

Unions in a Frictional Labor Market*

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Abstract

We analyze a labor market with search and matching frictions where wage setting is controlled by a monopoly union. Frictions render existing matches a form of firm-specific capital which is subject to a hold-up problem in a unionized labor market. We study how this hold-up problem manifests itself in a dynamic infinite horizon model, with fully rational agents. We find that wage solidarity, seemingly an important norm governing union operations, leaves the unionized labor market vulnerable to potentially substantial distortions due to hold-up. Introducing a tenure premium in wages may allow the union to avoid the problem entirely, however, potentially allowing efficient hiring. Under an egalitarian wage policy, the degree of commitment to future wages is important for outcomes: with full commitment to future wages, the union achieves efficient hiring in the long run, but hikes up wages in the short run to appropriate rents from firms. Without commitment, and in a Markov-perfect equilibrium, hiring is well below its efficient level both in the short and the long run. We demonstrate the quantitative impact of the union in an extended model with partial union coverage and multi-period union contracting.

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1 Introduction

Labor unions play an important role in many labor markets in many countries. There is also a large literature within labor economics studying how union presence influences labor market outcomes. Yet there is relatively little work studying the impact of this institution on the labor market when this market is described as having frictions and featuring unemployment

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due to these frictions. Since search and matching models have come to play a central role as a workhorse for macroeconomic labor market analyses, this gap in the literature leaves open important questions. What is the impact of unions on unemployment and wages? How do unions affect how strongly unemployment varies over the business cycle? What institutional settings are desirable, when considering implementing rules regarding union coverage?

Our model can be interpreted as representing either the aggregate labor market or an industry labor market, but in either case we consider the case of a “large” union, which has monopoly power over some group of workers. This case is of particular relevance for many European economies, where there is a nationwide union or cooperation/agreements among unions representing different industries. It is also of relevance in other settings where workers cannot easily move across industries, and competition among different unions within an industry is limited. We assume the union to be fully rational, taking job creation into account when making its wage demands, and its objective to be the welfare of all workers covered by union wages.

In our model, all workers have the same productivity and fulfil equally productive jobs. Our starting point is a view that union operations are governed by a norm of solidarity and egalitarianism among workers, which leads us to the assumption that unions impose identical wages across these workers. This view can be motivated in part by the broad empirical evidence documenting that unions compress the distribution of wages. We find that such fairness comes at a non-trivial cost, however, as it leaves the unionized labor market vulnerable to a potentially severe hold-up problem, which leads to inefficiently high wages and low job creation.

Under the egalitarian wage policy, the degree to which the union can commit to future wages becomes qualitatively and quantitatively important for outcomes.¹ If the union is able to fully commit to future wages, it attains an efficient level of unemployment in the long run. In the short run, however, unemployment is inefficiently high because the union uses its market power to raise current wages above the efficient level, in order to extract rents from firms with pre-existing matches. Specifically, we show that labor market tightness is inefficiently low in the initial period, but efficient from then on. These elements give rise to a time inconsistency: if a union had decided on a commitment plan yesterday, but had the opportunity to revise it today, it would indeed revise it to benefit again from pre-existing matches.

What would happen if the union did not have commitment to future wages? How large would the effects on the labor market be? We answer this question by analyzing Markov-perfect equilibria.² In a calibrated model, the presence of the union raises wages by 12.5%, consequently raising unemployment from 5% to 16.5%, and reducing output by 12%, relative to efficient outcomes. The distortions associated with the union diminish as the duration

¹The degree of commitment to wages is important in hold-up problems in general, with full commitment potentially avoiding the hold-up problem entirely. In the dynamic model with an egalitarian wage policy the situation is more involved, however, because even in the union problem with full commitment there are some workers who were hired in the past and whose wages will, in part, be set after they have already been hired.

²Our focus is on differentiable Markov-perfect equilibria. Thus we do not consider other equilibria where history matters, such as sustainable plans equilibria (Chari and Kehoe 1990).

of union contracts increases, but this effect appears quantitatively weak: the effects remain very similar as we vary duration from one to three years, viewed as the empirically relevant range of union contract durations (Taylor 1983).

In a classic paper, Calmfors and Driffill (1988) reconsidered the impact of unions on the level of aggregate economic activity. It has long been recognized that unions, through their monopoly power in the labor market, tend to raise wages above their competitive levels, suggesting that a greater union presence in the labor market has a primarily negative impact on economic activity. Calmfors and Driffill (1988) propose an additional factor for understanding the cross-country evidence on unions: they argue that the degree of coordination in union bargaining works to counteract the negative effects of monopoly power. Our model generates a related hump-shaped relationship, which we illustrate in the quantitative section, where we allow for partial union coverage of the workforce. Because union wages tend to be higher than non-union wages, greater union coverage tends to lead to higher unemployment in our model as well. But greater union coverage also increases the extent to which the union takes into account the effects of its wage demands on hiring, borne by union and non-union workers alike, leading to moderation in union wage setting. As we increase union coverage, the second effect eventually takes over the first, leading to a hump-shaped relationship.

An important motivation for macroeconomists to think about unions has been the idea that union wages are less responsive to shocks, potentially helping understand the observed variability of employment (see, e.g., Blanchard and Fischer 1989). The model we study in the quantitative section builds in significant stickiness in wages, due to the union recontracting only every one to three years. We demonstrate the substantial impact this has on shock propagation in the model, with amplification in the responses of vacancy creation, employment, and output to shocks.

Finally, while we view egalitarianism a characteristic of union operations, we also show that relaxing the egalitarian wage policy, for example by allowing a tenure premium in union wages, can provide the union sufficient instruments to avoid the holdup problem, even entirely. In this case the union extract rents from firms with high wages for senior workers, while setting the wages of junior workers low enough to nevertheless encourage hiring. Unless the union runs into a binding constraint on how low the wages of junior workers can be (possibly negative), efficient hiring attains. The model thus implies a rationale for a tenure premium in union wages.

Related literature There exists a set of papers developing extensions of the Mortensen-Pissarides model with a union/unions governing wage determination. Perhaps closest in spirit to our paper in this group is Pissarides (1986), which first introduces a monopoly union into the Pissarides (1985) framework, and studies the impact on equilibrium outcomes in the labor market. As the literature following it, that paper focuses on steady states, however, side-stepping the dynamic issues we highlight here. The more recent papers are more applied: Garibaldi and Violante (2005) and Boeri and Burda (2009) study the effects of employment protection policies, Ebell and Haefke (2006) the effects of product market regulation, Acikgoz and Kaymak (2014) the evolution of skill premia and unionization rates over time. These papers generally adopt frameworks imposing exogenous wage compression

into union wage setting, with the exception of Taschereau-Dumouchel (2011) who develops a framework where it is endogenous. Delacroix (2006) extends the framework of Ebell and Haefke (2006) to capture the U-shaped relationship between the degree of coordination in union bargaining and economic performance postulated by Calmfors and Driffill (1988).³

The paper is organized as follows. Section 2 begins with a brief overview of the empirical evidence on unions. Section 3 analyzes the benchmark model: first a one-period model to provide intuition, and then an infinite-horizon model with and without commitment. Section 4 turns to a quantitative illustration in the context of an extended model and Section 5 concludes.

2 Evidence on Unions, Wages and Unemployment

Most workers in the OECD, outside the US, have their wages determined by union agreements. This cross country evidence is discussed by Nickell and Layard (1999), who report that in most European countries the share of workers covered by union wages exceeds 70%. An important feature of the cross-country evidence to keep in mind is that union coverage rates – the share of the labor force whose wages are determined by union wage bargaining – generally exceed union membership rates outside of the US. Even in countries where union membership rates are low, such as France, within firms many non-union workers are paid the union wage, and in many countries union wages are legally extended to cover non-union firms as well.⁴

In terms of the effects of unions, Nickell and Layard (1999) show that a cross-country regression of unemployment on measures of union membership and coverage reveals a positive relationship between union presence and unemployment. But there is also significant heterogeneity across countries in the degree of centralization and coordination in union bargaining, as highlighted by Calmfors and Driffill (1988), and it turns out that this positive relationship between union presence and unemployment can partly be offset by measures of coordination in bargaining.

Nickell and Layard (1999) also report that union membership is associated with higher wages on the individual level across countries. A large literature has studied this union/non-union wage gap, using a variety of data sources and econometric approaches. Lewis (1986) reviews the literature for the US, concluding that the evidence points to an upper bound of 15% for the union wage gap. More recently, Blanchflower and Bryson (2003) confirm that the

³Lockwood and Manning (1989) and Modesto and Thomas (2001) have studied union wage-setting in labor markets where firms face adjustment costs to labor, developing the idea that dynamic concerns become important for thinking about union decision-making when labor markets are not fully frictionless. The simple partial equilibrium quadratic adjustment cost framework adopted in these papers affords closed-form results which speak to the level of union wage demands, as well as the speed of adjustment in firm-level employment. Our work brings these ideas into an equilibrium framework, which allows thinking about unemployment and vacancy creation as well.

⁴Visser (2003) also documents union membership and coverage rates across countries, reporting an average coverage rate of 73 % across European countries for the period 1985-97. While union membership has been on the decline in Europe as well as the US, coverage levels remain substantially higher in Europe.

estimates of the wage gap have remained relatively stable, with perhaps a modest decline over time. They also report estimates across countries, noting that in many European countries the extensive coverage of union wages reduces these gaps. An important concern with the estimates of the union wage gap in general involves selection on unobservables: it is likely that higher union wages attract better workers, but the data do not allow controlling for these differences properly, biasing the estimates of the wage gap. When DiNardo and Lee (2004) adopt a regression discontinuity design to get around some of the issues, they find a negligible wage gap, seemingly contradicting a large body of evidence.⁵

A robust finding appears to be that unions reduce wage inequality, compressing the distribution of wages (Card, Lemieux, and Riddell 2003). Do they compress wages across degrees of seniority as well? Certainly formal pay scales appear common in union compensation practices, but arguably wages rise with tenure in non-union settings as well. Perhaps because unions tend to compress the distribution of wages, a number of earlier studies have actually reported a stronger association between tenure and earnings in non-union settings. But properly estimating returns to tenure is challenging and the comparison confounded by the fact that the estimates tend to be biased by worker and job heterogeneity, generally found to be greater in non-union than union settings.⁶ Recognizing these challenges, Abraham and Farber (1988) find a stronger association between tenure and earnings in the unionized setting, supporting the idea that seniority plays an important role in union operations. At the same time, Topel (1991) finds no significant difference in returns to tenure based on union status. Again, data limitations leave us short of a conclusive answer, but the evidence in favor of overall wage compression does appear robust.

3 The model

This section begins by describing the simple Mortensen-Pissarides search and matching environment we base our analysis on. We then introduce a monopoly union into that framework, and characterize its behavior.

A frictional labor market Time is discrete and the horizon infinite. The economy is populated by a continuum of measure one identical workers, together with a continuum of identical capitalists who employ these workers. All agents have linear utility, and discount the future at rate $\beta < 1$. Capitalists have access to a linear production technology, producing z units of output per period for each worker employed. In addition to this market production technology, unemployed workers also have access to a home production technology, producing $b(< z)$ units of output per period.

The labor market is frictional, requiring capitalists seeking to hire workers to post vacancies.

⁵Their study focuses on close union election outcomes in the US. Of course it is possible that wage gaps at workplaces with close election outcomes are smaller than at those with clear cut ones, as well as that wages at newly unionized workplaces are different from those with an established union presence.

⁶The magnitude of returns to tenure is a debated topic, see for example Altonji and Williams (2005) and Buchinsky, Fougere, Kramarz, and Tchernis (2010).

The measure of matches in the beginning of the period is denoted by $n \in [0, 1]$, leaving $1 - n$ workers searching for jobs. Searching workers and posted vacancies are matched according to a constant-returns-to-scale matching function $m(v, 1 - n)$, where v is the measure of vacancies. With this, the probability with which a searching worker finds a job within a period can be written $\mu(\theta) = m(\theta, 1)$, and the probability with which a vacancy is filled $q(\theta) = m(1, 1/\theta)$, where $\theta = v/(1 - n)$ is the labor market tightness. We assume that $\mu'(\theta)$ is positive and decreasing and $q'(\theta)$ negative and increasing. With this, employment equals n plus the measure of new matches, $\mu(\theta)(1 - n)$. Jobs are destroyed each period with probability δ . Thus, the measure of matches evolves over time according to the law of motion

$$n_{t+1} = (1 - \delta) \underbrace{(n_t + \mu(\theta_t)(1 - n_t))}_{\text{employed}_t}. \quad (1)$$

Firms Capitalists operate production through firms, and these firms need to post vacancies in order to find workers, at a cost κ per vacancy. Competition drives profits from vacancy creation to zero, with firms taking into account the union wage-setting behavior today and in the future. The zero-profit condition thus determines the current market tightness according to current and future wages $\{w_{t+s}\}_{s=0}^{\infty}$ as follows:

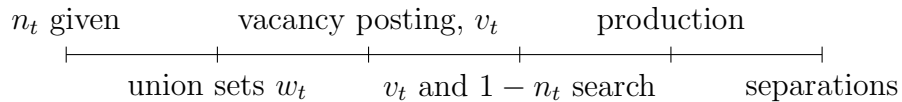
$$\kappa = q(\theta_t) \sum_{s=0}^{\infty} \beta^s (1 - \delta)^s [z - w_{t+s}]. \quad (2)$$

Union Wages are set unilaterally by a labor union, with universal coverage. The union sets wages to maximize the welfare of all workers, with equal pay for all those employed.⁷ The union objective thus becomes

$$\sum_{t=0}^{\infty} \beta^t \left[\underbrace{(n_t + \mu(\theta_t)(1 - n_t))}_{\text{employed}_t} w_t + \underbrace{(1 - n_t)(1 - \mu(\theta_t))}_{\text{unemployed}_t} b \right]. \quad (3)$$

The union takes as given the evolution of employment according to equation (1). It also internalizes the effect of its wage-setting decisions on hiring. Therefore, the union's problem is to choose a sequence of wages $\{w_t\}_{t=0}^{\infty}$ to maximize the objective (3) subject to the law of motion (1) and zero profit condition (2). The union must also respect the constraint that the firms, at each point in time, make a non-negative present value of profits on existing matches, as they could simply end them otherwise. This is implied by positive vacancy posting, however, because if firms posting vacancies break even, existing matches must have strictly positive value.

Summarizing the events in period t , we have



⁷Note that if we normalize $b = 0$, then the union objective becomes the total wage bill.

Given the path of wages $\{w_t\}_{t=0}^\infty$, then, equation (2) determines the path of market tightness $\{\theta_t\}_{t=0}^\infty$, which in turn determines the evolution of employment.

3.1 One-period example

To illustrate key forces at play, we first consider the impact of the union in a very simple setting: a one-period version of the above economy. Many of the features present here will be present in the subsequent analysis.

Planner A natural starting point is the efficient benchmark—the output-maximizing level of vacancy creation a social planner would choose. Here the planner solves the problem

$$\max_{\theta} \left(\underbrace{n + \mu(\theta)(1 - n)}_{\text{employed}} \right) z + \underbrace{(1 - n)(1 - \mu(\theta))}_{\text{unemployed}} b - \underbrace{\theta(1 - n)}_{\text{vacancies}} \kappa,$$

taking as given pre-existing matches n . The planner's optimum is characterized by the first-order condition $-\kappa + \mu'(\theta)(z - b) = 0$, which pins down θ independent of n . For concreteness, consider the matching function $m(v, u) = vu/(v + u)$, such that $\mu(\theta) = \theta/(1 + \theta)$. In this case the planner's optimum is given by $\theta^p = \sqrt{(z - b)/\kappa} - 1$, with market tightness an increasing function of market productivity. Of course, we must have $z - b > \kappa$ for vacancy creation to be optimal.

Union The union instead aims to maximize the welfare of workers

$$\left(\underbrace{n + \mu(\theta)(1 - n)}_{\text{employed}} \right) w + \underbrace{(1 - n)(1 - \mu(\theta))}_{\text{unemployed}} b,$$

by choice of w and θ , subject to the zero-profit condition: $\kappa = q(\theta)(z - w)$. The tradeoff the union faces here is that while higher wages increase the welfare of employed workers, they also reduce the job finding probability, because of reduced job creation.

To see how this problem relates to the planner's problem, we can use the zero-profit condition to solve for the wage, as $w = z - \kappa/q(\theta)$, and substitute into the union objective to yield a maximization problem in θ only:

$$\begin{aligned} & \max_{\theta} \left(\underbrace{n + \mu(\theta)(1 - n)}_{\text{employed}} \right) \left(z - \frac{\kappa}{q(\theta)} \right) + \underbrace{(1 - n)(1 - \mu(\theta))}_{\text{unemployed}} b \\ & = \max_{\theta} - \underbrace{\frac{n\kappa}{q(\theta)}}_{\text{capitalists' share}} + \underbrace{(n + \mu(\theta)(1 - n))z + (1 - n)(1 - \mu(\theta))b - \theta(1 - n)\kappa}_{\text{planner's objective}}, \end{aligned}$$

also taking as given n .⁸ From the second line, we see that the union objective differs from the planner's objective only by the term $-\frac{n\kappa}{q(\theta)}$. To understand how the two objectives relate

⁸This substitution assumes some vacancy creation is optimal. The union could also opt to simply set $w = z$ in the original problem, achieving the value $b + n(z - b)$ for the objective (forgoing vacancy costs entirely). To ensure the solution in the text is optimal, it is necessary to make sure the value of the objective exceeds this value.

to each other, recall that while the planner cares about all agents in the economy, the union only cares about workers. The union objective thus equals the planner's objective less the capitalists' share of total output: the profits on existing matches $n(z - w) = \frac{n\kappa}{q(\theta)}$, where the equality follows from the zero-profit condition.

An interior union optimum is characterized by the first-order condition $-\kappa + \kappa \frac{n}{1-n} \frac{q'(\theta)}{q(\theta)^2} + \mu'(\theta)(z - b) = 0$, which implies that the union's choice of θ *does depend on* n . In our example, an interior union optimum is given by $\theta = \sqrt{1 - n} \sqrt{(z - b)/\kappa} - 1$. Labor-market tightness is thus again an increasing function of market productivity, but now decreases in pre-existing matches. Clearly the union implements the socially optimal level of vacancy creation if $n = 0$. But if $n > 0$, the union has an incentive to raise wages above the efficient level, in order to appropriate surpluses from firms with existing matches.

Finally, note that a non-egalitarian union would instead solve the problem

$$\begin{aligned} \max_{\theta, w^e, w^n} \quad & n w^e + \mu(\theta)(1 - n)w^n + (1 - \mu(\theta))(1 - n)b \\ \text{s.t.} \quad & q(\theta)(z - w^n) = \kappa, \\ & w^e \leq z, \end{aligned}$$

where we allow the union to pay different wages to newly hired workers, w^n , and workers in existing matches, w^e . If we allow $w^e \neq w^n$, it is immediately optimal to set $w^e = z$. Substituting this into the union objective then yields the planner objective above, along with the same condition for optimal hiring: $-\kappa + \mu'(\theta)(z - b) = 0$. With this market tightness, the wage in new matches is then given by $w^n = z - \kappa/q(\theta)$, implying a tenure premium in union wages: $w^n < w^e$.

This non-egalitarian case demonstrates that the inefficiency in the initial union problem stems from the constraint to treat workers identically.⁹ The theory thus implies a rationale for tenure premia in union wages, which could—in the absence of a binding lower bound on the wages of junior workers—even allow the union to attain efficient hiring.

Next, we return to the dynamic infinite horizon setting, where the measure of initial matches is endogenous.

⁹These distortions arise because search frictions render existing matches a form of firm-specific capital which is subject to a hold-up problem. As is typically the case, the degree of commitment to wages is important for the severity of the hold-up problem. In the extreme case, if wages were set after vacancy creation takes place (rather than before), the union would simply set (both) wages equal to z , with no new hiring taking place. The timing here allows the union to commit to wages before vacancy creation, however, making outcomes less severe.

3.2 Efficient outcomes

To characterize union wage-setting when the time horizon is infinite, we again begin with the efficient benchmark. The planner now chooses a sequence $\{\theta_t\}_{t=0}^{\infty}$, with $\theta_t \geq 0$, to maximize

$$\sum_{t=0}^{\infty} \beta^t \left[\underbrace{(n_t + \mu(\theta_t)(1 - n_t))}_{\text{employed}_t} z + \underbrace{(1 - n_t)(1 - \mu(\theta_t))}_{\text{unemployed}_t} b - \underbrace{\theta_t(1 - n_t)}_{\text{vacancies}_t} \kappa \right]$$

s.t. $n_{t+1} = (1 - \delta) \underbrace{(n_t + \mu(\theta_t)(1 - n_t))}_{\text{employed}_t},$

with n_0 given.

For what comes later it will be useful to formulate problems recursively. Thus, we begin by writing the planner's problem recursively, and discussing efficient vacancy creation in that context. The recursive form for the planner's problem reads

$$V^p(n) = \max_{\theta} (n + \mu(\theta)(1 - n))z + (1 - n)(1 - \mu(\theta))b - \theta(1 - n)\kappa + \beta V^p(N(n, \theta)), \quad (4)$$

where $N(n, \theta) \equiv (1 - \delta)(n + \mu(\theta)(1 - n))$. Notice that the state variable is n , the number of matches at the beginning of the period, and that the control variable—market tightness θ —determines n' according to the law of motion $N(n, \theta)$.

The first-order condition, assuming an interior solution, is

$$\kappa = \mu'(\theta)(z - b + \beta(1 - \delta)V^{p'}(n')). \quad (5)$$

It equalizes the cost of an additional vacancy, κ , to its benefits: an increase in matches of $\mu'(\theta)$, with each new worker delivering the flow surplus $z - b$ today, together with a continuation value reflecting future flow surpluses.

The envelope condition gives the value of an additional beginning-of-period match, as

$$V^{p'}(n) = (1 - \mu(\theta) + \theta\mu'(\theta))(z - b + \beta(1 - \delta)V^{p'}(n')). \quad (6)$$

This value takes into account that the increase in initial matches hampers current hiring by shrinking the pool of searching workers. To see this in the expression, note that the derivative of the matching function with respect to unemployment, $m_u(\theta, 1)$, equals $\mu(\theta) - \theta\mu'(\theta)$.

Eliminating the derivative of the value function in (5), we arrive at the Euler equation

$$\frac{\kappa}{\mu'(\theta)} = z - b + \beta(1 - \delta)(1 - \mu(\theta') + \theta'\mu'(\theta'))\frac{\kappa}{\mu'(\theta')}. \quad (7)$$

This equation states the efficiency condition for the Mortensen-Pissarides model, solving a tradeoff between the costs and benefits of creating a new match today. The cost of an additional match today is $\kappa/\mu'(\theta)$: the cost of a vacancy, κ , times the measure of vacancies required for one match.¹⁰ The benefits of an additional match include the flow surplus $z - b$

¹⁰Since a unit increase in vacancies increases market tightness by $1/(1 - n)$ units, and a unit increase in market tightness yields $(1 - n)\mu'(\theta)$ new matches, one new vacancy creates $\mu'(\theta)$ new matches.

today, together with the expected value of the match next period. The expected value takes into account that the match survives to the next period with probability $1 - \delta$, and that the increase in matches shrinks the pool of searching workers tomorrow, so that any planned vacancy creation next period will yield fewer matches, leading to a net increase in matches of $1 - \mu(\theta') + \theta'\mu'(\theta')$. Finally, the value of a match tomorrow is again given by $\kappa/\mu'(\theta')$.

Note that the planner's Euler equation does not feature the state variable n explicitly at all, so a natural guess for the solution is a constant tightness independent of n . It is straightforward to show that the planner's value function is linear in n , and the efficient allocation thus characterized by a constant market tightness $\theta_t = \theta^p$, for all $t \geq 0$.

3.3 A union with commitment

Turning to the unionized labor market, consider the problem of the egalitarian union choosing a sequence of wages $\{w_t\}_{t=0}^{\infty}$ to maximize the objective (3) subject to the law of motion (1) and zero profit condition (2) holding at each point in time.

In order to relate the union problem to the planner's problem, we again use the zero profit conditions to rewrite the union objective. To this end, note that the union's choice of a sequence of wages determines, at each instant, the expected present value of union wages paid out over the course of an employment relationship: $W_t = \sum_{s=0}^{\infty} \beta^s (1 - \delta)^s w_{t+s}$. The sequence $\{W_t\}_{t=0}^{\infty}$ further pins down the sequence $\{\theta_t\}_{t=0}^{\infty}$ through the zero-profit conditions, assuming some vacancy creation occurs each period. Conversely, given a sequence $\{\theta_t\}_{t=0}^{\infty}$, one can back out per-period wages by first using the zero-profit condition to find W_t each period, and then computing wages as $w_t = W_t - \beta(1 - \delta)W_{t+1}$.

Using the zero-profit condition to eliminate wages, the union objective (3) can be written as:

$$-\frac{n_0 \kappa}{q(\theta_0)} + \sum_{t=0}^{\infty} \beta^t [(n_t + \mu(\theta_t)(1 - n_t))z + (1 - n_t)(1 - \mu(\theta_t))b - \theta_t(1 - n_t)\kappa], \quad (8)$$

revealing an identical objective to that of the planner except for the first term.¹¹ This term—familiar from the one-period example—reflects the share of the present discounted value of output accruing to capitalists. To see this, note that the capitalists' share, i.e., the present value of profits to firms, can be written as

$$n_0 \sum_{t=0}^{\infty} \beta^t (1 - \delta)^t [z - w_t] + \sum_{t=0}^{\infty} \beta^t [\mu(\theta_t)(1 - n_t) \sum_{s=0}^{\infty} \beta^s (1 - \delta)^s [z - w_{t+s}] - \theta_t(1 - n_t)\kappa]. \quad (9)$$

Here the first term captures the present value of profits on existing matches, and the second those on new vacancies created in periods $t = 0, 1, \dots$. The expression reduces to representing initial matches only, however, as free entry drives the present value of profits to new vacancies to zero.¹² Pre-existing matches, on the other hand, are due a strictly positive present value

¹¹See Appendix A.

¹²We can write the second term in equation (9) as $\sum_{t=0}^{\infty} \beta^t (1 - n_t) \theta_t [q(\theta_t) \sum_{s=0}^{\infty} \beta^s (1 - \delta)^s [z - w_{t+s}] - \kappa]$, which equals zero due to the free entry condition (2).

of profits, because these firms paid the vacancy cost in the past, anticipating positive profits in the future to make up for it. Using the zero-profit condition, this remaining present value can be expressed as $n_0\kappa/q(\theta_0)$.

The union objective (8) reflects the fact that while the planner maximizes the present discounted value of output, the union only cares about the workers' share of it. As a result, the union will have an incentive to appropriate some of this present value from capitalists by raising wages above the efficient level—and this is exactly how the solutions to the two problems will differ.

Proposition 1. *If the union is able to commit to future wages, hiring is efficient after the initial period. In the initial period, hiring is efficient if $n_0 = 0$ and below efficient if $n_0 > 0$.*

Note that after the initial period, the union effectively solves the planner's problem (4), and consequently chooses the planner's solution $\theta_t = \theta^p \forall t \geq 1$. In the initial period, however, the union chooses θ_0 to maximize

$$-\frac{n_0\kappa}{q(\theta_0)} + (n_0 + \mu(\theta_0)(1 - n_0))z + (1 - n_0)(1 - \mu(\theta_0))b - \theta_0(1 - n_0)\kappa + \beta V^p(N(n_0, \theta_0)), \quad (10)$$

where n_0 is given, and V^p solves the planner's problem (4).¹³

Deriving the optimality condition for this initial period is straightforward using the same methods as above. It becomes

$$\left[1 - \frac{n_0}{1 - n_0} \frac{q'(\theta_0)}{q(\theta_0)^2}\right] \frac{\kappa}{\mu'(\theta_0)} = z - b + \beta(1 - \delta)(1 - \mu(\theta^p) + \theta^p \mu'(\theta^p)) \frac{\kappa}{\mu'(\theta^p)}, \quad (11)$$

where we have used the fact that in subsequent periods we will have the efficient market tightness θ^p . Comparing to the efficiency condition (7), the cost of creating an additional match today (on the left) is higher for the union than for the planner. This occurs because in order to increase hiring, the union must lower wages, giving up some of the surplus it could have appropriated from firms with existing matches. Moreover, the more existing matches there are, the greater this additional cost.

Using the efficiency condition (7), we can further rewrite equation (11) as

$$\left[1 - \frac{n_0}{1 - n_0} \frac{q'(\theta_0)}{q(\theta_0)^2}\right] \frac{1}{\mu'(\theta_0)} = \frac{1}{\mu'(\theta^p)}.$$

Because $q'(\theta) < 0$ and $\mu'(\theta)$ is decreasing, this equation implies that the market tightness will generally be lower in the initial period than the efficient value it takes on after that, and the more initial matches, the lower its initial value. Thus, as in the one-period example, the initial market tightness depends negatively on the measure of existing matches. This is

¹³Again, using the zero-profit condition to substitute out wages assumes positive vacancy creation each period. The union could, as an alternative, also choose to set the initial present value of wages so high as to shut down hiring in the first period entirely, allowing matches to depreciate. This becomes more attractive when initial matches are plentiful.

a key feature of the model, which becomes even more important when the union does not have commitment.

That the outcome in the initial period differs from later periods reflects a time inconsistency issue in the union wage-setting problem. If the union were to re-optimize after the initial period, it would face a different objective and choose a different path of wages. While the union can thus get relatively close to the efficient outcome when it can commit, this immediate time inconsistency begs the question: what happens if the union cannot commit to future actions? To study time-consistent union decision making we next turn to a game-theoretic setting, which will be based on the recursive formulation of the union problem we set up above.

3.4 A union without commitment

The union problem (10) suggests that if the union were to re-optimize at any date, its choice of initial θ would depend on n , the measure of matches in the beginning of the period. In particular, a higher n should imply a lower θ . How would outcomes change if the union could not commit to not re-optimizing? We study this question by focusing on (differentiable) Markov-perfect equilibria with n as a state variable. That n is a payoff- and action-relevant state variable should be clear from the problem under commitment.¹⁴ In a Markov-perfect equilibrium, the union anticipates its future choices of θ to depend (negatively) on n , a relationship we label $\Theta(n)$. Our task, then, is to characterize $\Theta(n)$.

The function $\Theta(n)$ solves a problem similar to (10), namely

$$\Theta(n) \equiv \arg \max_{\theta} -\frac{n\kappa}{q(\theta)} + (n + \mu(\theta)(1 - n))z + (1 - n)(1 - \mu(\theta))b - \theta(1 - n)\kappa + \beta V(N(n, \theta)), \quad (12)$$

where the continuation value V satisfies the recursive equation

$$V(n) = (n + \mu(\Theta(n))(1 - n))z + (1 - n)(1 - \mu(\Theta(n)))b - \Theta(n)(1 - n)\kappa + \beta V(N(n, \Theta(n))). \quad (13)$$

Here, the union recognizes that its future actions will follow $\Theta(n)$, and this is reflected in the continuation value $V(n)$. Because $\Theta(n)$ will generally not be efficient, $V(n)$ will not equal $V^P(n)$, the continuation value under commitment.

A *Markov-perfect equilibrium* is defined as a pair of functions $\Theta(n)$ and $V(n)$ solving (12)–(13) for all n . We will assume that these functions are differentiable and characterize equilibria based on this assumption.

From equation (12), the first-order condition for market tightness becomes

$$\left[1 - \frac{n}{1 - n} \frac{q'(\theta)}{q(\theta)^2}\right] \kappa = \mu'(\theta)(z - b + \beta(1 - \delta)V'(n')), \quad (14)$$

¹⁴One can add states, representing histories of past behavior, but we do not consider such equilibria here.

and the equation paralleling the envelope condition—now not formally an envelope condition since the union does not agree with its future decisions—becomes

$$V'(n) = (1 - \mu(\theta) + \theta\mu'(\theta))(z - b + \beta(1 - \delta)V'(n')) \\ + \mu'(\theta)(\Theta'(n)(1 - n) - \theta) \left(-\frac{n}{1 - n} \frac{q'(\theta)}{q(\theta)^2} \frac{\kappa}{\mu'(\theta)} \right). \quad (15)$$

Equation (15) is derived by differentiating equation (13) and using equation (14) to arrive at a formulation close to the equivalent condition (6) for the planner. Compared to the planner's envelope condition, this equation includes some additional terms, which appear because the envelope theorem does not hold. These terms work to reduce the value of additional initial matches n , as the union sets the market tightness too low—following $\Theta(n)$ —and to an extent that increases in n .

We can further combine the above two equations to eliminate V' , obtaining

$$\underbrace{\left[1 - \frac{n}{1 - n} \frac{q'(\theta)}{q(\theta)^2}\right] \frac{\kappa}{\mu'(\theta)}}_{\text{cost of match today}} = z - b + \beta(1 - \delta) \left[\underbrace{(1 - \mu(\theta') + \theta'\mu'(\theta'))}_{\text{value of match tomorrow}} \left[1 - \frac{n'}{1 - n'} \frac{q'(\theta')}{q(\theta')^2}\right] \frac{\kappa}{\mu'(\theta')} \right. \\ \left. + \underbrace{\mu'(\theta')(\Theta'(n')(1 - n') - \theta') \left(-\frac{n'}{1 - n'} \frac{q'(\theta')}{q(\theta')^2}\right) \frac{\kappa}{\mu'(\theta')}}_{\text{loss in value from lack of commitment}} \right], \quad (16)$$

which is a *generalized Euler equation*. It is a functional equation in the unknown policy function Θ , where the derivative of Θ appears. The equation is written in a short-hand way: θ is short for $\Theta(n)$, θ' is short for $\Theta(N(n, \Theta(n)))$, and n' is short for $N(n, \Theta(n))$. The task is to find a function Θ that solves this equation for all n . Note that in contrast to the planner's Euler equation, n appears non-trivially in this equation and will generally matter for the tightness—it is easily verified that a constant Θ will not solve the equation.

Equation (16), like the planner's Euler equation (7), represents the tradeoff between the costs and benefits of creating matches today. The cost of an additional match for the union exceeds the cost for the planner, however, because in addition to the increase in vacancy costs $\kappa/\mu'(\theta)$, the union also takes into account that increasing hiring requires reducing wages, thereby giving up some of the surplus it could have appropriated from firms, captured by the term: $-\frac{n}{1 - n} \frac{q'(\theta)}{q(\theta)^2} \frac{\kappa}{\mu'(\theta)}$. This additional cost appears also in the Euler equation (11) for the union with commitment, but here it appears both today and tomorrow symmetrically, unlike in the commitment solution where tomorrow's union simply carries out today's plan. Beyond this difference, the union also takes into account its inability to commit to future wages: creating more matches today will reduce hiring tomorrow, as tomorrow's union will raise wages to exploit those matches. A marginal increase in matches reduces hiring by $\mu'(\theta)(\Theta'(n)(1 - n) - \Theta(n))$, with each lost worker valued at the size of the distortion in the union objective—the marginal surplus appropriated from capitalists.¹⁵

¹⁵To see this, note that the measure of vacancies can be written as $\Theta(n)(1 - n)$ and its derivative with respect to initial matches n as $\Theta'(n)(1 - n) - \Theta(n)$.

Note that equation (16) differs from standard Euler equations in that the derivative of the function Θ appears in the equation. This means that even solving for a steady state will be more complicated than usual, requiring information about the shape of the Θ function. By a steady state we mean a level of initial matches n and corresponding market tightness $\theta = \Theta(n)$ such that the law of motion maintains the same level of matches: $N(n, \Theta(n)) = n$. In this case we cannot simply use equation (16) together with the law of motion to solve for a steady state (n, θ) -pair, because the derivative appears as an additional unknown.

It is hard to establish theoretically that $\Theta(n)$ is indeed decreasing. In the one-period example of Section 3.1 we saw that Θ becomes a decreasing function of n , and in our numerically solved examples below this also holds. What is possible to show for the infinite-horizon case, however, is that whenever $\Theta(n)$ is decreasing, steady-state market tightness is strictly below its efficient level.

Proposition 2. *If $\Theta(n)$ is decreasing in n , then the steady-state market tightness, θ , in the unionized labor market (without commitment) is strictly below its efficient level.*

It follows that steady-state unemployment in the unionized labor market is strictly above its efficient level.

3.5 A non-egalitarian union

If we relax the equal pay constraint by allowing the union to pay different wages to newly hired workers (w_t^n) and workers in existing matches (w_t^e), then the union objective becomes

$$\sum_{t=0}^{\infty} \beta^t [n_t w_t^e + \mu(\theta_t)(1 - n_t)w_t^n + (1 - n_t)(1 - \mu(\theta_t))b], \quad (17)$$

and the zero profit condition

$$\kappa = q(\theta_t) \left[z - w_t^n + \sum_{s=1}^{\infty} \beta^s (1 - \delta)^s (z - w_{t+s}^e) \right]. \quad (18)$$

In this case we must also impose a separate condition ensuring firms make a non-negative present value of profits on existing workers:

$$\sum_{s=0}^{\infty} \beta^s (1 - \delta)^s (z - w_{t+s}^e) \geq 0, \forall t \geq 0. \quad (19)$$

The non-egalitarian union chooses two sequences of wages, $\{w_t^n\}_{t=0}^{\infty}$ and $\{w_t^e\}_{t=0}^{\infty}$, to maximize the objective (17) subject to the law of motion (1), zero profit conditions (18), and constraints (19) holding at each point in time.

In setting the wages of existing workers, the best the union can do is to set $w_t^e = z$ each period, leaving firms with zero surplus on existing matches. From the zero profit condition then, we have that $w_t^n = z - \kappa/q(\theta_t)$, $\forall t \geq 0$. Using this expression to substitute out wages in

the union objective, it is easy to see that the union problem becomes identical to the planner problem, thus leading to efficient hiring: $\theta_t = \theta^p, \forall t \geq 0$. The solution therefore involves a constant and efficient market tightness over time, as well as constant wages which exhibit a tenure premium: $w_t^n = z - \kappa/q(\theta^p)$ and $w_t^e = z \forall t \geq 0$.

We thus conclude that in the infinite horizon setting as well, the union may be able to attain efficient hiring through a wage tenure premium. A potential concern is that the implied wages of new workers may be quite low—they need to be low enough to allow firms to make the entire present value of profits associated with efficient hiring in the first period of the match. In the presence of a binding lower bound on the wages of junior workers, the union wage policy will still involve a tenure premium, but the market tightness will be distorted down.

In sum, wage solidarity comes at a cost in this economy, suggesting a role for tenure premia in union wages as a means to avoid the resulting distortions in hiring. And yet, the empirical evidence does not point to clearly greater returns to tenure in unionized settings than otherwise. Is this simply due to the measurement problems involved in the empirical work? Or are the distortions perhaps too insignificant in magnitude to warrant giving up (the benefits underlying) wage solidarity? To shed light on this question, the next section turns to a quantitative illustration looking at the impact of the egalitarian union on labor market outcomes.

4 Quantitative illustration

The presence of an egalitarian union affects the levels and dynamics of wages, unemployment, and output in the economy. In this section we illustrate these effects, in the context of an extended model.

4.1 Extended model

For added realism, we first lay out an extended model which incorporates partial unionization of the labor market, and multi-period union contracting. To this end, we assume that: i) a fraction α of workers are covered by union wages, with a worker's union status fixed over time, while the rest bargain their wages individually, and ii) instead of the union recontracting each period, it recontracts in any given period with probability λ , implying that contracts are expected to last $1/\lambda$ periods.¹⁶

For the non-union workers in the labor market, we can write standard Bellman equations, which can then be used to derive the following equation for the match surplus:

$$S_t = z - b + \beta(1 - \delta)(1 - \mu(\theta_{t+1})\gamma)S_{t+1}. \quad (20)$$

¹⁶We assume search is undirected, an assumption that plays a key role for the discussion in Section 4.4. If search were fully directed based on union status, the market would separate into two independent parts: one that follows the full unionization model and one following the standard Mortensen-Pissarides model.

The equation uses the fact that non-union workers bargain their wages individually, such that the bargaining outcome divides the match surplus according to the workers' bargaining power γ : workers get γS_t and firms $(1 - \gamma)S_t$.¹⁷ Note that the surplus equation (20) depends on the union's actions only through the market tightness.

The firms' zero profit condition can then be written, to reflect the presence of both union and non-union workers in the labor market, as

$$\kappa = q(\theta_t) \left[\alpha \left(\frac{z}{1 - \beta(1 - \delta)} - W_t \right) + (1 - \alpha)(1 - \gamma)S_t \right]. \quad (21)$$

As the right hand side states, firms expect a present value of profit of $(1 - \gamma)S_t$ on the $1 - \alpha$ non-union workers, and a present value of profit of $z/(1 - \beta(1 - \delta)) - W_t$ on the α union workers. The latter hinges on the expected present value of union wages paid out over the course of an employment relationship: $W_t = \sum_{s=0}^{\infty} \beta^s (1 - \delta)^s w_{t+s}$.

Turning then to thinking about how union wages $\{w_t\}_{t=0}^{\infty}$ are determined, we return to the union objective in equation (3). As before, we can rewrite this objective using the zero-profit condition (21), arriving at the expression

$$\begin{aligned} & \sum_{t=0}^{\infty} \beta^t [(n_t + \mu(\theta_t)(1 - n_t))z + (1 - \mu(\theta_t))(1 - n_t)b - \theta_t(1 - n_t)\frac{\kappa}{\alpha} + \frac{1 - \alpha}{\alpha}(1 - \gamma)\mu(\theta_t)(1 - n_t)S_t] \\ & - \frac{n_0\kappa}{\alpha q(\theta_0)} + \frac{1 - \alpha}{\alpha}(1 - \gamma)n_0S_0. \end{aligned} \quad (22)$$

Comparing this expression to the corresponding expression (8) earlier, note that with partial unionization, the non-union surpluses enter into the union objective because of their impact on vacancy creation.

Next, we would like to implement multi-period contracting in this setting, aiming for a recursive representation we could use to solve the model, as before. Note that as far as union wages are concerned, the object of interest for both the union and the firms is the expected present value of wages paid out over the course of an employment relationship, W_t . This present value determines the profitability of hiring union workers, governing vacancy creation through equation (21). In this sense, the allocative measure of wages here is W_t . What we would like to do, then, is to specify that in periods when the union does not recontract, W_t is held fixed, while in periods when the union does recontract, W_t is re-optimized. With full unionization this would imply that in periods when the union does not recontract, θ_t remains fixed, while in periods when the union does recontract, θ_t adjusts (due to equation (21)). With partial unionization, this need not hold exactly, because of the presence of the non-union surpluses in the zero profit condition. However, it turns out to be clearly simpler to solve the partial union model under the specification that what the union holds fixed in non-recontracting periods is θ_t directly.¹⁸ This also appears a reasonable approximation to

¹⁷See Appendix B for a derivation.

¹⁸Solving the partial union model with W_t held fixed leads to systems of non-linear equations for the non-union surpluses and their derivatives, while the current specification instead yields linear equations allowing analytical solutions, which is attractive from the point of view of minimizing error associated with numerical complexity.

holding W_t fixed, in the sense that changes in W_t during non-recontracting periods appear minor compared to the adjustments upon recontracting. With these concerns in mind, we proceed under the specification that what is held fixed in periods when the union does not recontract, is θ_t .¹⁹

To arrive at a recursive representation characterising labor market outcomes, then, we first write down recursive versions of the equations for the non-union surpluses. Based on equation (20), for periods when the union recontracts, we have:

$$S^r(n) = z - b + \beta(1 - \delta)[\lambda(1 - \mu(\Theta(N(n), \Theta(n))))\gamma]S^r(N(n), \Theta(n)) \\ + (1 - \lambda)(1 - \mu(\Theta(n))\gamma)S^f(N(n), \Theta(n)), \quad (23)$$

and for periods when the union does not recontract, we have:

$$S^f(n, \theta) = z - b + \beta(1 - \delta)[\lambda(1 - \mu(\Theta(N(n), \theta)))\gamma]S^r(N(n), \theta) \\ + (1 - \lambda)(1 - \mu(\theta)\gamma)S^f(N(n), \theta). \quad (24)$$

Note that in periods when the union does not recontract, the market tightness is held fixed, while in periods when the union does recontract, the tightness is determined via the equilibrium function $\Theta(n)$. Union decision-making in recontracting periods then determines the function $\Theta(n)$ as the solution to the problem:

$$\Theta(n) \equiv \arg \max_{\theta} (n + \mu(\theta)(1 - n))z + (1 - \mu(\theta))(1 - n)b - \theta(1 - n)\frac{\kappa}{\alpha} - \frac{n\kappa}{\alpha q(\theta)} \\ + \frac{1 - \alpha}{\alpha}(1 - \gamma)(n + \mu(\theta)(1 - n))S^r(n) + \beta\lambda V^r(N(n), \theta) + \beta(1 - \lambda)V^f(N(n), \theta), \quad (25)$$

where the union value satisfies

$$V^r(n) = (n + \mu(\Theta(n))(1 - n))z + (1 - \mu(\Theta(n)))(1 - n)b - \Theta(n)(1 - n)\frac{\kappa}{\alpha} \\ + \frac{1 - \alpha}{\alpha}(1 - \gamma)\mu(\Theta(n))(1 - n)S^r(n) + \beta\lambda V^r(N(n), \Theta(n)) + \beta(1 - \lambda)V^f(N(n), \Theta(n)) \quad (26)$$

in recontracting periods, and

$$V^f(n, \theta) = (n + \mu(\theta)(1 - n))z + (1 - \mu(\theta))(1 - n)b - \theta(1 - n)\frac{\kappa}{\alpha} \\ + \frac{1 - \alpha}{\alpha}(1 - \gamma)\mu(\theta)(1 - n)S^f(n, \theta) + \beta\lambda V^r(N(n), \theta) + \beta(1 - \lambda)V^f(N(n), \theta) \quad (27)$$

in non-recontracting periods. These equations follow from the union objective (22) as before.

Next, we move on to calibrating and illustrating the impact of unions in the context of this model. The focus will, for the most part, be on steady states: a level of initial matches n and a corresponding tightness $\theta = \Theta(n)$, such that $N(n, \theta) = n$. With this level of initial matches, if the union recontracts today it will keep the market tightness unchanged, leading to the same level of initial matches next period.

¹⁹This distinction matters only for Section 4.4, which allows partial union coverage.

4.2 Calibration and solution approach

We parameterize the model such that the efficient outcome corresponds to the US labor market, and study how introducing the union changes outcomes in this market.²⁰ The period length is set to one month, and the discount rate to correspond to a 5 percent annual rate of return, with $\beta = 1.05^{-12}$. We normalize labor productivity to $z = 1$ and set $b = 0.4$.²¹ We adopt the matching function $m(v, u) = \mu_0 v u / (v + u)$, as in den Haan, Ramey, and Watson (2000). We pin down the remaining parameters: δ , κ , and μ_0 , as follows. First, attaining an average duration of employment of 2.5 years requires a separation rate of $\delta = 0.033$. Second, to also be consistent with a steady-state unemployment rate of 5 percent, the average job-finding rate must be $\mu(\theta) = 0.388$. Finally, to also match the slope of the Beveridge curve, documented by Shimer (2007) to equal -1 , this requires setting $\mu_0 = 0.652$ and a steady-state value of $\theta = 1.47$. The latter can be achieved by setting $\kappa = 0.109$.

The basic Mortensen-Pissarides model is straightforward to solve, as is the planner problem discussed. The union problem without commitment is clearly more challenging, however. Issues to bear in mind include that there are few results on the existence of equilibrium for differentiable Markov-perfect equilibria, that these equilibria may not be unique, and that non-differentiable equilibria may exist as well.²² In solving for a differentiable equilibrium, a natural starting point would be the generalized Euler equation of the problem. In our case, the complexity of the system (23)-(27) does not allow deriving such an equation explicitly, but it turns out that we can proceed along the same lines also without this formal step. The focus will be on steady states and to solve for them we adopt the approach of Krusell, Kuruscu, and Smith (2002), which looks for a Taylor expansion approximation to the unknown function $\Theta(n)$ around the steady state. The approach involves solving successively larger systems of equations based on the first order condition (and successive derivatives of the first order condition) of problem (25), looking for convergence in the coefficients of the polynomial as the order increases. A description of how we implement this approach here can be found in Appendix C. The next sections describe the results.

4.3 Level effects

We begin by looking at the impact that introducing the union has on the levels of wages, unemployment and output, relative to the efficient outcome, in the case of full coverage.

According to the theory, the duration of union contracts should play a role in determining how large the distortions associated with the union are. Available evidence seems to point

²⁰The parametrization strategy follows that in Shimer (2005), aside from adopting a matching function which is better suited for a discrete time model. He calibrates a decentralized labor market to the US labor market, but the calibration strategy implies that the equilibrium outcome coincides with the socially optimal one.

²¹The results do not change substantially if we raise this to $b = 0.75$.

²²For examples where no differentiable equilibria exist but a non-differentiable equilibrium does, see Krusell, Martin, and Rios-Rull (2010), and for examples with a continuum of non-differentiable equilibria along with one or more differentiable ones, see Krusell and Smith (2003). Phelps and Pollak (1968) focus on differentiable equilibria and find multiplicity as well.

to one to three years as the relevant range of union contract durations, so we begin by setting $\lambda = 1/24$, implying an expected duration of union contracts of two years.²³ With this contract duration, introducing the union into the labor market raises wages by 12.5%, leading to an increase in unemployment from 5% to 16.5%, and a reduction in output of 12%, relative to efficient outcomes. As expected, wages and unemployment thus rise, leading to lower output, but the calculation reveals the quantitative impact to be substantial as well.

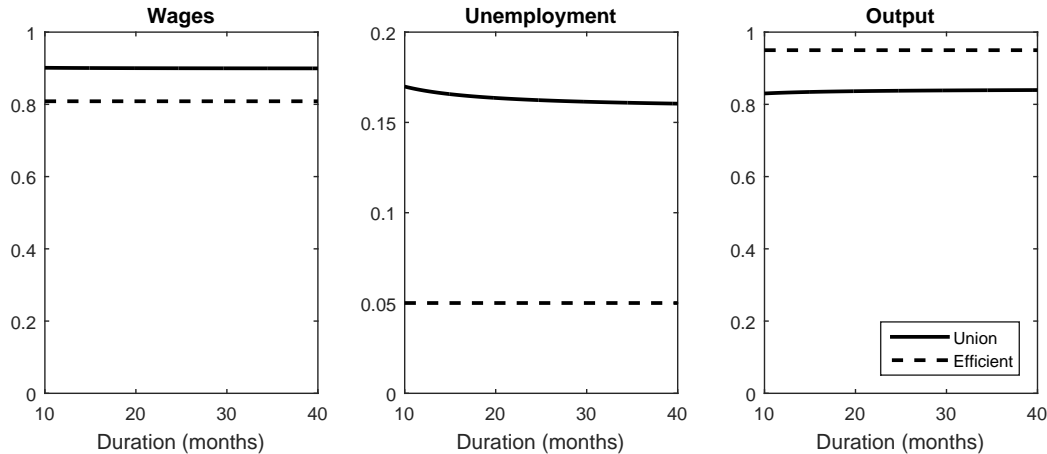


Figure 1: Level effect of union

Notes: The figure plots steady-state wages, unemployment, and output, as a function of the expected duration of a union contract $1/\lambda$.

To see how the union impact depends on contract duration, Figure 1 plots the steady-state levels of wages, unemployment and output as a function of the expected duration, $1/\lambda$. The benchmark in the figure—the efficient outcome—is naturally independent of λ . The figure shows that the impact of the union diminishes as contract duration increases, as we would expect. But the figure also reveals that for the relevant range of contract durations this effect turns out to be rather weak. Even though there is a visible decrease in unemployment as contract duration increases from one to three years, the magnitude of this decrease is overshadowed by the overall level effect associated with the union.²⁴

Finally, recall that the decentralized outcome in the Mortensen-Pissarides model is efficient only if the private bargaining power of workers coincides with that implementing efficient allocations (Hosios 1990). The value of the bargaining power parameter is difficult to infer from available data, however, so even though many researchers have chosen to calibrate it to guarantee efficient allocations, we do not have a good sense of the correct value of this

²³For example for the US, Taylor (1983) considers one to three years as the relevant range of union contracts, Card (1990) documents an average contract duration of 26 months, and Rich and Tracy (2004) a median duration of 36 months. Fregert and Jonung (2006) document similar durations for Sweden, and Avouyi-Dovi, Fougere, and Gautier (2013) an average duration of just under a year for France.

²⁴Note that there is no reason to expect the union outcome to converge to the efficient one as the duration of contracts approaches infinity: Recall that in the commitment union problem analyzed in Section 3.3, the union distorts θ down in the initial period but attains the efficient θ thereafter. This multi-period contracting specification, on the other hand, constrains θ to remain fixed between recontracting periods. Thus, it would seem natural for the union to set this fixed tightness above the efficient level when recontracting.

parameter. And if the private bargaining power turns out to be high relative to that implementing efficient allocations, unemployment will be above efficient also in a decentralized market without a union. We return to this issue in the next section, which turns to the case of partial union coverage.

4.4 Union coverage

In a classic paper, Calmfors and Driffill (1988) reconsider the impact of unions on the level of economic activity. It has long been recognized that unions, through their monopoly power in the labor market, tend to raise wages above their competitive levels. This suggests that a greater union presence in the labor market has a primarily negative impact on economic activity, as high union wages lead to higher unemployment. Calmfors and Driffill (1988) propose an additional factor for understanding the cross-country evidence on unions: they argue that the degree of coordination in union bargaining works to counteract the negative effects of monopoly power. A related hump-shaped relationship emerges in our model as well, as we vary the coverage of union wages across the work force.

Two competing forces come to play in the model as we vary union coverage: First of all, because union wages tend to be higher than non-union wages, greater union coverage tends to lead to higher unemployment in our model as well. But at the same time, limiting union coverage also introduces an additional distortion into the model, as the union no longer fully internalizes the effects of its wage demands on hiring, borne by union and non-union workers alike. Greater union coverage increases the extent to which the union takes into account the effects of its wage demands on hiring, leading to moderation in union wage setting. As we increase union coverage, the second effect eventually takes over the first, leading to a hump-shaped relationship between coverage and unemployment.

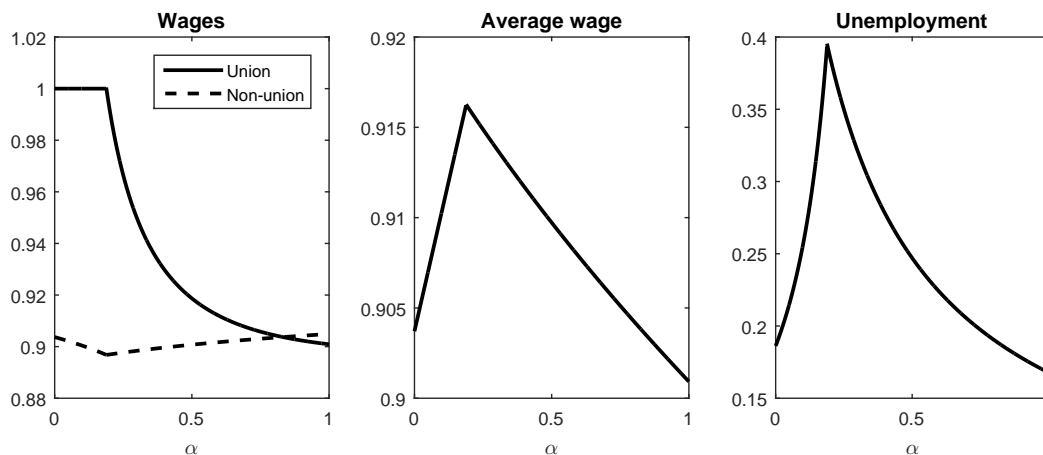


Figure 2: Role of unionization rate when non-union workers are strong bargainers

Notes: The figure plots union and non-union wages, the average wage, and unemployment as a function of union coverage α . The non-union bargaining power is set to $\gamma = 0.8$.

While intermediate coverage levels come with increased unemployment, which of the two extremes is better than the other depends on the bargaining power of non-union workers in

their private wage bargains. To illustrate, consider first a case where non-union workers are strong bargainers, setting $\gamma = 0.8$. Figure 2 plots the steady-state levels of wages and unemployment in this case. The panel on the left first shows how union and non-union wages vary with union coverage. As union coverage falls, union wages rise until they equal productivity, and cannot rise further.²⁵ In the meantime the wages of non-union workers remain mostly unaffected, although reflect changes in the outside options of these workers, which are worse at intermediate levels of coverage. What enters into firm profits is the weighted average of these wages across the pool of unemployed, shown in the middle panel. Averaging across union and non-union workers yields a hump-shaped relationship between union coverage and the average wage, which further gives rise to the hump-shaped relationship between union coverage and unemployment shown on the right.

Note that unemployment well exceeds the efficient level of 5% here even without the union, due to the high private bargaining power of workers, and that introducing the union can actually improve outcomes over that alternative, if the coverage is high enough. We could also ask what level of union coverage would be expected to emerge if workers got to choose (in the beginning of time) whether to be union or non-union. From the figure, it would appear there exists an interior union coverage level where workers would be indifferent between being union vs. non-union, in terms of the wages being equal. At that coverage level, unemployment is lower than it would be if unions were outlawed completely, but higher than with universal coverage of union wages.

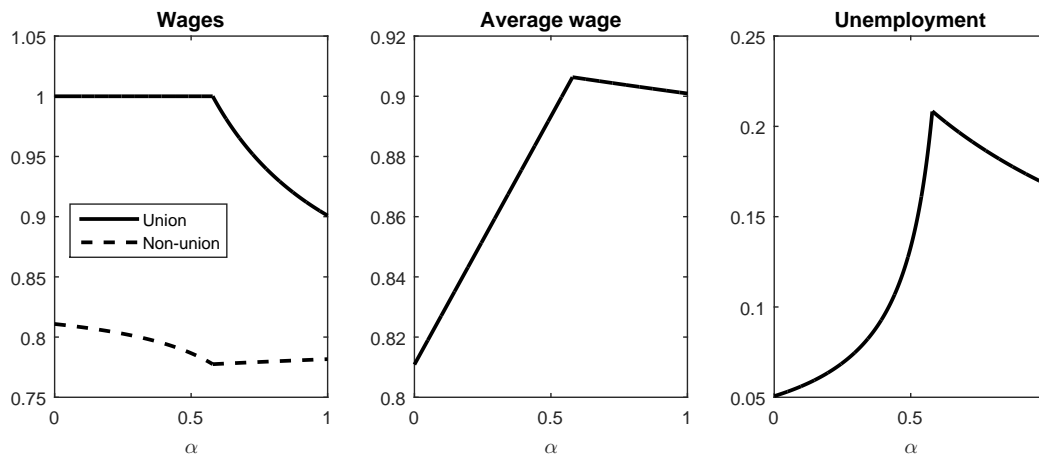


Figure 3: Role of unionization rate

Notes: The figure plots union and non-union wages, the average wage, and unemployment as a function of union coverage α . The non-union bargaining power is set to $\gamma = 0.6$.

To see how the picture changes when workers are weaker bargainers, Figure 3 looks at the case where the worker bargaining power yields efficient outcomes (here $\gamma = 0.6$). The figure is qualitatively similar, but in this case unemployment is always higher in the unionized labor market than it would be without the union. Union wages also always exceed non-union wages, and by a clear margin. Given a choice, all workers would prefer to be in the union, but it would be welfare improving to outlaw the union instead.

²⁵Wages cannot rise further or the firms would shut down.

4.5 Shock propagation

An important reason macroeconomists have been interested in labor unions is the notion that unions create rigidity in wages, affecting how the economy responds to shocks (Taylor 1980, Blanchard and Fischer 1989). We now turn to illustrating the union impact on shock propagation in our model, in the context of full unionization.

We consider the effects of a one-time, unanticipated, permanent increase in labor productivity. We first solve for the steady state before the shock, and then look at how the transition to higher productivity plays out when the expected duration of wage contracts is two years.²⁶ Figure 4 plots the responses, comparing the unionized labor market (solid line) to the efficient (dashed line), as well as fully fixed wages (dotted line). In the efficient response, the wage and market tightness adjust immediately to their new steady-state levels. With fixed wages, the market tightness also adjusts immediately to its new steady-state level, although in this case larger than what is efficient. The union response lies between these two extremes, but also differs in exhibiting significant inertia due to the multi-period union contracting.²⁷

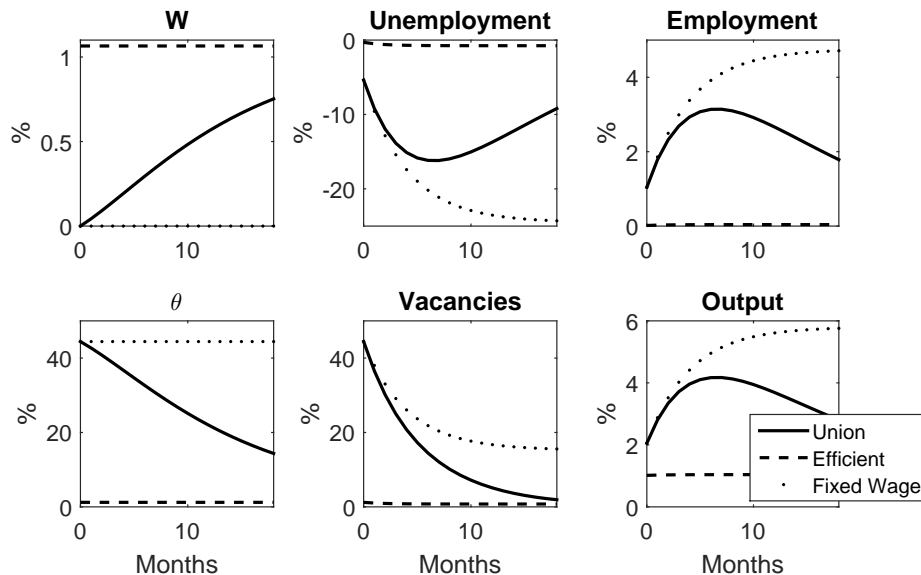


Figure 4: Impulse responses

Notes: The figure plots the responses of the present value of wages, market tightness, unemployment, vacancies, employment and output to a one percent unanticipated permanent increase in productivity. The figure shows the response for the economy with full coverage of union wages with two year contracts, the efficient response, and the response with fully fixed wages. What is plotted are expected values in each period after the increase in productivity, across possible realizations of the recontracting shock.

In terms of the magnitudes of these responses, the efficient response reflects a sizable on-

²⁶Recall that, with full unionization, our solution approach holds W_t fixed in non-recontracting periods. The solution to system (23)-(27) gives us a steady-state function $\Theta(n)$, which can be used to solve for a corresponding function $W(n) = z/(1 - \beta(1 - \delta)) - \kappa/q(\Theta(n))$, characterising the value W will take when the union recontracts. When the shock hits, we can use the old values of W, n together with the post shock function $W(n)$ to solve for the evolution of wages.

²⁷Taylor (1980) has highlighted the role of unions in generating persistence in aggregate dynamics due to long-term contracts.

impact response of the wage to the shock, which leads to small responses in quantities. This is the unemployment volatility puzzle discussed by Shimer (2005): the magnitude of these responses is an order of magnitude lower than what would be needed to be consistent with business cycle fluctuations in the data. If wages are fixed in response to the shock, quantities respond substantially more strongly, as highlighted by Hall (2005), allowing the model to match the magnitude of fluctuations observed. The stickiness in union wages, with two-year contracting, increases the volatility of quantities substantially relative to the efficient responses.

Underlying the stickiness in wages due to multi-period contracting, there is also an additional mechanism generating endogenous real wage rigidity in the model: Wages rise with some delay because the union distortion is weaker in the immediate aftermath of the shock when initial matches are relatively low. Wages rise to their full post-shock values only as matches accumulate to reflect the new, higher productivity. In the figure, the impact of this endogenous rigidity is overwhelmed by that of the stickiness associated with multi-period contracting, however.

5 Conclusions

This paper highlights a holdup problem that emerges when an egalitarian union sets wages in a frictional labor market. After demonstrating the issue in a theoretical setting, we study the severity of the holdup problem quantitatively in an extended model with partial unionization and long-term union contracting. We show that it raises wages and unemployment significantly above their efficient levels. The relationship between union coverage and unemployment is hump-shaped in the model, with intermediate levels of coverage featuring higher unemployment than either very low or very high coverage, and the bargaining power of non-union workers playing a key role in determining which of the two extremes is closer to efficient allocations. Long-term union contracts generate significant stickiness in the response of wages to shocks. Finally, the theory implies a rationale for tenure premia in union wages, as a means of avoiding the distortions associated with holdup.

The analysis is conducted in a stylized setting, to isolate key forces at play, but many extensions would seem natural, such as incorporating market power/decreasing returns, physical capital, worker heterogeneity, an insider-outsider wedge, as well as thinking more about the decisions of workers to join vs. leave the union in a dynamic setting.

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A Proofs

Proof of relationship between union and planner objectives For the benchmark model with egalitarian wages, we need to show that

$$\sum_{t=0}^{\infty} \beta^t (n_t + h_t) w_t = \sum_{t=0}^{\infty} \beta^t [(n_t + h_t)z - \theta_t(1 - n_t)\kappa] - \frac{n_0\kappa}{q(\theta_0)}, \quad (28)$$

where h_t stands for newly hired workers, i.e., $h_t = \mu(\theta_t)(1 - n_t)$.

First, note that the law of motion for employment implies that $n_t = (1 - \delta)^t n_0 + \sum_{k=0}^{t-1} (1 - \delta)^{t-k} h_k$, so we can write $n_t + h_t = (1 - \delta)^t n_0 + \sum_{k=0}^t (1 - \delta)^{t-k} h_k$. Using this identity, the left hand side of equation (28) can then be written as

$$\sum_{t=0}^{\infty} \beta^t (n_t + h_t) w_t = n_0 \sum_{t=0}^{\infty} \beta^t (1 - \delta)^t w_t + \sum_{t=0}^{\infty} \beta^t \sum_{k=0}^t (1 - \delta)^{t-k} h_k w_t. \quad (29)$$

The first term on the right of equation (29) can be written, using the zero-profit condition, as

$$-\frac{n_0\kappa}{q(\theta_0)} + n_0 \sum_{t=0}^{\infty} \beta^t (1 - \delta)^t z.$$

The second term can be written, rearranging and using the zero-profit condition, as

$$\begin{aligned} \sum_{t=0}^{\infty} \beta^t \sum_{k=0}^t (1 - \delta)^{t-k} h_k w_t &= \sum_{k=0}^{\infty} \beta^k h_k \sum_{t=k}^{\infty} \beta^{t-k} (1 - \delta)^{t-k} w_t \\ &= - \sum_{k=0}^{\infty} \beta^k h_k \frac{\kappa}{q(\theta_k)} + \sum_{k=0}^{\infty} \beta^k h_k \sum_{t=k}^{\infty} \beta^{t-k} (1 - \delta)^{t-k} z \\ &= - \sum_{t=0}^{\infty} \beta^t h_t \frac{\kappa}{q(\theta_t)} + \sum_{t=0}^{\infty} \beta^t \sum_{k=0}^t (1 - \delta)^{t-k} h_k z. \end{aligned}$$

These two terms combine into

$$\sum_{t=0}^{\infty} \beta^t [(n_t + h_t)z - \theta_t(1 - n_t)\kappa] - \frac{n_0\kappa}{q(\theta_0)}$$

i.e., the right hand side of equation (28). To see this, note that $h_t/q(\theta_t) = \theta_t(1 - n_t)$, and

$$n_0 \sum_{t=0}^{\infty} \beta^t (1 - \delta)^t z + \sum_{t=0}^{\infty} \beta^t \sum_{k=0}^t (1 - \delta)^{t-k} h_k z = \sum_{t=0}^{\infty} \beta^t (n_t + h_t) z.$$

With partial unionization, the zero-profit condition changes, affecting this derivation. The zero-profit condition now implies that the present value of wages W_t satisfy

$$W_t = \sum_{k=0}^{\infty} \beta^k (1 - \delta)^k z - \frac{\kappa}{\alpha q(\theta_t)} + \frac{1 - \alpha}{\alpha} (1 - \gamma) S_t.$$

Using this new zero-profit condition, the first and second terms on the right of equation (29) can be written, respectively, as

$$-\frac{n_0 \kappa}{\alpha q(\theta_0)} + n_0 \sum_{t=0}^{\infty} \beta^t (1 - \delta)^t z + n_0 \frac{1 - \alpha}{\alpha} (1 - \gamma) S_0,$$

and

$$-\sum_{t=0}^{\infty} \beta^t h_t \frac{\kappa}{\alpha q(\theta_t)} + \sum_{t=0}^{\infty} \beta^t \sum_{k=0}^t (1 - \delta)^{t-k} h_k z + \frac{1 - \alpha}{\alpha} (1 - \gamma) \sum_{t=0}^{\infty} \beta^t h_t S_t.$$

These terms now combine into

$$\sum_{t=0}^{\infty} \beta^t [(n_t + h_t) z - \theta_t (1 - n_t) \frac{\kappa}{\alpha} + \frac{1 - \alpha}{\alpha} (1 - \gamma) h_t S_t] - \frac{n_0 \kappa}{\alpha q(\theta_0)} + \frac{1 - \alpha}{\alpha} (1 - \gamma) n_0 S_0.$$

□

Proof of Proposition 1 The union objective can be written in terms of the planner's value function, as in equation (10). The planner problem is standard, and known to have a linear solution $V(n)$, with the planner's choice of θ constant, independent of n . The union objective differs in the initial period by the $-n_0 \kappa / q(\theta_0)$ term, however, which implies that the initial θ_0 is below the planner's choice, and this difference is greater the greater is n_0 . □

Proof of Proposition 2 Consider a steady state of the unionized economy, where $\Theta'(n) = -c$ for some $c > 0$. Using this fact, steady-state employment can be written as $n = (1 - \delta)\mu(\theta) / (1 - (1 - \delta)(1 - \mu(\theta)))$, equation (16) implies that the steady-state θ satisfies the equation

$$1 = \mu'(\theta) \frac{z - b}{\kappa} + \beta(1 - \delta)(1 - \mu(\theta) + \theta \mu'(\theta)) - \Delta(\theta), \quad (30)$$

where $\Delta(\theta) \equiv 0$ in the efficient outcome, and

$$\Delta(\theta) \equiv -\frac{1 - \delta}{\delta} \mu(\theta) \frac{q'(\theta)}{q(\theta)^2} \left[1 - \beta(1 - \delta)(1 - \mu(\theta) - \frac{\mu'(\theta) \delta c}{1 - (1 - \delta)(1 - \mu(\theta))}) \right]$$

in the unionized economy. The term $\Delta(\theta)$ thus captures the union distortion. Under efficiency, the right-hand side of equation (30) is strictly decreasing in θ pinning down a unique steady-state θ (as long as $\mu'(0) \frac{z-b}{\kappa} + \beta(1 - \delta) > 1$).²⁸ Because the union distortion $\Delta(\theta)$ is strictly positive for any $\theta > 0$, the unionized economy must have lower steady-state θ . □

Proof of Proposition 3 Similarly to Proposition 1, this result follows from writing the union problem in terms of the planner's value function, as in the text. □

²⁸Note that $m_u(v, u) = \mu(\theta) - \mu'(\theta)\theta$, an expression which is reasonable to assume to be increasing in θ .

B Bellman equations for extended model

The Bellman equations for the non-union workers in the extended model can be written as follows. For the value of unemployment and employment we have

$$\begin{aligned} U_t &= \mu(\theta_t)E_t + (1 - \mu(\theta_t))(b + \beta U_{t+1}), \\ E_t &= w_t + \beta\delta U_{t+1} + \beta(1 - \delta)E_{t+1}, \end{aligned}$$

where U_t is the value of an unemployed worker, E_t the value of an employed worker, and w_t the wage of a non-union worker. Recall that the union status of a worker is fixed over time.

For the value of a job filled with a non-union worker we have

$$J_t = z - w_t + \beta(1 - \delta)J_{t+1}.$$

Defining the match surplus as $S_t \equiv E_t + J_t - b - \beta U_{t+1}$, and requiring that the private wage bargains divide the surplus according to $J_t = (1 - \gamma)S_t$, and $E_t - b - \beta U_{t+1} = \gamma S_t$, the match surplus can be shown to satisfy equation (20) in the text.

C Numerical approach

As discussed in Section 4.2, we adopt the solution method outlined in Krusell, Kuruscu, and Smith (2002) for the generalized Euler equation. Given the complexity of the system (23)-(27), we begin by describing the approach in the context of equation (16), before proceeding to the extended model.

The generalized Euler equation (16) is a functional equation in $\Theta(n)$, defined over a range of values of n encompassing the steady state. The idea is to calculate a Taylor polynomial approximating $\Theta(n)$ around its steady state. Calculating a k^{th} order polynomial involves first analytically differentiating the Euler equation k times with respect to n , acknowledging that $\Theta(n)$ and $N(n, \Theta(n))$ are functions of n . We then evaluate the resulting system of $k + 1$ equations in steady state, setting Θ and its derivatives to their steady-state values $\theta, \theta', \dots, \theta^{k+1}$, as well as replacing n with its steady-state value $\mu(\theta)(1 - \delta)/(\delta + \mu(\theta)(1 - \delta))$. This yields a system of $k + 1$ equations in the unknowns $\theta, \theta', \dots, \theta^{k+1}$. We then set θ^{k+1} to zero, to arrive at a system of $k + 1$ equations in $k + 1$ unknowns—the coefficients of the polynomial approximating $\Theta(n)$.

We implement this approach in two stages. The first stage involves using an analytical solver, like Mathematica, to calculate the analytical derivatives and perform the substitutions needed to arrive at the system of equations in $\theta, \theta', \dots, \theta^k$. The second stage involves using a numerical solver, like Matlab, to solve this non-linear system of equations. Solving the system can, in practice, require a good initial guess, so we approach the problem iteratively. We start with a 0^{th} order Taylor polynomial and proceed to successively higher-order polynomials using the results from the previous step as initial guesses, looking for convergence in the coefficients of the polynomial as we proceed.

Turning to the extended model, then, we apply the same approach directly to the system (23)-(27). Instead of taking derivatives of an Euler equation, we take derivatives of the first

order condition of the union objective in equation (25). These derivatives will involve the values and derivatives of the functions $V^f(n, \theta)$, $V^r(n)$, $S^f(n, \theta)$, $S^r(n)$ as well, since they were not eliminated from the first order condition in deriving an Euler equation. We therefore need to calculate these values and derivatives in a separate step, in order to incorporate them into the equations.

Again, we implement the solution approach in two stages. In the analytical derivation stage, we tackle the system (23)-(27) starting from the surplus equations (23)-(24), then the value equations (26)-(27), and finally the first order condition for equation (25).

Beginning with the system (23)-(24): To solve for the steady-state levels of surpluses $S^f(n, \theta)$, $S^r(n)$, we simply solve the system analytically, expressing the steady-state levels of $S^f(n, \theta)$, $S^r(n)$ as functions of steady-state θ . To solve for first derivatives of $S^f(n, \theta)$, $S^r(n)$, we differentiate the system (23)-(24) with respect to (n, θ) (acknowledging that the recontracting tightness $\Theta(n)$ and $N(n, \Theta(n))$ are functions of n), evaluate the resulting equations at steady state, and solve the resulting system for steady-state values of the derivatives $S_n^f(n, \theta)$, $S_\theta^f(n, \theta)$, $S_n^r(n)$ as functions of the steady-state values and derivatives of $\Theta(n)$ and values of $S^f(n, \theta)$, $S^r(n)$. To solve for higher order derivatives of $S^f(n, \theta)$, $S^r(n)$, proceed along the same lines. The equations allow analytical solutions in each step.

We then turn to system (26)-(27), and proceed with the same approach to arrive at expressions for the steady-state values and derivatives of $V^f(n, \theta)$, $V^r(n)$ as functions of the steady-state values and derivatives of $\Theta(n)$, values and (lower order) derivatives of $V^f(n, \theta)$, $V^r(n)$, as well as values and derivatives of $S^f(n, \theta)$, $S^r(n)$. Again, these equations allow analytical solutions in each step.

Finally, we turn to the first order condition for equation (25), proceeding to differentiate the equation k times, and evaluate the resulting equations in steady state. This yields $k + 1$ equations in the steady-state value and derivatives of $\Theta(n)$, which depend also on those of $S^f(n, \theta)$, $V^f(n, \theta)$, $V^r(n)$. To eliminate the latter from the equations, we use the expressions derived above. This yields $k + 1$ equations representing the $0^{th} - k^{th}$ derivatives of the first order condition for equation (25) in the unknowns $\theta, \theta', \dots, \theta^{k+1}$. We then set θ^{k+1} to zero, to arrive at a system of $k + 1$ equations in $k + 1$ unknowns—the coefficients of the polynomial approximating $\Theta(n)$.

In the numerical stage, we solve this non-linear system of equations. In practice, solving the system can require a good initial guess, so we approach the problem iteratively. We start with a 0^{th} order Taylor polynomial and proceed to successively higher-order polynomials using the results from the previous step as initial guesses, looking for convergence in the coefficients of the polynomial as we proceed.