important effect, because a reduction in the price level (for any reason) starts a self-reinforcing process that deepens recessions. In particular, the reduction in the price level puts upward pressure on real wages, which reduces profits. The fall in profits reduces investment in new jobs, which in turn reduces employment.\textsuperscript{55} Since this reduction in employment is persistent, employment prospects worsen. With elevated risk there is a further increase in the demand for money, which in turn puts additional upward pressure on the price level, and so on. The impulse responses show that this mechanism is powerful enough to completely overturn the dampening effect that sticky nominal wages have in an economy with complete risk sharing.

Although this is a powerful mechanism, there is a counterforce. In particular, a higher unemployment rate increases the probability that a firm finds a worker, even at a given level of investment. For the results reported here, this counterforce is strong enough to ensure stability. For some parameter values, the fluctuations could very well become so large that no non-explosive solution exists.\textsuperscript{56}

6.2 Role of imperfect insurance when nominal wages are not sticky

In this section, we discuss business cycle properties when changes in the price level leave real wages unaffected, that is, $\omega_P = 1$. Figure 11 plots the IRFs for the heterogeneous-agent economy and the IRFs for the corresponding representative-agent economy. There are several similarities with our benchmark results, but also one essential difference. We start with the similarities.

\textsuperscript{55}The negative productivity shock still has a direct negative effect on real wages. Which effect is stronger depends on parameter values. In our benchmark economy, the impulse response function of real wages is positive for the first two years and then turns negative.

\textsuperscript{56}In particular, changes in parameter values that substantially enhance the deflationary mechanism make it computationally challenging, or even impossible, to find an accurate solution. This does not prove that a stationary solution does not exist, but it would be consistent with this hypothesis. If such a solution would exist, however, it is likely to have complex nonlinear features. This result is in contrast to standard perturbation methods which impose that aggregate shocks of any size will not destabilize the economy as long as arbitrarily small shocks do not. For example, the technique developed in Reiter (2009) to solve models with heterogeneous agents relies on a perturbation solution for changes in the aggregate shock, which implies that the solution is imposed to be stable – for shocks of any size – as long as the Blanchard-Kahn conditions are satisfied. Our experience suggests that this may impose stability where there is none.
Figure 11: Impulse responses with flexible nominal wages

Notes. These graphs illustrate the difference between the expected time path when the economy is in an expansion in period 0 and the expected time path conditional on being in a recession in period 1. $\omega_p = 1$, that is, nominal wages respond 1-for-1 to price changes.

A negative productivity shock still has a direct negative effect on profits, which leads to a reduced demand for equity (firm ownership), which in turn means that fewer jobs are created. Also, increased concerns about employment prospects still induce agents in the heterogeneous-agents economy to increase their demand for money holdings, which again is strong enough to push the price level down, while it increases in the representative-agent economy.

There is also a striking difference. In the economy with flexible nominal wages, recessions are less severe when agents cannot insure themselves against unemployment risk.
The reason is the following. Increased uncertainty, alongside with an expected reduction in individual consumption, increase the expected value of the marginal rate of substitution. This affects the first-order condition of the liquid asset as well as the first-order condition of the productive investment, at least if each agent’s future revenues are discounted by their own individual MRS. Since wages are flexible, the associated rise in the price level bears no consequence on the return on equity, and the chain of events underlying the deflationary spiral breaks down.

Thus, if the rise in precautionary savings is partially used as productive investments, then this would dampen the reduction in the demand for equity induced by the direct negative effect of the productivity shock on profits. The IRFs document that this is indeed the case when nominal wages respond one-for-one to changes in the price level. The magnitude of the dampening effect is nontrivial. Whereas the biggest drop in employment is 3.3ppt in the representative-agent economy, it is equal to 2.7ppt in the heterogeneous-agent economy. These results make clear that a researcher would bias the model predictions if this dampening aspect of precautionary savings is not allowed to operate, for example, because there is communal firm ownership.\footnote{See footnote 6 for a list of papers following this approach.}

In our benchmark economy, we allow this channel to operate, but the effect is dominated by the interaction between sticky nominal wages and uninsurable unemployment risk. Increased uncertainty may increase the demand for equity, but it will also increase the demand for money. Increased money demand depresses the price level, which, provided that nominal wages are somewhat sticky, increases real wages. The rise in real wages reduces profits, which in turn lowers the demand for equity. This channel dominates any positive effect that precautionary savings may have on the demand for equity.

**Robustness of the dampening effect.** In all cases considered, we find that recessions are less severe in the heterogeneous-agent economy than in the representative-agent economy if nominal wages respond one-for-one to change in the price level. That is, this dampening effect is very robust. During the nineties, several papers argued that an increase in idiosyncratic risk could lead to a reduction in the demand for a risky asset when investors can save through both a risky and a risk-free asset even though it would increase total savings. This effect is referred to as temperance.\footnote{See Kimball (1990), Kimball (1992), Gollier and Pratt (1996), and Elmendorf and Kimball (2000).} We find that this result is quite fragile for several reasons.

The first is that it is a partial equilibrium result. In general equilibrium prices adjust. This is important. Suppose that the economy as a whole can increase savings through the
risky investment, but not through the risk-free investment. Then the relative price of the risk-free asset would increase making the riskier asset more attractive. This plays a role in our economy, because the only way the economy as a whole can do something now to have more goods in the future is by investing more in the productive asset, that is, in the risky asset. There are several other features, typically present in macroeconomic models, that make temperance less likely. One is that the temperance result relies on idiosyncratic risk to be sufficiently independent of investment risk. In macroeconomic models, that is not the case. The amount of idiosyncratic risk depends on the level of the wage rate.\footnote{In the extreme case when the wage rate is zero or equal to the value of home production, there is no unemployment risk.} But the level of the wage rate is often correlated with the return of the risky asset, since both are affected by the same shocks.\footnote{In the model, considered here they are both directly affected by \( z_t \).} In appendix D, we show that this feature alone can overturn the temperance result even in a partial equilibrium setting. Another feature that works against the temperance result is the short-sale constraint on equity, which directly prevents a reduction in the demand for equity, at least for some agents. In our model, diminishing returns on the transactions aspect of money also work against temperance. This makes increased investment in the risk-free asset less attractive relative to a framework in which the return remains fixed. In appendix D, we show that this can overturn the temperance result by itself even if the transactions component is very small and there is little convexity in the marginal return on money balances. Finally, in our model, money is not nearly as risky as equity, but it is also not completely risk-free.

It may be the case that temperance can be generated in models with different utility functions, for example, if the utility function is such that the price of risk increases during recessions.\footnote{We considered models with different degrees of risk aversion, but this does not seem to matter for this particular issue.} We leave this for future research.

\section{Government policy}

In this section, we discuss the two components of government policy in this model: unemployment insurance and monetary policy.

\subsection{Unemployment-insurance (UI) policies}

In this section, we analyze the impact of alternative unemployment-insurance policies. In our model, changes in such policies affect the economy quite differently than in many other

\footnotesize
\begin{itemize}
\item \footnote{In the extreme case when the wage rate is zero or equal to the value of home production, there is no unemployment risk.}
\item \footnote{In the model, considered here they are both directly affected by \( z_t \).}
\item \footnote{We considered models with different degrees of risk aversion, but this does not seem to matter for this particular issue.}
\end{itemize}
models. Our results do not only differ from those of the standard labor search business cycle model with a representative agent, but also from those with heterogeneous agents, such as the models of Krusell, Mukoyama, and Sahin (2010) and McKay and Reis (2013).

The experiments we consider are straightforward. We solve the model for a range of values of the replacement rate, $\mu$, and simulate the economy over the course of a long horizon. For each value of $\mu$, we report the resulting effect on the average employment rate conditional on the economy being in an expansion and in a recession, as well as its unconditional, or expected, value. These experiments are conducted both for sticky ($\omega_P = 0.7$) and flexible ($\omega_P = 1$) wages, as well when changes in the replacement rate affect the wage rule, and when they do not. As a comparison, we also show the results of the experiments for the economy without aggregate risk.

7.1.1 Unemployment insurance when nominal wages are sticky, $\omega_P = 0.7$.

Figure 12 illustrates the results of these exercises. The value of the replacement rate, $\mu$, is provided on the $x$-axis, and the resulting employment rate on the $y$-axis. The top row shows the results with sticky wages, both when changes in the replacement rate does (left graph) and does not (right graph) affect the wage-setting rule.

Changes in the replacement rate when the wage setting rule is not affected. First, consider the case without aggregate uncertainty. An increase in the replacement rate means that agents are better insured against idiosyncratic risk, which lowers the expected value of their MRS. The latter triggers a decrease in precautionary savings, which decreases investment and employment. At low values of $\mu$, however, changes in the replacement rate have virtually no effect on the employment level. The reason is that the presence of money puts a lower bound on the expected return on equity, and therefore an upper bound on the expected MRS. As a consequence, equity prices are bounded from above, which – through the free-entry condition – implies that employment is as well.

For the case with aggregate uncertainty, we report results for values of $\mu$ equal to and above 0.5. For values of $\mu$ above 0.6, the case with aggregate uncertainty is very similar to the case without aggregate uncertainty. The "expansion" and "recession" employment levels form a band around the no-aggregate-uncertainty employment level with a roughly

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62 We discuss how changes in the replacement rate may change the wage-setting rule on page 39.

63 In the model without aggregate uncertainty, the presence of money implies that the real return on firm ownership cannot be less than minus the (constant) inflation rate.

64 At lower levels, the deflationary mechanism becomes very strong. This makes it harder to obtain an accurate solution and it might even be the case that at a sufficiently low value for $\mu$ there is no stable solution to the model.
constant width. As the replacement rate increases beyond 0.6, all three employment levels decrease.

\[ \omega_p = 0.7, \mu \text{ does not affect wages} \]

\[ \omega_p = 0.7, \mu \text{ affects wages} \]

\[ \omega_p = 1, \mu \text{ does not affect wages} \]

\[ \omega_p = 1, \mu \text{ affects wages} \]

**Figure 12: Average employment and replacement rates.**

*Notes.* The left column displays the results when wage setting is not affected by changes in the replacement rate, \( \mu \). The right column displays the results when wages setting is such that the implied average Nash-bargaining weights are kept constant when \( \mu \) changes. The top row presents the results when nominal wages do not fully respond to changes in the price level and the bottom row presents results when they do.

When \( \mu \) is between 0.5 and 0.6, however, our deflationary mechanism is quantitatively important and an increase in the replacement rate leads to a sharp decrease in aggregate volatility, that is, the band narrows substantially. For example, consider a rise in \( \mu \) from 0.5 to 0.55. The increase in the replacement rate leads to a 50.3% decrease in the standard
deviation of the employment rate. The reason for this decline is that improved insurance lowers the strength of the deflationary mechanism. Indeed, the standard deviation of individual consumption is reduced by 15.3%.

The figure also documents that the increase in $\mu$ not only decreases aggregate volatility, it can also increase the average employment rate. In particular, the increase of $\mu$ from 0.5 to 0.55 increases the average employment rate with 0.31ppt. By contrast, in the version of our model without aggregate uncertainty the same increase in the replacement rate leads to a decrease in average employment of 0.52ppt. Such comparative statics typically result in similar answers for economies with and without aggregate uncertainty, because aggregate uncertainty is relatively small. Volatility of the only aggregate exogenous random variable, productivity, is indeed modest in our model. Nevertheless, the economy with aggregate uncertainty responds to a change in the replacement rate quite differently than the economy without aggregate uncertainty.

Lastly, it is interesting to note the highly nonlinear effects on employment in recessions for moderate, and for large increases of the replacement rate. In particular, for moderate changes (i.e. for $\Delta \mu < 0.1$), the employment rate in recessions is increasing in $\mu$. For larger changes, however, the employment rate is instead decreasing. Nevertheless, it should be noted that the employment rate in recessions is no lower than its benchmark value, even for values of $\mu$ as high as 0.8.

In the remainder of this section, we explain why an increase in unemployment insurance can lead to an increase in the average employment rate, when the value of the replacement rate is such that the deflationary mechanism is quantitatively important. In particular, in the economy with aggregate uncertainty, there are two effects associated with an increase in the replacement rate that increases the demand for equity and, thus, job creation. The first effect is that more insurance reduces the risk of holding equity, because an increase in the replacement rate not only reduces the volatility of real activity, but also leads to a substantial reduction in stock price volatility. In fact, if $\mu$ increases from 0.5 to 0.55 the standard deviation of the real equity price drops by 49.8%. This reduction in risk and increased demand for equity leads to more job creation and an increase in average employment of 0.42ppt. The second effect is related to the nonlinearity of the matching process; that is, increases in equity prices have a smaller effect on job creation.

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65We calculate this as follows. If there is no aggregate uncertainty, then the increase in $\mu$ leads to a decrease in employment of 0.52ppt and a decrease in real equity value of 5%. If there is aggregate uncertainty, then the same change in $\mu$ leads to a decrease in the average real equity value by only 1%. This lower drop in equity value is due to the fact that there also is a decrease in aggregate uncertainty. Assuming that these effects are linear, the difference between the 5% and the 1% drop corresponds to an increase in average employment of 0.416ppt ($= 4/5 \times 0.52$).
than decreases.\textsuperscript{66} For the same change in the replacement rate as before, the decrease in the volatility of the real equity price increases average employment with 0.41ppt.\textsuperscript{67} We now have all the ingredients to explain why the employment rate increases with 0.31ppt. If we add the 0.41ppt increase due to the nonlinearity of the matching function to the 0.42ppt increase due to a reduction in risk, we get an increase in the employment rate of 0.83ppt. If we subtract the 0.52ppt decrease due to the reduction in savings because of better individual insurance, we get an increase in average employment of 0.31ppt.\textsuperscript{68}

A direct effect of an increase in $\mu$ is an increase in the tax rate. This direct effect is counteracted by the increase in the tax base.\textsuperscript{69} The indirect effect is not strong enough to decrease average tax rates, but it is strong enough to decrease the tax rate during recessions. The reason is that the reduction in employment volatility and the higher average employment rate imply that the tax base is reduced by less when the economy enters a recession. If tax rates were distortionary – which they are not in our model – then lower tax rates during recessions could lead to a further dampening of business cycle fluctuations.

**Changes in the replacement rate when the wage setting rule is affected.** The discussion above considered an increase in unemployment insurance while leaving the wage setting rule unchanged. This is not unreasonable given that several empirical papers find that UI benefits do not have a significant effect on wages.\textsuperscript{70} However, not all papers reach this conclusion. Schmieder, von Wachter, and Bender (2014) find that more generous UI benefits have a significant negative effect on wages. This could happen if a higher level

\begin{equation}
\psi_{1/(1-\eta)} \left( \frac{\eta}{1-\eta} \right) (1-q) \approx \delta q.
\end{equation}

Since $\eta = \frac{1}{2}$, we get that

$$q \approx 1 - \frac{\delta}{\delta + q^2 J/P}.$$  

Thus, $q$ is a concave function of $J/P$.

\textsuperscript{66}Ignoring transitions – which occur fast in this model – Equation (13) implies that

\begin{equation}
\psi_{1/(1-\eta)} \left( \frac{\eta}{1-\eta} \right) (1-q) \approx \delta q.
\end{equation}

\textsuperscript{67}We calculate this as follows. When $\mu = 0.5$, then the introduction of aggregate uncertainty leads to a reduction in employment of 1.01ppt of which 46ppt can be explained by the reduction in the average equity price. The remainder of 0.55ppt is, thus, due to the nonlinearity of the matching function. When $\mu = 0.55$, then this nonlinearity effect is only 0.14ppt. Thus, when $\mu$ increases from 0.5 to 0.55 in the economy with aggregate uncertainty, then there is a reduction of the impact of the nonlinearity on average employment of 0.55ppt−0.14ppt=0.41ppt.

\textsuperscript{68}The fact that the numbers add up means that interaction of the different effects either cancels out or is negligible.

\textsuperscript{69}In our model, taxes are only used to finance unemployment benefits and are, thus, very low. The increase in revenues caused by an increase in the tax base would be higher when average tax rates are higher.

\textsuperscript{70}See, for example, Card, Chetty, and Weber (2007), Lalove (2007), van Ours and Vodopivec (2008), and Le Barbanchon (2012).
of UI benefits prolongs unemployment spells and increases skill loss. Nekoei and Weber (2015) find that UI benefits have a positive effect on re-employment wages. This could happen because an increase in UI benefits increases workers’ outside options, or because it allows workers to find better matches. If it is the former, then higher UI benefits would decrease the surplus of the match and the share that accrues to firm owners, which in turn would negatively affect job creation.

Even though the empirical evidence is inconclusive, it is interesting to see how the results change if wages do adjust following an increase in the replacement rate. In our next exercise, we use the same wage setting rule as before but let $\omega_0$ – and thus average wages – increase when $\mu$ increases to ensure that the average Nash bargaining weight implied by our wage rule remains unaffected.\(^71\) We leave $\omega_P$ unchanged, which implies that wages remain sticky with respect to the aggregate price level. As pointed out in Hall and Milgrom (2008), Nash bargaining may overstate the importance of fluctuations in the value of unemployment, because the worker’s threat in bargaining is typically not leaving the relationship and becoming unemployed, but prolonging negotiations. Consequently, our procedure may overstate the upward pressure on wages following an increase in $\mu$. By considering the case when wages do not respond at all, as well as the case when wages possibly respond too much, we can bound likely outcomes of increases in the replacement rate.

The top right graphs of figure 12 shows the results of the experiments when the wage rule is affected by changes in the replacement rate. First consider the case without aggregate uncertainty. The results are similar to the case when the replacement rate does not affect wage setting. One difference is that the employment level decreases by much more when the replacement rate increases. The reason is that an increase in $\mu$ now has a direct negative effect on firm profits as overall wages are higher.\(^72\) Next, consider the case with aggregate uncertainty. Qualitatively the results do not depend on whether $\omega_0$ adjusts or not. Quantitatively, however, they differ. Consider again an increase in $\mu$ from 0.5 to 0.55. When the increase in the replacement rate goes together with an increase in $\omega_0$, then the standard deviation of individual consumption drops by 10.2% instead of 15.3%,

\(^{71}\)Let $U_e(y + (1 - \tau) w)$ be the expected utility of an employed worker with financial assets worth $y$ and a current wage rate equal to $w$. Other state variables are suppressed. Also, $U_u(y + (1 - \tau) \mu \bar{w})$ is the expected utility of an unemployed worker. His utility depends on the market wage rate, $\bar{w}$. Firm value minus the wage payment is equal to $(1 - \delta) J/P + z - w$. The implied Nash bargaining weight is then equal to

$$\frac{U_e(y + (1 - \tau) w) - U_u(y + (1 - \tau) \mu \bar{w})}{U_e(y + (1 - \tau) w) - U_u(y + (1 - \tau) \mu \bar{w}) + \frac{\partial U_e(y + (1 - \tau) w)}{\partial \mu}((1 - \delta) J/P + z - w)}$$

\(^{72}\)For the same reason, the effect of $\mu$ on the employment level does not flatten out at low levels of $\mu$. 

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and the standard deviation of aggregate employment drops by 42.1% instead of 50.3%.\textsuperscript{73} The reason these volatilities do not drop by more is because the increase in $\omega_0$ lowers average profits which makes profits more sensitive to changes in productivity. As a result, job creation and employment become more volatile. In addition, the decrease in average profits lowers average employment substantially. In particular, average employment decreases with 0.31ppt when $\omega_0$ changes, whereas average employment increases with 0.31ppt when $\omega_0$ remains constant.

7.1.2 Unemployment insurance when nominal wages are flexible, $\omega_P = 1$

The bottom two graphs of figure 12 display the results when nominal wages respond one-for-one to changes in the price level and the deflationary mechanism is, thus, not present. The left graph displays the results when the replacement rate does not affect wage setting, and the right graph displays the results when wage setting is affected. The consequences of an increase in $\mu$ are now quite different. In particular, increases in the replacement rate never increase employment and never reduce volatility of aggregate variables. In fact, aggregate volatility always increases. This happens even if the replacement rate does not affect wages. The reason is the following. As discussed in section 6.2, imperfect insurance dampens business cycles when wages are flexible (i.e. $\omega_P = 1$). Precautionary savings increase when unemployment risk increases, which dampens the reduction in investment in job creation. As the replacement rate increases, this effect becomes less important and business cycles therefore become more volatile.

The change in $\mu$ from 0.5 to 0.55 makes the role of $\omega_P$ very clear. When wages are flexible, this increase in the replacement rate leads to an increase in the standard deviation of the aggregate employment rate of 9.3%. In contrast, when wages are sticky the same standard deviation drops by more than 50%. The results regarding risk sharing are also different. When wages are flexible, the increase in $\mu$ leads to a decrease in standard deviation of individual log consumption of only 8.4%. With sticky nominal wages the drop is equal to 15.3%.

7.1.3 Welfare consequences of UI changes

In this subsection, we document how changes in the replacement rate affect agents’ welfare. The experiments we consider are similar, but not identical, to those of the previous subsection. We solve the model for range of values of the replacement rate. For each value of $\mu$, we calculate the effect on welfare of unexpectedly and permanently switching from

\textsuperscript{73}The value of $\omega_0$ increases to 0.9722, which implies a 7% reduction in firm profits.
\( \mu = 0.5 \) to this new level of insurance at two different stages: (i) when the economy enters a recession at an employment rate equal to its peak during the expansionary phase; and (ii) when the economy enters an expansion at an employment rate equal to its trough during the recessionary phase.\(^{74}\) These experiments are, again, conducted both for sticky (\( \omega_P = 0.7 \)) and flexible (\( \omega_P = 1 \)) wages, as well when changes in the replacement rate affect the wage rule, and when they do not. As a comparison, we also show the results of the latter experiment for the economy without aggregate risk. The change in welfare is calculated as follows. Starting with our benchmark economy, we calculate the cash-equivalent for each agent of changing the replacement rate. That is, we calculate the change in cash on hand required to render an agent indifferent between the change in \( \mu \) and leaving it unchanged at its benchmark value of 0.5. A positive value means that the agent is better off. Our calculations take into account the expected transition associated with the change in \( \mu \).

Figure 13 reports the average of the cash-equivalents across all agents, relative to output in period of implementation. The two top graphs show the results when \( \omega_P = 0.7 \) and when \( \mu \) does and does not affect wage setting. The bottom two graphs illustrate the same results when \( \omega_P = 1 \). In the next subsection, we will show how these results depend on an agent’s individual wealth and employment status.

First, consider the results in the top-left graph, which corresponds to the case when nominal wages are sticky and \( \mu \) does not affect the wage setting rule. In the version of the model without aggregate uncertainty, our benchmark value for the replacement rate of 0.5 happens to be optimal according to this average welfare criterion. Agents benefit from an increase in \( \mu \) because it lowers the volatility of individual consumption, but this is offset by the decrease in average employment induced by a decrease in precautionary savings.

The results are quite different when aggregate uncertainty is present. An increase of \( \mu \) from 0.5 to 0.55 during a recession corresponds to an average utility gain that is equivalent to 115\% of quarterly per capita output. The increase in unemployment insurance is welfare improving because it leads to a decrease in the volatility of individual consumption, a decrease in aggregate volatility, and an increase in the average employment rate. Why are the welfare gains so large? A key reason is that agents are not well insured in this economy. During the first year of an unemployment spell, consumption drops on average by 34\%, which equals its empirical counterpart. At some point, unemployed workers become hand-to-mouth consumers. In such a world, the level of unemployment benefits

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\(^{74}\) Although the results depend on the current values of aggregate productivity and employment, they are very similar for different histories of productivity leading to these values. Since productivity follows a two-state Markov process, and since transitional dynamics are quite fast, the employment rate which we consider in each experiment is close to its typical value at the first period of a transition.
matters tremendously; when $\mu$ increases from 0.5 to 0.55, consumption volatility declines by 44.5%. The other reason the numbers are large is that they are expressed as one lump-sum payment. If we use 0.18% as the discount rate – i.e., the model’s average quarterly return on equity –

![Graph](image)

**Figure 13: Average welfare and replacement rates.**

*Notes.* This figure displays the average change in our welfare measure when the replacement rate, $\mu$, changes from our benchmark value of 0.5 to the indicated value on the $x$-axis. The left column displays the results when wage setting is not affected by changes in the replacement rate, $\mu$. The right column displays the results when wages setting is such that the implied average Nash-bargaining weights are kept constant when $\mu$ changes. The top row presents the results when nominal wages do not fully respond to changes in the price level and the bottom row presents results when they do.
then a lump sum payment equal to 115% of quarterly per capita output corresponds to a permanent increase of 0.20% of quarterly per capita output.

The top-right graph of figure 13 documents that the results are qualitatively similar if the change in the replacement rate affects wage setting. Increasing the replacement rate still increases our welfare measure, but by less, especially if the increase occurs during an expansion. Moreover, increasing $\mu$ to levels above 0.66 during a recession renders negative average cash-equivalents, whereas the average cash-equivalents never turn negative when changes in the replacement rate do not affect wage setting. The reason behind the attenuated welfare gains of higher unemployment insurance is that the associated increase in wages lowers the level of employment. For the case without aggregate uncertainty – in which changes in the replacement rate cannot alter the business cycle properties – an increase in $\mu$ corresponds to a decrease in our average welfare measure.

The bottom two graphs display the results if there is no nominal wage stickiness; that is, if the deflationary mechanism is not present. If there is no aggregate uncertainty, then the price level is constant, which implies that the value of $\omega_P$ does not matter. Moreover, without nominal wage stickiness the results with and without aggregate uncertainty are very similar. It should be noted that average welfare always declines for any increase in $\mu$ beyond its benchmark value.

7.1.4 Who benefits from a change in unemployment insurance?

In this subsection, we discuss how an increase in the replacement rate affects different agents. We focus again on an increase in $\mu$ from 0.5 to 0.55 at the beginning of a recession or at the beginning of an expansion. This increase is, again, completely unexpected and believed to be permanent. When evaluating the change, the agents take into account the transitional dynamics.

Changes in the replacement rate affect different agents for different reasons. Unemployed workers benefit immediately from the increase in unemployment benefits, since it is a direct transfer of resources from the employed to the unemployed. But employed agents benefit too. They benefit because: (i) the dampening of the downturn increases the value of their asset holdings; (ii) a higher replacement rate provides better insurance against a shortfall in income should they become unemployed; and (iii) average employment increases when $\mu$ does not affect wages, which means that all workers can expect to be less affected by unemployment. Although the increase in the replacement rate increases average tax rates, it lowers the tax rate during recessions, provided that $\mu$ does not affect wages.

Figure 14 displays the cash equivalent ($y$-axis) of the proposed change in the replace-
ment rate as a function of the agent’s beginning-of-period cash-on-hand level (x-axis) and employment status. The figure documents that all unemployed and all employed agents prefer the switch to the higher level of unemployment insurance, irrespective of whether wages, \(\omega_0\), adjust. For the same cash-on-hand levels, an unemployed worker benefits more than an employed worker. This is not surprising given that an unemployed worker benefits directly from higher unemployment insurance. More surprisingly, rich agents benefit more than poor agents. One reason is that they hold more equity and, thus, benefit from the fact that stock prices drop by less when \(\mu\) is increased at the onset of a recession.

![Figure 14: Increasing \(\mu\) in the first period of a recession.](image)

**Notes.** This figure plots the welfare gains (measured as cash-on-hand equivalents relative to output) for the four possible labor market transitions. Since the change in \(\mu\) affects asset prices individual portfolio shares matters, which implies that the results also depend on last period’s employment status. The label “EE”, for instance, indicates an agent who was employed in the preceding period, and remained employed in the current period, etc. The average welfare gain with \(\omega_0\) unchanged equals 1.146. The average welfare gain with \(\omega_0 = 0.9722\) equals 0.562.

All agents benefit less from an increase in the replacement rate when the increase is associated with higher wages. Why are the general equilibrium effects such that even employed workers who hold no equity benefit less? The associated increase in \(\omega_0\) implies that the rise in the replacement rate does not dampen the downturn in real activity, nor the drop in stock prices, by as much. The benefit of an increase in the replacement rate

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\(^{75}\)Cash on hand is measured at the point when it is known that the economy has entered a recession, but before it is known that \(\mu\) has changed.
for a poor employed worker is not affected by these different responses, since the agent is employed and does not hold equity. However, the same agent is affected by worsened future employment prospects, which are important enough to offset the increase in the agent’s wage rate.

If the rise in the replacement rate occurs at the beginning of an expansion, the calculated cash-equivalents are lower than those previously reported. The reason is that an increase in the replacement rate not only dampens recessions, it also dampens expansions, since the upward pressure on prices induced by a reduction in precautionary savings is smaller. It is still the case, however that all workers prefer the increase in unemployment insurance when wages are not affected. When wages do increase, however, then both the richer employed and the richer unemployed workers do not prefer to increase the replacement rate from the status quo.

### 7.1.5 Aggregate transition dynamics when the replacement rate increases

Figure 15 displays the time paths for employment when the economy moves from an expansion to a recession and back to an expansion. It plots the series when the replacement rate remains unchanged throughout, and when it unexpectedly and permanently increases to 0.55 at the onset of a recession. The results above made it clear that this increase in the replacement rate leads to smaller fluctuations and, if wage setting is not affected, also to a higher average employment rate. Consequently, employment should drop by less if $\mu$ is increased at the start of a recession. The same turns out to be true if the increase in $\mu$ is associated with an increase in wages. That is, the negative effect of the induced increase in wages on average employment is smaller than the dampening effect of the increase in $\mu$ on business cycle fluctuations.

When the economy leaves the recessionary phase, however, the recovery is dampened by the higher unemployment benefits, irrespective of whether $\mu$ affects wages. When wages are not affected, the employment level associated with a higher replacement rate is above the employment level associated with the benchmark value, throughout the recession and during the first two periods of the recovery. If the elevated replacement rate does affect wages, however, the employment level during the recovery is already lower than the benchmark after two periods of the recovery. The result that higher unemployment benefits can be harmful for a recovery is consistent with the results in Hagedorn, Karahan, Manovskii, and Mitman (2015), who argue that the extension of unemployment benefits in the US increased unemployment in 2011 – when the US recovery had started – by 2.5
percentage points.\textsuperscript{76}

![Graph](image)

**Figure 15**: Switch to higher $\mu$ at the start of the recession.

*Notes*. This figure compares the benchmark time path of beginning-of-period employment with the time path when $\mu$ increases (unexpectedly and permanently) to 0.55, both when $\omega_0$ does and when $\omega_0$ does not adjust upwards.

### 7.1.6 Unemployment benefits and unemployment duration

As discussed above, it is not clear from empirical studies whether changes in unemployment benefits affect wages. There is much more empirical support for the hypothesis that more generous benefits increase unemployment duration (see, for instance, Le Barbanchon (2012) for an overview). Several of these studies identify the effect of unemployment benefits on unemployment duration by considering changes in benefits that affect workers differently. These results may, thus, not be relevant for our general equilibrium experiment in which *everybody* is affected by the same increase in the replacement rate. If a large share of the unemployed search less intensely, this provides improved opportunities of finding a job for those who actively search.\textsuperscript{77}

In response to a 10% increase in the replacement rate, $\mu$, from 0.5 to 0.55, our framework generates an increase in average unemployment duration of 1.7% when wages respond to

\textsuperscript{76}Amaral and Ice (2014), in contrast, argue that the extension in benefits only had a minor impact and that part of the increase in the unemployment rate was due to a reduction in the number of unemployed leaving the labor force.

\textsuperscript{77}Lalive, Landais, and Zweimller (2015) argue that these externalities are quantitatively important.
the increase in $\mu$.\textsuperscript{78} Krueger and Meyer (2002) report that 0.5 is not an unreasonable rough summary of empirical estimates for the elasticity of unemployment duration with respect to unemployment benefits, but estimates vary. So even though search intensity is constant in our model and an increase in unemployment insurance leads to a sharp decrease in unemployment duration during recessions, our model can still explain a substantial part of the observed relationship between unemployment benefits and unemployment durations.\textsuperscript{79}

### 7.1.7 Relation to the literature

In the standard search-and-matching model with a representative agent, an increase in the replacement rate that is accompanied by an increase in wages would, (i) lower average employment because firm profits fall; and (ii) increase aggregate volatility because higher wages increases the sensitivity of firm profits to shocks.

In our benchmark model, in which the deflationary mechanism operates, we find the opposite: an increase in the replacement rate increases average employment and decreases aggregate volatility. If the deflationary mechanism does not operate, then our framework’s predictions correspond to those of the representative-agent version. This is most clear in the bottom right graph of figure 12 which shows the employment rate as a function of $\mu$ when wages are not sticky ($\omega_p = 1$) and wages are affected. The graph shows that increases in the replacement rate always lower average employment and always increase aggregate volatility.

Our results also differ substantially from those in Krusell, Mukoyama, and Sahin (2010), who also look at changes in the replacement rate in a model with incomplete risk sharing and labor market frictions. They show that 92.1% of all agents would prefer a reduction in the replacement rate from 0.4 to 0.04. As previously shown, in our benchmark economy with a replacement rate equal to 0.5, all agents would prefer a 10% increase in the replacement rate. The key difference is that we look at an economy with aggregate uncertainty. Moreover, our parameter values are such that there is a strong interaction between sticky nominal wages and imperfect insurance, which results in a deflationary mechanism that increases the volatility of business cycles and asset prices. An increase in the replacement rate weakens the deflationary mechanism and has the capacity to make

\textsuperscript{78}Our framework can also generate an increase in average unemployment duration following an increase in $\mu$ when wages are not affected by changes in $\mu$, but only when $\mu$ is above 0.6.

\textsuperscript{79}The empirical literature focuses on changes in UI benefits on individual workers and changes in search effort are thought to be behind changes in unemployment duration. In our model, search effort is constant and the increase in unemployment duration is due to a reduction in the job creation, either because wages increase or because precautionary savings decrease.
all agents better off. In the version of our model without aggregate uncertainty, agents also prefer a reduction in the replacement rate as long as wages are affected through Nash bargaining, which also is the case in Krusell, Mukoyama, and Sahin (2010). Similarly, if our deflationary mechanism is not present – for example when \( \omega_P = 1 \), or if there is no aggregate uncertainty – then agents also prefer values of the replacement rate that are below our benchmark value, provided that wages are affected.\(^{80}\)

McKay and Reis (2013) consider the effects of changes in unemployment benefits on aggregate volatility in an economy with imperfect risk sharing. They find that a reduction in transfers has a close-to-zero effect on the average level of output, and actually lowers the volatility of aggregate consumption.\(^{81}\) Their approach differs from ours in that it does not include a frictional labor market and – more importantly – also no sticky nominal wages. Consequently, imperfect risk sharing does not interact with sticky nominal wages, which are the key ingredients that generate the powerful deflationary mechanism studied in this paper. Indeed, it is that precise mechanism that underlies our finding that an increase in unemployment insurance leads to a sharp decrease in aggregate volatility.

### 7.2 Monetary policy

Could the severity of the recessions that occur in our model possibly be alleviated by monetary policy?\(^{82}\) If the central bank could respond to changes in productivity instantaneously, and if the central bank could increase the money supply by “helicopter drops”, then the central bank could prevent the deflationary pressure on the price level and the ensuing upward pressure on real wages. In practice, however, there are several reasons why it may not be that easy for central banks to offset the harmful effects caused by the interaction of precautionary savings and sticky nominal wages.

---

80By contrast to Krusell, Mukoyama, and Sahin (2010), it never is the case in any of the models considered that lots of agents prefer extremely low replacement rates. In fact, even if \( \mu \) affects wages and in the absence of nominal wage stickiness, we find that only 40% prefer a reduction in \( \mu \) from 0.5 to 0.35 when implemented in a recession. Moreover, this fraction drops sharply for further reductions; no agent prefers a reduction of \( \mu \) from 0.5 to 0.32. These numbers are for the case with aggregate uncertainty. Without aggregate uncertainty, the fraction of agents that prefer low replacement rates is even smaller.

81In their model, a reduction in transfers lead to an increase the volatility of output and hours. Their effects are small relative to ours. They report that an 80% reduction in transfers leads to an 8% increase in the variance of hours, whereas our increase in the replacement rate from 0.5 to 0.55 leads to a 47.9% decrease in the standard deviation of the employment rate.

82Several papers with heterogeneous-agent models adopt the cashless-limit approach. In a representative-agent economy, there are different ways to motivate a cashless economy. One is to assume that the level of real money balances does not interact with the real economy. If that is the case, then all that matters is the policy rate and one does not have to model how open market operations affect the policy rate. Such a motivation for the cashless economy is unlikely to carry over to a model with heterogeneous agents, since open market operations are likely to affect the distribution of money holdings, which will affect real activity.
In our heterogeneous-agent model, a negative shock to productivity leads to a drop in the price level followed by a period of higher-than-usual expected inflation (see figure 10). Suppose that the central bank cannot prevent this initial drop because it cannot respond instantaneously to the decline in productivity. To undo the harmful effects of nominal wage stickiness, the central bank would have to increase inflation at a point when expected inflation is already higher than normal. That is, the central bank would have to adopt a regime of price-level targeting instead of a regime of inflation targeting. But price-level targeting may not be the best policy when the economy faces other problems. So the first reason why monetary policy may not eliminate the channel identified in this paper is that they may not be willing to adopt a price-level targeting regime.

Another aspect is, of course, that helicopter drops of money are not part of the usual set of central bank instruments. The typical way for a central bank to increase liquidity in the economy is to purchase government bonds from banks. This increases the liquidity position of banks. If the additional liquidity induces banks to issue more loans, then bank deposits will increase. That is, money holdings of the non-financial private sector will increase. Note, however, that the liabilities of this sector must have increased by the same amount. It is possible that this combined increase of liquid assets and debt eases workers concerns about future unemployment, for example, because the loans are (perceived to be) long-term loans. If workers care about their net-liquidity position, however, then this monetary stimulus would not undo the workers’ desire to hold more money balances, and there would still be downward pressure on the price level during recessions. This latter case would be especially relevant if loans cannot be rolled over if a worker becomes unemployed.

Another factor that is likely to make it difficult to find the right monetary response is a lack of information about the state of the economy, and a lack of information about the “true” model. In our economy, changes in the price-level deepen the recession when nominal wages are sticky and there is limited insurance against unemployment risk. When these two features are not present, or when they are not strong enough, then changes in the price-level dampen recessions. Since the strength of different channels may change over time, it would not be clear whether it is always preferred for the central bank to try to undo any changes in the price level.

Finally, monetary policy that undoes changes in the price level to offset nominal wage stickiness will have distributional consequences, which – depending on parameter values – may make it difficult to determine the appropriate policy.
8 Concluding comments

The properties of our model depend crucially on whether the deflationary mechanism is sufficiently powerful. If it is not powerful enough, then the model properties are close to the outcomes of a representative-agent version of the model. In particular, the presence of nominal sticky wages would then *dampen* the effects of productivity shocks, and an increase in the replacement rate would *decrease* the average employment rate. If the deflationary mechanism is strong enough, however, then our model predicts the opposite. In so far as the conditions that affect the strength of the deflationary mechanism vary across time and place, one can also expect business cycle properties to vary across time and place. The same is true for the effects of changes in unemployment insurance. Whether the deflationary mechanism is operative or not may depend on relatively small changes. For example, the mechanism is quantitatively very important when the replacement rate is equal to its benchmark value of 50%, but not when the replacement rate exceeds 60%. The message is that even if one is confident that a particular model describes the data well, it may still be difficult to predict business cycle behavior and the consequences of policy changes.
A Data Sources

- Eurozone GDP implicit price deflators are from the Federal Reserve Economic Data (FRED). Data are seasonally adjusted. Here the Eurozone consists of the 18 countries that were members in 2014.

- Eurozone private sector hourly earnings are from OECD.STATExtracts (MEI). The target series for hourly earnings correspond to seasonally adjusted average total earnings paid per employed person per hour, including overtime pay and regularly recurring cash supplements. Data are seasonally adjusted.

- Unit labor costs are from OECD.STATExtracts. Data are for the total economy. Unit labour costs are calculated as the ratio of total labour costs to real output. Data are seasonally adjusted.

- Average unemployment rate: Average unemployment rate for the four large Eurozone economies, France, Germany, Italy, and Spain. Data is from OECD.STATExtracts (ALFS).

- Average unemployment duration: Average unemployment duration for Europe is from OECD.StatExtracts. This is annual data. The data series for Europe is used because no data for the Eurozone is available, nor data for the big Eurozone countries. Starting in 1992, separate data is given for Europe, the European Union with 21 countries, and the European Union with 28 countries, and the series are quite similar over this sample period.

B Solution algorithm

B.1 Solution algorithm for representative-agent model

B.1.1 Algorithm

To solve the representative-agent models, we use a standard projection method, which solves for \( q_t \) and \( P_t \) on a grid and approximates the outcomes in-between gridpoints with piecewise linear interpolation.

B.1.2 Accuracy

Petrosky-Nadeau and Zhang (2013) consider a search and matching model with a representative agent. They show that it is not a trivial exercise to solve this model accurately, even though fluctuations are limited. Our representative-agent model is even simpler than the one considered in Petrosky-Nadeau and Zhang (2013). Nevertheless, we document here that both the linear and the log-linear perturbation solution are clearly not accurate. We also document that the projection solution is accurate.
To establish accuracy, we use the dynamic Euler-equation errors described in Den Haan (2010). The test compares simulated time series generated by the numerical solution for the policy rules with alternative time series. The alternative time paths are calculated using the \textit{exact} equations of the model in each period; the approximation is not used, except when evaluating next period’s choices inside the expectations operator. This test is similar to the standard Euler equation test, but reveals better whether (small) errors accumulate over time. If a numerical solution is accurate, then the two procedures generate very similar time paths.

Figure 16 displays part of the generated time series and clearly documents that the linear perturbation solution has a substantial systematic error, whereas our projection solution does not. Table 1 provides a more complete picture.

B.2 Solution algorithm for heterogeneous-agent model

In appendix B.2.1, we document how we solve the individual problem taking as given perceived laws of motion for prices and aggregate state variables. In appendix B.2.2, we document how to generate time series for the variables of this economy, including the complete cross-sectional distribution, taking the individual policy rules as given. The simulation is needed to update the laws of motion for the aggregate variables and to characterize the properties of the model. We make a particularly strong effort in ensuring that markets clear \textit{exactly} such that there is no “leakage” during the simulation. This is important since simulations play a key role in finding the numerical solution and in characterizing model properties.\footnote{If the equilibrium does not hold exactly, then the extent to which there is a disequilibrium is likely to accumulate over time, unless the inaccuracy would happen to be \textit{exactly} zero on average. Such accumulation is problematic, since long time series are needed to obtain accurate representations of model properties.}

B.2.1 Solving for individual policy functions

When solving for the individual policy functions, aggregate laws of motion as specified in appendix B.2.2 are taken as given. Let $\bar{x}_i$ denote an individuals cash on hand at the perceived prices. That is,

$$
\bar{x}_i = e_i(1-\tau)\frac{\bar{W}}{P} + (1-e_i)\mu(1-\tau)\frac{\bar{W}}{P} + q_i \left(\frac{\bar{D}}{P} + (1-\delta)\frac{\bar{J}}{P}\right) + \frac{M_i}{P}.
$$

(15)

Individual policy functions for equity, $q'_i = q(\bar{x}_i, e_i, q, z)$, and money, $M'_i = M(\bar{x}_i, e_i, q, z)$, are obtained by iteration:

\begin{enumerate}
  \item Using initial guesses for $q'_i$ and $M'_i$, a policy function for consumption can be calculated from the agent’s budget constraint:

$$
c(\bar{x}_i, e_i, q, z) = \bar{x}_i - \frac{q'_i\bar{J} + M'_i}{P}.
$$
\end{enumerate}
Conditional on the realizations of the aggregate shock and the agent’s employment state, cash on hand and consumption in the next period can be calculated:

\[
\tilde{x}'(e_i', z') = e_i'(1 - \tau') \frac{\tilde{W}'}{\tilde{P}'} + (1 - e_i') \mu(1 - \tau') \frac{\tilde{W}'}{\tilde{P}'} + q_i' \left( \frac{\tilde{D}'}{\tilde{P}'} + (1 - \delta) \frac{\tilde{T}'}{\tilde{P}'} \right) + \frac{M_i'}{\tilde{P}'},
\]

\[
c'(e_i', z') = c(\tilde{x}'(e_i', z'), e_i', q_i', z').
\]

Using the individual and aggregate transition probabilities, the expectations \(E \left[ c' - \gamma \tilde{P} \right]\) and \(E \left[ c' - \gamma \frac{\tilde{D}'}{f'} + (1 - \delta) \frac{\tilde{T}'}{f'} \tilde{P}' \right]\), in the first-order conditions (3) and (4) can be calculated. Then, the first-order condition for equity holdings gives an updated guess for consumption of agents holding positive amounts of equity:

\[
c^{new}(\tilde{x}_i, e_i, q, z) = \left( \beta E \left[ c' - \gamma \tilde{P} \right] \right)^{-\frac{1}{\gamma}}.
\]

The first-order condition for money gives an updated policy function for money:

\[
M^{new}(\tilde{x}_i, e_i, q, z) = \tilde{P} \chi \left( c^{new}(\tilde{x}_i, e_i, q, z) - \beta E \left[ c' - \gamma \tilde{P} \right] \right)^{-\frac{1}{\gamma}}.
\]

The budget constraint in the current period gives the updated policy function for equity:

\[
q^{new}(\tilde{x}_i, e_i, q, z) = \max \left( 0, \frac{\tilde{x}_i \tilde{P} - c^{new}(\tilde{x}_i, e_i, q, z) \tilde{P} - M^{new}(\tilde{x}_i, e_i, q, z)}{f} \right).
\]

For agents with a binding short-sale constraint, updated policy functions for consumption and money are instead calculated using only the first-order condition for money and the budget constraint:

\[
c^{new, constraint}(\tilde{x}_i, e_i, q, z) = \left( \beta E \left[ c' - \gamma \tilde{P} \right] \right)^{-\frac{1}{\gamma}},
\]

\[
M^{new, constraint}(\tilde{x}_i, e_i, q, z) = \tilde{x}_i \tilde{P} - c^{new, constraint}(\tilde{x}_i, e_i, q, z) \tilde{P}.
\]

A weighted average of the initial guesses and the new policy functions is used to update the initial guesses. The procedure is repeated from step (i) until the differences between initial and updated policy functions become sufficiently small.
B.2.2 Simulation and solving for laws of motion of key aggregate variables

The perceived laws of motion for the real stock price, $\tilde{J}/\tilde{P}$ and the price level, $\tilde{P}$, are given by the following two polynomials (using a total of 12 coefficients):

\begin{align*}
\ln \frac{\tilde{J}}{\tilde{P}} &= a_0(z) + a_1(z) \ln q + a_2(z) (\ln q)^2, \quad (20) \\
\ln \tilde{P} &= b_0(z) + b_1(z) \ln q + b_2(z) (\ln q)^2. \quad (21)
\end{align*}

Note that $q$ is not only the level of employment, but also the number of firms, and the aggregate amount of equity shares held. We only use the first moment of the distribution of equity holdings, as in Krusell and Smith (1997), but we use a nonlinear function.\(^{84}\) To update the coefficients of this law of motion, we run a regression using simulated data. In this appendix, we describe how to simulate this economy taking the policy rules of the individual agents as given. We start by describing the general idea and then turn to the particulars.

**General idea of the simulation part of the algorithm.** Policy functions are typically functions of the state variables, that is, functions of predetermined endogenous variables and exogenous random variables. These functions incorporate the effect that prices have on agents’ choices, but this formulation does not allow for prices to adjust if equilibrium does not hold exactly when choices of the individuals are aggregated. If we used the true policy functions, then the equilibrium would hold exactly by definition. Unfortunately, this will not be true for numerical approximations, not even for very accurate ones. Since long simulations are needed, errors accumulate, driving supply and demand further apart, unless these errors happen to be exactly zero on average. Our simulation procedure is such that equilibrium does hold exactly. The cost of achieving this is that actual prices, $J$ and $P$, will be different from perceived prices, $\tilde{J}$ and $\tilde{P}$ and some of the actual individual choices will be different from those according to the original policy functions.\(^{85}\) These are errors too, but there is no reason that these will accumulate. In fact, we will document that perceived prices are close to actual prices in appendix B.2.3.

**Preliminaries.** To simulate this economy, we need laws of motions for perceived prices, $\tilde{J}(q, z)$ and $\tilde{P}(q, z)$, as well as individual policy functions, $q_i' = q(\tilde{x}_i, e_i, q, z)$ and $M_i' = M(\tilde{x}_i, e_i, q, z)$. At the beginning of each period, we would also need the joint distribution of employment status, $e_i$, and cash on hand, $x_i$. This distribution is given by $\psi(\tilde{x}_i, e_i)$, where the tilde indicates that cash on hand is evaluated at perceived prices. The distribution is such that,

\[ \int_{e_i} \int_{\tilde{x}_i} d\psi_i = zq + (1 - \delta)q \frac{\tilde{J}}{\tilde{P}} + \frac{\tilde{M}}{\tilde{P}}, \quad (22) \]

\(^{84}\)Note that the first-moment of money holdings is constant, since money supply is constant. \(^{85}\)Throughout this appendix, perceived variables have a tilde and actual outcomes do not.
where the dependence of prices on the aggregate state variables has been suppressed. Below, we discuss how we construct a histogram for the cross-sectional distribution each period and show that this property is satisfied. We do not specify a joint distribution of equity and money holdings. As discussed below, we do know each agent’s level of beginning-of-period equity holdings, $q_i$, and money holdings, $m_i$. 

A household’s cash-on-hand level is given by

$$\tilde{x}_i = e_i (1 - \tau) \frac{\bar{W}}{\bar{P}} + (1 - e_i) \mu (1 - \tau) \frac{\bar{W}}{\bar{P}} + q_i \left( \frac{\bar{D}}{\bar{P}} + (1 - \delta) \frac{\bar{J}}{\bar{P}} \right) + \frac{M_i}{\bar{P}},$$  \hspace{1cm} (23)

and the household can spend this on consumption and asset purchases, that is,

$$\bar{x}_i = c_i + \frac{\tilde{J}}{\bar{P}} + \frac{M_i'}{\bar{P}}.$$  \hspace{1cm} (24)

The government has a balanced budget each period, that is,

$$\tau = \mu \frac{1 - q}{q + \mu (1 - q)}.$$  \hspace{1cm} (25)

Even if the numerical solutions for $q_i', M_i', \tilde{J}$, and $\bar{P}$ are very accurate, it is unlikely that equilibrium is exactly satisfied if we aggregate $q_i'$ and $M_i'$ across agents. To impose equilibrium exactly, we modify the numerical approximations for equity and money holdings such that they are no longer completely pinned down by exogenous random variables and predetermined variables, but instead depend directly – to at least some extent – on prices. In the remainder of this section, we explain how we do this and how we solve for equilibrium prices.

**Modification and imposing equilibrium.** To impose equilibrium we adjust $q_i', M_i', \tilde{J}$, and $\bar{P}$. The equilibrium outcomes are denoted by $q_i, M_i, \tilde{J}$, and $\bar{P}$. The individual’s demand for assets is modified as follows:

$$q_{i+1} = \tilde{J}/\bar{P} q_i'$$  \hspace{1cm} (26)

$$M_{i+1} = \frac{P}{\bar{P}} M_i'.$$  \hspace{1cm} (27)

We will first discuss how equilibrium prices are determined and then discuss why this is a sensible modification. An important accuracy criterion is that this modification of the policy functions is small, that is, actual and perceived laws of motions are very similar.\(^{86}\)

\(^{86}\)The policy functions $q(\tilde{x}_i, \epsilon_i, q, z)$ and $M(\tilde{x}_i, \epsilon_i, q, z)$ do depend on prices, but this dependence is captured by the aggregate state variables.

\(^{87}\)As explained above, it is important to do a modification like this to ensure that equilibrium holds exactly, even if the solution is very accurate and the modification small.
We solve for the actual law of motion for employment, \( q_{+1} \), the number of new firms created, \( h \), the amount spent on creating new firms in real terms, \( v = hJ/P \), the market clearing asset price, \( J \), and the market clearing price level, \( P \), from the following equations:\(^{88}\)

\[
\begin{align*}
q_{+1} &= (1 - \delta) q + h, \\
h &= \psi v \eta (1 - q)^{1 - \eta}, \\
v &= hJ/P,
\end{align*}
\]

\[
h = \int \int_{\tilde{x}_i, \tilde{\eta}_i} \left( q_{i+1} (\tilde{x}_i, e_i, q, z) - (1 - \delta) q_i \right) d\psi_i
\]

\[
= \int \int_{\tilde{x}_i, \tilde{\eta}_i} \left( \bar{P} / \bar{J} \right) q(\tilde{x}_i, e_i, q, z) - (1 - \delta) q_i \right) d\psi_i,
\]

\[
\bar{M} = \int \int_{\tilde{x}_i, \tilde{\eta}_i} M_{i+1} (\tilde{x}_i, e_i, q, z) d\psi_i = \int \int_{\tilde{x}_i, \tilde{\eta}_i} \frac{P}{\bar{P}} M(\tilde{x}_i, e_i, q, z) d\psi_i.
\]

In particular, the distribution satisfies

\[
\int \int_{\tilde{x}_i, \tilde{\eta}_i} q_{i+1} d\psi_i = q_{+1}.
\]

**Logic behind the modification.** Recall that \( q(\tilde{x}_i, e_i, q, z) \) and \( m(\tilde{x}_i, e_i, q, z) \) are derived using perceived prices, \( \bar{J}(q, z) \) and \( \bar{P}(q, z) \). Now suppose that – in a particular period – aggregation of \( q(\tilde{x}_i, e_i, q, z) \) indicates that the demand for equity exceeds the supply for equity. This indicates that \( \bar{J}(q, z) \) is too low in that period. By exactly imposing equilibrium, we increase the asset price and lower the demand for equity. Note that our modification is such that any possible misperception on prices does not affect the real amount each agent spends, but only the number of assets bought.

Throughout this section, the value of cash on hand that is used as the argument of the policy functions is constructed using *perceived* prices. In principle, the equilibrium prices that have been obtained could be used to update the definition of cash on hand and one could iterate on this until convergence. This would make the simulation more expensive. Moreover, our converged solutions are such that perceived and actual prices are close to each other, which means that this iterative procedure would not add much.

**Equilibrium in the goods market.** It remains to show that our modification is such that the goods market is in equilibrium as well. That is, Walras’ law is not wrecked by our modification.

---

\(^{88}\)Recall that we define variables slightly different and \( v \) is not the number of vacancies, but the amount spent on creating new firms.
From the budget constraint we get that actual resources of agent \( i \) are equal to
\[
x_i = e_i (1 - \tau) \frac{W}{P} + (1 - e_i) \mu (1 - \tau) \frac{W}{P} + \left( \frac{D}{P} + (1 - \delta) \frac{I}{P} \right) q_i + M_i \tag{34}
\]
and actual expenditures are equal to
\[
x_i = c_i + \frac{I}{P} q_{i+1} + \frac{M_{i+1}}{P}. \tag{35}
\]
The value of \( c_i \) adjusts to ensure this equation holds. Aggregation gives
\[
x = zq + \frac{I}{P} (1 - \delta) q + \frac{M}{P} \tag{36}
\]
and
\[
x = c + \frac{I}{P} \int q_{i+1} d\psi_i + \frac{\int M_{i+1}}{P} = c + \frac{I}{P} q_{i+1} + \frac{M}{P}. \tag{37}
\]
Equation (36) uses the definition of dividends together with equation (33). Equation (37) follows from the construction of \( J \) and \( P \).

Since
\[
\frac{I}{P} q' - \frac{I}{P} (1 - \delta) q = v, \tag{38}
\]
we get
\[
zq = c + v, \tag{39}
\]
which means that we have goods market clearing in each and every time period.

**Implementation.** To simulate the economy, we use the “non-stochastic simulation method” developed in Young (2010). This procedure characterizes the cross-sectional distribution of agents’ characteristics with a histogram. This procedure would be computer intensive if we characterized the cross-sectional distribution of both equity and bond holdings. Instead, we just characterize the cross-sectional distribution of cash-on-hand for the employed and unemployed. Let \( \psi(\tilde{x}, e) \) denote last period’s cross-sectional distribution of the cash-on-hand level and employment status. The objective is to calculate \( \psi(\tilde{x}, e) \).

(i) As discussed above, given \( \psi(\tilde{x}, e) \) and the policy functions, we can calculate last period’s equilibrium outcome for the total number of firms (jobs) carried into the current period, \( q \); the job-finding rate, \( h_{-1} / (1 - q_{-1}) \); last period’s prices, \( I_{-1} \) and \( P_{-1} \); and for each individual the equilibrium asset holdings brought into the current period, \( q_{i} (\tilde{x}_{i-1}, e_{i-1}) \) and \( M_{i} (\tilde{x}_{i-1}, e_{i-1}) \).

(ii) Current employment, \( q \), together with the current technology shock, \( z \), allows us to calculate
perceived prices $\tilde{f}$ and $\tilde{P}$.

(iii) Using the perceived prices together with the asset holdings $q_i$ and $M_i$, we calculate perceived cash on hand conditional on last-period’s cash-on-hand level and both the past and the present employment status. That is,

$$\tilde{x}(e_i, \tilde{x}_{i-1}, e_{i-1}) = e_i(1 - \tau) \frac{\tilde{W}}{P} + (1 - e_i)\mu(1 - \tau) \frac{\tilde{W}}{P}$$

$$+ q_i(\tilde{x}_{i-1}, e_{i-1}) \left( \frac{\tilde{D}}{P} + (1 - \delta) \frac{\tilde{f}}{P} \right) + M(\tilde{x}_{i-1}, e_{i-1}).$$

(iv) Using last period’s distribution $\psi(\tilde{x}_{i-1}, e_{i-1})$ together with last-period’s transition probabilities, we can calculate the joint distribution of current perceived cash on hand, $\tilde{x}_i$, past employment status, and present employment status, $\tilde{\psi}(\tilde{x}_i, e_i, e_{i-1})$.

(v) Next, we retrieve the current period’s distribution as

$$\psi(\tilde{x}_i, 1) = \tilde{\psi}(\tilde{x}_i, 1, 1) + \tilde{\psi}(\tilde{x}_i, 1, 0), \quad (40)$$

$$\psi(\tilde{x}_i, 0) = \tilde{\psi}(\tilde{x}_i, 0, 1) + \tilde{\psi}(\tilde{x}_i, 0, 0). \quad (41)$$

(vi) Even though we never explicitly calculate a multi-dimensional histogram, in each period we do have information on the joint cross-sectional distribution of cash on hand at perceived prices and asset holdings.

Details. Our wage-setting rule (7), contains $\tilde{P}$, an indicator for the average price level. For convenience, we use the average between the long-run expansion and the long-run recession value.\textsuperscript{89} Since it is a constant, it could be combined with the scaling factor, $\omega_0$. The properties of the algorithm are improved by including $\tilde{P}$. If a term like $\tilde{P}$ would not be included, then average wages would change across iteration steps. Moreover, without such a term, then recalibrating $\omega_0$ would be more involved, for example, if one compares the case with and the case without aggregate uncertainty. We use a simulation of 2,000 observations to estimate the coefficients of the laws of motion for aggregate variables. The first 200 observations are dropped to ensure the results are not affected by the specification of the initial state. The histogram that we use to track the cross-sectional distribution has 2,000 grid points. Statistics reported in the main text that are obtained by simulation are from a sample of 100,000 observations.

\textsuperscript{89}This actually is a good approximation of the average price level.
B.2.3 Accuracy

Conditional on perceived laws of motion for the price level and the employment rate, individual policy rules can be solved accurately using common numerical tools even though the presence of a portfolio problem makes the individual optimization problem a bit more complex than the standard setup in heterogeneous-agent models. The key measure of accuracy is, therefore, whether the perceived laws of motion for the price level and the employment rate coincide with the corresponding laws of motion that are implied by the individual policy rules and market clearing. Figure 17 shows that both perceived laws of motion track the implied market clearing outcome very closely.

C Equivalence with standard matching framework.

In the standard matching framework, new firms are created by “entrepreneurs” who post vacancies, $\tilde{v}_t$, at a cost equal to $\kappa$ per vacancy. The number of vacancies is pinned down by a free-entry condition. In the description of the model above, such additional agents are not introduced. Instead, creation of new firms is carried out by investors wanting to increase their equity holdings.

Although, the “story” we tell is somewhat different, our equations can be shown to be identical to those of the standard matching model. The free-entry condition in the standard matching model is given by

$$\kappa = \frac{\tilde{h}_t}{\tilde{v}_t} \tilde{J}_t$$

(42)

where

$$\tilde{h}_t = \tilde{\psi} \tilde{v}_t^{\eta} u_t^{1-\eta}.$$  (43)

Each vacancy leads to the creation of $\tilde{h}_t / \tilde{v}_t$ new firms, which can be sold to households at price $\tilde{J}_t$.

Equilibrium in the equity market requires that the net demand for equity by households is equal to the supply of new equity by entrepreneurs, that is

$$\int_i (q(e_i, q_i, M_i; s_t) - (1 - \delta) q_i) dF_i (e_i, q_i, M_i) = \tilde{v}_t \tilde{\psi} \tilde{v}_t^{\eta} u_t^{1-\eta}.$$  (44)

Using equations (42) and (43), this equation can be rewritten as

$$\int_i (q(e_i, q_i, M_i; s_t) - (1 - \delta) q_i) dF_i (e_i, q_i, M_i) = \tilde{\psi}^{1/(1-\eta)} \left( \frac{\tilde{J}_t}{\kappa} \right)^{\eta/(1-\eta)} u_t.$$  (45)

This is equivalent to equation (13) if

$$\tilde{\psi} = \psi \kappa^{\eta}.$$  (46)
It only remains to establish that the number of new jobs created is the same in the two setups, that is,

\[ h_t = \tilde{h}_t \]  

(47)

or

\[ \psi \nu_{1}^\eta u_{1}^{1-\eta} = \tilde{\psi} \nu_{1}^\eta u_{1}^{1-\eta}. \]  

(48)

From equations (42) and (43), we get that

\[ \tilde{v}_t = \left( \frac{\psi J_t \kappa \tilde{v}_t}{\kappa \tilde{v}_t} \right)^{1/(1-\eta)} u_t. \]  

(49)

Substituting this expression for \( \tilde{v}_t \) and the expression from equation (11) for \( v_t \) into equation (48) gives indeed that \( h_t = \tilde{h}_t \). Moreover, the total amount spent on creating new firms in our representation, \( v_t \), is equal to the number of vacancies times the posting cost in the traditional representation, \( \kappa \tilde{v}_t \).

The focus of this paper is on the effect of negative shocks on the savings and investment behavior of agents in the economy when markets are incomplete. We think that our way of telling the story behind the equations has the following two advantages. First, there is only one type of investor, namely, the household and there are no additional investors such as zombie entrepreneurs (poor souls who get no positive benefits out of fulfilling a crucial role in the economy). Second, all agents have access to investment in the same two assets, namely equity and the liquid asset, whereas in the standard labor market model there are households and entrepreneurs and they have different investment opportunities.

D Idiosyncratic labor income risk and demand for risky assets

Here we prove that an increase in idiosyncratic risk increases the demand for equity when – as is the case in typical macroeconomic models – the wage rate and the return on investment are affected by the same factor.

\[
\max_{c_1, c_2, \tilde{b}, \tilde{q}} \frac{c_1^{1-\gamma} - 1}{1 - \gamma} + \beta \mathbb{E} \left[ \frac{c_2^{1-\gamma} - 1}{1 - \gamma} \right]
\]

s.t.

\[ c_1 + p_q \tilde{q} + p_b \tilde{b} = y_{1}, \]  

(50)

\[ c_{2,i} = \tilde{q} \tilde{y}_q + b + \tilde{y}_i \left( 1 + \sigma_{\eta} \tilde{\eta}_i \right), \]  

(51)

\[ \tilde{\eta}_i \]

One could argue that entrepreneurs are part of the household, but with heterogeneous households the question arises which households they belong to.
where $\eta_i$ is an idiosyncratic component that is i.i.d. distributed.

The Euler equations are given by

\begin{align}
 p_b c_1^{-\gamma} &= \beta \mathbb{E} \left[ c_2^{-\gamma} \right], \\
p_q c_1^{-\gamma} &= \beta \mathbb{E} \left[ c_2^{-\gamma} \tilde{y}_q \right].
\end{align}

Kimball (1992) considers the case in which the common component of labor income, $\tilde{y}_l$, is constant and, thus, not correlated with the return of the risky investment. He shows that an increase in idiosyncratic uncertainty leads to a decrease in the amount in invested in the risky asset even though total savings increase. Here we make the assumption that labor income and the return on the risky investment are correlated, because average labor income, $\tilde{y}_l$ is correlated with the return on the risky investment. In particular, we assume that

\begin{align}
 \tilde{y}_q &= a \tilde{y}, \\
 \tilde{y}_l &= (1 - a) \tilde{y}.
\end{align}

Also,

\begin{align}
 \mathbb{E} [\tilde{\eta}_i] &= 0, \quad \mathbb{E} [\tilde{\eta}_i^2] = 1, \\
 \mathbb{E} [\tilde{y}_q] &= 1.
\end{align}

**Proposition.** Suppose that the random variables satisfy equations (54) through (57) and agent’s choices are determined by equations (50) through (53). Let $\bar{b}$ and $\bar{q}$ denote the values for $b$ and $q$ when $\sigma_\eta = \sigma_\bar{\eta}$. Prices are such that $\bar{b} = 0$. Let $\bar{b}$ and $\bar{q}$ denote the values for $b$ and $q$ when $\sigma_\eta = \sigma_\bar{\eta}$. If

\begin{equation}
 \sigma_\eta > \sigma_\bar{\eta}
\end{equation}

then

\begin{align}
 \hat{b} &= \bar{b} = 0, \\
 \hat{q} &> \bar{q}.
\end{align}

**Proof.** Since $\bar{b}$ and $\bar{q}$ satisfy the agent’s first-order conditions and $\bar{b} = 0$, we know that the following two equations hold:

\begin{align}
 p_b (y_1 - p_q \tilde{y})^{-\gamma} &= \beta \mathbb{E} \left[ (\tilde{q} \tilde{y}_q + \tilde{y}_l (1 + \sigma_\eta \tilde{\eta}_i))^{-\gamma} \right], \\
p_q (y_1 - p_q \tilde{y})^{-\gamma} &= \beta \mathbb{E} \left[ (\tilde{q} \tilde{y}_q + \tilde{y}_l (1 + \sigma_\eta \tilde{\eta}_i))^{-\gamma} \tilde{y}_q \right].
\end{align}
Using equations (54) and (55) and the fact that \( \tilde{\eta}_i \) is an idiosyncratic random variable and, thus, not correlated with \( \tilde{y} \), we can rewrite these two equations as

\[
\begin{align*}
  & p_b \left( y_1 - p_q \tilde{\eta} \right)^{-\gamma} = \beta \mathbb{E} \left[ (\tilde{q}\alpha + (1 - \alpha) (1 + \tilde{\sigma}_\eta \tilde{\eta}_i))^{-\gamma} \right] \mathbb{E}[\tilde{y}_q^{-\gamma}], \quad (63) \\
  & p_q \left( y_1 - p_q \tilde{\eta} \right)^{-\gamma} = \beta \mathbb{E} \left[ (\tilde{q}\alpha + (1 - \alpha) (1 + \tilde{\sigma}_\eta \tilde{\eta}_i))^{-\gamma} \right] \mathbb{E}[\tilde{y}_q^{-\gamma}]. \quad (64)
\end{align*}
\]

Combining gives

\[
  p_b = p_q \frac{\mathbb{E}[\tilde{y}^{-\gamma}]}{\mathbb{E}[\tilde{y}^1-\gamma]}.
\]

\( \hat{b} \) and \( \hat{q} \) satisfy the following two equations:

\[
\begin{align*}
  & p_b \left( y_1 - p_q \hat{\eta} - p_b \hat{b} \right)^{-\gamma} = \beta \mathbb{E} \left[ (\hat{q}\tilde{y}_q + \hat{y}_l (1 + \hat{\sigma}_\eta \tilde{\eta}_i))^{-\gamma} \right], \quad (66) \\
  & p_q \left( y_1 - p_q \hat{\eta} - p_b \hat{b} \right)^{-\gamma} = \beta \mathbb{E} \left[ (\hat{q}\tilde{y}_q + \hat{y}_l (1 + \hat{\sigma}_\eta \tilde{\eta}_i))^{-\gamma} \tilde{y}_q \right]. \quad (67)
\end{align*}
\]

To check whether \( \hat{b} = 0 \) is also the solution when \( \sigma_\eta = \hat{\sigma}_\eta \), we substitute \( \hat{b} = 0 \) in the two equations above and check whether both equations would give the same solution for \( \hat{q} \). Substituting \( \hat{b} = 0 \) gives

\[
\begin{align*}
  & p_b \left( y_1 - p_q \hat{\eta} \right)^{-\gamma} = \beta \mathbb{E} \left[ (\hat{q}\tilde{y}_q + \tilde{y}_l (1 + \hat{\sigma}_\eta \tilde{\eta}_i))^{-\gamma} \right], \quad (68) \\
  & p_q \left( y_1 - p_q \hat{\eta} \right)^{-\gamma} = \beta \mathbb{E} \left[ (\hat{q}\tilde{y}_q + \tilde{y}_l (1 + \hat{\sigma}_\eta \tilde{\eta}_i))^{-\gamma} \tilde{y}_q \right], \quad (69)
\end{align*}
\]

which can be rewritten as

\[
\begin{align*}
  & p_b \left( y_1 - p_q \hat{\eta} \right)^{-\gamma} = \beta \mathbb{E} \left[ (\hat{q}\tilde{\eta} + (1 - \alpha) (1 + \hat{\sigma}_\eta \tilde{\eta}_i))^{-\gamma} \right] \mathbb{E}[\tilde{y}_q^{-\gamma}], \quad (70) \\
  & p_q \left( y_1 - p_q \hat{\eta} \right)^{-\gamma} = \beta \mathbb{E} \left[ (\hat{q}\tilde{\eta} + (1 - \alpha) (1 + \hat{\sigma}_\eta \tilde{\eta}_i))^{-\gamma} \tilde{y}_q \right]. \quad (71)
\end{align*}
\]

If we use equation (65), we get that both equations are identical and, thus, would give the same solution for \( \hat{q} \).

It remains to show that \( \hat{q} > q \). An increase in \( \sigma_\eta \) means that the right-hand side of both Euler equations increases. If \( q \) would decrease, then the right-hand sides would increase further and the left-hand sides would decrease, which clearly could not lead to a solution.
E Information about accuracy

Figure 16: Accuracy representative-agent solution.

Notes. These graphs plot the time series for the employment rate generated with the indicated solution method and the exact solution according to the Euler equation when the approximation is only used to evaluate next period’s choices.
Table 1: Accuracy comparison - Representative Agent Model

<table>
<thead>
<tr>
<th></th>
<th>projection</th>
<th>linear perturbation</th>
<th>log-linear perturbation</th>
</tr>
</thead>
<tbody>
<tr>
<td>average error (%)</td>
<td>$0.84 \times 10^{-5}$</td>
<td>1.01</td>
<td>0.43</td>
</tr>
<tr>
<td>maximum error (%)</td>
<td>$0.28 \times 10^{-4}$</td>
<td>1.76</td>
<td>0.74</td>
</tr>
<tr>
<td>average unemployment rate (%)</td>
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<td>10.7</td>
<td>10.7</td>
</tr>
<tr>
<td>standard deviation employment</td>
<td>2.91</td>
<td>2.63</td>
<td>2.87</td>
</tr>
</tbody>
</table>

Notes: These results are based on a sample of 100,000 observations.

Figure 17: Accuracy heterogeneous-agent solution.

Notes. These graphs plot for the indicated variable the timeseries according to the perceived law of motion (used to solve for the individual policy rules) and the actual outcomes consistent with market clearing.
References


