TIME-CONSISTENT FISCAL POLICY IN A DEBT CRISIS

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Abstract

We analyze time-consistent fiscal policy in a sovereign debt model. We consider a production economy that incorporates feedback from policy to output through employment, features inequality through unemployment, and in which the government lacks a commitment technology. The government’s optimal policies play off wedges due to the lack of lump-sum taxes and the distortions that taxes and transfers introduce on employment. Lack of commitment matters during a debt crises – episodes where the price of debt reacts elastically to the issuance of new debt. In normal times, the government sets procyclical taxes, transfers and public goods provision but in crisis times it is optimal to implement austerity policies which minimize the distortions deriving from default premia. Could a third party provide a commitment technology, austerity is no longer optimal.

JEL: E20, E62, F34, F41

Keywords: Time-consistent fiscal policy, sovereign debt, debt crisis, austerity

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1 Introduction

How should fiscal policy be adjusted during episodes that involve both deep recessions and debt crises, such as the circumstances that took place in Europe in 2009 onwards? Under such circumstances, is it optimal to implement stimuli measures aimed at addressing the recession but at the cost of increasing government borrowing, or should governments introduce austerity measures to tackle the debt crisis at the cost of potentially worsening the recession and its consequences? This paper examines optimal fiscal policy design in a sovereign debt model in which the government lacks commitment to all of its instruments, in which there is feedback from the policy instruments to the state of the economy and which features concerns about inequality and redistribution. Recessions are modeled as the result of productivity slowdowns but also of fiscal policies. We find that commitment problems during debt crises motivate austerity measures which involve the government sacrificing social insurance in order to access international financial markets. The crux of the commitment problem is that governments have an incentive to renege on debt repayments in order to stabilize the economy and provide insurance post-default. Could a third party impose fiscal rules that constrain the government’s ability to implement a post-default fiscal stimulus, austerity measures are less likely to be optimal.

We study a small open economy of sovereign default in the Eaton and Gersovitz (1981) tradition in which the government issues non-state-contingent debt purchased by international lenders. Lenders can punish the sovereign for default through exclusion from financial markets and through sanctions that inhibit productivity. We consider a production economy in which output is produced by one-worker firms subject to stochastic productivity shocks common across firms. The labor market is frictional and jobs are created by matching workers searching for employment and vacancies created by firms. We introduce inequality across households by assuming that households are subject to uninsurable idiosyncratic income shocks that derive from unemployment risk. Households cannot save and employment contracts last for a single period so that agents are ex-ante identical at the beginning of each period but ex-post heterogeneous due to unemployment risk. We allow for a rich set of fiscal instruments consisting of unemployment welfare benefits, income taxes, public goods, and debt policy. However, we do not allow the gov-

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1See Aguiar et al (2016) for a brilliant survey and critical evaluation of the sovereign default literature.
ernment to collect revenues using lump-sum taxes. The richness of the fiscal policy instruments is key for capturing major trade-offs when setting the optimal policy mix.

The government has a utilitarian objective function but cannot commit to the future path of any of its policy instruments and we focus on Markov-Perfect equilibria. In traditional sovereign debt models, the government is concerned about providing intertemporal insurance only. In the current setup, the government also needs to address inequality and therefore engages also in intratemporal insurance. Moreover, fiscal policies affect the economy through their impact on job creation and on job search. The government uses income taxes to stabilize employed workers’ consumption streams in response to wage changes caused by productivity shocks (the standard intertemporal insurance channel). Income taxes, however, are distortionary and reduce workers’ incentive to search for employment. Welfare benefits provide income for the unemployed workers and the utilitarian objective function gives the government an incentive to redistribute income from employed to unemployed households. However, the lack of lump-sum taxes makes it costly to raise revenues and welfare benefits also distort workers’ incentive to search for employment. Finally, the government would optimally want to provide the public good so as to implement the Samuelson condition but refrains from fully doing so because of the lack of lump-sum taxes.

We show that optimal fiscal policies depend crucially on the debt regime and distinguish between “normal times,” financial autarky, and “debt crisis.” In financial autarky (which occurs after a default) and in “normal times” (when the default premium is moderate and stable), the government sets procyclical tax rates, welfare benefits and government purchases. Procyclical taxes help stabilize employed workers consumption but it is not optimal to provide complete consumption insurance because of the negative impact on search effort. This leaves employed workers’ consumption streams procyclical and for that reason it is optimal to provide procyclical welfare benefits and government purchases. However, when default premia are sensitive to debt issuance, a situation that we will refer to as a “debt crisis,” lack of commitment becomes an issue for the government and it introduces an additional wedge. In the crisis zone, it becomes optimal to implement austerity policies that sacrifice household consumption relative to the level that the government would provide if it were to default. The austerity measures involve the government hiking income tax rates and cutting government purchases and welfare transfers.

There are two underlying sources of the optimality of austerity policies. When the default
premium is high, it is expensive to issue debt and this motivates austerity measures by itself if the default premium is sufficiently sensitive to debt issuance. But there is also a strategic motive due to the lack of commitment. In particular, when debt is high, lenders realize that the sovereign has a strong motive to free up resources to stimulate the economy in a recession by defaulting on their existing debt which paves the way for a post-default fiscal stimulus. Lenders therefore set the debt price structure to induce the government to sacrifice household consumption in the crisis zone. This policy makes it expensive for the sovereign to take out a loan and thus gives an incentive to reduce debt issuance through austerity. We show that if a third party could enforce commitment to either fixed tax rates or welfare benefits, austerity expressed in terms of sacrificing household consumption is no longer optimal in a debt crisis while the pure fiscal motive remains. Thus, our results indicate that the extent to which countries can credibly commit to fiscal rules – and in particular to not abandon these should they default – is crucial for the design of fiscal policy in a debt crisis.

Our paper is related to Arellano and Bai (2014) and Cuadra, Sanchez and Sapriza (2011) who also consider a production economy formulation of the Eaton and Gersovitz (1981) model. These authors assume Walrasian labor markets. Arellano and Bai (2014) examine a model in which the government provides intertemporal insurance through lump-sum transfers and assume that the government can commit to taxes (which are fixed) on labor income and on consumption. They distinguish between fiscal and aggregate defaults, defaults that occur either because the government is unable to service its debt or because debt is unsustainable. They argue that austerity during crisis times (increasing the tax rate) may not prevent default because of the distortions on labor supply induced by taxes. Cuadra, Sanchez and Sapriza (2011) focus on a smaller set of fiscal instruments, consumption taxes, government purchases, and foreign borrowing and lending, and, like us, remove commitment from all of the instruments. They show that this model can replicate emerging markets evidence that government spending is increased in booms while taxes are hiked in recessions and argue that this is due to costs of debt being high in recessions. Our analysis allows for a richer set of fiscal instruments than these contributions and shows that optimal austerity packages are designed to promote employment by manipulating both taxes and transfers to induce higher search effort. More fundamentally, our analysis incorporates both intertemporal and intratemporal insurance channels and shows how commit-
ment impacts on optimal policies. Bi (2011) looks at a model with similar features to Arellano and Bai (2014) but models default as non-strategic and deriving from stochastic fiscal limits. He shows that austerity measures may be insufficient to avoid default in this setup. Conesa, Kehoe and Ruhl (2016) also examine the role of commitment for austerity policies in a sovereign debt model. They study a production economy version of the celebrated Cole and Kehoe (2000) multiple equilibria model of sovereign debt crisis. Their focus is on the impact of allowing the government to commit to taxes within a period, i.e. whether tax revenues are decided upon prior or posterior to the government auctioning of debt and making its default decision. They show that once a debt crisis starts, austerity policies are optimal while allowing the government to commit to taxes within the period implies that austerity policies are implemented also when the equilibrium is unique (in order to prevent debt crises from arising). Niemann and Pichler (2016) also study a Cole and Kehoe (2000) style model with production in which default costs are modeled through disruptions of intermediate goods import and analyze optimal fiscal policy. As in Conesa, Kehoe and Ruhl (2016), these authors find that austerity policies may be optimal in the crisis zone. Our analysis differs significantly from these contributions in the modeling of the default incentive, the fiscal instruments, and the introduction of inequality concerns.

The remainder of the paper is organized as follows. Section 2 describes the model and the equilibrium. Section 3 contains the numerical analysis of the model. We calibrate the model and look at its implications for optimal fiscal policy. In Section 4 we apply the model to the Greek crisis. Section 5 concludes and summarizes.

2 The Model

We formulate a sovereign debt model that extends the existing literature in several dimensions. First, we introduce inequality across households which derives from idiosyncratic uninsurable unemployment risk. Second, because of inequality, the government engages in both intratemporal redistribution across agents as well as the standard intertemporal redistribution usually considered in the sovereign debt literature. Third, our model features feedback from fiscal policy

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2 D’Erasmo and Mendoza (2016) also consider the impact of inequality and distributional issues but in a simple two-period model of sovereign domestic default.
instruments to the state of the economy. This is key for understanding important aspects of the optimal fiscal policy design. Finally, we assume lack of commitment to all fiscal instruments.

2.1 Environment

Households. There is a continuum of mass 1 of infinitely-lived households indexed by \( i \in [0, 1] \). Households have rational expectations, maximize intertemporal preferences and derive utility from consumption of private goods, \( c_{i,s} \), and of publicly provided goods, \( G_s \). Households exert costly search effort, \( e_{i,s} \), to find employment. Employed households work full time and there is a utility cost of working, \( \kappa \geq 0 \).

Preferences are given as:

\[
U_{i,t} = \mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \left[ u(c_{i,s}, e_{i,s}, G_s) - \kappa n_{i,s} \right]
\]  

(1)

where \( \mathbb{E}_t \) is the mathematical conditional expectations operator, \( \beta \in (0, 1) \) is the subjective discount factor, and \( u \) is the instantaneous felicity function which is assumed strictly increasing and concave in \( c \), non-decreasing in \( G \), and strictly decreasing and convex in \( e \). \( n_{i,s} \) indicates the household’s labor market status:

\[
n_{i,s} = \begin{cases} 
0 & \text{if household } i \text{ is unemployed in period } s \\
1 & \text{if household } i \text{ is employed in period } s 
\end{cases}
\]  

(2)

Households search for jobs in a frictional matching market. \( e \) units of search effort produce an employment probability \( pe \in [0, 1] \) where \( p \in [0, 1] \) is the job finding rate per unit of search intensity which households take as given. Employment contracts last a single period.

Households live in single agent entities and cannot save. An employed household earns a stochastic wage \( w \in R_{++} \) and pays a proportional income tax \( \tau w \), \( \tau \in [0, 1) \). Unemployed households receive a government provided welfare benefit \( \mu \in R_{++} \). In equilibrium all employed households are subject to common wage risk due to productivity shocks. Idiosyncratic earnings risk derives from the presence of unemployment risk. Hence, there is within-period income inequality but it is not transmitted over time due to the lack of savings and the one-period nature of employment contracts. Households also own the firms and may receive dividends \( \pi \) which are shared equally across households.
From now on we remove time indices unless needed. The budget constraints facing households imply that:
\[
c_i^w = (1 - \tau) w + \pi
\]
where \( c_i^w \) and \( c_i^u \) denote consumption of an employed and an unemployed household, respectively.

The lack of opportunities for intertemporal smoothing implies that households choose optimal search effort as the solution to the static optimization problem:
\[
\max_{e_i} p e_i \left[ u(e_i^w, e_i, G) - \kappa \right] + (1 - p e_i) \left[ u(e_i^u, e_i, G) \right]
\]
subject to (3) – (4). The first-order necessary condition for search effort can be expressed:
\[
e_i = \frac{1}{p} \frac{u_e(c_i^w, e_i, G)}{u_e(c_i^u, e_i, G)} + \frac{p}{u_e(c_i^u, e_i, G)} \left[ u(c_i^w, e_i, G) - u(c_i^w, e_i, G) - \kappa \right] u_e(c_i^w, e_i, G) - u_e(c_i^u, e_i, G)
\]

Since households are ex-ante identical, they choose the same search effort, \( e^* \), and aggregate employment is given as:
\[
n = \int_i n_i di = p e^*
\]

**Firms.** A continuum of identical competitive firms indexed by \( j \in [0, J] \) produce a single identical good. Entrants to the industry post vacancies, \( v_j \), and, if filled, produce \( x \) units of output employing a single worker. The profits of a firm with a filled job are:
\[
\pi_j = x(z, h') - w - a v_j
\]
where \( a \in \mathbb{R}_{++} \) is a vacancy posting cost. We assume \( x \) depends non-negatively on an exogenous aggregate productivity shock, \( z \), and on the government’s end of period credit history \( h' \):
\[
h' = \begin{cases} 
0 & \text{if the country has access to financial markets} \\
1 & \text{if the country is in autarky}
\end{cases}
\]
The aggregate productivity shock, \( z \), is generated by a continuous first-order autoregressive process with persistence \( \rho_z \) and innovation variance \( \sigma^2_z \). We assume that \( x(z, 0) \geq x(z, 1) \). One might think of this impact of the credit history on productivity as reflecting trade sanctions imposed after a sovereign default because high quality imported intermediary goods are substituted with lower quality domestic alternatives, see e.g. Mendoza and Yue (2012).
The value of a filled job, $R^f$, and the value of a vacancy, $R^v$, are given as, respectively:

$$R^f = x(z, h') - w$$  \hspace{1cm} (7)$$

$$R^v = qR^f(w, z, h') - a$$  \hspace{1cm} (8)$$

To fill one position, an entrant needs to post $1/q$ vacancies. Free entry therefore implies:

$$x(z, h') - w = a/q$$  \hspace{1cm} (9)$$

It follows that the value of a filled job equals the expected cost of hiring a worker in a free entry equilibrium.

**Labor Market.** The measure of new matches between workers and firms is determined by a Cobb-Douglas matching function. Since matches last only a single period, the measure of new matches equals aggregate employment, $n$:

$$n = \psi e^{\phi} v^{1-\phi}$$  \hspace{1cm} (10)$$

where $v = \int v_j dj$ is the aggregate measure of vacancies, $e = \int e_i di$ is the measure of aggregate search effort and $\phi, \psi \in \mathbb{R}_{++}$ are constant parameters. The job finding and job filling rates are given as:

$$p = \frac{n}{e} = \psi \left( \frac{v}{e} \right)^{1-\phi}$$  \hspace{1cm} (11)$$

$$q = \frac{n}{v} = \psi \left( \frac{v}{e} \right)^{-\phi}$$  \hspace{1cm} (12)$$

Wages are determined according to a non-cooperative Nash bargaining game between workers and firms. The equilibrium wage is the solution to:

$$w = \arg \max \left( S^w \right)^{\lambda} \left( R^f \right)^{1-\lambda}$$  \hspace{1cm} (13)$$

where $\lambda \in (0, 1)$ denotes the households’ bargaining power. $S^w$ denotes the surplus to a worker of finding a job given as:

$$S^w = u(c^w, e, G) - u(c^u, e, G) - \kappa$$  \hspace{1cm} (14)$$

Due to free entry, the surplus to firms from having a filled job is given by the within-period return $R^f$, see equation (7). It follows that the equilibrium wage satisfies the first order necessary condition:

$$w = x(z, h') - \frac{1 - \lambda u(c^u, e^*, G) - u(c^u, e^*, G) - \kappa}{\lambda (1 - \tau) u_e(c^w, e^*, G)}$$  \hspace{1cm} (15)$$
The ex-ante homogeneity of workers imply that match surpluses are identical across worker-firm matches. Wages therefore equalize across matches but may vary over time due to productivity shocks and due to government policies.

**Lenders.** There are many identical risk-neutral international lenders and free entry to this sector. The total borrowing of the sovereign is denoted $B'$. An individual lender purchases $b'$ bonds from the sovereign at the price $q(B', z)$ and receives $b'$ in the subsequent period unless the sovereign defaults.\(^3\) We assume that lenders can alternatively invest in a risk-free asset which delivers a real return $1 + r$.

The expected present value of lending $b'$ to the sovereign is:

$$\Lambda = -R(B', z)b' + \mathbb{E}\frac{1 - d'}{1 + r}b'$$

Free entry implies the expected payoff from lending to the government must equal the risk free rate:

$$R(B', z) = \frac{1 - \mathbb{E}d'}{1 + r} \quad (16)$$

The expected return on sovereign lending equals the default-risk adjusted risk-free rate. Note that when default is inevitable, $\mathbb{E}d' = 1$, the bond price falls to zero and there is de facto exclusion from international debt markets.

**Government.** The government chooses the income tax rate, $\tau$, the level of transfers to unemployed households, $\mu$, public goods provision, $G$, and government debt, $B'$, in order to maximize social welfare. It maximizes a utilitarian social welfare function:

$$\mathcal{U}_t^G = \mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} u^G(c^w, c^u, e, n, G) \quad (17)$$

$$u^G(c^w, c^u, e, n, G) = n[u(e^w, e, G) - \kappa] + (1 - n)u(e^u, e, G) \quad (18)$$

The timing is as follows. The government enters a period with $B$ units of inherited debt and a credit history $h$. The productivity shock, $z$, then realizes. Thereafter, nature decides whether

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\(^3\)Note that the debt price depends only on debt issuance, not on existing debt. This derives from the sovereign only accessing the debt market once per period. It would be potentially interesting to relax this and allow for multiple rounds of debt financing each period in which case the price of debt will depend also on existing debt, see e.g. Lorenzoni and Werning (2014).
a country with a bad credit record, \( h = 1 \), is readmitted to financial markets. This event occurs with probability \((1 - \alpha) \in [0, 1]\). If a country with a bad credit record is readmitted to financial markets, its initial debt level is equal to zero, \( B = 0 \), and it is free to issue sovereign debt.

The government then chooses policies. We assume that it chooses both taxes, welfare transfers, public goods provision and debt policy at the same time.\(^4\) If it has access to international financial markets, the government needs to decide whether or not to default on its outstanding debt.\(^5\) If it does not default, \( d = 0 \), the government can issue new debt \( B' \) at the price \( R(B', z) \) and its end of period credit score will be \( h' = 0 \). If the country defaults, it cannot issue new debt this period, may experience a drop in productivity, and its end of period credit score deteriorates, \( h' = 1 \).\(^6\) A country with a bad beginning of period credit score that is not readmitted to international financial markets is in autarky and must run a balanced budget.

Define the aggregate state vector as \( S = (z, B, h) \). The government’s policy vector is \( \Omega(S) = [\tau, \mu, G, d, B'] \). We can write social welfare as \( u^G(Y, \Omega, S) \) where \( Y = (e, c^w, v) \) and other endogenous variables can be expressed as implicit functions of \( (Y, \Omega, S) \). The government has to observe a budget constraint, a resource constraint, and implementability constraints:

\[
G + (1 - n(Y, \Omega, S)) \mu + (1 - h') B = \tau w(Y, \Omega, S) n(Y, \Omega, S) + (1 - h') R(B', z) B' \tag{19}
\]

\[
n(Y, \Omega, S) c^w + (1 - n(Y, \Omega, S)) c^u(Y, \Omega, S) + G = \left( x(z, h') - \frac{\alpha}{q(Y, \Omega, S)} \right) n(Y, \Omega, S) + (1 - h') (R(B', z) B' - B) \tag{20}
\]

\[
u(e^w, e, G) - u(c^u(Y, \Omega, S), e, G) - \kappa = e u_e(c^w, e, G) + \frac{(1 - p(Y, \Omega, S) e)}{p(Y, \Omega, S)} u_e(c^u(Y, \Omega, S), e, G) \tag{21}
\]

\(^4\)Conesa, Kehoe and Ruhl (2016) instead examine the impact of within-period lack of commitment in a Cole and Kehoe (1996) setting with multiple equilibria. They focus upon the timing of tax (and spending) policy choices relative to the revelation of information and the timing of the debt issuance decision.

\(^5\)We assume full default but allowing for partial default as in Yue (2010) does not materially impact on our main results.

\(^6\)Note that a country which has just been readmitted to financial markets will never default since it has no debt. The timing is important for the properties and uniqueness of the equilibrium for well known reasons, see e.g. Aguiar et al. (2016) or Ayres et al. (2015).
\[ w(Y, \Omega, S) = x(z, h') - \frac{1 - \lambda}{\lambda} u\left(c^w, e, G\right) - \frac{u\left(c^u(Y, \Omega, S), e, G\right) - \kappa}{(1 - \tau) u_c(c^w, e, G)} \]

(22)

\[ w(Y, \Omega, S) = x(z, h') - \frac{a}{q(Y, \Omega, S)} \]

(23)

where debt prices have to fulfill condition (16).

(19) is the government budget constraint which sets total spending (the LHS – the sum of spending on public goods, \(G\), welfare transfers, \((1 - n) \mu\), and debt repayment, \((1 - h') B\)) equal to total revenue (income tax collection, \(\tau wn\), plus new debt issuance, \((1 - h') RB'\)). Equation (20) is the economy-wide resource constraint where the left hand side is absorption and the right hand side is domestic output net of hiring costs plus net imports. (21) – (23) are the implementability restrictions that follow from the government having to observe private sector behavior. (21) is the first-order condition for optimal search effort, (22) is the Nash bargaining solution for the real wage, and (23) is the free entry condition in the goods market.

A government with a good credit history \(h = 0\) decides whether or not to honor its debt:

\[ Q_i(B, z) = \max_{d \in (0, 1)} (1 - d) Q^{nd}(B, z) + dQ^d(B, z) \]

(24)

where \(Q^{nd}(B, z)\) is the value of the government’s objective when choosing not to default and \(Q^d(B, z)\) is the value when it chooses to default. The value of not defaulting is:

\[ Q^{nd}(B, z) = \max_{Y, \Omega_0} u^G(Y, \Omega_0, S) + \beta E Q_i^i(B', z') \]

subject to (19)-(23) setting \(h' = 0\). We use \(\Omega_0\) to indicate that \(h' = 0\) since the government chose \(d = 0\). Hence, when the government does not default it can issue new debt, retains high productivity (relative to autarky), and keeps open the option of borrowing next period.

The value of default \(Q^d(B, z)\) is given as:

\[ Q^d(B, z) = \max_{Y, \Omega_1} u^G(Y, \Omega_1, S) + \beta E \left[ \alpha Q^d(z') + (1 - \alpha) Q^i(0, z') \right] \]

subject to the government budget constraint, to the aggregate resource constraint, and to the implementability conditions (21) – (23) setting \(h' = 1\) due to the default. We use \(\Omega_1\) to indicate that \(h' = 1\). Thus, while the government does not pay its current creditors, it cannot issue new debt, it may experience a drop in productivity, and remains in autarky next period with probability \(\alpha\).
2.2 Equilibrium

The government lacks commitment to all of its instruments and we focus on Markov-Perfect equilibria. The policy maker will therefore have to set policies that are self-reinforcing in a game between its current self, the future government and foreign lenders:

**Definition 1** A Markov Perfect equilibrium is a set of policies $\Omega(S)$, an allocation $Y(S,\Omega)$ and a set of future policies $\Omega'(S)$ such that (i) the policies and the allocations solve (24) – (26), (ii) the bond price is given by (16), and (iii) $\Omega(S) = \Omega'(S)$.

The equilibrium concept imposes a sequential structure on the policy maker’s problem. The only policies that can be credibly announced by the current policy maker are those that are also ex-post optimal choices for the future policy maker. Current and future policy makers are linked through the debt accumulation and the nature of the commitment problem depends crucially on whether the price of debt depends elastically on debt issuance. When the debt price is sensitive to debt issuance, i.e. if the likelihood of default states is significantly affected by current borrowing, we will say that the economy is in a *debt crisis*.

The government faces several trade-offs when setting the optimal policy. Given the utilitarian preferences, the government has an incentive to equalize marginal utility of consumption across agents which implies providing *intratemporal insurance* against idiosyncratic unemployment shocks. This redistributive motive, however, needs to be weighed up against the disincentive it provides to exerting search effort. Moreover, high welfare transfers depress job creation since they improve workers’ outside option thereby putting upward pressure on wages and decreasing firm entry. Finally, higher welfare transfers will eventually have to be paid for by increasing the income tax rate or by cutting spending on public goods both of which distort.

Furthermore, since households cannot smooth their consumption stream, the government has an incentive to provide *intertemporal insurance* against aggregate productivity shocks. This can be accomplished by adjusting taxes (and welfare benefits) over states of nature in order to equalize the intertemporal marginal rate of substitution of consumption to the real interest rates. Such a policy requires cutting income taxes when wages are low and increasing taxes in times of high wages. High taxes in periods of high productivity, however, hamper job creation because
of their impact on search effort, therefore introducing a wedge on the intertemporal insurance provided.

Finally, since the resource costs of private and public consumption are identical, the government has an incentive to equalize the marginal utility of private and public consumption, i.e. to implement the Samuelson condition. This means increasing (lowering) public goods provision in good (bad) times when private consumption is high (low). However, the lack of lump-sum taxes implies that the government will provide less public goods than dictated by the Samuelson condition.

Each of these wedges are static in nature and the lack of commitment therefore has no impact on them. However, there is an additional dynamic intertemporal wedge that is affected by lack of commitment. When implementing these policies, the government smooths deficits in response to shocks to the economy by borrowing and lending in the international financial market. However, an additional dynamic wedge arises in crisis periods because debt issuance then impacts on debt prices. Importantly, the dynamic nature of this mechanism implies that lack of commitment matters for all policy instruments in crisis periods. In such episodes, international borrowers realize that promises of future repayment of government debt may not be fully credible. We show below that fiscal policy switches from being procyclical – which induces intertemporal and intratemporal insurance – in normal times to austerity in debt crisis and that this derives from two distinct forces one of which is lack of commitment to instruments other than debt.

3 Quantitative Analysis

We solve the model numerically using a collocation method, see Appendix 7.1 for details. We first present the calibration and then discuss policy functions and the implications of the model for debt crises.

3.1 Calibration

One period corresponds to a quarter and the model is calibrated for developed economies such as those in Southern Europe that were affected by the European debt crisis. We use outside
estimates whenever possible and calibrate the remaining parameters using indirect inference targeting a set of moments discussed further below. We give the targets equal weight in the penalty function which we specify as a quadratic form. Tables 1 and 2 summarize the calibrated parameters and the targets used in the indirect inference step.

Preferences. We specify the household utility function as:

\[ u(c, e, G) = \frac{c^{1-\sigma_c} - 1}{1 - \sigma_c} - \theta \frac{c^{1+\sigma_e} - 1}{1 + \sigma_e} + \xi \log G \quad (27) \]

\( \sigma_c > 0 \) determines the intertemporal elasticity of substitution of consumption. We assume \( \sigma_c = 2 \) which is in line with a large amount of empirical estimates using either household data or aggregate data, see e.g. Attanasio and Weber (1995) or Eichenbaum, Hansen, and Singleton (1988).

Given (27) the optimal level of search effort is given as:

\[ e = \left( \frac{p}{\theta} \right)^{1/\sigma_e} \left( \frac{(c^u)^{1-\sigma_c} - (c^u)^{1-\sigma_e}}{1 - \sigma_c} - \kappa \right)^{1/\sigma_e} \quad (28) \]

\( 1/\sigma_e \) is therefore the elasticity of search effort to variations in the job finding rate. We follow Costain and Reiter (2008), Mukoyama et al. (2014) and others, and assume that the search effort elasticity is relatively small and set \( \sigma_e = 3 \).

\((\beta, \theta, \kappa, \xi)\) are estimated by indirect inference. The main target for estimating the intertemporal discount factor, \( \beta \), is the default probability which we set equal to three percent annually so that a country defaults on average every 30 years approximately.\(^7\) This frequency is consistent with the observation that Greece has defaulted six times since 1829 and it is a value that has been used by others in the literature, see e.g. Arellano (2008). In conjunction with other parameters, we find an estimate of \( \beta \) of 0.90.

We estimate \( \theta \), the weight on search effort in the utility function, by targeting an unemployment rate of eleven percent in the long run, a value that is empirically relevant for Southern Europe. This implies \( \theta = 0.02 \). \( \kappa \), the utility cost of working, is calibrate by targeting an average net welfare benefit replacement rate of 58 percent. Our estimate of \( \kappa \) is 1.03.

\(^7\)Because of the non-linearity of the moments in the parameters, there is no exact one-to-one mapping from targets to parameters individually. However, some targets are more important for some parameters than others and this is the logic of our discussion.
Finally, we calibrate $\xi$ by targeting the average value of the ratio of government purchases of goods and services to private sector consumption. We use a target of 33 percent for this ratio, a value that is close to the average observed across the OECD in the post-war period. Given this target, we estimate $\xi = 0.54$.

**Labor Market**: The matching function elasticity, $\phi$, is calibrated following Merz (1995), $\phi = 0.4$. This implies that job finding and the job filling probabilities, $p$ and $q$, depends on labor market tightness relatively elastically. Above, we have assumed that the value of unemployment is low and we therefore adopt $\lambda = 0.4$.\(^8\)

**Technology**: We follow much of the labor market matching literature and assume that vacancy posting costs correspond to 4.5 percent of quarterly wages. Given this target, we find an estimate of the vacancy posting cost of $a = 0.04$.

We assume that aggregate productivity follows a homogenous Markov process targeting an autoregressive process for the logarithm of $z$ with a persistence of 88 percent per quarter and a standard deviation of the (normally distributed, mean zero) innovation of three percent. These moments for the productivity process match those observed for developed economies. In the computation we limit the innovations to lie within the 0.00001 and 0.99999 interior of the normal cdf that we split into 200 equi-spaced grid points and recover the shock values using the inverse cdf on this grid. We compute expectations using a cubic spline to evaluate the expected value functions at these 200 points and weigh them by the probability mass around the shocks.

We follow Arellano (2008) and assume that $x(z, h')$ is given as:

$$x(z, h') = \begin{cases} 
  z & \text{if } h' = 0 \\
  z & \text{if } h' = 1 \land z < \hat{z} \\
  \hat{z} & \text{if } h' = 1 \land z \geq \hat{z}
\end{cases} \quad (29)$$

Equation (29) implies that the productivity loss from default is weakly increasing in productivity with no loss for low levels of productivity. We estimate $\hat{z}$ by targeting an output loss of five percent for countries in default. Given this target we find $\hat{z} = 0.97$. Therefore, during a default

\(^8\)Alternatively, one might consider adopting a Hagedorn-Manovskii calibration by assuming a much higher replacement rate.
productivity is capped at approximately three percent below its unconditional mean implying that high productivity economies have a strong incentive not to default.

**International Lenders:** We calibrate the risk free rate to four percent annually. The parameter $\alpha$ which determines the probability that a country with a bad credit history is readmitted to international financial markets is calibrated to 8.25 percent per quarter which implies that the expected duration of exclusion is 12 quarters. This number matches the estimates of Das and Richmond (2009) who report that defaults on average leads to three years lack of access to international lending.

### 3.2 Policy Functions

Figure 1 illustrates the sovereign bond spread, the optimal default decision, and the government debt issuance policy plotted against initial debt and productivity (assuming that the government has a good credit history). These policy functions are important for understanding many other properties of the model.

When productivity is sufficiently high and government debt sufficiently low, it is optimal for the government to honor its debt obligations and there is little likelihood that the economy would be hit by a sufficiently bad productivity shock that it would choose to default next period. The sovereign debt premium is therefore low and the country can issue debt at favorable prices. The significant difference between the intertemporal discount in utility and the risk-free rate induces a strong argument to accumulate debt when default premia are small, an incentive that stabilizes as default premia rise due to debt accumulation. We will refer to the part of the state space where default premia are moderate and relative insensitive to debt and productivity as normal times.

As productivity falls and/or debt increases, the sovereign debt premium rises and eventually becomes very sensitive to further changes in productivity or government debt. At high levels of the default premium the government makes a strong effort at stabilizing its debt dynamics

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$^9$Given the government’s incentive to accumulate debt induced by the difference between the risk-free rate and the subjective discount factor, we concentrate on the relevant part of the state space where net foreign assets are negative.
because bad productivity draws reduce the debt price making it hard for the government to avoid default. Thus, in this zone there is strong feedback from borrowing decisions to the debt price and we use the phrase “crisis zone” to refer to the subset of the state space where the default premium is high and reacts elastically to changes in debt and productivity (but for which it is still optimal for the government not to default).

Should productivity fall even further, the country will default. The “default zone” is the combination of productivity and initial debt levels for which \( d(B, z) = 1 \):

\[
\text{def}_{z} (B) = \{(z, B) \in Z \times B : d(B, z) = 1\}
\]

This set is non-empty and \( \text{def}_{z} (B_2) \subseteq \text{def}_{z} (B_1) \) for \( B_2 \leq B_1 \), i.e. countries with higher debt have a lower tolerance for productivity falls. Intuitively, at low levels of productivity, default is more tempting because the utility cost of servicing the debt is higher when few resources are available. Once a country enters \( \text{def}_{z} (B) \) and defaults, its debt will fall to zero and it remains in autarky for \( 1/\alpha \) periods on average (three years in our calibration).

Figures 2 and 3 illustrate other policy functions in two dimensional plots.\(^{10}\) Figure 2 shows various policy functions plotted against productivity for three different levels of initial debt (high, medium and low). Figure 3 instead shows the policy functions plotted against initial debt for different levels of productivity (mean productivity, and at the mean \( \pm \) two standard deviations). In each of these we demonstrate the default zone with vertical lines.

The properties of the optimal fiscal policies are very sensitive to the debt regime. When productivity is high and government debt is low, the optimal income tax rate is procyclical as are income tax revenues. Such procyclical income tax rates help stabilize employed agents’ consumption streams and implement the intertemporal smoothing mechanism stressed in much of the sovereign debt literature. The government does not, however, fully stabilize employed agents’ consumption streams in response to productivity shocks because it needs to incentivize workers to search for jobs, a mechanism that is absent in endowment economies. The search incentive is induced by increasing employed workers’ consumption in high productivity states. Because of the lack of full stabilization of employed workers’ consumption streams, the government’s utilitarian preferences imply that the optimal welfare benefit policy is also procyclical.

\(^{10}\) The Appendix contains plots of the three dimensional policy functions.
subject to retaining a search incentive. Finally, since private consumption is procyclical, the government also implements procyclical government public goods provision. The budget cost of higher welfare spending in higher productivity states is not very large given that unemployment is countercyclical while higher public goods provision induces procyclicality of total spending. Optimal fiscal policy is therefore procyclical in normal times.

Figure 3 shows that the feedback from the level of debt to welfare benefits and government spending is small but negative while higher debt also induces marginally higher income tax rates. However, this feedback is very limited as long as the sovereign debt premium is sufficiently small. The source of this impact of government debt on the fiscal instruments is a purely fiscal cost channel indicating that higher debt needs to be financed through primary surpluses but this is done by spreading the burden over time.

As the economy moves into the crisis zone and the sovereign debt spread rises, the optimal government debt policy involves reducing the debt burden. The debt reduction is implemented by rolling out austerity policies. The optimal fiscal policy mix involves a hike in the income tax rate, a sharp drop in unemployment benefits, both of which reduce private sector consumption, and a drastic cut in public goods provision. In combination, these policies produce primary surpluses and reduce foreign debt. The government implements austerity measures in a debt crisis in two distinct senses. First, as the economy enters the crisis zone, the government reduces expenditures and raises tax revenues significantly and thereby sacrifices its aim for providing insurance in an attempt to stabilize the debt dynamics. Second, the rise in taxes and the reduction in expenditure exceed the adjustments that would take place if the economy were to default. The austerity package actually implies a drop in the flow utility in the crisis zone relative to the flow utility that the government could obtain were it to default. Another interesting feature of the austerity policies is that the hike in taxes and drop in unemployment benefits is designed to stimulate search intensity during crisis time so that employment rises as productivity falls in this zone. This aspect of the optimal policy has the consequence of reducing the welfare bill (due to declining unemployment), therefore enabling slightly less severe austerity measures than

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11Recall that the utilitarian preferences gives the government a fundamental incentive to equalize marginal utility of consumption across agents.

12We show these policy functions in the Appendix.
otherwise needed.

In the default zone, the government returns to a procyclical fiscal policy regime. In this regime the government balances the budget since it lacks access to international financial markets. Hence the choice of optimal fiscal policy reduces down to trading off the static wedges. In this regime, the government therefore sets procyclical income tax rates, welfare benefits and public goods provision. The lack of opportunity to borrow on international financial markets implies that the government is less able to smooth household consumption streams in this regime than in “normal times.”

It follows that there are sharp differences in optimal fiscal policy depending on the debt regime. In normal times, the government can smooth deficits, engage in redistribution across agents and across states of nature by varying tax rates and welfare transfers subject to taking account of the distortions to search effort provision and firm creation created by these policies. When the economy enters the crisis zone, the optimal policy choice involves abandoning insurance policies and cut spending and increase tax rates as the price of debt declines. Should the government choose to default, it returns to procyclical fiscal policy but without the option of smoothing deficits offered by international lenders in normal times.

### 3.3 Debt Crises and Optimal Fiscal Policy

The policy functions are instructive about the design of optimal policy in response to productivity shocks depending on initial debt. However, they are not informative about what actually typically happens during crisis periods because of the dynamics induced by debt accumulation and by persistent productivity shocks. In order to understand better the dynamics of crisis and policy, we use simulation techniques. We evaluate the dynamics of the economy obtained from stochastic simulations of the model for 1,000,000 periods. We draw random productivity shocks and discard the first 100 periods starting the simulations from the mean productivity level and zero asset level assuming that it starts with a good credit history. We then compute the paths of the variables of interest in 4 year windows around two different types of debt crisis. Figure 4 reports the dynamics of the economy around default episodes and we center the plot around the period where the default occurs. Figure 5 instead examines instances where sovereign spreads rose persistently but where the government avoided a default, episodes that we will refer to as “debt
crises.” We define these as episodes where the mean spread of 4 consecutive quarters is not less than 5 percent and the spread of the succeeding quarter exceeded 5 percent and where there is no default in the 4 year window. For crisis episodes, “0” is the final quarter where the spread exceeds 5 percent in each of the episodes. In both cases we report the median paths and 66 percent confidence intervals.

There is a striking difference between default episodes and debt crisis both in terms of their nature and in terms of the fiscal policy response. The typical default occurs when a period of steady growth is followed by a sudden and large drop in productivity. In our simulations, productivity falls on average six percent in the six months prior to the default but the drop is much larger in some cases. The suddenness of the recession and its size imply that austerity policies are ineffective in preventing default in these instances. In contrast, debt crises involve long slow declines in productivity that see the government rolling out austerity measures which manage to stabilize the debt burden and avoid a default. Once the economy moves out of the debt crisis, productivity recovers usually within a year of default premia falling below five percent.

To understand these differences, consider first the median default episode. According to Figure 4, defaults occur when periods of positive growth in productivity which spurs output and employment growth are interrupted by a dramatic drop in productivity that sends the economy into a recession. Prior to the default, moderate default premia lead the government to accumulate debt and the government debt-to-GDP ratio therefore typically rises prior to the crisis. Given the high persistence of productivity growth it is very unlikely that productivity suddenly falls abruptly and optimal policy is designed on the basis of the more likely events. When a sudden TFP reversal does occur, it therefore triggers a default if it is sufficiently dramatic. The rapid nature of the productivity loss prevents the government from anticipating the debt crisis and the low likelihood of the event makes it too costly for the government to prevent the default. The costs of a default materialize post-default where productivity falls persistently and the economy enters an extended recession. The persistent nature of the productivity collapse derives from \( z > \hat{z} \) at the time of the typical default so that productivity only recovers once the economy re-enters international financial markets. Because of the default, output and employment drop at the time of the default which leads to a sharp reduction in tax revenues and since the government must balance its budget post-default, government spending on public goods and on welfare benefits
are reduced significantly.

Things are very different in debt crises, see Figure 5. Debt crises typically occur as a result of a long and moderate fall in productivity. The slow (but significant) decline in productivity leads the economy into the crisis zone and the government initiates austerity policies which involve a hike in tax rates and cuts in welfare transfers and in government purchases. In combination, these policies produce a primary budget surplus in the range of one to three percent of GDP, depending on the severity of the crisis. The austerity measures stabilize the drop in employment as we have discussed before but sacrifice current consumption which drops by six percent on average during such episodes. The typical debt crisis ends when the productivity drop reverses and produces a much stronger recovery in the economy than default episodes.

To sum up, optimal fiscal policy during a debt crisis when there is lack of commitment involves governments using austerity measures to avoid defaults. This is a successful strategy when a country faces a moderate sequence of adverse productivity shocks but not in the case of a more abrupt productivity drop. The austerity measures involve cuts in transfers and government spending on public goods implemented such that they provide an incentive to search harder for jobs when sovereign default premia rise.

3.4 The Role of Lack of Commitment

One reason why the government implements austerity measures in a debt crisis is that government debt prices are low which makes it very expensive to issue debt. However, default incentives also matter since the government may want renege on its promise to honor its existing debt obligations in order to create the fiscal room to stimulate the economy and provide income insurance to households in a debt crisis. We now examine the importance of the incentive problem by introducing a partial commitment technology: Suppose a third party imposes hard fiscal rules on the government, how does this impact on the optimal fiscal policies?

We assume that the government can commit to either fixed tax rates or to fixed levels of welfare transfers.\textsuperscript{13} We fix each of these instruments one-by-one at their average value in the

\textsuperscript{13}In principle it would be interesting to study the impact of introducing commitment technologies but here we will take an easier route and simply assume that the government can commit to constant paths of either taxes or transfers.
stationary distribution. We then solve for the MPE assuming lack of commitment to all other instruments and compare the properties of these alternative economies with the benchmark model discussed above.

Figure 6 shows the policy functions when introducing commitment to either tax rates or the level of welfare transfers at the mean productivity level. Partial commitment – commitment to either taxes or transfers – allows countries to delay the default decision slightly but the impact is rather minor. Introducing partial commitment has instead fundamental consequences for the optimality of austerity policy. When the government can commit to fixing the income tax rate, it is no longer optimal to cut welfare transfers in the crisis zone and the government no longer sacrifices private sector consumption when default premia rise and productivity falls. The reason is that commitment to taxes removes the government’s ability to stimulate the economy post-default by cutting taxes and therefore makes default less palatable. This, in turn, means that creditors no longer need to ask the sovereign to implement austerity in the crisis zone. Allowing the government instead to commit to the level of welfare transfers increases the procyclicality of income tax rates and it is no longer optimal to hike tax rates in the crisis zone. In this scenario, the government exploits the fiscal space deriving from the default to cut income tax rates. Committing to either taxes or to transfers does not remove the incentive to cut government purchases during a debt crisis but the government no longer needs to sacrifice private sector consumption.

It follows that there are two underlying reasons for why the government chooses to implement austerity during a debt crisis. One is a pure budgetary incentive not to issue debt when its price is low and falling. The other derives from the government’s lack of commitment when it faces the incentive to stimulate the economy in default. Investors realize this incentive and ‘force’ the government to implement a severe austerity plan that involves sacrificing household consumption to keep the debt price low. The post-default stimulus is harder to implement when a partial commitment technology is available because the government is unable to cut taxes or increase transfers. Investors therefore do not use the debt price to manipulate the crisis zone consumption drop that we found in the benchmark model.
4 A Greek Experiment

It is interesting to investigate how the model allows for a better understanding of default and crisis episodes in practice. Figure 7 reports the results for simulating the model for a “Greek-style” debt crisis experiment. Greek output grew by more than 50 percent from the mid-1990’s to late 2007. The financial crisis reversed this strong growth performance and by 2015 Greek output had fallen back almost 30 percent relative to its pre-recession peak, a recession that exceeds that of any other OECD country during this period. The drop in output was also reflected in a huge decline in private sector consumption which fell marginally less than output during this period. At the same time, the unemployment rate in Greece shot up from 10 percent to more than 25 percent and has since been stuck at very high levels. The dramatic fall in output and the increase in debt produced a stark rise in the debt-to-GDP ratio and this sparked a severe rise in the sovereign bond spread which went above 25 percent (annually) in 2012. The recession triggered a huge decline in government revenues while spending on goods and on transfers rose until 2008/09 after which they have been reduced significantly.

In order to contrast the Greek experience with the prediction of the model we simulate a “Greek experiment”. We back out a sequence of productivity shocks that allow us to match the path of Greek GDP over the sample period 1995-2015. We then normalize foreign debt to zero at the beginning of the sample and simulate the model in response to the estimated productivity shocks.

We fit the output path almost perfectly (by construction) but, given the output path, the model is also consistent with many other features of the Greek experience over the post 2007/08 period. As in the data, the model implies that aggregate consumption falls sharply along with the decline in activity. The timing and size of the consumption drop in the data and in the model are very similar. We also find a very similar shape of the path of unemployment. In the data, the unemployment rate more than doubled from less than 10 percent in 2009 to above 25 percent by 2013. The model generates a very similar timing of the rise in unemployment but is off in terms of the size of the rise in the unemployment rate. This latter feature is hardly surprising given the simplifying assumptions we have made (especially, the one-period employment contracts).

The most interesting features, however, concern the adjustment of fiscal policies. The model
implies very similar paths of government revenues and of government spending to those observed in the Greek data. In the data, government revenues grew strongly in the pre-crisis boom period and then fell strongly as the recession set in. The model is consistent with this both in terms of timing and in terms of the size of the fall in government revenues. This implies that the tax policy generated by the model shares important features with the tax rate implemented by Greek governments. The similarity between model and data is equally strong for government spending which rises substantially (in the model and in the data) in the pre-crisis boom and then falls back significantly.

The model, however, is not consistent with the path of welfare transfers observed in Greece. In the data, spending on welfare transfers (as a percentage of GDP) increased a lot in the pre-crisis period but then declined strongly as unemployment started to rise. The model instead implies a very stable path of welfare spending over the sample that we examine. It follows that the Greek government undertook much stronger austerity measures than implied by the model. Moreover, while the austerity measures implemented by the government in the model economy manage to bring down the default spread, the even stronger austerity measures in the actual data failed at doing so.

Thus, the model can account for why Greece implemented austerity measures but not for why these did not succeed in reducing the sovereign debt premium.

5 Conclusions

In this paper we have constructed a model of sovereign default that extends the classic Eaton and Gersovitz model in several dimensions and used this model to consider the design of optimal fiscal policy in a crisis that involves both recessions and falling sovereign debt prices. The model features endogenous output determination in order to allow for feedback from policy to the state of the economy. We also introduce inequality that arises because of unemployment risk. The government therefore needs to cater both for intertemporal insurance against productivity shocks and for intratemporal insurance against unemployment risk. Furthermore, we allow for a rich set of fiscal instruments and we remove the government’s ability to commit.

We have shown within this framework that optimal fiscal policy is intrinsically linked to the
When default premia are low and do not react much to debt issuance, optimal fiscal policy is designed to provide intertemporal and intratemporal insurance while at the same time addressing a number of static wedges which prevent the government from implementing the first-best allocation. The government provides intertemporal insurance by increasing income tax rates with productivity but without fully insuring employed workers’ consumption streams. This induces procyclical consumption of employed workers which leads the government to provide procyclical welfare benefits and government purchases. The same is the case in default but while a government with a good credit record can smooth deficits, this is not possible in default. In a debt crisis, a situation that occurs when the economy suffers low productivity and has sufficiently high debt that default premia become very sensitive to debt issuance, it is instead optimal to implement austerity policies. These consist of hiking taxes and strongly cutting back spending both on public goods and welfare transfers. These policies induce the government to sacrifice private sector consumption in order to access international financial markets. The austerity policies are optimal both for purely fiscal reasons – it is expensive to issue debt – and for strategic reasons – the government cannot commit. We show that introducing commitment to either taxes or welfare transfers, the strategic reason for austerity disappears and the government no longer sacrifices private sector consumption in a debt crises. The fiscal reason remains but induces only a strong cut in public goods provision.

Stochastic simulations of the model showed that defaults are typically not preceded by austerity. The reason is that governments default after sharp falls in productivity which austerity cannot address. Instead austerity is implemented in a less gradual crisis which sees an extended period of poor growth. In such circumstances, governments roll out austerity policies.

It would be interesting to extend our analysis in a number of different dimensions. Our analysis does not allow for private savings. To the extent that the domestic private sector holds sovereign bonds, it would be interesting to consider the impact that this would have on optimal fiscal policy. Perez (2015) provides an interesting analysis of the link between sovereign default and banking crises in this dimension. We also assume that labor contracts last only a single period and therefore do not allow for persistent unemployment. An important aspect of deep recessions is that the duration of unemployment often increases significantly which could have important consequences for fiscal policy design. Balke (2016) analyzes the interaction between
sovereign default incentives and labor market dynamics with persistent unemployment. Another important concern is the term structure of public debt. We have assumed that debt is one-period but it would be interesting to consider the impact of long-term debt and possibly allowing the government to choose whether to issue short- or long-term debt.

6 References


Lorenzoni, Guido and Ivan Werning. 2014. “Slow Moving Debt Crises.” Manuscript, MIT.


7 Appendix

7.1 Computation

We use collocation methods to solve for the value functions $Q^i$, $Q^{nd}$ and $Q^d$ on a grid for $(z, B)$. We also approximate the expected value functions and the price function $q$. The solution
algorithm involves the following steps:

1. Grid
   The collocation nodes over the state \((z, B)\) have grid size \(N_z = 50\) and \(N_B = 50\). Parameter values are set. The limits for productivity are set such that the lower bound on the probability of \(z\) is 0.00001 and the nodes have a higher coverage in the higher probability regions for \(z\). The values of assets \(B\) lie between 0 and -0.6, ensuring that the lower limit is not hit on the ergodic set. The grid points for \(B\) are equi-spaced. Note that the range of grids is parameter-dependent.

2. Pre-computation
   We solve the optimal intratemporal allocation given productivity \(z\) and capital inflow using \textit{knitro}. We approximate the flow utility that is associated with the allocation.

3. Initialization
   Initial guesses for value functions are set to the flow utilities at zero new debt issuance. This initial value approximates the last period of a finite period limit. The maximum of repayment and default values determines the initial debt price schedule that is also approximated.

4. Solving of value and policy functions
   We approximate each value function by solving for the \(N_s = N_z \times N_B\) coefficients using cubic splines. Given \(N_s\) coefficients and \(N_s\) Bellman equations for each value function we have a just-identified set of equations. In each step, we solve for the best debt policy \(B'\) of the government using golden search. This implements a continuous optimal debt choice. In this step, for each possible \(B'\) the price function delivers \(q\) and the flow utility is obtained from the pre-computational approximation. When taking expectations we limit the innovations to lie within the 0.00001 and 0.99999 interior of the normal cdf that is split into 200 equi-spaced grid points and we recover the shock values using the inverse cdf on this grid. Each shock value is associated with a certain productivity level given by the AR(1) process of \(z\). We compute expectations using a linear spline to evaluate the expected value functions at these 200 productivity levels and weigh them by the probability mass around the shocks.
5. Iterate until convergence

We compare the coefficients of the new value functions to the previous ones from step 4. If convergence is achieved we stop. Otherwise we take the maximum over repayment and default states to update the price function and go back to step 4. We start with a few value function iterations and then use Newton method to update guesses.

7.2 Tables and Figures

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<th>Table 1. Calibrated Parameters</th>
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<td>Parameter</td>
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<th>Table 2. Parameters estimated with indirect inference</th>
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<td>Parameter</td>
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<td>$\beta$ (discount factor)</td>
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<td>$a$ (vacancy costs)</td>
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<td>$\vartheta$ (pref. weight)</td>
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<td>$\kappa$ (pref. cost)</td>
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<td>$\xi$ (pref. weight)</td>
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<td>$\hat{z}$ (prod. ceiling)</td>
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Figure 1: Policy Functions for Debt, Sovereign Spread and Default Decision.
Figure 2: Policy Choices and Productivity.

Debt Policy $B_{t+1}$

Tax Policy $\tau$

Transfer Policy $T$

Spending Policy $G$

Interest Rate Spread

Total Expenditure

Tax Revenue

Primary Deficit to GDP

GDP $Y$

Search Effort $E$

Consumption $C$

Consumption $C^w$

Search Effort $E$

Vacancies $V$

low $z$ ($-2\sigma_\epsilon$)

mean $z$

high $z$ ($+2\sigma_\epsilon$)
Figure 3: Policy Choices and Debt.
Figure 4: Default Episodes.
Figure 5: Debt Crises.
Figure 6: The Role of Commitment.
Figure 7: Greek Experiment.