The Fundamental Nature of HARA Utility

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Many models in Macroeconomics and Finance posit that consumers/investors seek to maximize or optimize a utility function. This is a way of modelling behavior, whereby individuals want to maximize their well-being, as captured by the utility function. Typically the form of the utility function used in these fields is a particular mathematical formulation called the Hyperbolic Absolute Risk Aversion (HARA). This mathematical form has utility being a power function of a linear function of wealth. The HARA form is prevalent because it is theoretically convenient and has been relatively successful empirically.

This paper shows that HARA utility is more fundamental to economic analysis. It shows that the HARA functional form is the unique form which satisfies basic economic principles in an optimization context. Using HARA is therefore not just a matter of convenience or tractability but rather emerges from economic reasoning, i.e., it is inherent in the economic optimization problem.

The paper applies a mathematical technique, widely used in Physics, called Lie symmetries, to the optimality equation of Nobel Laureate Robert Merton's canonical model of the consumer-investor. It does so in order to show the inherent nature of the HARA utility function. Lie symmetries derive the conditions whereby the optimal solution retains the same form under (scale) transformations of wealth. Such transformations may arise as the result of growth of wealth over time or due to the effects of policy. For example, taxes change individuals' levels of wealth and engender such transformations. The symmetries place restrictions on the model, with the key one being the need to use of HARA utility. This means that if one wants to get sensible results of consumer/investor optimization when wealth is subject to the afore-cited transformations, one **has** to use the HARA form.

The paper shows that this property – scale invariance of agents' wealth – implies linear optimal solutions to consumption and portfolio allocation and linear risk tolerance.

The results have broad implications, as the model studied is a fundamental one in Macroeconomics and Finance. The paper demonstrates the use of Lie symmetries as a powerful tool to deal with economic optimization problems.