Nominal rigidities in debt and product markets*

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Abstract

Standard models used for monetary policy analysis rely on sticky prices. Recently, the literature started to explore also nominal debt contracts. Focusing on mortgages, this paper compares the two channels of transmission within a common framework. The sticky price channel is dominant when shocks to the policy interest rate are temporary, the mortgage channel is important when the shocks are persistent. The first channel has significant aggregate effects but small redistributive effects. The opposite holds for the second channel. Using yield curve data decomposed into temporary and persistent components, the redistributive and aggregate consequences are found to be quantitatively comparable.

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1 Introduction

Models of the real effects of monetary policy typically rely on nominal rigidities in product markets (sticky prices). The New-Keynesian literature—represented by, e.g., Woodford (2003) and Galí (2015)—has devoted significant effort into understanding the monetary transmission mechanism operating through this type of frictions. The focus has been mainly on aggregate effects; i.e., on how changes in the nominal interest rate—a monetary policy instrument—affect aggregate output, consumption, and other key macro variables. Following Doepke and Schneider (2006a), another strand of the literature on the real effects of monetary policy has emerged. This literature, for most part, maintains the assumption of flexible prices but pays attention to nominal rigidities in debt markets (Meh, Rios-Rull and Terajima, 2010; Sheedy, 2014; Doepke, Schneider and Selezneva, 2015; Garriga, Kydland and Šustek, 2016; Sterk and Tenreyro, 2016, among others).[1] In particular, this line of research recognizes that most debt contracts specify cash flows between borrowers and lenders in nominal terms. As a result, by ultimately affecting inflation, monetary policy affects the real value of these payments and thus, under incomplete asset markets, disposable income of borrowers and lenders. By the very nature of the problem, this research can address redistributive consequences of monetary policy, in addition to aggregate effects.[2]

The aim of this paper is to explore the interaction between the two rigidities and compare their quantitative importance in transmitting nominal shocks into the real economy in light of the empirical movements of the nominal interest rate set by monetary policy. For a number of countries, fluctuations in the nominal interest rate are fully characterized by two orthogonal

[1]This literature is distinct from the literature on the credit channel of monetary policy, represented by, e.g., Bernanke, Gertler and Gilchrist (1999).

[2]Another recent literature (e.g., Kaplan, Moll and Violante, 2016; Den Haan, Rendahl and Reigler, 2016) explores the interaction between sticky prices and/or wages and household heterogeneity (see, e.g., Gornemann, Kuester and Nakajima, 2016, for a brief review). As such, these studies can also address redistribution. They typically feature incomplete asset markets and rich household heterogeneity, often arising due to endogenous labor market frictions, in addition to standard New-Keynesian assumptions. This literature, however, abstracts from long-term nominal debt. Auclert (2014) explores redistributive effects of monetary policy operating through a traditional real rate channel. Redistributive consequences of inflation working through other channels than nominal debt markets have been addressed by Imrohoroğlu (1992), Erosa and Ventura (2002), and Albanesi (2007). Redistributive effects of money injections arise naturally in limited participation models (e.g., Williamson, 2008) and search models of money (e.g., Rocheteau, Weill and Wong, 2015).

components, one fairly temporary, the other highly persistent. It is therefore plausible to hypothesize that changes in the nominal interest rate, and thus inflation, interact with the two frictions differently, depending on their persistence. Especially, if debt contacts are long-term contracts. Focusing on mortgage debt, we therefore ask the following question: suppose the only impulses for the movements of the nominal interest rate were nominal (monetary policy) shocks, what would be the quantitative importance of each friction, sticky prices and mortgage contracts, in transmitting these shocks into real variables? To this end we construct a dynamic stochastic general equilibrium model that contains both nominal rigidities.

Our focus on mortgages is motivated by the observation that for most households the main asset is a house and the main liability is a mortgage (Campbell and Cocco, 2003). Mortgages carry significant financial commitments. For example, in the United States over the past forty years or so, mortgage payments were, on average, equivalent to 15 to 20% of homeowners' income, depending on the data used; similar magnitudes are observed also for other countries. In addition, mortgage loans have a longer term than other types of debt, with the typical term being 15-30 years. This feature, together with the fact that their payments are set in nominal terms, makes mortgages especially exposed to inflation risk, thus making them a source of potentially important redistributive effects.[3] An issue, however, arising with mortgages is that their form differs across countries. Most developed economies have variants of adjustable rate mortgages (ARM), whereby the interest rate the homeowner is charged adjusts more or less in line with short-term nominal interest rates. In some countries, however, the typical mortgage is a fixed rate mortgage (FRM), with the interest rate fixed for at least 10 years and often for the entire term of the loan (Belgium, France, Germany, Denmark, and the United States). The two contracts thus have very different exposures to nominal interest rates and inflation. Scanlon and Whitehead

[3] Even though mortgages can be pre-paid or refinanced, the extent to which this is possible varies across countries. While in the United States homeowners enjoy a lot of flexibility in this respect, in some other developed economies legal constraints and monetary costs restrict the extent to which this can be done easily (see, e.g., Scanlon and Whitehead, 2004; Green and Wachter, 2005; European Mortgage Federation, 2012). For example, Villar Burke (2015) documents that in France and Germany the interest rate on outstanding mortgage debt stayed essentially unchanged following the Euro Area policy rate cut to near zero in 2008/09, suggesting little refinancing activity, despite large drops in new mortgage rates and stable house prices.

(2004), Green and Wachter (2005), European Mortgage Federation (2012), and Campbell (2013) provide details of these institutional arrangements, but a theory of such cross-country differences is yet to be developed. We therefore consider both mortgage types.

Our analysis is based on a variant of the model of Garriga et al. (2016), who use it to study the transmission mechanism of monetary policy in the housing market. The model consists of two agent types, homeowners and mortgage investors. Homeowners invest in housing, financing their investment with mortgages, while mortgage investors provide mortgage loans and directly invest in productive capital. Asset markets are incomplete in the sense that a full set of state-contingent securities does not exist. Instead, the only other financial instrument, besides the mortgage, the two agent types can trade is a noncontingent one-period bond.[4] Here we extend the model by introducing standard New-Keynesian features, and elastic labor supply by both agent types, and use it as a laboratory for our question.

Monetary policy is modeled as an interest rate feedback rule with two shocks. The persistence of the shocks is calibrated so that, in equilibrium, the model is consistent with both, the standard New-Keynesian responses to monetary policy shocks and the observed persistence of the FRM rate. This makes one shock fairly temporary and the other shock highly persistent. The temporary shock generates the New-Keynesian responses, moving nominal interest rates and inflation (as well as output) in opposite directions as much as in VAR studies. In contrast, the persistent shock makes nominal interest rates positively correlated with inflation through the Fisher effect.[5] As the first shock affects the long-short spread, whereas the second shock affects all interest rates more or less equally, we use information from the nominal yield curve to gauge the relative seizes of the two shocks. Specifically, we decompose post-war yield curve data for a sample of developed economies into principal components. Like in the case of the United States (e.g., Piazzesi, 2006), we

[4]Garriga et al. (2016) show that the model has a number of desirable properties in terms of matching business cycle moments, responses under FRM and ARM of housing investment to shocks studied in the VAR literature, and marginal propensities to consume of homeowners documented in micro-level studies for changes in income due to ARM resets.

[5]Recently, the Fisher effect came to prominence, under the name 'Neo-Fisherism', in the context of the low inflation period following the interest rate cuts by central banks around the world to close to zero (Williamson, 2016).

find that for each country in the sample the first principal component accounts for over 95% of the movements in nominal interest rates across maturities, including the short rate effectively controlled by monetary policy. This component closely resembles the persistent shock in the model (in the yield curve terminology, it is referred to as the 'level factor'). The second principal component essentially accounts for the remaining movements in the data and is much less persistent than the first principal component (it is referred to as the 'slope factor'). We associate the second component with the movements in nominal interest rates occurring in the model due to the temporary shock.[6]

The main takeaway from the paper is that, in light of the statistical properties of the fluctuations in the policy rate, the redistributive consequences of monetary policy operating through debt markets are of similar magnitudes as the standard aggregate consequences operating through sticky prices. In more detail, the findings can be summarized in three points: (i) The sticky price channel mainly transmits the temporary shock, the mortgage channel is important only for the transmission of the persistent shock. (ii) The first channel has significant aggregate effects but small redistributive effects. The opposite is true for the second channel. (iii) Once the sizes of the shocks are calibrated from the principal components, the redistributive effects are somewhat larger than the aggregate effects, when measured by the unconditional volatility of percentage deviations of aggregate and individual consumption from steady state. The size of redistribution is similar under FRM and ARM, albeit the timing and direction is different, and consumption of homeowners is affected significantly more than consumption of lenders.

Our main contribution is the quantitative comparison of the nominal debt (mortgage) channel with the traditional sticky price channel in light of the empirical properties of nominal interest rates. By abstracting from other frictions, the paper provides a clean account of the relative importance of these two rigidities. While more narrow in the coverage of

[6]Of course, the second component in the data is affected by other factors, such as time-varying risk premia, than just the temporary nominal shock as in the model. Indeed, the second component is more persistent than the movements in the long-short spread implied by the shock in the model. By attributing all of the movements in the second component to the effects of the temporary shock is overstating the shock's importance, thus maximizing the relative importance of the sticky price channel.

nominal debt and heterogeneity than some parts of the literature on the debt channel, in exchange the paper brings to the literature explicit treatment of FRM and ARM. Modeling both FRM and ARM as long-term loans is critical when studying the real effects of monetary policy working through mortgage debt. Approximating ARM loans with one-period bonds, as is often done in the macro literature, misses the nominal rigidity built into these contracts and thus its interaction with the nominal interest rate and inflation. This point is especially relevant as most countries have mortgage markets that are closer to ARM than FRM.

The literature on nominal debt and monetary policy effectively started with the work of Doepke and Schneider (2006a). Their study provides a comprehensive accounting exercise of net nominal positions (nominal assets less nominal liabilities) of different sectors and types of households in the United States. It then demonstrates how the distribution of wealth, in present value real terms, is affected by a surprise increase in inflation. Building on a theoretical framework of Doepke and Schneider (2006b), the subsequent studies by Doepke and Schneider (2006c) and Doepke et al. (2015) feed these present value changes in wealth into a heterogenous agent model to study their implications for household decisions and the economy. Accounting exercises have been also conducted for Canada (Meh and Terajima, 2011) and the Euro Area (Adam and Zhu, 2016).[7] Meh et al. (2010) and Sheedy (2014) evaluate alternative monetary policy rules (inflation, price level, nominal GDP targeting) in the presence of nominal debt contracts. Sterk and Tenreyro (2016) build a model in which nominal debt plays a key role in the implementation of monetary policy through open market operations.[8] Gomes, Jermann and Schmid (2013) focus on how monetary policy and nominal corporate debt affect firms' decisions. Krause and Moyen (forthcoming) and Hedlund (2016) study the consequences of inflating away nominal debt burden of government and FRM debt respectively.[9]

[7] A more general exploration of redistributive effects of monetary policy, beyond focusing on the nominal debt channel, is conducted by Coibion, Gorodnichenko, Kueng and Silvia (2012) on the basis of VAR analysis. In line with our results, they find that persistent monetary policy shocks have larger effects on redistribution than do standard monetary policy shocks.

[8] The role of nominal debt in the conduct of monetary policy has been also explored at the zero lower bound (e.g., Azariadis, Bullard, Singh and Suda, 2015; Braun and Oda, 2010).

[9] The question of deflating government debt is also studied by Hilscher, Raviv and Reis (2014).

The paper proceeds as follows. Section 2 lays down the model. Section 3 explains the channels of transmission. Section 4 describes calibration. Section 5 reports quantitative findings. Section 6 concludes. A supplementary material contains two appendixes containing some auxiliary derivations.

2 The model

The economy's population is split into two groups, 'homeowners' and 'capital owners', with measures Ψ and $(1-\Psi)$, respectively. Within each group, agents are identical. Homeowners own the economy's housing stock whereas capital owners own the economy's capital stock. Both agent types supply labor. This abstraction is motivated by cross-sectional observations by Campbell and Cocco (2003): The typical homeowner is a middle class household in the wealth distribution, with one major asset, a house, and almost no corporate equity. This is in contrast to households in the top quintile of the wealth distribution, who own the entire corporate equity in the economy and housing makes up a small fraction of their assets.[10]

Homeowners finance housing investment through mortgages with a given loan-to-value ratio. Mortgages are modeled as long-term loans specifying the *nominal* payments that homeowners have to make throughout the life of the loan (the model abstracts from default). By being long-term loans for house purchase, mortgages in the model resemble first mortgages, as opposed to home equity lines of credit, which are closer to the short-term loans in Iacoviello (2005). The model economy operates under either ARM contracts (like, e.g., Australia) or FRM contracts (like, e.g., Germany). Our focus is on modeling the key characteristics of these two basic mortgage contracts, rather than specific institutional details.[11]

[10]The lowest two quintiles in the data are renters with little assets and little debt. These agents are not included in the model.

[11]Garriga et al. (2016) consider a richer mortgage market structure, allowing for refinancing and mortgage choice between FRM and ARM contracts. But when they calibrate their model to the data, these additional features turn out not to affect the responses of the model economy to shocks in a substantial way. This is due to the fact that, even though the composition of *new loans* is sensitive to economic shocks, this translates to only small changes in the composition of the *outstanding stock* of debt, either in terms of ARM vs. FRM or refinanced loans. And it is the composition of the stock that predominantly matters for the behavior of the economy.

Capital owners are the mortgage investors in the model and price mortgages competitively by arbitrage. Financial markets are incomplete in the sense that the full set of state-contingent securities does not exist. The only other financial instrument available, apart from the mortgage, that the two agent types can trade is a noncontingent one-period bond. Due to the market incompleteness, the stochastic discount factors of the two agent types are not equalized state by state and risk sharing is limited.

The production side of the economy has standard New-Keynesian features. In fact, the model collapses into a standard representative agent New-Keynesian model with endogenous capital once homeowners (and thus also housing and mortgage markets) are removed. Monopolistic intermediate good producers combine capital and labor according to a common constant returns to scale (CRS) production function to produce goods that are used as inputs by perfectly competitive CRS final good producers. The intermediate good producers set prices in nominal terms, subject to price adjustment costs. Output of the final good can be used for consumption, investment in capital, and investment in housing, subject to a concave production possibilities frontier (PPF). The concavity of the PPF plays a similar role as investment adjustment costs used in New-Keynesian models. Monetary policy follows an interest rate feedback rule. Finally, taxes, transfers, and government expenditures are introduced into the model to ensure a sensible calibration, as explained in Section 4.

2.1 Capital owners

A representative capital owner (agent 1) maximizes expected life-time utility

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_{1t}, n_{1t}), \quad \beta \in (0,1),$$

where $u(.,.)$ has the standard properties guaranteeing a unique interior solution, subject to a sequence of constraints

$$c_{1t}+q_{Kt}x_{Kt}+\frac{b_{1,t+1}}{p_t}+\frac{l_{1t}}{p_t}=\left[(1-\tau_K)r_t+\tau_K\delta_K\right]k_t+(1-\tau_N)\epsilon_w w_t n_{1t}+(1+i_{t-1})\frac{b_{1t}}{p_t}+\frac{m_{1t}}{p_t}+\tau_{1t}+\Pi_t, \tag{1}$$

$$k_{t+1}=(1-\delta_K)k_t+x_{Kt}.$$

Here, c_{1t} is consumption, n_{1t} is labor, x_{Kt} is investment in capital, q_{Kt} is a relative price, $b_{1,t+1}$ is holdings of the one-period nominal bond between periods t and $t+1$, p_t is the nominal price of the final good, l_{1t} is nominal mortgage lending, τ_K is a capital income tax rate, r_t is a real capital rental rate, δ_K is a capital depreciation rate, k_t is capital, τ_N is a labor income tax rate, ϵ_w is the relative productivity of capital owners (a parameter), w_t is the aggregate real wage rate, i_{t-1} is the nominal interest rate on the one-period bond bought in the previous period, m_{1t} is nominal payments from a pool of outstanding mortgages, τ_{1t} is government transfers, and Π_t is profits of the intermediate good producers, assumed to be owned by the capital owner. The determination of mortgage payments is discussed in Section 2.3.

2.2 Homeowners

A representative homeowner (agent 2) maximizes expected life-time utility

$$E_0\sum_{t=0}^{\infty}\beta^t v(c_{2t},h_t,n_{2t}),$$

where $v(.,.,.)$ also has the standard properties, subject to a sequence of constraints

$$c_{2t}+q_{Ht}x_{Ht}+\frac{b_{2,t+1}}{p_t}=(1-\tau_N)w_t n_{2t}+(1+i_{t-1}+\Upsilon_{t-1})\frac{b_{2t}}{p_t}+\frac{l_{2t}}{p_t}-\frac{m_{2t}}{p_t}+\tau_{2t}, \tag{2}$$

$$\frac{l_{2t}}{p_t}=\theta q_{Ht}x_{Ht},$$

$$h_{t+1} = (1 - \delta_H)h_t + x_{Ht}.$$

Here, c_{2t} is consumption, h_t is housing stock, n_{2t} is labor, x_{Ht} is housing investment, q_{Ht} is its relative price, $b_{2,t+1}$ is holdings of the one-period nominal bond between periods t and $t+1$, l_{2t} is new nominal mortgage borrowing, m_{2t} is nominal mortgage payments on outstanding debt, τ_{2t} is government transfers, θ is a loan-to-value ratio, and δ_H is a housing depreciation rate. Further, Υ_{t-1} is the homeowner's cost of participating in the bond market, taking the form of a spread over the market interest rate i_{t-1}. The cost is governed by a function $\Upsilon(-\widetilde{b}_{2t})$, where $\widetilde{b}_{2t} \equiv b_{2t}/p_{t-1}$. The function $\Upsilon(.)$ is assumed to be increasing and convex and satisfy the following additional properties: $\Upsilon(.) = 0$ when $\widetilde{b}_{2t} = 0$, $\Upsilon(.) > 0$ when $\widetilde{b}_{2t} < 0$ (the homeowner is borrowing), and $\Upsilon(.) < 0$ when $\widetilde{b}_{2t} > 0$ (the homeowner is saving). We think of $\Upsilon(.) > 0$ as capturing a premium for unsecured consumer credit, which is increasing in the amount borrowed. $\Upsilon(.) < 0$ can be interpreted as intermediation costs that reduce the homeowner's returns on savings below those of capital owners. The bond market cost function controls the extent to which the homeowner can use the bond market to smooth out fluctuations in income.[12]

2.3 Mortgages

Mortgages are modeled using the approximation of Kydland, Rupert and Šustek (forthcoming). Mortgage loans—like the agents—live forever, but their payment schedules resemble those of standard 30-year mortgages. Denoting by d_{1t} the period-t stock of outstanding nominal mortgage debt owed to the capital owner, the nominal mortgage payments received by the capital owner in period t are

$$m_{1t} = (R_{1t} + \gamma_{1t})d_{1t}.$$

[12] A technical role of the cost function is that, as in two-country business cycle models with incomplete asset markets, it prevents the one-period debt from becoming a random walk in a log-linear solution of the model. In other words, it keeps the log-linearized model stationary. In order to avoid the cost affecting the definition of aggregate output, it is rebated to the homeowner in a lump-sum way as a part of τ_{2t}.

Here, R_{1t} and γ_{1t} are, respectively, the interest and amortization rates of the outstanding stock of debt. The variables comprising m_{1t} are state variables evolving as

$$d_{1,t+1} = (1 - \gamma_{1t})d_{1t} + l_{1t}, \tag{3}$$

$$\gamma_{1,t+1} = (1 - \phi_{1t})\,(\gamma_{1t})^{\alpha} + \phi_{1t}\kappa, \tag{4}$$

$$R_{1,t+1} = \begin{cases} (1 - \phi_{1t})R_{1t} + \phi_{1t}i_t^F, & \text{if FRM,} \\ i_t, & \text{if ARM,} \end{cases} \tag{5}$$

where i_t^F is the interest rate on new FRM loans and

$$\phi_{1t} \equiv \frac{l_{1t}}{d_{1,t+1}}$$

is the fraction of new loans in the outstanding mortgage debt next period. The amortization rate $\gamma_{1,t+1}$ and (in the FRM case) the interest rate $R_{1,t+1}$ thus evolve as weighted averages of the amortization and interest rates, respectively, of the existing stock and new loans. In equation (4), $\kappa, \alpha \in (0, 1)$ are parameters. Specifically, κ is the initial amortization rate of a new loan and α controls the evolution of the amortization rate over time.[13]

In the FRM case, a first-order condition for l_{1t} pins down an arbitrage-free i_t^F. Under such a mortgage interest rate, the capital owner is indifferent between extending new mortgage loans and rolling over the one-period bond from period t on. Under ARM, the nominal interest rate of the one-period bond, i_t, is an arbitrage-free mortgage rate in the above sense. These properties are discussed further in Section 3. Under both contracts, as a result of the arbitrage free pricing, the capital owner is indifferent across investing in mortgages, bonds,

[13]Even though each new loan has an infinite life, it shares under an appropriate choice of κ and α the following features with standard mortgages. It gets essentially repayed within 30 years (120 periods, if the model is quarterly). The nominal mortgage payments are approximately constant for most of these 30 years (provided the loan's interest rate does not change). And at the start of the life of the loan most of the mortgage payments consist of interest payments, whereas towards the end of its life most of the payments consist of amortization payments. See Kydland et al. (forthcoming) for details. The adopted modeling of mortgages is convenient, as both the agents and the loans have an infinite life, thus allowing a simple recursive representation of the model with only a few state variables.

and capital—in real terms, the present value of future cash flows from one unit of any of these assets is equal to one unit of current consumption. The capital owner's composition of period-t investment (in terms of x_{Kt}, $b_{1,t+1}$, and l_{1t}) is pinned down by homeowners' demand for new mortgages and the one-period bond.

The evolution of mortgage payments that the homeowner has to make is governed by similar laws of motion as in the case of the capital owner:

$$m_{2t} = (R_{2t} + \gamma_{2t})d_{2t},$$

where

$$d_{2,t+1} = (1 - \gamma_{2t})d_{2t} + l_{2t}, \tag{6}$$

$$\gamma_{2,t+1} = (1 - \phi_{2t})\left(\gamma_{2t}\right)^{\alpha} + \phi_{2t}\kappa, \tag{7}$$

$$R_{2,t+1} = \begin{cases} (1 - \phi_{2t})R_{2t} + \phi_{2t}i_t^F, & \text{if FRM,} \\ i_t, & \text{if ARM,} \end{cases} \tag{8}$$

with $\phi_{2t} \equiv l_{2t}/d_{2,t+1}$. Demand for new mortgages is determined by the homeowner's choice of x_{Ht} and the financing constraint $l_{2t} = \theta p_t q_{Ht} x_{Ht}$.

2.4 Production

Perfectly competitive final good producers, of which there is a measure one, produce a single good Y_t using as inputs a continuum of goods $y_t(j)$, $j \in [0, 1]$. The representative producer solves a static profit maximization problem

$$\max_{Y_t, \{y_t(j)\}_0^1} p_t Y_t - \int_0^1 p_t(j) y_t(j) dj \quad \text{subject to} \quad Y_t = \left[\int_0^1 y_t(j)^{\varepsilon} dj\right]^{1/\varepsilon},$$

where $p_t(j)$ is the nominal price of an intermediate good j and $\varepsilon \in (0, 1]$. As all final good producers are the same, and there is a measure one of them, Y_t is also aggregate output. A first-order condition of this problem gives a demand function for good j

$$y_t(j) = \left[\frac{p_t}{p_t(j)}\right]^{\frac{1}{1-\varepsilon}} Y_t. \tag{9}$$

The producer of the intermediate good j is a monopolist in market j. It faces the Calvo-style price stickiness and, if allowed to change its price in period t, solves the dynamic maximization problem

$$\max_{p_t(j)} E_t \sum_{\iota=0}^{\infty} \psi^{\iota} Q_{1,t+\iota} \left[\frac{p_t(j)}{p_{t+\iota}} y_{t+\iota}(j) - \chi_{t+\iota} y_{t+\iota}(j)\right], \quad j \in [0,1], \tag{10}$$

where $Q_{1,t+\iota} \equiv \beta u_{c,t+\iota}/u_{ct}$ is the stochastic discount factor of the capital owner, $\chi_{t+\iota}$ is a real marginal cost, and $y_{t+\iota}(j)$ is given by the demand function (9), with $p_{t+\iota}(j) = p_t(j)\ \forall \iota$.[14] The expression in the square brackets is the per-period profit and $\psi \in [0,1]$ is the probability that the producer will not be able to change its price in a given period. By the law of large numbers, it is equal to the fraction of producers not changing prices.

The real marginal cost χ_t is given by a linear cost function of a static cost minimization problem

$$\chi_t y_t(j) = \min_{k_t(j), n_t(j)} r_t k_t(j) + w_t n_t(j) \quad \text{subject to} \quad A k_t(j)^{\varsigma} n_t(j)^{1-\varsigma} - \Delta = y_t(j).$$

Here, A is a constant technology level and $k_t(j)$ and $n_t(j)$ are capital and labor, respectively, used by producer j.[15] Further, Δ is a fixed cost, which is a common feature of New-Keynesian models with capital, ensuring that profits in steady state are equal to zero. This is relevant for mapping the parameter ς to National Income and Product Accounts. The first-order

[14]Notation such as u_{ct} means the first derivative of the function u with respect to argument c, evaluated in period t.

[15]In this paper we focus only on the real effects of nominal shocks, so TFP shocks or any other real shocks are abstracted from. A is therefore just a parameter. In Garriga et al. (2016) we subject the model (a version without the New-Keynesian features) to multiple shocks, including TFP shocks, and compare the model's business cycle properties with the data, as a form of model cross-validation.

condition of the cost minimization problem is

$$\frac{w_t}{r_t} = \left(\frac{1-\varsigma}{\varsigma}\right)\frac{k_t(j)}{n_t(j)}, \tag{11}$$

which sets relative factor prices equal to the marginal rate of technological substitution. The cost function then yields $\chi_t \equiv A^{-1}(r_t/\varsigma)^{\varsigma}[w_t/(1-\varsigma)]^{1-\varsigma}$. When this expression for the marginal cost is combined with the above first-order condition (11), we get

$$\chi_t = \frac{1}{A(1-\varsigma)}\left[\frac{n_t(j)}{k_t(j)}\right]^{\varsigma} w_t = \frac{1}{A(1-\varsigma)}\left[\frac{\overline{y}_t(j)}{Ak_t(j)}\right]^{\frac{\varsigma}{1-\varsigma}} w_t, \tag{12}$$

where $\overline{y}_t(j) \equiv y_t(j) + \Delta$. The second equality follows by substituting in for $n_t(j)$ from the production function. This expression will be relevant in Section 3.

The aggregate PPF is assumed to be nonlinear. Specifically,

$$C_t + q_{Kt}X_{Kt} + q_{Ht}X_{Ht} + G = Y_t, \tag{13}$$

where $C_t \equiv (1-\Psi)c_{1t}+\Psi c_{2t}$, $X_{Kt} \equiv (1-\Psi)x_{Kt}$, $X_{Ht} \equiv \Psi x_{Ht}$, and G is (constant) government expenditures. Further, q_{Kt} is the marginal rate of transformation between consumption and capital investment and q_{Ht} is the marginal rate of transformation between consumption and housing investment (in steady state, the rates of transformation are normalized to be equal to one). Under perfect competition, the rates of transformation are equal to relative prices of capital and housing investment in terms of consumption, as has already been assumed in the budget constraints. The rates of transformation are given by strictly increasing convex functions $q(X_{Kt})$ and $q(X_H)$, which make the economy's PPF concave. This specification is akin to that of Fisher (1997) and Huffman and Wynne (1999) and is meant to capture, in a reduced-form way, the costs of moving factors of production across different sectors (e.g., between construction and nondurable goods). As noted above, the concavity of the PPF works in a similar way as investment adjustment costs, which are a standard feature of

New-Keynesian models with capital (the reason why will become apparent below).

2.5 Monetary policy

Monetary policy is modeled as an interest rate feedback rule with two shocks, μ_t and η_t,

$$i_t = i + \mu_t - \pi + \nu_\pi(\pi_t - \mu_t) + \eta_t, \quad \nu_\pi > 1. \tag{14}$$

Here, i and π are the steady-state short-term nominal interest and inflation rates, respectively, $\pi_t \equiv p_t/p_{t-1} - 1$ is the inflation rate between periods t and $t-1$, and ν_π is a weight on deviations of the inflation rate from a stochastic inflation target μ_t. The inflation target has an unconditional mean equal to π and follows a stationary AR(1) process $\mu_{t+1} = (1-\rho_\mu)\pi + \rho_\mu \mu_t + \xi_{\mu,t+1}$, where $\xi_{\mu,t+1}$ is a mean-zero innovation with standard deviation σ_μ. The other shock has an unconditional mean equal to zero and follows a stationary AR(1) process $\eta_{t+1} = \rho_\eta \eta_t + \xi_{\eta,t+1}$, where $\xi_{\eta,t+1}$ is a mean-zero innovation with standard deviation σ_η. Both shocks are observed by the agents.[16]

Inflation target shocks have been considered by, e.g., Smets and Wouters (2003), Ireland (2007), Atkeson and Kehoe (2009), and Krause and Moyen (forthcoming).[17] Assuming the inflation target shock is highly persistent, it plays a role of a 'level factor shock', shifting short- and long-term nominal interest rates approximately equally, as discussed below. The second shock is a 'standard monetary policy shock' studied in the New-Keynesian literature (e.g., Galí, 2015, among many others). In order to generate the typical New-Keynesian responses, the persistence of this shock has to be fairly low. It thus essentially only affects the short rate and thus the long-short spread. Together, the two shocks allow the model to be consistent with both, the New-Keynesian responses identified in VARs and the empirical

[16] The specification of the policy rule abstracts from responding to fluctuations in output and from interest rate smoothing (a weight on past nominal interest rates). We have experimented with these features but found them to have only a limited effect on the results. In the interest of a more transparent exposition, these features have therefore been dropped from the model.

[17] See Ireland (2007) for further discussion.

persistence of the FRM rate.[18]

2.6 Equilibrium

In equilibrium, the following conditions are satisfied: (i) the capital owner and the homeowner solve their respective maximization problems, choosing contingency plans for c_{1t}, n_{1t}, x_{Kt}, k_{t+1}, $b_{1,t+1}$, and l_{1t} (capital owner) and for c_{2t}, n_{2t}, x_{Ht}, h_{t+1}, $b_{2,t+1}$, and l_{2t} (homeowner); (ii) intermediate good producers solve their respective optimization problems, choosing $k_t(j)$ and $n_t(j)$ and, if allowed, $p_t(j)$; (iii) the relative prices q_{Kt} and q_{Ht} are given by the respective marginal rates of transformation; (iv) monetary policy follows the interest rate rule; and (v) mortgage, bond, labor, capital, and goods markets clear:

$$(1-\Psi)l_{1t} = \Psi l_{2t},$$

$$(1-\Psi)b_{1,t+1} + \Psi b_{2,t+1} = 0,$$

$$\int_0^1 n_t(j) = \epsilon_w N_{1t} + N_{2t} \equiv N_t,$$

$$\int_0^1 k_t(j) = K_t,$$

$$C_t + q_{Kt}X_{Kt} + q_{Ht}X_{Ht} + G = Y_t.$$

In the above, $N_{1t} \equiv (1-\Psi)n_{1t}$, $N_{2t} \equiv \Psi n_{2t}$, and $K_t \equiv (1-\Psi)k_t$. As capital owners' and homeowners' labor inputs are perfect substitutes, capital owners' wage rate is $\epsilon_w w_t$, whereas homeowners' wage rate is w_t, as has already been assumed in the respective budget constraints. Aggregate consistency further implies: $(1-\Psi)d_{1t} = \Psi d_{2t}$, $\gamma_{1t} = \gamma_{2t}$, and $R_{1t} = R_{2t}$. As a consequence, $(1-\Psi)m_{1t} = \Psi m_{2t}$.[19] For the quantitative experiments, the

[18] Through out the paper, we use the terms 'persistent shock' and 'level factor shock' and the terms 'temporary shock' and 'standard monetary policy shock' interchangeably.

[19] The government budget constraint is given by $G+(1-\Psi)\tau_{1t}+\Psi\overline{\tau}_2 = \tau_K(r_t-\delta_K)K_t+\tau_N w_t(\epsilon_w N_{1t}+N_{2t})$. It holds by Walras' law. Here, $\overline{\tau}_2$ is a parameter and τ_{1t} takes up the slack to ensure that the budget constraint is satisfied state-by-state. Transfers to the homeowner are given by $\tau_{2t} = \overline{\tau}_2 - (b_{2t}/p_t)\Upsilon_{t-1}$; i.e., the participation cost is rebated back to the homeowner in a lump-sum way in order not to affect aggregate output. In steady state, the participation cost is equal to zero.

equilibrium is computed using standard log-linearization methods.

3 The channels of real effects

Nominal rigidities in the model come from two sources: sticky prices and mortgage contracts. In this section we discuss the equilibrium consequences of each rigidity in isolation in order to facilitate the interpretation of the quantitative findings later on. First, however, it is instructive to partially characterize the equilibrium mappings from the two shocks into the nominal interest rate and inflation.

3.1 Nominal shocks, nominal interest rate, and inflation

Without the loss of generality, in the following discussion it is useful to abstract from the capital income tax rate to simplify notation. The capital owner's first-order conditions for $b_{1,t+1}$ and x_{Kt} yield

$$1 = E_t\left(Q_{1,t+1}\frac{1+i_t}{1+\pi_{t+1}}\right) \quad \text{and} \quad 1 = E_t\left[Q_{1,t+1}\left(\frac{r_{t+1}}{q_{Kt}} + \frac{q_{K,t+1}(1-\delta_K)}{q_{Kt}}\right)\right]. \tag{15}$$

In the second equation, the first term in the inner brackets can be interpreted as a dividend yield, while the second term as a capital gain. Once log-linearized around a steady state, the two equations yield the Fisher equation

$$i_t - E_t\pi_{t+1} \approx E_t\left[r_{t+1} + (1-\delta_K)q_{K,t+1} - q_{Kt}\right] \equiv r_t^*, \tag{16}$$

where r_t^* is the ex-ante real interest rate and (abusing notation) all variables are in percentage point deviations from steady state. Combining equation (16) with the policy rule (14), assuming ρ_μ close to one and excluding explosive paths for inflation, yields

$$i_t \approx \sum_{\iota=0}^{\infty}\left(\frac{1}{\nu_\pi}\right)^{\iota} E_t r_{t+\iota}^* - \frac{\rho_\eta}{\nu_\pi - \rho_\eta}\eta_t + \mu_t. \tag{17}$$

Observe that unless the effect of μ_t is sufficiently offset by an endogenous response of the future path of the real rate, the μ_t shock generates almost permanent one-for-one changes in i_t. It thus affects not only the short rate but also the long rate (i_t^F) and, in this sense, works like a level factor shock. Substituting equation (17) into the policy rule (14) provides an analogous expression for the inflation rate

$$\pi_t \approx \frac{1}{\nu_\pi} \sum_{\iota=0}^{\infty} \left(\frac{1}{\nu_\pi} \right)^{\iota} E_t r^*_{t+\iota} - \frac{1}{\nu_\pi - \rho_\eta} \eta_t + \mu_t, \tag{18}$$

where the effect of μ_t is the same as on the nominal interest rate.

From equations (17) and (18) follows that the effect of the standard monetary policy shock η_t on both the short-term nominal interest rate and inflation is, ceteris paribus, negative. In order to generate the typical New-Keynesian response of the two variables to a positive η_t shock—i.e., a decline in π_t but an increase in i_t—the ex-ante real rate has to increase: observe that the real rate has a larger positive effect on the nominal interest rate than on inflation, whereas η_t has a larger direct negative effect on inflation than on the nominal interest rate. Observe further that the negative effect of the shock increases with its persistence. Thus, in order to produce an increase in the nominal interest rate alongside a decline in inflation, the persistence of the shock cannot be too high. Otherwise, the direct negative effect of the shock on the nominal interest rate may outweight any positive effect coming from an increase in the real rate.

3.2 Sticky price channel

As noted above, if homeowners are removed ($\Psi = 0$), the model collapses into a standard representative agent New-Keynesian model with endogenous capital. If homeowners are present but mortgages are removed ($\theta = 0$), the model becomes a two-agent New-Keynesian model with endogenous capital and housing, in which housing investment is equity financed. All aspects of the model related to price stickiness are contained in the optimization problem (10). As demonstrated in numerous texts (e.g., Galí, 2015), the log-linearized version of the first-order condition for this problem, once aggregation is imposed, yields the New-Keynesian Phillips curve (NKPC)

$$\pi_t = \frac{(1-\psi)(1-\beta\psi)}{\psi}\Theta\widehat{\chi}_t + \beta E_t\pi_{t+1}, \tag{19}$$

where $\Theta \equiv (1-\varsigma)/[1-\varsigma+\varsigma/(1-\varepsilon)] \geq 0$ and $\widehat{\chi}_t$ is the percentage deviation of the marginal cost from steady state.[20] This equilibrium condition embodies the nominal rigidity in the model due to sticky prices. For β close to one, it provides a negative relationship between an expected change in the inflation rate, $E_t\pi_{t+1} - \pi_t$, and the real marginal cost, $\widehat{\chi}_t$. For a highly persistent inflation rate, $E_t\pi_{t+1} - \pi_t$ is close to zero, implying $\widehat{\chi}_t \approx 0$. In this case, monetary policy has almost no real effects. If, in contrast, the inflation rate is not very persistent, then $E_t\pi_{t+1} - \pi_t \neq 0$ and $\widehat{\chi}_t \neq 0$. In this case, monetary policy has real effects.

Appendix A.1 establishes that percentage deviations of the marginal cost are positively related to percentage deviations of aggregate output, $\widehat{Y}_t$.[21] Equation (19) thus provides a negative relationship between $E_t\pi_{t+1} - \pi_t$ and $\widehat{Y}_t$. As a result, a shock that *temporarily* reduces inflation, thus generating $E_t\pi_{t+1} - \pi_t > 0$, produces a decline in output, $\widehat{Y}_t < 0$. In the face of the output drop, consumption smoothing by capital owners requires a drop in capital investment, which leads to a decline in q_{Kt} and thus positive expected capital gains, $E_t(1-\delta_K)q_{K,t+1} - q_t > 0$.[22] A sufficiently large increase in capital gains then leads to an increase in the ex-ante real interest rate r_t^*, as follows from equation (16). The greater is the curvature of the PPF, the less can consumption be smoothed out in equilibrium. Therefore, the greater is the increase in expected capital gains, and thus in the ex-ante real interest rate. This mechanism generates the typical New-Keynesian response to a temporary

[20]Equation (19) is derived under the common assumption that the steady-state inflation rate is equal to zero. This assumption provides a more elegant expression for the linearized NKPC than would otherwise be the case. For expositional purposes, this section therefore proceeds under this common assumption, even though the model is computed under a calibrated non-zero steady-state inflation rate.

[21]A positive relationship between $\widehat{\chi}_t$ and $\widehat{Y}_t$ is easier to derive in the textbook New-Keynesian model without capital, in which $\widehat{C}_t = \widehat{Y}_t$.

[22]A drop in capital investment can occur through a direct channel, by capital owners reducing capital investment for given holdings of bonds, and through an indirect channel, by homeowners reducing holdings of the bonds, whose proceeds could otherwise be used to support capital investment by capital owners.

monetary policy shock; i.e., the ex-ante real rate increases while output and inflation fall, with a sufficiently large increase in the real rate producing also an increase in the nominal rate, as discussed above. As this is an aggregate effect (i.e., aggregate output falls), the decline in output is born by both agent types, albeit to a possibly different extent.[23]

3.3 Mortgage channel

To highlight the role of mortgages, nominal prices in this section are assumed to be fully flexible (i.e., $\psi = 0$). The NKPC (19) then implies $\widehat{\chi}_t = 0$ (i.e., $\chi_t = \chi$). That is, the marginal cost is constant, equal to its steady-state value, which is given by a standard static profit maximization condition of a monopolist, $(1/\varepsilon)\chi = p(j)/p$; i.e., the relative price of good j is set as a constant markup over marginal costs. When $\varepsilon = 1$, this condition yields $\chi = 1$. The marginal cost is equal to the relative price of good j, which is equal to one, as all goods are perfectly substitutable; a standard profit maximization condition under perfect competition. As there are no monopoly profits, we set $\Delta = 0$. Equation (12), with $\chi_t = 1$, then yields $w_t = (1 - \varsigma)AK_t^{\varsigma}N_t^{-\varsigma}$. Combining this expression with the cost minimization condition (11) gives $r_t = \varsigma AK_t^{\varsigma-1}N_t^{1-\varsigma}$. Thus, under perfect competition, the wage rate and the rental rate are equalized with the respective marginal products of labor and capital.

Mortgages introduce a nominal rigidity into the model due to the multi-period term over which homeowners make nominal payments. The nominal rigidity shows up in two places: as an income effect in the budget constraints of the two agents and as a price effect in a first-order condition of the homeowner for housing. The income effect occurs due to the effects of inflation surprises on the real value of payments on *outstanding* mortgage debt, while the price effect concerns the effects of expected future inflation on the cost of *new* mortgage borrowing.[24]

[23] Again, these responses are easier to establish in the textbook New-Keynesian model without capital.

[24] Bernanke and Gertler (1995) and Mishkin (2007) refer to the income effect also as a 'cash flow' or 'household balance sheet' effect.

3.3.1 Income effect

It is convenient for this and the next section to write the real mortgage payments in the budget constraints (1) and (2) as

$$\frac{m_{et}}{p_t} \equiv \widetilde{m}_{et} = \frac{R_{et} + \gamma_{et}}{1 + \pi_t} \widetilde{d}_{et} \tag{20}$$

where $\widetilde{d}_{et} \equiv d_{et}/p_{t-1}$ and $e \in \{1, 2\}$. Recall that in period t, the variables R_{et}, γ_{et}, and d_{et} that make up the nominal payments are pre-determined and from period t on evolve according to the laws of motion (3)-(5), for $e = 1$, and (6)-(8), for $e = 2$. To focus on outstanding debt, let us set $l_{e,t+\iota} = 0$, for $\iota = 0, 1, 2, \ldots$

It is clear from equation (20) that, as the numerator is predetermined in period t, an unexpected increase in π_t has a standard income effect under both FRM and ARM. It reduces the real value of mortgage payments in period t and thus redistributes income from capital owners to homeowners.

Suppose, however, that the increase in the inflation rate is persistent and assume there are no further inflation surprises. From period $t + 1$ on, the effects of higher inflation are different under FRM and ARM. Under FRM, the sequence of real mortgage payments is

$$\widetilde{m}_{e,t+1} = \frac{R_{et} + \gamma_{e,t+1}}{(1 + \pi_{t+1})(1 + \pi_t)} (1 - \gamma_{et}) \widetilde{d}_{et},$$

$$\widetilde{m}_{e,t+2} = \frac{R_{et} + \gamma_{e,t+2}}{(1 + \pi_{t+2})(1 + \pi_{t+1})(1 + \pi_t)} (1 - \gamma_{e,t+1})(1 - \gamma_{et}) \widetilde{d}_{et}, \quad \text{etc.},$$

where R_{et} is constant and $\gamma_{e,t+\iota}$ converges to one over time.[25] Higher inflation thus reduces the real value of mortgage payments under FRM and, through accumulated inflation, the size of this effect increases over time.

[25] $\gamma_{e,t+\iota}$ converges to one because $\gamma_{et} \in (0, 1)$ and $\alpha \in (0, 1)$; see the law of motion (4) or (7) for $l_{e,t+\iota} = 0$, $\iota = 0, 1, 2, \ldots$.

Under ARM, the sequence of real mortgage payments is

$$\widetilde{m}_{e,t+1} = \frac{i_t + \gamma_{e,t+1}}{(1+\pi_{t+1})(1+\pi_t)}(1-\gamma_{et})\widetilde{d}_{et}, \tag{21}$$

$$\widetilde{m}_{e,t+2} = \frac{i_{t+1} + \gamma_{e,t+2}}{(1+\pi_{t+2})(1+\pi_{t+1})(1+\pi_t)}(1-\gamma_{e,t+1})(1-\gamma_{et})\widetilde{d}_{et}, \quad \text{etc.},$$

The difference, compared with FRM, is that the mortgage rate of the outstanding debt is equal to the short-term nominal interest rate, which can change over time. To demonstrate the consequence of this aspect of ARM loans, let us first focus on $\widetilde{m}_{e,t+1}$. Holding the ex-ante real rate constant, a higher π_{t+1} translates through the Fisher equation (16) into equiproportionally higher i_t. As a result, and in contrast to the FRM case, $\widetilde{m}_{e,t+1}$ increases. To see this, focus on the ratio in equation (21), which can be written as

$$\frac{i_t + \gamma_{e,t+1}}{(1+\pi_{t+1})(1+\pi_t)} \approx \frac{i_t + \gamma_{e,t+1}}{1+\pi_{t+1}+\pi_t} \approx i_t + \gamma_{e,t+1} \approx r^* + \pi_{t+1} + \gamma_{e,t+1},$$

where the first two approximations hold for sufficiently small inflation rates and $\gamma_{e,t+1}$ sufficiently smaller than one. Thus, in contrast to FRM, a higher π_{t+1} leads to a higher $\widetilde{m}_{e,t+1}$. This front-end property reflects the fact that at the early stages in the life of a mortgage, a bulk of the payments are interest payments. Over time, however, the effects of accumulated inflation get stronger. To see this back-end property of the loan, notice that for a sufficiently high ι, the ratio can be written as

$$\frac{i_{t+\iota} + \gamma_{e,t+\iota}}{1+\pi_{t+\iota}+\ldots\pi_{t+1}+\pi_t} \approx \frac{r^* + \pi_{t+\iota} + \gamma_{e,t+\iota}}{1+\pi_{t+\iota}+\ldots\pi_{t+1}+\pi_t} \approx \frac{\gamma_{e,t+\iota}}{1+\pi_{t+\iota}+\ldots\pi_{t+1}+\pi_t},$$

where the last approximation is due to $\gamma_{t+\iota} \to 1$ and r^* and $\pi_{t+\iota}$ being assumed to be relatively small. Observe that for $\gamma = 1$ (i.e., a one-period loan, a short cut often taken in the literature to model ARM), neither the front-end nor the back-end property of ARM is present and the only effect of inflation is the standard income effect on $\widetilde{m}_{et}$ in equation (20).

To sum up the income effect: After the initial period t, higher inflation reduces real mortgage payments under FRM, but increases real mortgage payments under ARM, at least in the short run. While the reduction under FRM is gradual, the increase under ARM is immediate. Over time, however, as the loan gets amortized and interest payments become a small fraction of mortgage payments, the income effect under ARM starts to resemble the income effect under FRM.

3.3.2 Price effect

The price effect concerns the cost of new mortgage borrowing and thus the effective price of housing investment. The first-order condition for x_{Ht} takes the form

$$v_{ct} q_{Ht}(1+\tau_{Ht}) = \beta E_t V_{h,t+1}, \tag{22}$$

where $V_{h,t+1}$ is the derivative of the homeowner's value function with respect to h_{t+1} in a recursive formulation of the problem and τ_{Ht} is a wedge, discussed below, summarizing the effect of mortgage finance on the optimal choice of x_{Ht}. Notice that the wedge affects the first-order condition in a similar way as the relative price of new housing q_{Ht}, hence the term 'price effect'. To see how nominal interest rates and inflation affect the real cost of a new mortgage loan in isolation, it is instructive to consider a once-and-for-all housing investment decision in period t, without any outstanding debt. That is, assume $d_{2t} = 0$, $x_{Ht} > 0$, and $x_{H,t+\iota} = 0$ for $\iota = 1, 2, \ldots$. In this case, the wedge is[26]

$$\tau_{Ht} \equiv \theta \left\{ -1 + E_t \left[Q_{2,t+1} \frac{i^M_{t+1} + \gamma_{2,t+1}}{1+\pi_{t+1}} + Q_{2,t+2} \frac{(i^M_{t+2} + \gamma_{2,t+2})(1-\gamma_{2,t+1})}{(1+\pi_{t+1})(1+\pi_{t+2})} + \ldots \right] \right\}. \tag{23}$$

Here, $Q_{2,t+\iota} \equiv \beta v_{c,t+\iota}/v_{ct}$ is the stochastic discount factor of the homeowner and $i^M_{t+\iota} = i^F_t$ under FRM and $i^M_{t+\iota} = i_{t+\iota-1}$ under ARM. Observe that the term inside the square brackets is a present value of real mortgage payments from the homeowner's perspective (i.e., the payments are discounted with the homeowner's stochastic discount factor).

[26] See Appendix for derivation.

The FRM interest rate is determined by a first-order condition of the capital owner with respect to l_{1t}, which takes the form

$$1 = E_t \left[Q_{1,t+1} \frac{i_t^F + \gamma_{1,t+1}}{1+\pi_{t+1}} + Q_{1,t+2} \frac{(i_t^F + \gamma_{1,t+2})(1-\gamma_{1,t+1})}{(1+\pi_{t+1})(1+\pi_{t+2})} + ... \right], \tag{24}$$

where $Q_{1,t+\iota} \equiv \beta u_{c,t+\iota}/u_{ct}$. It is straightforward to also verify that the following holds in the case of ARM

$$1 = E_t \left[Q_{1,t+1} \frac{i_t + \gamma_{1,t+1}}{1+\pi_{t+1}} + Q_{1,t+2} \frac{(i_{t+1} + \gamma_{1,t+2})(1-\gamma_{1,t+1})}{(1+\pi_{t+1})(1+\pi_{t+2})} + ... \right]. \tag{25}$$

These two conditions state that, from the capital owner's perspective, the present value of real mortgage payments on a one dollar loan has to be equal to one dollar. These two conditions are the mortgage counterparts to the no-arbitrage conditions for bonds and capital (15).

Observe that if asset markets were complete ($Q_{1,t+\iota} = Q_{2,t+\iota}$) then the present value in equation (23) would be equal to one and the wedge would be equal to zero. Under incomplete markets, $Q_{1,t+\iota} \neq Q_{2,t+\iota}$ and the wedge in general is not equal to zero and depends on nominal variables. To see how the price effect works, assume again that the real rate r^* is constant and that there is no uncertainty about future inflation (the case of perfect foresight is the easiest case in which to explain, without the loss of generality, the price effect).

It is convenient to start with the ARM case. Suppose π_{t+1} increases. Through the Fisher effect, this leads to an equiproportional increase in i_t. As a result, the real mortgage payment in period $t+1$ increases, since as in the case of the income effect, the dominant effect is the interest rate effect. The same argument applies for other periods $t+\iota$ if the inflation rate increases persistently. However, as in the case of the income effect, there is again an ι such that the effect of accumulated inflation starts to dominate the effect of higher nominal interest rates. But if this occurs in a sufficiently distant future, so that those future payments are sufficiently discounted, the wedge increases, making housing investment more expensive.

In the FRM case, the pricing equation (24) shows that, for a given sequence of $Q_{1,t+\iota}$, the mortgage rate i_t^F depends positively on future inflation. Higher expected future inflation thus increases i_t^F. Similar arguments as in the ARM case therefore apply, at least qualitatively, and higher inflation makes new FRM loans more expensive to the homeowner. Thus, in contrast to the income effect, the price effect works qualitatively in the same direction under FRM and ARM.

3.3.3 Summary of the mortgage channel

To summarize the mortgage channel, it operates by affecting the relative price of new housing and the distribution of current and expected future disposable income. Unlike the sticky price channel, it does not directly affect producers. Under ARM, both the price and income effects hurt homeowners when inflation increases. Under FRM, the price effect hurts homeowners while the income effect benefits them. In contrast to the sticky price channel, the size of the price and income effects increases with inflation persistence.

4 Calibration

The calibration is based on U.S. targets, details of which can be found in Garriga et al. (2016). The New-Keynesian parameters are the standard ones in the literature. The mechanism under investigation, however, is not specific to the U.S. economy and applies more generally. The U.S. calibration simply provides an example of a reasonable parameterization of the model. Most of the targets are based on data for the post-war period, until 2007, and come from National Income and Product Accounts (NIPA) and the Survey of Consumer Finances (SCF). One period in the model corresponds to one quarter.

4.1 Functional forms

The capital owner's per-period utility function is $u(c_1, n_1) = \log c_1 - [\omega_1/(1+\sigma_1)]n_1^{(1+\sigma_1)}$, where $\omega_1 > 0$ and $\sigma_1 > -1$. Such specification is common in the New-Keynesian literature.

The homeowner's utility function is analogous, except that it also depends on housing: $u(c_2, h, n_2) = \varrho \log c_2 + (1-\varrho) \log h - [\omega_2/(1+\sigma_2)] n_2^{(1+\sigma_2)}$, with $\omega_2 > 0$, $\sigma_2 > -1$, and $\varrho \in (0,1)$. The production function $AK^\varsigma N^{1-\varsigma}$ is also standard. The function governing the curvature of the production possibilities frontier is $q_H(X_{Ht}) = \exp(\zeta_H(X_{Ht} - X_H))$, where $\zeta_H > 0$ and X_H is the steady-state ratio of housing investment to output (output is normalized to be equal to one in steady state). Analogously, $q_K(X_{Kt}) = \exp(\zeta_K(X_{Kt} - X_K))$, where $\zeta_K > 0$ and X_K is the steady-state ratio of capital investment to output. Finally, $\Upsilon(-\widetilde{B_t}) = \exp(-\vartheta \widetilde{B}_t) - 1$, where $\vartheta > 0$ and in steady-state $\widetilde{B} = 0$. All the functional forms satisfy the properties assumed in the description of the model.

4.2 Parameter values

The parameter values are listed in Table 1, where they are organized into nine categories: Ψ (population); β, σ_1, σ_2, ω_1, ω_2, ϱ (preferences); ς, Δ, δ_K, δ_H, ϵ_w, ζ_K, ζ_H (technology); G, τ_N, τ_K, $\overline{\tau}_2$ (fiscal); ε, ψ (goods market); θ, κ, α (mortgage market); ϑ (bond market); $\overline{\pi}$, ν_π (monetary policy); and ρ_μ, ρ_η, σ_μ, σ_η (stochastic processes). Most parameters can be assigned values independently, without solving a system of steady-state equations. Six parameters (ω_1, ω_2, ϱ, ϵ_w, τ_K, $\overline{\tau}_2$) have to be obtained jointly from such steady-state relations. And another six parameters (ζ_K, ζ_H, ρ_μ, ρ_η, σ_μ, σ_η) are assigned values on the basis of the dynamic properties of the model; these last six parameters do not affect the steady state and thus the values of the other parameters.

4.2.1 Parameters calibrated independently

We start with a description of the parameters in the first group. The population parameter Ψ is set equal to 2/3. This corresponds to the notion that the typical homeowner comes from the middle class, the 3rd and 4th quintiles of the wealth distribution, whereas the typical owner of capital comes from the 5th quintile (Campbell and Cocco, 2003). The parameter controlling the elasticity of labor supply is treated symmetrically across homeowners and capital owners.

Guided by the New-Keynesian literature, $\sigma_1 = \sigma_2 = 1$. Regarding β, the Euler equation for l_{1t} constrains i^F to equal to i in steady state. The Euler equation for $b_{1,t+1}$ then relates i and π to β. Using $i^F = 0.0233$ and $\pi = 0.0113$, implies $\beta = 0.9883$. The parameter ς corresponds to the NIPA share of capital income in output and is set equal to 0.283. As in the New-Keynesian literature, the fixed cost is set so as to ensure zero steady-state profits. This requires $\Delta = 0.2048$. The depreciation rates δ_K and δ_H are set equal to 0.02225 and 0.01021, respectively, to be consistent with the average flow-stock ratios for capital and housing, X_K/K and X_H/H. Based on NIPA, the appropriate counterpart to G makes up on average 0.138 of output and the aggregate labor income tax rate τ_N is 0.235. The parameter ε governing the goods elasticity of substitution and the Calvo parameter ψ (the fraction of firms not adjusting prices) are set equal to 0.83 and 0.7, respectively—standard values in the New-Keynesian literature.[27] The loan-to-value (LTV) ratio θ is set equal to 0.6. This is based on the long-run average of the cross-sectional mean LTV ratio for newly-built home mortgages and the share of conventional mortgages in total new loans. The amortization parameters κ and α are set equal to 0.00162 and 0.9946, respectively. These values provide a reasonable approximation of the payment schedule for a 30-year mortgage. The bond market parameter ϑ is set equal to 0.035, in order to replicate an interest premium schedule for unsecured credit estimated by Chatterjee, Corbae, Nakajima and Rios-Rull (2007). The steady-state inflation rate π is set equal to the aforementioned average of 0.0113. The weight on inflation ν_π in the monetary policy rule is set equal to 1.5, a standard value in the New-Keynesian literature.

4.2.2 Parameters calibrated jointly

Given the values of the parameters in the first set, the values of the six parameters in the second set $(\omega_1, \omega_2, \varrho, \epsilon_w, \tau_K, \overline{\tau}_2)$ are determined by matching, in steady state, six targets: the observed average capital-to-output ratio ($K = 7.06$); housing stock-to-output ratio ($H =$

[27] According to this parameterization, the average price duration is $(1-\psi)^{-1} = 3.33$ quarters, about 10 months.

5.28); the aggregate hours worked ($N = 0.255$); capital owners' income share from labor ($\epsilon_w wn_1/income_1 = 0.53$), mortgage debt servicing costs of homeowners ($\widetilde{m}_2/income_2 = 0.15$); and homeowners' income share from transfers ($\tau_2/income_2 = 0.12$). Here, $income_1 = (rk + \widetilde{m}_1) + \epsilon_w wn_1 + \tau_1$, $income_2 = wn_2 + \overline{\tau}_2$, and $\widetilde{m}_1 \equiv m_1/p$, $\widetilde{m}_2 \equiv m_2/p$, with p normalized in steady state to equal to one. The expressions for income are consistent with the way income is defined in SCF. These targets yield $\omega_1 = 8.1616$, $\omega_2 = 13.004$, $\varrho = 0.6183$, $\epsilon_w = 2.4$, $\tau_K = 0.3362$, and $\overline{\tau}_2 = 0.0589$. Roughly speaking, K identifies τ_K, H identifies ϱ, homeowners' income share from transfers identifies $\overline{\tau}_2$, and the aggregate labor N, capital owners' income share from labor, and mortgage debt servicing costs of homeowners identify the labor supply variables ω_1, ϵ_w, and ω_2.

4.2.3 Discussion: the role of fiscal parameters

It is appropriate at this stage to explain why taxes and government expenditures are included in the model. Without taxes on capital and labor, positive transfers to homeowners would have to be financed by negative transfers to capital owners, which is inconsistent with the SCF data. Government expenditures in the model then ensure that, given the revenues from capital and labor taxes, the transfers to the two agents are not too large and thus do not account for too large shares of their income. Lining up the sources of income in the model with the data allows for realistic margins of income adjustment in smoothing out the effects of the real value of mortgage payments on disposable income.

4.2.4 Calibration based on model dynamics

Six parameters remain to be assigned values: ζ_K, ζ_H, ρ_μ, ρ_η, σ_μ, σ_η. These are calibrated on the basis of the model dynamics. Recall that we require the model to be consistent with both, the standard New-Keynesian responses to a monetary policy shock, discussed in Sections 3.1 and 3.2, and the empirical persistence of the FRM rate. The parameter ρ_μ is chosen so as to replicate the latter.[28] This yields $\rho_\mu = 0.99$. The parameters ζ_K and ρ_η

[28] 10-year government bond yield, rather than the 30-year FRM rate, is used due to longer data availability.

are chosen so as to replicate the typical New-Keynesian responses. In particular, the model is required to generate a one-percentage point (annualized) increase in the nominal interest rate accompanied with a -0.5 percent decline in output. Such quantitative responses, based on standard VARs, seem to roughly hold in both the United States and Eurozone data (e.g., Peersman and Smets, 2001). This strategy yields $\zeta_K = 4.5$ and $\rho_\eta = 0.3$. The value of ρ_η is within the bounds of the persistence of standard monetary policy shocks, from 0 to 0.5, reported in the literature, depending on the model and the specification of the interest rate feedback rule.[29] The parameter ζ_H is then chosen so as to make housing investment about twice as volatile as capital investment, roughly in line with the data. This yields $\zeta_H = 5.0$. The calibration of σ_μ and σ_η is postponed until Section 5.2.

4.3 Steady-state implications

Table 2 reports the steady-state values of the model's endogenous variables and, where possible, the long-run averages of their data counterparts. The first panel lists the variables used as calibration targets, while the second panel lists implications of the parameterization for other variables. As can be seen from the second panel, despite the stylized nature of the model, the steady state is broadly consistent with a number of moments not targeted in calibration. In particular, the model is consistent with the net rate of return on capital, the share of asset income in total income of capital owners, the share of labor income in total income of homeowners, and the distribution of earnings. Income distribution in the model prescribes somewhat larger share to capital owners than in the data. We also calculate mortgage payments, received (capital owner) or paid (homeowner), as a fraction of the agents' post-tax income. This fraction is much higher for the homeowner, 0.19, than for the capital owner, 0.07.

[29]The model is not rich enough to replicate the exact shape of the responses to the standard monetary policy shock obtained from the VARs. The calibration target is simply the sign and the relative size of the responses of the nominal interest rate and output. In the data, the decline in output is somewhat delayed, whereas in the model it is immediate.

5 Findings

The presentation of findings consists of two steps. First, we present the responses of the model economy to a one-percentage point (annualized) increase in the short-term nominal interest rate occurring due to either (i) the temporary or (ii) the persistent shock. In each case the responses are decomposed into the individual contributions of sticky prices (i.e., mortgages are removed by setting $\theta = 0$) and mortgages (i.e., prices are made fully flexible by setting $\psi = 0$). Second, we calibrate the relative sizes of the two shocks from yield curve data and use this information to assess the relative importance of the two shocks and the two frictions for the economy.

5.1 A one-percentage point increase in the short rate

The results of the first set of experiments confirm the arguments regarding the interaction between the frictions and the persistence of the shocks, developed analytically in Section 3, and their consequences for the aggregates and redistribution. These findings are presented in Figures 1-4.

Figure 1 shows the responses to the temporary shock under ARM. Under the baseline scenario with both sticky prices and mortgages, we can see the typical New-Keynesian responses that the model was calibrated to generate: the nominal interest rate increases while output and inflation fall, with the decline in output being larger than the decline in inflation. The decline in output is distributed across all of its components: consumption of both homeowners and capital owners, housing investment, and capital investment all decline in response to the shock. When the responses are decomposed into the effects of the individual frictions, it becomes apparent that they are driven by sticky prices. Mortgages are almost irrelevant. Their presence essentially only leads to a short-lived increase in real mortgage payments and thus somewhat stronger decline in consumption of homeowners than is the case otherwise. Effectively the same message comes out also from the responses under FRM, as Figure 2 shows. Here, even the response of homeowners' consumption is almost identical with or without mortgages, as the temporary shock under FRM has very limited effect on real mortgage payments.

Figures 3 and 4 report the responses, under ARM and FRM, to the level-factor shock. First, observe that in both cases, by the nature of the shock, the nominal interest rate and inflation increase almost one-for-one and that their responses are highly persistent. Further, in line with our discussion in Section 3, real mortgage payments increase immediately and persistently under ARM, whereas under FRM they exhibit a protracted decline. Notice also that, in accordance with the intuition developed in Section 3, the size of the initial increase in real mortgage payments under ARM is essentially the same (about 6%) as in the case of the temporary shock. In both cases the nominal interest rate increases, on impact, by one percentage point per annum. In the previous case this was due to an increase in the real rate, whereas in the present case it is due to an increase in the inflation rate. In the present case, however, the increase in real mortgage payments is substantially more persistent.

The two figures also show the redistributive nature of the shock. Under ARM, in response to the sharp increase in real mortgage payments, consumption of homeowners declines. Housing investment, which is in addition negatively affected by more expensive new loans (the price effect), also declines, thus reducing future housing services. In contrast, consumption of capital owners, as well as capital investment, increase. Aggregate responses, measured by the responses of aggregate output and consumption, are, however, small (as total investment is the difference between output and total consumption, it also responds only a little). Decomposition into the contribution of the individual frictions shows that most of the responses of consumption by the two agents (as well as of housing investment) are due to mortgages. In fact, the redistributive consequences for homeowners would be even larger if sticky prices were not present. This is because positive inflation under sticky prices somewhat increases output and thus also homeowners' income and consumption.

The message under FRM (Figure 4) is similar to that under ARM. The main effect of the level-factor shock is redistributive and redistribution occurs due to mortgages. The difference, compared with the ARM case, is that the redistribution is in favor of homeowners and that the redistributive effects are gradual, as expected from our discussion in Section 3.

5.2 Quantitative assessment

The movements in nominal interest rates in the model induced by the two shocks take the form of either shifts in the level of all nominal interest rates or changes in the long-short spread, $i_t^F - i_t$. We therefore bring in the principal component (PC) analysis of the nominal yield curve in the data to gauge the relative sizes of the two shocks. The PC analysis reveals that, like for the United States (e.g., Piazzesi, 2006), for a number of developed economies two factors are sufficient to describe all of the movements of nominal interest rates across maturities, including the short rate controlled by monetary policy. The quantitatively more important factor closely resembles the persistent shock in the model.

5.2.1 Mapping the interest rate into principal components

Specifically, PC analysis decomposes fluctuations in J yields into, at the most, J orthogonal principal components.[30] Let $\mathbf{Y}_t$ be a vector of J nominal yields at time t and $\text{var}(\mathbf{Y}_t) = \Omega\Lambda\Omega^\top$ be its variance-covariance matrix, where Λ is a diagonal matrix of eigenvalues and Ω is a matrix of the associated eigenvectors. A $J \times 1$ vector of principal components is then given as $\mathbf{pc}_t = \Omega^{-1}(\mathbf{Y}_t - \mathbf{Y})$, where $\mathbf{Y}$ is the unconditional mean of the vector $\mathbf{Y}_t$. The variance of the jth principal component is equal to the jth element of the matrix Λ and $\text{tr}(\Lambda) = \text{tr}(\text{var}(\mathbf{Y}_t))$; i.e., the sum of the variances of the principal components is equal to the sum of the variances of the individual yields.

We carry out the PC analysis for Australia, Canada, Germany, Japan, and the United States. These countries include both FRM and ARM countries. The sample is limited by our requirement of data availability for at least three maturities of government bonds for each

[30]While the movements in the level of interest rates and the long-short spread in the model are not strictly speaking orthogonal to each other, due to endogenous responses of the real rate in equation (17) to both shocks, their correlation is essentially zero, as the persistent shock has only a tiny effect on the real rate (it moves nominal interest rates and inflation roughly by the same amount—refer back to Figures 3 and 4).

country going back to at least mid 1970s. A selection of the yields for each country is plotted in the left-hand side columns of Figures 5 and 6.[31] Clearly, for all countries, the yields across maturities tend to move together. This is reflected in the first principal components plotted in the right-hand side columns of Figures 5 and 6. The second principal component, also plotted, accounts for the differences between the long and short yields.

Table 3 summarizes the statistical properties of the first principal component. As would be expected from the figures, the first principal component is much more volatile than the second principal component (measured by the ratio of their standard deviations) and accounts for a bulk of the volatility across maturities. Over 95%, and in most countries 98%, of the volatility is due to this factor; this percentage is for $\Lambda_1 / \operatorname{tr}(\Lambda)$. The second principal component essentially accounts for the remainder of the volatility; the other components have negligible effect. Furthermore, the first principal component is highly persistent (autocorrelation of around 0.98) and highly positively correlated with both short and long yields (0.93-0.98 and 0.97-0.99, respectively). Its correlation with inflation is also high (0.67-0.80).[32] All these properties are broadly in line with the properties of the level factor shock in the model.

Let us rewrite equation (17) as

$$i_t \approx \mu_t + \left[\sum_{\iota=0}^{\infty} \left(\frac{1}{\nu_\pi} \right)^{\iota} E_t r^*_{t+\iota} - \frac{\rho_\eta}{\nu_\pi - \rho_\eta} \eta_t \right] \equiv \mu_t + slope_t. \tag{26}$$

Recall that all variables are expressed as percentage point deviations from steady state. Further,

$$\operatorname{var}(i_t) \approx \operatorname{var}(\mu_t) + \operatorname{var}(slope_t).$$

It is possible to decompose the variance of i_t this way as the μ_t shock has only a small effect

[31] All data used in the PC analysis are from Haver: AUS (3M, 5YR, 10YR for 1972.Q1-2016.Q1); CAN (3M, 1-3YR, 3-5YR, 5-10YR, 10+YR for 1962.Q1-2015.Q1); GER (3M, 1YR, 2YR, 3YR, 4YR, 5YR, 6YR, 7YR, 8YR, 9YR, 10YR for 1972.Q4-2012.Q1); JAP (3M, 3YR, 5YR, 7YR, 9YR for 1975.Q4-2014.Q4); and US (3M, 1YR, 3YR, 5YR, 10YR, 20YR for 1953.Q2-2016.Q1). While the set of maturities and the sample period differ across countries, we chose to maximize the number of observations over sample consistency.

[32] The data for inflation are for quarterly year-on-year changes in CPI. Source: FRED.

on the real rate $r_{t+\iota}$ (it moves the nominal interest rate and inflation more or less by the same amount; see Figures 3 and 4), thus making μ_t and $slope_t$ approximately orthogonal to each other. From the PC analysis, the short rate is given as

$$\widehat{y}_{1t} = \Omega_{11} pc_{1t} + \Omega_{12} pc_{2t} + o_t \approx \Omega_{11} pc_{1t} + \Omega_{12} pc_{2t}, \tag{27}$$

where $\widehat{y}_{1t} = y_{1t} - y_1$, Ω_{11} and Ω_{12} are factor loadings (elements of the matrix Ω), and o_t denotes the effect of higher principal components.

As stated in the Introduction, we ask the hypothetical question: Suppose the observed movements of nominal interest rates were due to only nominal (monetary policy) shocks. What would be the quantitative importance of each friction, sticky prices and mortgage contracts, in transmitting these shocks into real variables? Given the resemblance of the level-factor shock in the model to the first principal component in the data, we put equality between the two. We then assign all of the remaining volatility in the short rate in the data to the standard monetary policy shock. Equations (26) and (27) thus provide the following mapping between the data and the model

$$\Omega_{11} pc_{1t} = \mu_t \quad \text{and} \quad \Omega_{12} pc_{2t} = slope_t.$$

Abstracting from other factors behind the movements in the second principal component, such as time-varying risk premia, overstates the contribution of the standard monetary policy shock to the movements in the short rate. As this shock transmits only through sticky prices, this assumption maximizes the quantitative importance of the sticky price channel in affecting the real variables in the model.

Taking the U.S data as representative of the data in Figures 5 and 6, the variance of the short rate is equal to 5.897e-5 (this is for the data at the quarterly rate; i.e., the annual percentage rate divided by 400). Using the calibrated persistence of the two shocks, 0.99 for the level-factor shock and 0.3 for the temporary shock, the standard deviations of the respective innovations are chosen so that μ_t accounts for 95% and $slope_t$ for 5% of the volatility of i_t, while matching the volatility of i_t in the data.[33] This yields, $\sigma_\mu = 0.0011$ and $\sigma_\eta = 0.0025$. Because the nominal interest rate responds to the two shocks very similarly under both ARM and FRM (refer back to Figures 1-4), the same standard deviations (up to the fourth decimal place) are obtained in the two cases.

5.2.2 Findings from simulations

Using the above values of σ_μ and σ_η, the model is simulated to obtain the stationary distribution of the endogenous variables under (i) both shocks, (ii) only the level factor shock, and (iii) only the standard monetary policy shock. The purpose of these simulations is to see the relative magnitude of the real effects of the two shocks and hence the two frictions. Recall that the level factor shock transmits mainly through mortgages, whereas the standard monetary policy shock transmits only through sticky prices. The stationary distribution is normal as the shocks are normally distributed and the approximate solution of the model is linear. The mean of the stationary distribution is the deterministic steady state. We use both standard deviations and variances to report the real effects. Standard deviations are easier to interpret, but variances allow for decomposition. Table 4 contains the findings. In the simulations, like in the impulse responses, the interest rate is measured as percentage point deviations from steady state, quantities as percentage deviations from steady state. Aggregate and individual quantities are thus measured in the same unit. In contrast to the impulse responses, the interest rate is APR/400.

The first line of Table 4 simply reports that the level factor shock indeed accounts for 95% of the movements of the short rate. Next, observe that under both shocks, individual consumption of the two types is more volatile than aggregate consumption under both ARM and FRM (indeed, C_1 and C_2 are negatively correlated; see the bottom line of the table). The standard deviations are similar under the two contracts.[34] Once the volatility (i.e.,

[33] Our main results stay basically the same if we use instead the persistence of the first principal component in the data, equal to 0.98, instead of the calibrated value 0.99 we have worked with so far.

[34] These are unconditional moments and so the timing of the movements in consumption reported in the

variance) of the consumption variables is decomposed into volatility due to each shock, the level factor shock accounts for a bulk of the volatility of individual consumption of the two agent types (80% and 83% of C_1 and C_2, respectively, under ARM and 82% and 93%, under FRM), whereas the standard monetary policy shock accounts for a bulk of the volatility of aggregate consumption (78% under ARM and 96% under FRM). The level factor shock is thus the main driving force behind individual consumption volatility, whereas the standard monetary policy shock is the main driving force behind aggregate consumption volatility. The redistributive vs. aggregate consequences of the shocks are also demonstrated by the correlations of C_1 with C_2. Under the level factor shock the correlations are -0.89 and -0.99 under ARM and FRM, respectively, whereas under the standard monetary policy shock the correlations are 0.65 and 0.77.[35]

With respect to the relative sizes of the effects of the two shocks on individual and aggregate consumption, the amount of the volatility of C_1 and C_2 attributed to the level factor shock is somewhat larger than the amount of the volatility of aggregate consumption attributed to the standard monetary policy shock: the level factor shock generates variances of, respectively, C_1 and C_2 equal to 1.69e-5 and 9.86e-5 under ARM and 1.97e-5 and 15.2e-5 under FRM, whereas the standard monetary policy shock generates variances of C equal to 0.6e-5 under both contracts. The redistributive consequences of the persistent shock to the policy rate are thus at least as important as the aggregate consequences of the temporary shock. Furthermore, as the variances show, these consequences are far more significant for the homeowner, than the capital owner.

For completeness, we also report the findings for the investment variables. The picture here is less clear-cut, but at least for housing investment, most of the volatility is due to the level factor shock (60% under ARM and 0.71% under FRM). Indeed, as the table shows, including housing into the measure of consumption of the homeowner, as $C_{2t}^{\varrho} H_t^{1-\varrho}$,

impulse responses are not of a first order importance.

[35] An idealized shock with purely redistributive consequences for consumption should generate perfect negative correlation and have no effect on aggregate consumption, whereas an idealized shocks with purely aggregate consequences for consumption should generate a perfect positive correlation.

increases the share of homeowners' individual consumption accounted for by this shock. As for aggregate output, most of the volatility is due to the standard monetary policy shock (60% under ARM and 76% under FRM), but the effects of the level factor shock are not negligible (40% under ARM and 24% under FRM).

6 Conclusion

The presence of nominal rigidities is an important element in the transmission mechanism of monetary policy. For a number of developed economies, yield curve data show that fluctuations in nominal interest rates, including the short rate that is under an effective control of monetary policy, are well captured by two distinct components. One is relatively temporary whereas the other is highly persistent. Such changes in the policy interest rate can potentially generate both aggregate as well as redistributive effects in the economy, in particular when borrowers and lenders use long-term nominal contracts, such as mortgages, and products markets are not fully flexible.

Using a dynamic stochastic general equilibrium model, we compare the quantitative importance of such nominal rigidities, sticky prices and long-term mortgage contracts, in transmitting temporary and persistent changes in the policy rate into the real economy. Sticky prices have been at the core of models used for monetary policy analysis for nearly two decades, while the interest in nominal debt contracts is more recent. Our model indicates that the sticky price channel is the more important transmission mechanism for temporary changes in the policy rate, whereas the mortgage channel is powerful when the changes are persistent. The real effects of the two channels, however, manifest themselves differently. The rigidities in product markets generate significant aggregate effects but small redistributive effects. The opposite holds for the transmission through mortgages. Simulating the economy shows that the redistributive consequences of monetary policy operating through the mortgage channel are of similar magnitudes as the standard aggregate consequences operating through the sticky price channel. The size of the redistribution is not affected by the nature of the debt contract (ARM vs. FRM), although the timing and direction is. Furthermore, consumption of homeowners (borrowers) is affected significantly more than consumption of lenders.

In terms of policy implications for central banks, the model suggests that while persistent changes in the policy rate have a small impact on aggregate economic activity, they generate sizeable redistributions in mortgage markets. This lesson is especially pertinent in the current policy environment, in which nominal interest rates have been kept at low levels for almost a decade. The purpose of such policies was to stimulate aggregate economic activity. According to our model, the initial cut in policy rates may have fulfilled this objective, to the extent it was expected to be temporary, but the subsequent policy of keeping rates low for a substantial period of time more likely led to income and consumption redistribution than to the desired aggregate effects. As inflation followed nominal interest rates to similarly low levels, based on our model, we can infer that lenders in FRM countries gained at the expense of borrowers due to persistently low inflation rates, while in ARM countries borrowers gained at the expense of lenders due to persistently low nominal interest rates.

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Table 1: Parameter values

Symbol	Value	Description
Population		
Ψ	2/3	Share of homeowners
Preferences		
β	0.9883	Discount factor
σ_1	1.0	Frisch elasticity (capital owner)
σ_2	1.0	Frisch elasticity (homeowner)
ω_1	8.1616	Disutility from labor (capital owner)
ω_2	13.004	Disutility from labor (homeowner)
ϱ	0.6183	Weight on consumption (homeowner)
Technology		
ς	0.283	Capital share of output
Δ	0.2048	Fixed cost
δ_K	0.02225	Depreciation rate of capital
δ_H	0.01021	Depreciation rate of housing
ϵ_w	2.4	Rel. productivity of cap. owners
ζ_K	4.5	Curvature of PPF (X_K)
ζ_H	5.0	Curvature of PPF (X_H)
Fiscal		
G	0.138	Government expenditures
τ_N	0.235	Labor income tax rate
τ_K	0.3362	Capital income tax rate
$\overline{\tau}_2$	0.0589	Transfer to homeowner
Goods market		
ε	0.83	Elasticity of substitution
ψ	0.7	Fraction not adjusting prices
Mortgage market		
θ	0.6	Loan-to-value ratio
κ	0.00162	Initial amortization rate
α	0.9946	Amortization adjustment factor
Bond market		
ϑ	0.035	Participation cost function
Monetary policy		
π	0.0113	Steady-state inflation rate
ν_π	1.5	Weight on inflation
Exogenous processes		
ρ_μ	0.99	Persistence of the level factor shock
ρ_η	0.3	Persistence of standard mon. pol. shock

Table 2: Nonstochastic steady state vs long-run averages of U.S. data

Symbol	Model	Data	Description
Targeted in calibration:			
i^M	0.0233	0.0233	Nominal mortgage rate
X_K	0.156	0.156	Capital investment
X_H	0.054	0.054	Housing investment
K	7.06	7.06	Capital stock
H	5.28	5.28	Housing stock
N	0.255	0.255	Aggregate hours worked
$\epsilon_w wn_1/income_1$	0.53	0.53¶	Labor income in cap. owners' income
$\widetilde{m}_2/(wn_2+\overline{\tau}_2)$	0.15	0.15	Debt-servicing costs (pre-tax)
$\overline{\tau}_2/(wn_2+\overline{\tau}_2)$	0.12	0.12¶	Transfers in homeowners' income
Not targeted:			
A. Capital owner's variables			
$(1-\tau_K)(r-\delta_K)$	0.012	0.013§	Net (post-tax) rate of return on capital
$[(r-\delta)k+\widetilde{m}_1]/income_1$	0.42	0.39¶,§§	Income from assets in total income
$\tau_1/income_1$	0.05	0.08	Transfers in total income
$\widetilde{m}_1/netincome_1$	0.07	N/A	Mortg. income in post-tax income
B. Homeowner's variables			
$wn_2/(wn_2+\tau_2)$	0.88	0.82¶	Labor income in total income
τ_H	0	N/A	Housing wedge
$\widetilde{m}_2/[(1-\tau_N)wn_2+\tau_2]$	0.19	N/A	Debt-servicing costs (post-tax)
C. Earnings distribution			
$\epsilon_w wN_1/[\epsilon_w wN_1+wN_2]$	0.60	0.54¶	Capital owners' share
$wN_2/[\epsilon_w wN_1+wN_2]$	0.40	0.46¶	Homeowners' share
D. Income distribution			
$Income_1/[Income_1+(wN_2+\Psi\tau_2)]$	0.70	0.60¶	Capital owners' share
$(wN_2+\Psi\tau_2)/[Income_1+(wN_2+\Psi\tau_2)]$	0.30	0.40¶	Homeowners' share

Notes. $Y=1$ in steady state. Capital owner's income: $income_1=(rk+\widetilde{m}_1)+\epsilon_w wn_1+\tau_1$; $Income_1=(1-\Psi)income_1$; and $netincome_1=((1-\tau_K)rk+\tau_K\delta_K k+\widetilde{m}_1)+(1-\tau_N)\epsilon_w wn_1+\tau_1$. Rates of return, interest, and amortization rates are expressed at quarterly rates.

¶ SCF; the model counterpart is defined so as to be consistent with the definition in SCF.

‡ Average for a standard 30-year mortgage.

§ NIPA-based estimate (Gomme, Ravikumar and Rupert, 2011).

§§ The sum of capital and business income in SCF, where capital income is income from all financial assets.

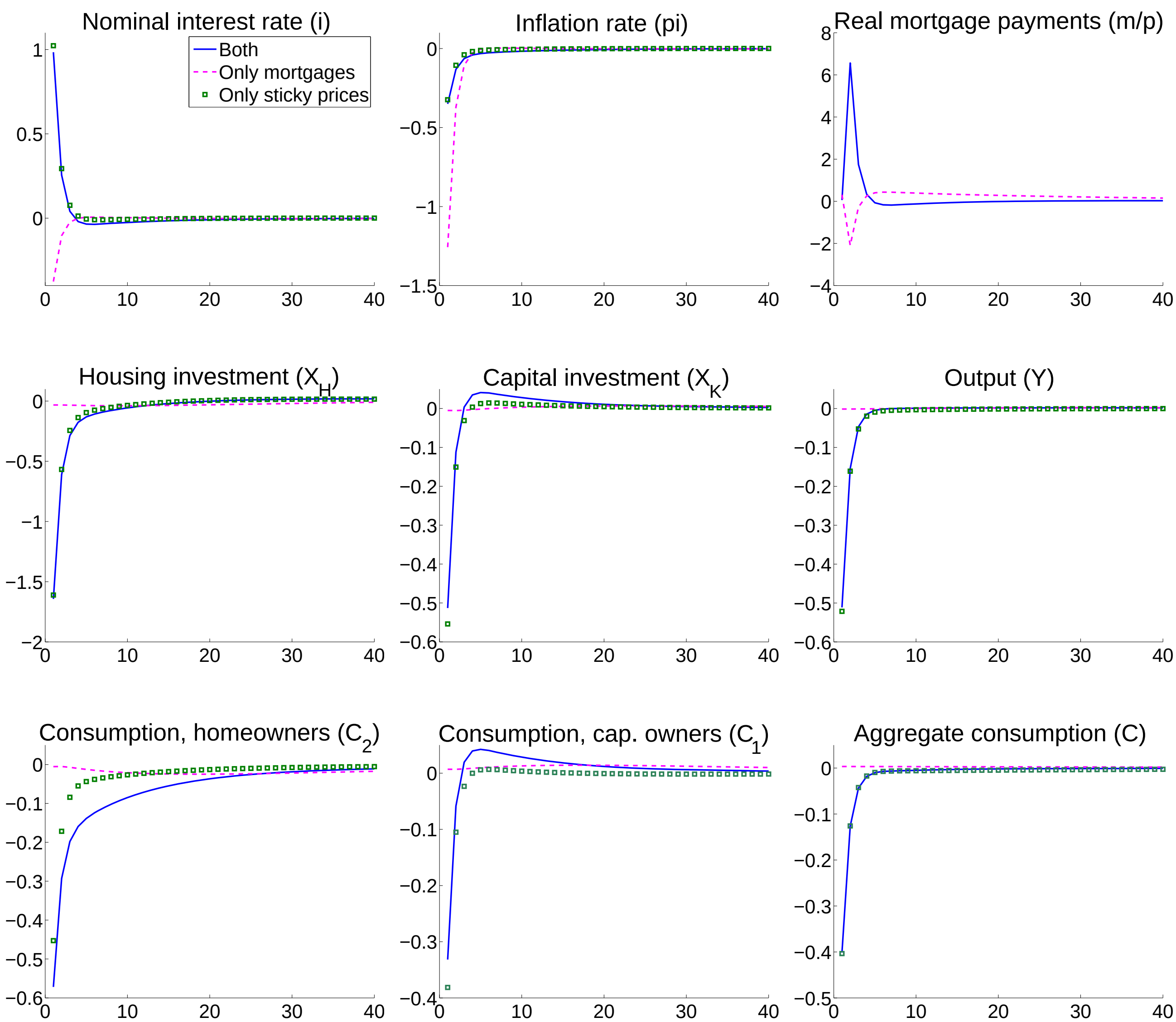


Figure 1: ARM; responses to the standard monetary policy shock. Interest rates and the inflation rate are measured as percentage point (annualized) deviations from steady state, quantities are in percentage deviations. One period = one quarter.

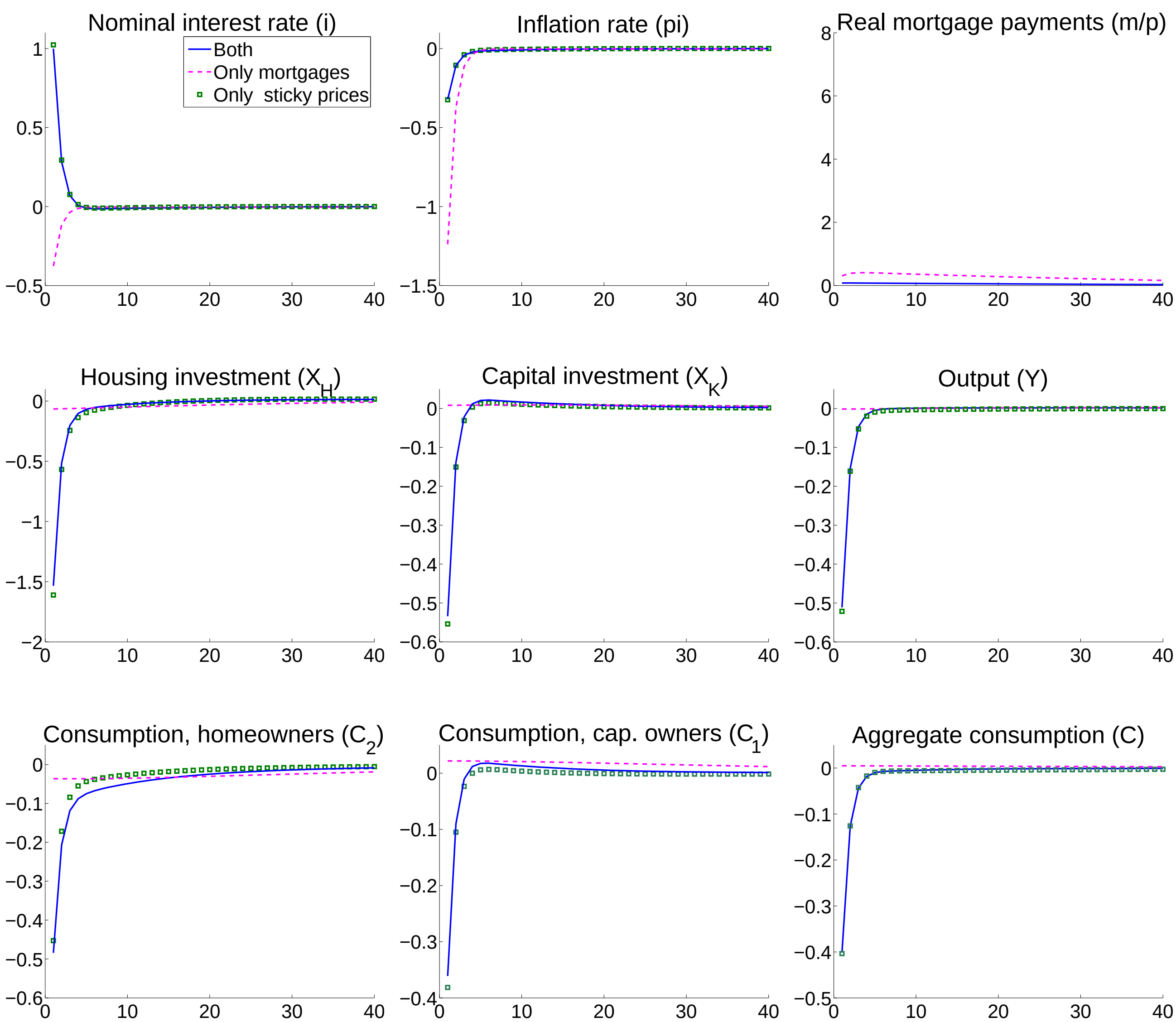


Figure 2: FRM; responses to the standard monetary policy shock. Interest rates and the inflation rate are measured as percentage point (annualized) deviations from steady state, quantities are in percentage deviations. One period = one quarter.

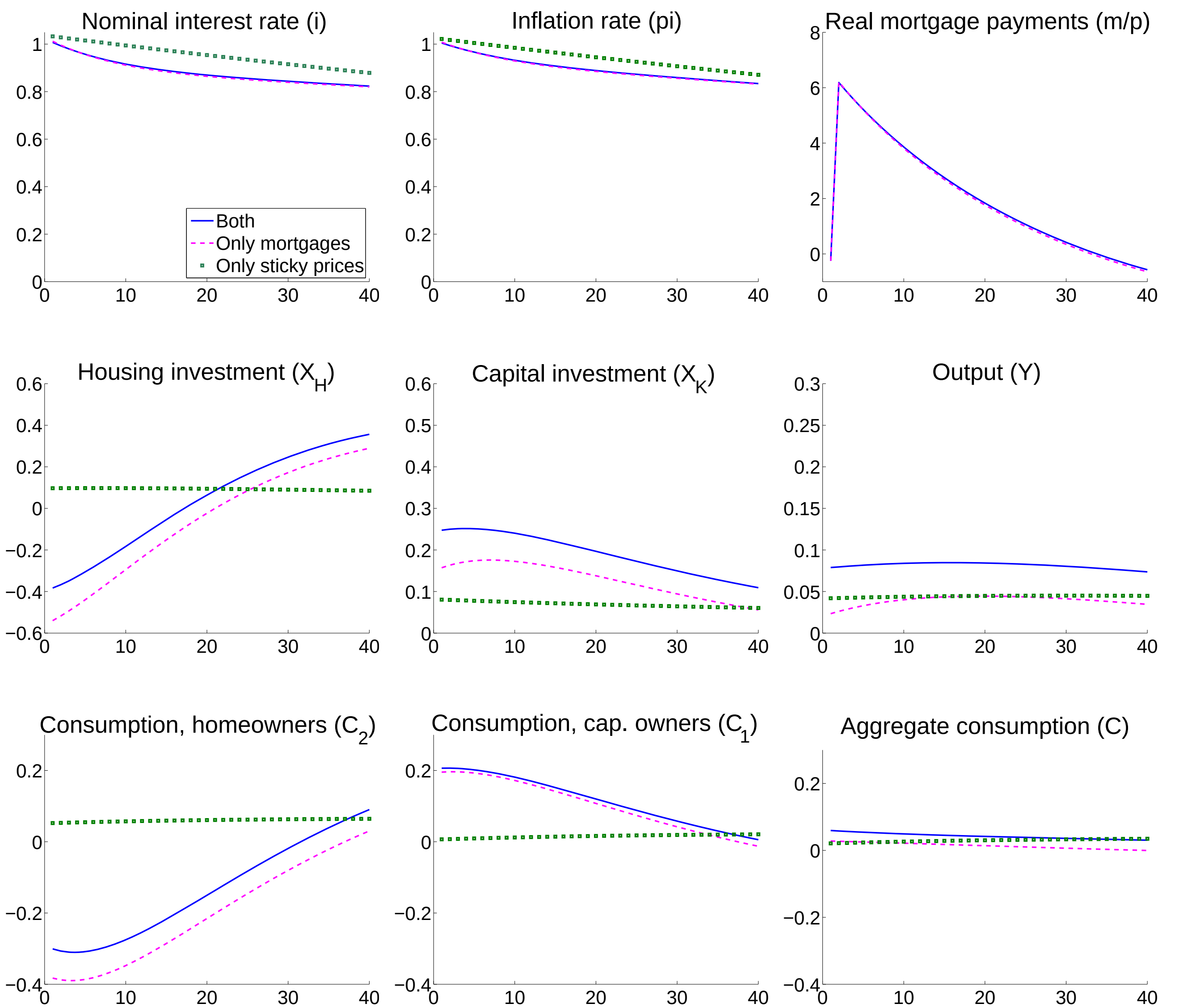


Figure 3: ARM; responses to the level factor shock. Interest rates and the inflation rate are measured as percentage point (annualized) deviations from steady state, quantities are in percentage deviations. One period = one quarter.

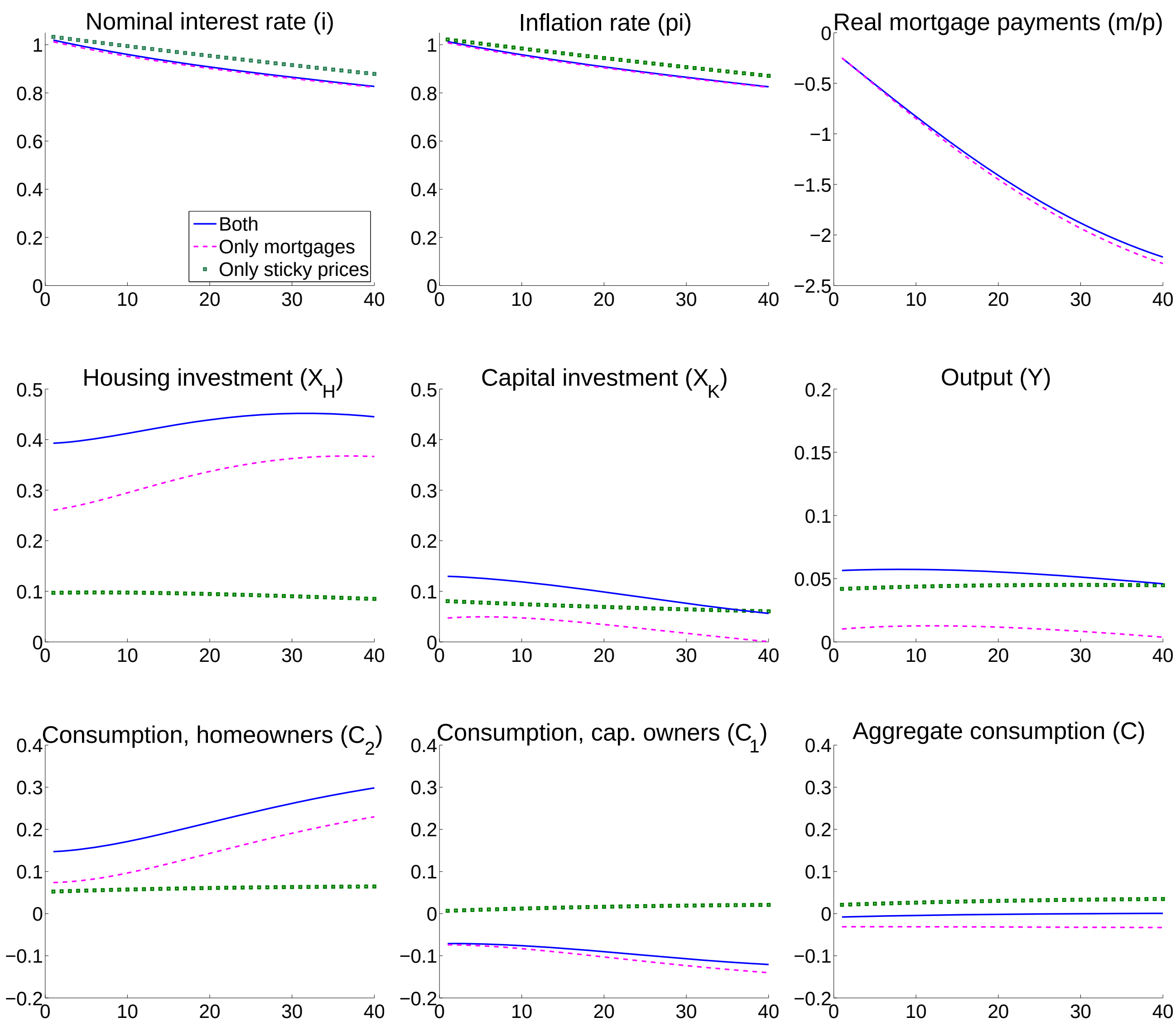
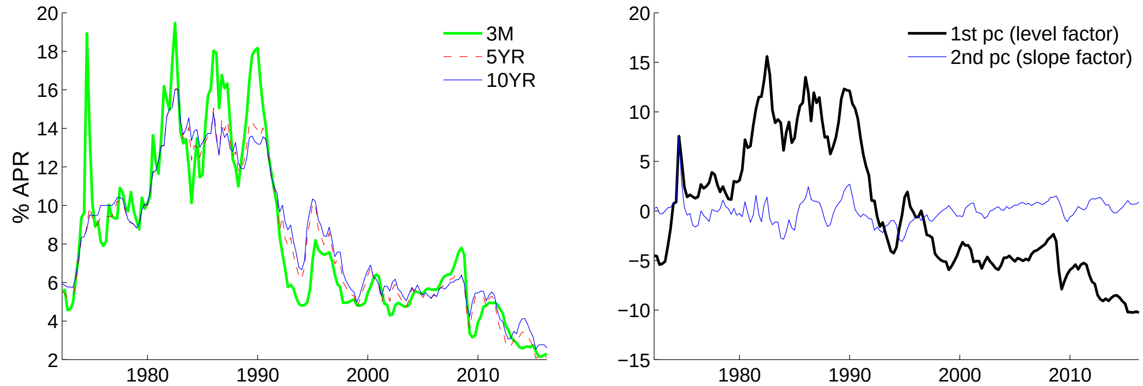
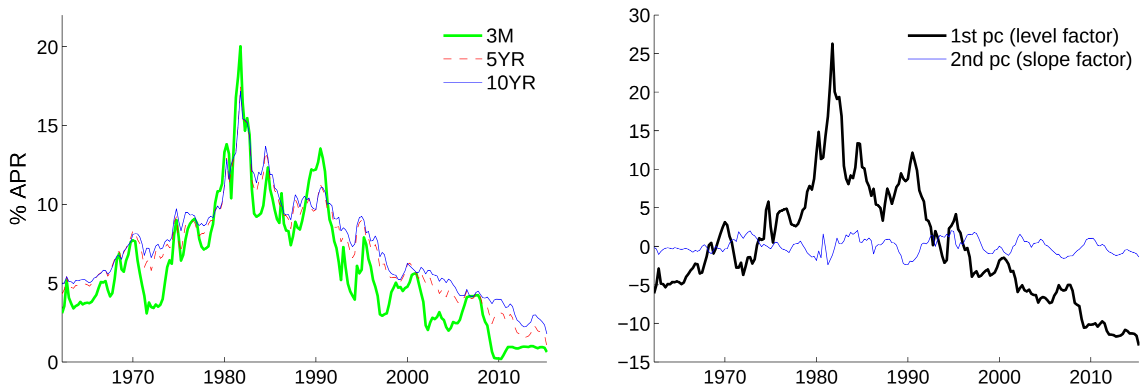


Figure 4: FRM; responses to the level factor shock. Interest rates and the inflation rate are measured as percentage point (annualized) deviations from steady state, quantities are in percentage deviations. One period = one quarter.

Australia



Canada



Germany

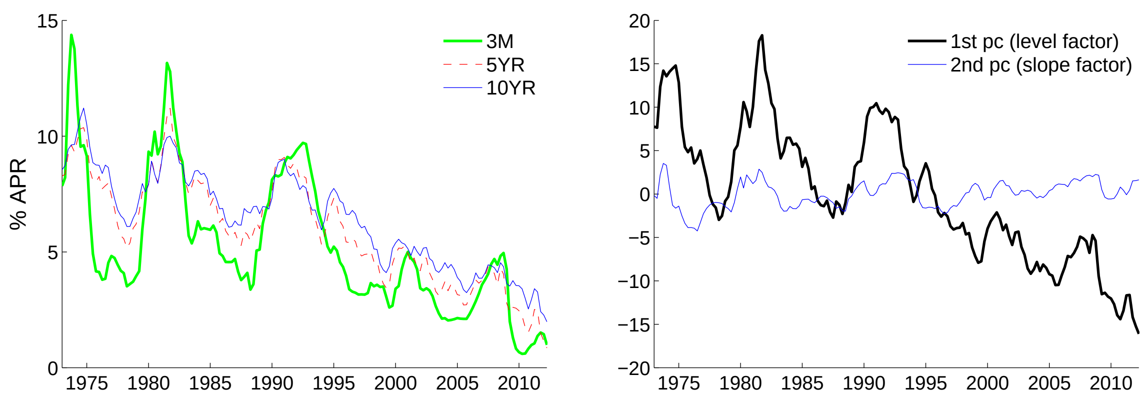


Figure 5: Nominal interest rates and principal components. Note: Only selected maturities used in the principal component analysis are plotted in the left-hand side charts (see the text for a complete list of maturities used).

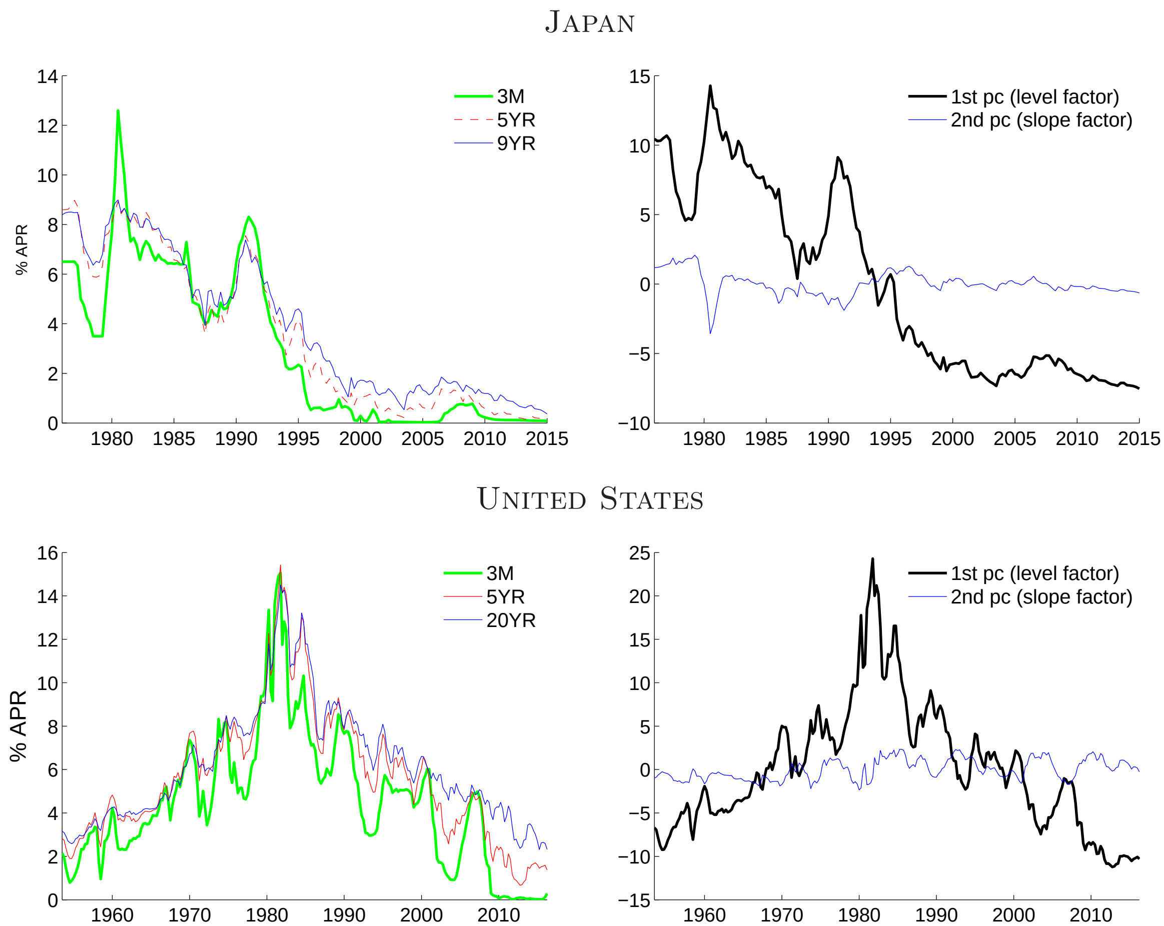


Figure 6: Nominal interest rates and principal components (continued). Note: Only selected maturities used in the principal component analysis are plotted in the left-hand side charts (see the text for a complete list of maturities used).

Table 3: Statistical properties of the 1st principal component (level factor)

	$\frac{\text{std(1st pc)}}{\text{std(2nd pc)}}$	% var($ylds$) expl.	acorr	corr w/ short	corr w/ long	corr w/ infl.
AUS	5.49	0.97	0.98	0.98	0.98	0.67
CAN	7.69	0.98	0.98	0.98	0.98	0.72
GER	5.02	0.96	0.98	0.93	0.97	0.80
JAP	7.92	0.98	0.99	0.97	0.99	0.77
US	6.22	0.97	0.98	0.98	0.97	0.73

Table 4: Quantitative assessment of the shocks/frictions

	ARM			FRM		
	100 × std μ and η	% var only μ	% var only η	100 × std μ and η	% var only μ	% var only η
i	0.77	0.95	0.05	0.77	0.95	0.05
C	0.32	0.22	0.78	0.25	0.04	0.96
C_1	0.52	0.80	0.20	0.54	0.82	0.18
C_2	1.21	0.83	0.17	1.33	0.93	0.07
$C_{2,total}$	0.97	0.88	0.12	1.12	0.96	0.04
X	1.22	0.55	0.45	1.08	0.53	0.47
X_K	0.66	0.72	0.28	0.43	0.39	0.61
X_H	1.79	0.60	0.40	1.85	0.71	0.29
Y	0.45	0.40	0.60	0.37	0.24	0.76
	corr μ and η	corr only μ	corr only η	corr μ and η	corr only μ	corr only η
C_1, C_2	-0.61	-0.89	0.65	-0.78	-0.99	0.77

Note. The moments are for the stationary distribution of the endogenous variables. The μ_t (level factor) shock transmits mainly through mortgages whereas the η_t (standard monetary policy) shock transmits only through sticky prices. The interest rate (APR/400) is measured as a percentage point deviation from steady state, quantities as percentage deviations from steady state. Agent 1 = capital owner, agent 2 = homeowner. $C_{2,total} = C_2^{\varrho} H^{1-\varrho}$.

Supplementary material: Appendixes

A.1. Real marginal costs and aggregate output

Given that the NKPC in the text is based on a log-linear approximation, our exposition uses log-linear approximations of the other relevant equations as well. First, a log-linear aggregate counterpart to equation (12) is

$$\widehat{\chi}_t = \frac{\varsigma}{1-\varsigma}\frac{1}{1+\Upsilon/Y}\widehat{Y}_t + \widehat{w}_t - \frac{\varsigma}{1-\varsigma}\widehat{K}_t, \tag{A.1}$$

where all variables are in percentage deviations from steady state.[36] For a given $\widehat{w}_t$ and the state variable $\widehat{K}_t$, this equation provides a positive relationship between aggregate output and the marginal cost. The wage rate can be eliminated from equation (A.1) by utilizing first-order conditions of the two agents for labor supply: $u_{ct}(1-\tau_N)\epsilon_w w_t + u_{nt} = 0$ and $v_{ct}(1-\tau_N)w_t + v_{nt} = 0$. At the aggregate level, in a log-linearized form, these conditions become

$$\Phi_{1n}\widehat{N}_{1t} = \Phi_{1w}\widehat{w}_t + \Phi_{1c}\widehat{C}_{1t} \quad \text{and} \quad \Phi_{2n}\widehat{N}_{2t} = \Phi_{2w}\widehat{w}_t + \Phi_{2c}\widehat{C}_{2t}, \tag{A.2}$$

where for standard utility functions (e.g., log additive) $\Phi_{1c} < 0$, $\Phi_{1w} > 0$, and $\Phi_{1n} > 0$, and similarly for the second agent. Further, from the production function of intermediate goods producers follows

$$(1+\Upsilon/Y)^{-1}\widehat{Y}_t = (1-\varsigma)[\upsilon\widehat{N}_{1t} + (1-\upsilon)\widehat{N}_{2t}] + \varsigma\widehat{K}_t, \tag{A.3}$$

where $\upsilon \equiv \epsilon_w N_1/N$.[37] For tractability, focus on immediate responses from steady state. This allows us to set $\widehat{K}_t = 0$. Furthermore, to simplify the exposition, assume that the agents can smooth out consumption reasonably well so that changes in consumption are small, $\widehat{C}_{1t} \approx 0$ and $\widehat{C}_{2t} \approx 0$. The above equations, once $\widehat{\chi}_t$, $\widehat{w}_t$, $\widehat{N}_{1t}$ and $\widehat{N}_{2t}$ are substituted out, then provide a negative relationship between $E_t\pi_{t+1} - \pi_t$ and $\widehat{Y}_t$ in the NKPC (19).

A.2. Derivation of the housing wedge

This appendix derives the expression for the housing wedge in the main text. The first-order condition for x_{Ht} is

$$v_{ct}q_{Ht}(1+\tau_{Ht}) = \beta E_t V_{h,t+1},$$

where

$$\tau_{Ht} = -\theta\left\{1 + \frac{\beta E_t \widetilde{V}_{d,t+1}}{v_{ct}} + \frac{\zeta_{Dt}(\kappa - \gamma_{2t}^{\alpha})\beta E_t V_{\gamma,t+1}}{v_{ct}} + \frac{\zeta_{Dt}(i_t^M - R_{2t})\beta E_t V_{R,t+1}}{v_{ct}}\right\}.$$

[36] Equation (A.1) contains the well-known result that an aggregation bias—occurring due to price dispersion across the intermediate good producers—disappears under log-linearization around a zero steady-state inflation rate.

[37] Equation (A.3) also contains the result that the Calvo aggregation bias due to price dispersion disappears under a log-linearization around zero steady-state inflation rate.

Here, $\widetilde{V}_{d,t+1} \equiv p_t V_{d,t+1}$, $V_{\gamma,t+1}$, and $V_{R,t+1}$ are the derivatives of the value function with respective to the variables in the subscript. Specifically, $V_{\gamma,t+1}$ and $V_{R,t+1}$ capture the marginal effects of new loans on the amortization and interest rates of the outstanding stock of debt next period, and thus on the life-time utility (notice that the latter effect is absent under ARM, as $R_{2t} = i_t^M$). The derivatives are given by Benveniste-Scheinkman conditions

$$\widetilde{V}_{dt} = -\upsilon_{ct}\left(\frac{R_{2t}+\gamma_{2t}}{1+\pi_t}\right) + \beta\left(\frac{1-\gamma_{2t}}{1+\pi_t}\right) E_t\left[\widetilde{V}_{d,t+1} + \zeta_{xt}(\gamma_{2t}^{\alpha}-\kappa)V_{\gamma,t+1} + \zeta_{xt}(R_{2t}-i_t^M)V_{R,t+1}\right],$$

$$V_{\gamma t} = -\upsilon_{ct}\left(\frac{\widetilde{d}_{2t}}{1+\pi_t}\right) + \left[\zeta_{xt}(\kappa-\gamma_{2t}^{\alpha}) + \frac{(1-\gamma_{2t})\alpha\gamma_{2t}^{\alpha-1}}{\frac{1-\gamma_{2t}}{1+\pi_t}\widetilde{d}_{2t}+\theta q_t x_{Ht}}\right]\left(\frac{\widetilde{d}_{2t}}{1+\pi_t}\right)\beta E_t V_{\gamma,t+1}$$
$$-\left(\frac{\widetilde{d}_{2t}}{1+\pi_t}\right)\beta E_t \widetilde{V}_{d,t+1} + \zeta_{xt}(i_t - R_{2t})\left(\frac{\widetilde{d}_{2t}}{1+\pi_t}\right)\beta E_t V_{R,t+1},$$

and

$$V_{Rt} = -\upsilon_{2t}\left(\frac{\widetilde{d}_{2t}}{1+\pi_t}\right) + \frac{\frac{1-\gamma_{2t}}{1+\pi_t}\widetilde{d}_{2t}}{\frac{1-\gamma_{2t}}{1+\pi_t}\widetilde{d}_{2t}+\theta q_t x_{Ht}}\beta E_t V_{R,t+1}.$$

Further, in terms of new notation

$$\zeta_{Dt} \equiv \left(\frac{1-\gamma_{2t}}{1+\pi_t}\widetilde{d}_{2t}\right) / \left(\frac{1-\gamma_{2t}}{1+\pi_t}\widetilde{d}_{2t} + \theta q_{Ht} x_{Ht}\right)^2$$

and

$$\zeta_{xt} \equiv \theta q_{Ht} x_{Ht} / \left(\frac{1-\gamma_{2t}}{1+\pi_t}\widetilde{d}_{2t} + \theta q_{Ht} x_{Ht}\right)^2.$$

Observe that when $d_{2t} = 0$ and $x_{H,t+\iota} = 0$ for $\iota = 1, 2, \ldots$, the above expressions, by forward substitutions, yield the wedge in the main text

$$\tau_{Ht} \equiv \theta\left\{-1 + E_t\left[Q_{2,t+1}\frac{i_t^M + \gamma_{2,t+1}}{1+\pi_{t+1}} + Q_{2,t+2}\frac{(i_{t+1}^M + \gamma_{2,t+2})(1-\gamma_{2,t+1})}{(1+\pi_{t+1})(1+\pi_{t+2})} + \ldots\right]\right\}.$$