# Is Inflation Default? The Role of Information in Debt Crises*

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**Abstract**

We consider a two-period Bayesian trading game where in each period informed agents decide whether to buy an asset ("government debt") after observing an idiosyncratic signal about the prospects of default. While second-period buyers only need to forecast default, first-period buyers pass the asset to the new agents in the secondary market, and thus need to form beliefs about the price that will prevail at that stage. We provide conditions such that coarser information in the hands of second-period agents makes the price of debt more resilient to bad shocks not only in the last period, but in the first one as well. We use this model to study the consequences of issuing debt denominated in domestic vs. foreign currency: we interpret the former as subject to inflation risk and the latter as subject to default risk, with inflation driven by the information of a less-sophisticated group of agents endowed with less precise information, and default by the information of sophisticated bond traders. Our results can be used to account for the behavior of debt prices across countries following the 2008 financial crisis, and also provide a theory of "original sin."

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# 1 Introduction

The sovereign borrowing experience of advanced economies in the aftermath of the financial crisis of 2008 has once again highlighted the important role of the currency in which debt is denominated. Countries which had control over their monetary policy, such as the United States, the United Kingdom, and Japan, were able to borrow at extremely low rates throughout the episode, even though they experienced very high deficit/GDP ratios (the UK) or debt/GDP ratios (Japan). In contrast, peripheral Eurozone countries were either unable to borrow from the market (Portugal, Ireland) or faced volatile interest rates when doing so (Italy, Spain).[1]

In previous crises, such as Latin America in the 1980s and Asia in 1998, currency mismatch was identified as a source of instability, and hence many authors have studied the role of the "original sin" or other causes of financial underdevelopment that led to the mismatch. In the presence of nominal rigidities, having an own currency may allow for a quick devaluation as a means to adjust to domestic shocks, preserving the country's economy and ability to repay its debt, but only if this debt is denominated in domestic currency.[2]

Compared to those crises, 2008 presents some important differences. First, financial underdevelopment of the debt market was not a cause of the Eurozone countries' difficulties, since they all had an ample and liquid market for government debt denominated in their home currency before joining the Euro. Second, it is not clear that the ability to devalue and thereby spare the economy from a deeper recession was a major factor in explaining the different behavior of interest rates: while it is true that the United Kingdom depreciated the Pound in the wake of the recession, the Yen appreciated substantially against the Euro, exacerbating the slump in Japan.

Our goal is to dig deeper in the source of frictions that may make the price of a country's debt less sensitive to adverse news on the government solvency. A premise of our analysis is

[1] See e.g. Plender [35] and De Grauwe [21].

[2] Krugman [30, 31] sketches a theory whereby an asymmetry arises because defaults would lead to larger real haircuts for bondholders than inflation. While it is true that a default is a discrete event and inflation erodes the value of repayments over time, it is not a priori obvious that the cumulative losses would be different in the two scenarios. We consider a benchmark in which losses are the same. Our mechanism would of course remain at work even if inflation were less costly for creditors, as the two channels would complement each other.

that a domestic currency partially insulates a country from default risk, as the government may be able to lean on the central bank to act as a residual claimant on government debt securities. However, the resulting increase in the money supply would be bound to generate inflation, so that default risk would be replaced by inflation risk and we might expect interest rates to spike similarly under the two scenarios. Yet in practice inflation expectations, as well as the behavior of actual inflation, are very sluggish compared to the speed with which default crises, such as Greece's, unfold.

To reconcile these facts, we study an economy where private agents have dispersed and heterogeneous information about the government's ability to repay its debt. Public debt is purchased by overlapping generations of "bond traders", a segment of the population which is more attentive to economic news. In contrast, a much larger fraction of the population abstains from trading in public debt, but uses nominal contracts in their everyday transactions. This larger class, which we call the "workers," are less sophisticated and receive noisier information about government finances. We contrast two economies: in the first one, contracts are denominated in an outside currency (the "Euro"), and the government is forced to outright default when its tax revenues fall short of debt promises, while in the second one a domestic currency is present (the "Yen"), and the government resorts to the printing press and eventual inflation to cover any shortfalls. Other than this difference, we impose as much symmetry as possible between the two economies: agents start with identical priors over government solvency, bond traders receive signals with equal precision across the two economies, and the haircut upon default is matched to the loss in value due to inflation. All these assumptions allow us to concentrate on the consequences of heterogeneous information. When debt is denominated in Euros, there is no interaction between bond traders and workers: when bond traders wish to sell their debt on the secondary market, they need to find other (relatively well-informed) traders to buy. In contrast, when debt is denominated in Yen, its nominal payoff is risk-free, and the relevant measure of risk is captured by the purchasing power of the Yen. Since workers are assumed to be a much larger group, they determine this price, based on their noisier information. In the special case in which past prices are unobserved to current strategic participants, it is straightforward to prove that noisier information implies that the debt price is less responsive to incoming information about government solvency, so Yen-denominated debt is more resilient to bad news. The anticipation of this resilience in the secondary market in turn spills over to the primary market as well: even well-informed traders are less responsive to their signals if they anticipate the future price to be more weakly affected by fundamentals. We then show that, with some qualifications, this result extends when the primary-market price is taken into account by future traders and workers.

In sum, our results confirm that heterogeneity between a small sophisticated group of bond traders and a large, less informed population that drives the aggregate price level can explain why domestic-currency debt may be less information-sensitive than foreign-currency debt (or debt denominated in a common currency not directly controlled by the domestic central bank). This result can account for why a country which starts from a favorable prior condition may be able to better withstand the arrival of bad news. Conversely, our results also suggest that a country who is perceived as very likely to default may find it easier to borrow in foreign currency in the few instances in which its fundamentals are comparatively more favorable: sophisticated bond traders would find it easier to spot the presence of such conditions, while a pessimistic population may immediately fear (and trigger) hyperinflation. This could be an alternative explanation for the "original sin." Finally, while less information sensitivity may be good when incoming news suggest worse fundamentals than prior information, ex ante this insurance comes at a cost: only under special conditions can we unambiguously establish that ex-ante expected interest rates are lower for countries issuing debt denominated in their domestic currency.

Our paper is related to the vast literature that has used the global-games approach pioneered by Carlsson and van Damme [17] to study the fragility of regimes subject to infrequent crises. Their methods were first applied to currency attacks by Morris and Shin [32]. The role of signaling in this environment has been studied by Angeletos, Hellwig, and Pavan [7], and the efficiency of information acquisition has been further analyzed by Angeletos and Pavan [9, 10]. Dasgupta [20] and Angeletos, Hellwig and Pavan [8] studied learning in dynamic global games. In a more general context of dispersed information, Amador and Weill [5, 6] considered learning from aggregate prices in stylized macroeconomic models. Allen, Morris, and Shin [4] studied an environment in which an asset goes through multiple rounds of trade, as is our case. They emphasized the dampening effect of higher-order beliefs on price movements and conversely the greater emphasis that public signals take in that context. In our environment, as in theirs, it is true that the response of the primary-market price to fundamentals is dampened by the presence of a second round of trading. However, we emphasize a different force: we take as given the presence of multiple rounds and consider the consequences of heterogeneous quality of information in later rounds.[3]

The structure of our model is closely related to Hellwig, Mukherji, and Tsyvinski [27] and Albagli, Hellwig, and Tsyvinski [3], where a flexible specification of noisy information aggregation in market prices is developed. Our paper considers a version of their model in which trade occurs repeatedly. Our theorems are also related to Iachan and Nenov [28], whose paper presents a systematic analysis of comparative statics results with respect to the precision of information in global games.

On the international-economics side, the role of currency mismatch has been studied extensively, particularly in the years that follow the 1998 Asian crisis. Eichengreen and Hausmann [23] review competing theories about the origins of the mismatch, with an eye towards its consequences and policies. Examples of theories of crises based on mismatch appear in Aghion, Bacchetta, and Banerjee [1] and Calvo, Izquierdo, and Talvi [16]. Particularly relevant for our analysis is Bordo and Meissner [13]: they show that currency mismatch and "original sin" are not necessarily harbingers of more frequent crises, provided fundamentals are managed correctly. This is reminiscent of our result, in which it is not necessarily the unconditional probability of eventual default or inflation that increases when debt is denominated in foreign currency: fragility manifests itself instead as a greater volatility of debt prices. While we are not aware of other papers linking imperfect information to the choice of denomination of government debt, imperfect information in sovereign debt markets plays an important role in Sandleris [36], where a default reveals adverse information about the state of the economy, with negative consequences for private investment, and in Gu and Stangebye [26], where endogenous time-varying informa-

[3]In a static context, Corsetti et al. [19] consider a global game with a single large player who may be differentially well informed from a continuum of small players.

tion precision generates variation in risk premia.

Finally, the information sensitivity of assets play a major role in the work of Gorton and Ordoñez [25]. While combining their forces and ours in a self-contained model is beyond the scope of our project, their theory and our work are complementary in accounting for sudden sovereign crises: as debt becomes more information-sensitive through the channels that we emphasize, Gorton and Ordoñez' forces would lead first-period agents to invest in even greater information acquisition, leading to further volatility and possibly market freezes.

We proceed by describing the setup in Section 2, which also shows that the economy maps into a two-stage Bayesian trading game. In Section 3 we analyze the simplest case: here, second-period buyers cannot observe the first-period price. In Section 4 we tackle the harder (but more realistic) case in which the first-period price is observed. Section 5 extends the result to cases in which the default threshold may depend on the price of debt in the primary market, and Section 6 concludes.

# 2 The Setup

We consider an economy that lasts for three periods. There is a single consumption good in each period. We consider two alternative scenarios: in the first one, the unit of account is exogenously fixed (the "Euro") and the price of the consumption good is normalized to 1. In the second case, the value of a unit of account (the "Yen") is endogenous.

The economy is populated by multiple generations of four types of agents: strategic workers, noise workers, strategic bond traders, and noise bond traders. In addition, a government is also present.

Workers are born in period 2 and die in period 3.[4] Strategic workers are endowed with one unit of the consumption good in period 2 and wish to consume in period 3; they are risk neutral and have access to a storage technology which has a yield normalized to zero. Negative storage is not allowed. Noise workers demand one unit of consumption in period 2, and can produce

[4]We could add workers that live in periods 1 and 2, but these would not interact with bond traders, and so their presence would not have any effect on our results.

exclusively in period 3. To consume, they trade with strategic workers using nominal contracts, denominated in Euros or Yen, depending on the regime.[5] The relative mass of noise vs. strategic workers is $\Phi(\epsilon_2^w)$, where $\Phi$ is the normal cumulative distribution function and $\epsilon_2^w$ is i.i.d. with a normal distribution having mean zero and variance $1/\psi_2^w$. Neither strategic workers nor noise workers have access to the bond market. Their asset position is limited to storage, trade credit with each other, and cash, which they may acquire from the bond traders.[6]

Under the Euro scenario, workers do not interact with bond traders, and their interaction with the government is limited to paying a lump-sum tax which is a negligible fraction of their endowment.

Bond traders live for two periods, and there will be overlapping generations of them. Their mass is negligible compared to workers; hence, when the two groups trade, the price is set by the workers. Bond traders are endowed with goods in the first period of their life,[7] which they want to consume in the second period. Strategic traders can store their endowment at a return normalized to 0. Alternatively, they can sell some of their endowment in exchange for a government bond, which in period 1 can be purchased from the primary market and in period 2 from the secondary market, soon to be described. To preserve tractability, we assume that holdings of government debt are limited to $\{0, 1\}$.[8] Noise traders do not get a choice; they absorb a fraction $\Phi(\epsilon_t^b)$ of the government bonds supplied to the market, where $\epsilon_t^b$ is i.i.d. with a normal distribution having mean zero and variance $1/\psi_t^b$.

We next describe the government. We normalize its positions in per capita terms with respect

[5] We do not model the reason why workers coordinate on nominal contracts. Euro contracts are equivalent to real contracts in our setup. Yen-denominated contracts favor strategic workers, as they can reap information rents at the expense of noise workers.

[6] The assumption that workers cannot buy government bonds could be justified by indivisibility constraints as in Wallace [38].

[7] We assume that their endowment is always sufficient to buy one unit of government bonds.

[8] The lower bound of 0 is equivalent to a short-selling constraint. Provided $\theta$ is sufficiently high, the upper bound is equivalent to an indivisibility assumption, which implies that traders cannot hold a non-integer position and do not have enough resources to buy two units. Consistently with the indivisibility assumption, we impose that their holdings must be either 0 or 1, but risk neutrality implies that the analysis is unchanged if traders are instead allowed any position in $[0, 1]$.

to one cohort of strategic bond traders. In the first period, the government issues nominal bonds, backed by taxes that will be collected in period 3.[9] Revenues from bond issuance are spent in a public good which does not affect the marginal utility of private consumption. When government bonds are denominated in Euros, they mature only in period 3, when the government has access to tax revenues. When instead the Yen is present, bonds are repaid in cash in period 2, and period-3 revenues are used to repurchase cash, as in Cochrane [18]. This arrangement corresponds to one of the important observations from which we started: that inflation is often sluggish in advanced countries and workers often do not realize immediately that the government is resorting to the printing press to cover its fiscal needs.[10] In period 1, the government auctions one unit of bonds with a promised repayment $\hat{s}(q_1)$ in period 3, where $q_1 := 1/(1+R_1)$ and $R_1$ is the nominal interest rate. Two examples of the function $\hat{s}$ are the following:

- $\hat{s}(q_1) \equiv \hat{s} \equiv 1$, corresponding to the Eaton-Gersovitz [22] timing, in which the government offers bonds making a fixed unit future repayment in period 3, and $q_1$ represents the first-period discount;

- $\hat{s}(q_1) \equiv 1/q_1$, corresponding to the Calvo [15] timing, in which the government offers bonds to raise a fixed amount of revenues (one) in period 1 and $1/q_1 - 1$ represents promised interest payments in period 3.

The ability of the government to raise revenues without a default in period 3 is limited by a single random variable $s$. If $s \geq \hat{s}(q_1)$, revenues from current and future taxes are sufficient to repay the debt in full (under the Euro interpretation) or to maintain the price of goods pegged at parity with the Yen (when the government has its own currency). When instead $s < \hat{s}(q_1)$, tax revenues are insufficient to avoid explicit default or inflation. In this case, we assume that

[9] Since the relative mass of traders is small compared to the mass of workers, the amount of these taxes per worker is negligible, and no issue about worker solvency arises.

[10] We view this assumption as particularly appropriate for a government who has in the past established a reputation for stability. There are examples in history where this assumption would be violated. Sargent [37] discusses cases in which inflation responded quickly to fiscal news, and other, more recent cases in which doubts about the fiscal stance led to sluggish adjustments.

the government imposes an exogenous haircut and only repays $\theta\hat{s}(q_1)$ units of the consumption good in period 3. When debt is denominated in Euros, this is implemented directly as a haircut upon default. When instead debt is denominated in Yen, the revenues $\theta\hat{s}(q_1)$ are available to repurchase Yen, implying that the price level at which Yen are withdrawn becomes $1/\theta$.

Nature draws $s$ from the prior distribution $N(\mu_0, 1/\alpha_0)$. Each strategic trader $i$ in period $t$ receives a private signal $x^b_{i,t} = s + \xi^b_{i,t}/\sqrt{\beta^b_t}$, where $\xi^b_{i,t}$ is distributed according to $N(0,1)$ for all $i,t$ pairs and we assume that a law of large numbers across agents applies as in Judd [29]. Similarly, each strategic worker receives a private signal $x^w_{i,t} = s + \xi^w_{i,t}/\sqrt{\beta^w_t}$, where $\xi^w_{i,t}$ has again a standard normal distribution.[11] Signals are independent of the number of noise traders present in the market. Strategic agents submit price-contingent demand schedules, so the equilibrium debt price in each period conveys information on the realization of the fundamental variable $s$.[12] Noise agents account for the additional, stochastic demand that is needed in rational-expectation models to have a non-degenerate equilibrium.

## 2.1 Trading in the Euro Economy

In the Euro economy, there is no uncertainty about the value of nominal contracts, which is fixed at 1. At these prices, strategic workers are indifferent between storing their endowment or lending it at a rate zero to the noise workers. Hence, they will absorb all of the demand $\Phi(\epsilon^w_2) \in (0,1)$ with no effect on their lending rate.

Next, we consider bond trading in the secondary market (period 2). Bond supply is fixed at one: both strategic and noise traders who purchased the bond in period 1 must sell it to consume.

Strategic bond traders born in period 2 must choose whether to store their entire endowment or purchase a government bond in the secondary market.[13] Defining $q_2 := 1/(1+R_2)$, where $R_2$

[11] We assume that the law of large numbers applies here too.

[12] Given that we assume risk neutrality, the optimal demand schedule will take the form of a reservation price, below which strategic agents are willing to buy government debt.

[13] They could also lend to noise workers at the same rate as storage; since their mass is negligible compared to workers, this would not affect the market-clearing condition for trade credit between periods 2 and 3.

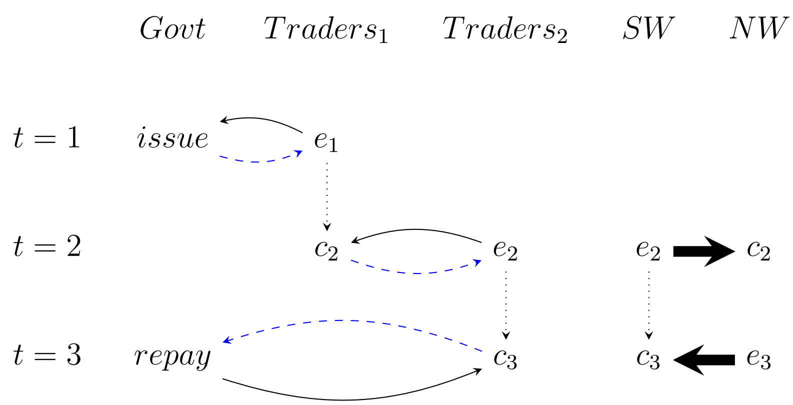


Figure 1: Markets in the Euro scenario. Goods (solid black); Bonds (dashed blue); Storage (dotted black). $e$ stands for endowment and $c$ for consumption.

is the nominal interest rate (yield to maturity) in the secondary market, the expected net profit from buying the bond is

$$\hat{s}(q_1)\left[\theta + (1-\theta)\mathbb{E}(1-\delta|\mathcal{I}^b_{i,2}) - q_2\right], \tag{1}$$

where $\delta = 1$ when $s < \hat{s}(q_1)$ (the states in which the government defaults) and $\mathcal{I}^b_{i,t}$ is the information available to bond trader $i$ in period $t$. We denote by $D^b_t$ the demand for bonds by strategic bond traders in period $t$; this demand depends on the price $q_t$, but also on the details of available information, which vary across the cases of Sections 3-5. Second-period strategic bond traders must absorb a fraction $1 - \Phi(\epsilon^b_2)$ of bonds in equilibrium, with the balance purchased by noise traders. Market clearing will then require

$$D^b_2 = 1 - \Phi(\epsilon^b_2). \tag{2}$$

Going back to period 1, strategic bond traders born at that time must choose whether to store their entire endowment or purchase a government bond in the primary market. The expected profit from buying a bond is

$$\hat{s}(q_1)\left\{\mathbb{E}[q_2|\mathcal{I}^b_{i,1}] - q_1\right\}.$$

Market clearing in the first period requires

$$D^b_1 = 1 - \Phi(\epsilon^b_1). \tag{3}$$

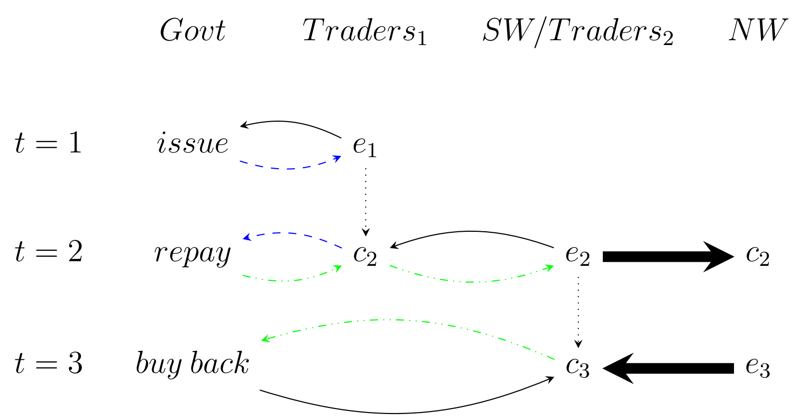


Figure 2: Markets in the Yen scenario. Goods (solid black); Bonds (dashed blue); Cash (dot-dashed green); Storage (dotted black). $e$ stands for endowment and $c$ for consumption.

The equilibrium is therefore characterized by the primary- and secondary-market interest rates on government debt, which are summarized by the discount factors $q_1$ and $q_2$.

## 2.2 Trading in the Yen Economy

In the Yen economy, there is no uncertainty about the nominal repayment from government bonds, which happens in cash in period 2. However, the terminal value of cash in period 3 depends on tax revenues. Strategic workers must decide whether to store their endowment until period 3 or to sell their goods in period 2 for cash or trade credit, at a price $P_2$. Noise workers will demand goods in period 2 in exchange for trade credit, in a fixed amount $\Phi(\epsilon_2^w) \in (0,1)$. Traders born in period 1 will also use their cash to buy goods in period 2; by assumption, their demand is negligible compared to that of the workers.

The payoff for a strategic worker of selling a unit of goods right away relative to storing it is

$$\mathbb{E}\left(\frac{1}{P_3}|\mathcal{I}_{i,2}^w\right) - \frac{1}{P_2}, \tag{4}$$

where $\mathcal{I}_{i,2}^w$ is the information available to the worker and $P_3$ is the nominal price in period 3, which is either 1 or $1/\theta$, depending on whether $s \geq \hat{s}(q_1)$. Hence, equation (4) becomes[14]

$$\theta + (1-\theta)\mathbb{E}(1-\delta|\mathcal{I}^w_{i,2}) - \frac{1}{P_2}. \tag{5}$$

Letting $D^w_2$ be the fraction of strategic workers selling the goods in period 2 (demanding cash or trade credit), market clearing in period 2 requires

$$D^w_2 = \Phi(\epsilon^w_2) = 1 - \Phi(-\epsilon^w_2). \tag{6}$$

Since there is no secondary market for government bonds in period 2, noise traders are not active.[15] Strategic traders face the same choice as the workers: either store their endowment or sell it for cash or trade credit. Since their mass is negligible relative to that of the workers, their choice has no effect on market clearing and prices.

Going back to period 1, the problem of strategic bond traders in period 1 is similar to the Euro economy, except that their payoff is now a fixed amount of Yen with uncertain value rather than an uncertain amount of Euros. The expected profit from buying a bond is

$$\hat{s}(q_1)\left\{\mathbb{E}[\frac{1}{P_2}|\mathcal{I}^b_{i,1}] - q_1\right\},$$

and market clearing is still given by (3).

The equilibrium is now characterized by the primary-market interest rate on government debt, summarized by the discount factor $q_1$, and the nominal price level $P_2$.

## 2.3 Comparing the Two Economies

The construction of an equilibrium in the two economies is very similar. The only difference between the two concerns the identity of the marginal agent in period 2. In the Euro scenario, this is a bond trader active in the secondary market, while in the case of Yen-denominated debt it is a worker selling her goods in exchange for nominal payments. This is seen comparing equations (1) and (2) for the Euro economy with equations (5) and (6) of the Yen economy.

[14] $\delta$ is the same indicator function as in the Euro model, except that now it indicates states of high inflation in period 3 rather than default.

[15] Recall that we assumed that the demand from noise traders is a fraction of the supply of bonds.

| | Euro | Lira |
|---|---|---|
| Identity of marginal buyer | bond trader | worker |
| Goods given up | $\hat{s}(q_1)q_2$ | 1 |
| Goods received: | | |
| w/o default/inflation: | $\hat{s}(q_1)$ | $P_2/P_3 = P_2$ |
| with default/inflation: | $\theta\hat{s}(q_1)$ | $P_2/P_3 = \theta P_2$ |
| Return: | | |
| w/o default/inflation: | $1/q_2$ | $P_2$ |
| with default/inflation: | $\theta/q_2$ | $\theta P_2$ |

Table 1: Comparison of the payoffs of strategic agents in period 2.

The parameters of interest are thus the relative information that workers and second-period traders have about the government's ability to raise taxes in the final period. Our key assumption is that bond traders are more informed than workers, that is, they have a more precise signal ($\beta_2^b > \beta_2^w$) and face less market noise ($\psi_2^b > \psi_2^w$).[16]

Table 1 highlights the symmetry between the two scenarios, which we exploit to collapse the two cases into a single problem. Accordingly, we drop the superscripts referring to workers and traders, we define $q_2 := 1/P_2$ in the case of the Yen, and we refer to "demand" by second-period strategic agents as their real demand for risky assets, which is their *supply* of goods: in the case of the Euro, traders acquire government bonds in the secondary market, whereas in the case of the Yen workers acquire cash or trade credit.[17]

[16] We state our results separately for $\beta_2$ and $\psi_2$, but in practice their effect is quite similar, and what matters for characterizing the equilibrium are only their products $\beta_2\psi_2$ and $\beta_2(1+\psi_2)$.

[17] As we discuss later, individual demand will take the form of a reservation price. This convention preserves the feature that strategic agents will want to "demand" the asset (and supply goods) when $q_2$ is low. In the case of the bond traders, $q_2$ is the price of the bond, which they want to acquire only below their reservation price; in the case of workers $q_2$ is the inverse of the price level, and workers choose to sell their goods for nominal claims when $P_2$ is sufficiently high relative to their expectations about $P_3$.

We thus proceed by analyzing a single problem, in which we drop the superscripts referring to workers and traders, and studying comparative statics with respect to $\beta_2$ and $\psi_2$.[18]

# 3 The Simplest Case: No Recall of Past Prices

In this section, we study the simpler case in which agents buying in period two do not have any information on the equilibrium price from period one and $\hat{s}(q_1) \equiv \hat{s}$ is constant. This allows us to derive particularly transparent intuition. In Section 4, we move to the case in which the first-period price is observable to second-period agents, and in Section 5 we further add the possibility that the default threshold depends on the interest rate paid by the government at issuance (letting $\hat{s}$ vary with $q_1$). Let $d(x_{i,t}, q_t)$ denote demand schedules in each period, forming a mapping $d : \mathbb{R}^2 \to \{0, 1\}$ from signal-price pairs $(x_{i,t}, q_t)$ into risky asset holdings. Given that we assume risk neutrality, the optimal demand schedule will take the form of a reservation price.

## 3.1 Strategies, Beliefs and Equilibrium

**Definition 1.** A Perfect Bayesian Equilibrium consists of bidding strategies $d(x_{i,t}, q_t)$ for strategic players, price functions $q_t(s, \epsilon_t)$ and posterior beliefs $p_t(x_{i,t}, q_t)$ such that

(i) $d(x_{i,t}, q_t)$ is optimal given beliefs $p_t(x_{i,t}, q_t)$,

(ii) $q_t(s, \epsilon_t)$ clears the market for all $(s, \epsilon_t)$, and

(iii) $p_t(x_{i,t}, q_t)$ satisfies Bayes' Law for all market clearing prices $q_t$.

To characterize the equilibrium we work backwards, starting from period 2. The derivation of the second-period equilibrium follows Albagli, Hellwig, and Tsyvinski [3]. Second-period agent $i$'s expected payoff of buying the risky asset is $\hat{s}[\theta + (1-\theta)\text{Prob}(s \geq \hat{s}|x_{i,2}, q_2) - q_2]$. Since posterior beliefs over $s$ are increasing in $x_{i,2}$ in the sense of first-order stochastic dominance,[19] agents'

[18]We exploit the symmetry of the normal distribution in equation (6) and renormalize $\epsilon_2 = -\epsilon_2^w$ in the case of the Yen economy.

[19]See Proposition 6 in the appendix.

expected payoffs are an increasing function of $x_{i,2}$. This implies that agents follow monotone strategies of the form

$$d(x_{i,2}, q_2) = \mathbb{1}[x_{i,2} \geq \hat{x}_2(q_2)], \tag{7}$$

where $\mathbb{1}$ is the indicator function and $\hat{x}_2(q_2)$ is a threshold which is endogenous to the equilibrium.

Integrating strategic players' demand schedules over the signal distribution, the market clearing condition in either period $t = 1, 2$ is

$$\int d(x, q_t)\sqrt{\beta_t}\phi[\sqrt{\beta_t}(x - s)]dx + \Phi(\epsilon_t) = 1, \tag{8}$$

where $\phi$ is the density of a standard normal distribution. In general, this equation characterizes the equilibrium price $q_t(s, \epsilon_t)$. Using equation (7), the aggregate demand of strategic agents is $\text{Prob}[x_{i,2} \geq \hat{x}_2(q_2)|s]$, and the market clearing condition becomes

$$z_2 := s + \frac{\epsilon_2}{\sqrt{\beta_2}} = \hat{x}_2(q_2). \tag{9}$$

Henceforth we will focus on equilibria where the price is a continuous function of $s$ and $\epsilon_2$. In this case, Proposition 7 proves that conditioning beliefs about $s$ (and other exogenous events) on $q_2$ is equivalent to conditioning them on $z_2$. This simplifies the analysis in that $z_2$ is itself exogenous. Second-period agents' posterior beliefs in an equilibrium are given by

$$s|x_{i,2}, z_2 \sim N\left(\frac{\alpha_0\mu_0 + \beta_2 x_{i,2} + \beta_2\psi_2 z_2}{\alpha_0 + \beta_2(1 + \psi_2)}, \frac{1}{\alpha_0 + \beta_2(1 + \psi_2)}\right). \tag{10}$$

An agent whose private signal is at the threshold $\hat{x}_2(q_2)$ must be indifferent in equilibrium between buying risky claims or storing. Combining this with equation (9), $q_2(z_2)$ must satisfy the indifference condition

$$q_2(z_2) = \theta + (1 - \theta)\text{Prob}(s \geq \hat{s}|x_{i,2} = z_2, z_2) = \theta + (1 - \theta)\Phi\left(\frac{(1 - w_S)\mu_0 + w_S z_2 - \hat{s}}{\sigma_S}\right), \tag{11}$$

where $w_S = \frac{\beta_2(1+\psi_2)}{\alpha_0+\beta_2(1+\psi_2)}$ is the Bayesian weight on $z_2$, that summarizes new private and public information for the marginal second-period agent, and $\sigma_S$ is the standard deviation of the conditional beliefs of secondary-market participants, which in this case is $(\alpha_0 + \beta_2(1 + \psi_2))^{-1/2}$ from equation (10). As it's clear from Equation (11), $q_2$ exists and is unique for all $z_2 \in \mathbb{R}$.[20]

[20] This will apply to the more general cases where second-period agents also observe $q_1$, and $\hat{s} = \hat{s}(q_1)$.

Having defined equilibrium price and strategies in the second period, we can move to the first period and derive strategic bond traders' behavior. The analysis follows that of period two quite closely. Traders $i$'s expected payoff of buying government bonds in period one is $\mathbb{E}[q_2(z_2)|x_{i,1}, q_1] - q_1$. Since $q_2(z_2)$ is increasing in $z_2$ and Proposition 6 applies to first-period agents' beliefs as well, they optimally follow monotone strategies which, given risk neutrality, will be described by a threshold signal of the form $d(x_{i,1}, q_1) = \mathbb{1}[x_{i,1} \geq \hat{x}_1(q_1)]$.

Repeating the steps that led to (9), the market clearing condition in the first period can be rewritten as

$$z_1 := s + \frac{\epsilon_1}{\sqrt{\beta_1}} = \hat{x}_1(q_1) \tag{12}$$

As in period two, we focus on equilibria where the price is a continuous function of $s$ and $\epsilon_1$, in which case conditioning on $q_1$ or the observable state variable $z_1$ is equivalent for forming beliefs about $s$. In any such equilibrium, traders' posterior beliefs on $s$ are given by

$$s|x_{i,1}, z_1 \sim N\left(\frac{\alpha_0\mu_0 + \beta_1 x_{i,1} + \beta_1\psi_1 z_1}{\alpha_0 + \beta_1(1+\psi_1)}, \frac{1}{\gamma_1} := \frac{1}{\alpha_0 + \beta_1(1+\psi_1)}\right). \tag{13}$$

However note that the payoff-relevant variable that traders need to predict is not just $s$, but $z_2$, because the latter is what determines the resale price in period two. Since $z_2|(x_{i,1}, z_1) = s|(x_{i,1}, z_1) + \epsilon_2/\sqrt{\beta_2}$, the marginal bond trader's posterior beliefs on $z_2$ are given by

$$z_2|(x_{i,1} = z_1, z_1) \sim N\left(\frac{\alpha_0\mu_0 + \beta_1(1+\psi_1)z_1}{\gamma_1}, \sigma^2_{S|B} := \frac{1}{\gamma_1} + \frac{1}{\psi_2\beta_2}\right) \tag{14}$$

where $\sigma^2_{S|B}$ is the variance of the second-period agents' sufficient statistic $z_2$ conditional on first-period bond traders' information.

The marginal agent whose private signal is at the threshold $\hat{x}_1(q_1)$ must be indifferent in equilibrium between buying government bonds or storage. Let us denote the Bayesian weight she puts on $z_1$ when forecasting $s$ as

$$w_B := \frac{\beta_1(1+\psi_1)}{\alpha_0 + \beta_1(1+\psi_1)}. \tag{15}$$

Then market clearing (12) and the indifference condition can be used to solve for $q_1$:

$$\begin{aligned} q_1(z_1) &= \mathbb{E}[q_2(z_2)|x_{i,1} = z_1, z_1] \\ &= \theta + (1-\theta)\int \Phi\left(\frac{(1-w_S)\mu_0 + w_S z_2 - \hat{s}}{\sigma_S}\right)\frac{1}{\sigma_{S|B}}\phi\left(\frac{z_2 - (1-w_B)\mu_0 + w_B z_1}{\sigma_{S|B}}\right)dz_2 \\ &= \theta + (1-\theta)\Phi\left[\frac{[1-w_S w_B]\mu_0 + w_S w_B z_1 - \hat{s}}{\sqrt{w_S^2\sigma_{S|B}^2 + \sigma_S^2}}\right]. \end{aligned} \tag{16}$$

Since we assume $\hat{s}$ exogenous, existence and uniqueness of $q_1(z_1)$ here are guaranteed.

## 3.2 Comparative Statics

We now expose our main result, that states that a government that faces a bad shock realization compared to its prior would benefit from a decrease in secondary agents' information precision. That is, the "Euro" scenario would prove more adverse in such a situation. In the case of the second-period price $q_2$, this result is straightforward from equation (11): the more informed are the second-period agents (higher $\beta_2$), the more they will trust their signal; furthermore, the more informed are their trading partners (by symmetry, this is also due to higher $\beta_2$) or the less market noise is present (higher $\psi_2$), the more the price will aggregate the strategic agents' information. Both of these forces lead the strategic agents to put less weight on the prior, so that their demand will be more responsive to incoming bad news. Mathematically, the result follows from two effects:

1. **second-period mean weight channel**: an increase in $\beta_2$ or $\psi_2$ increases the weight of $z_2$ in second-period agents' beliefs on $s$. This effect appears from the term $w_S$ at the numerator of (11).

2. **second-period information precision channel**: an increase in $\beta_2$ or $\psi_2$ decreases the noise over $s$ for second-period agents, thus making $q_2$ more responsive to the state because information on it is more precise. This effect appears from the term $\sigma_S$ in the denominator.

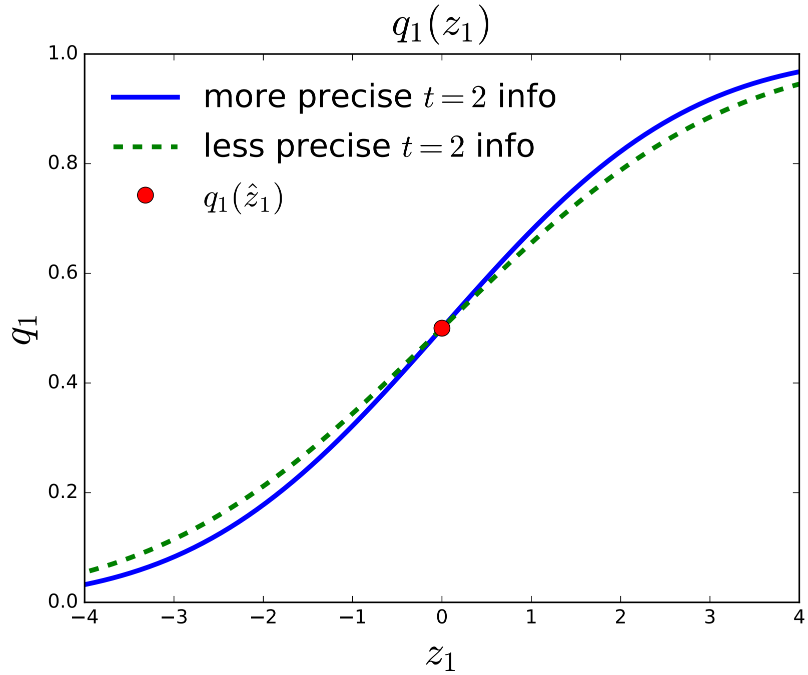


Figure 3: Effect of a Change in Second-Period Information Precision $\beta_2$ or $\psi_2$ on Debt Price $q_1$.

The more interesting result concerns the first period. Even when the second-period price is set by relatively uninformed agents, as it happens in our Yen scenario, bonds are still purchased by well-informed traders in the first period. What we need to show is that these sophisticated traders will also find it optimal to be less responsive to incoming news when they anticipate being able to offload their position onto a less-informed party. This is established by the following propositions:

**Proposition 1.** *There exists a cutoff level* $\hat{z}_1^\beta \in \mathbb{R}$ *such that when* $z_1 < \hat{z}_1^\beta$*, a decrease in* $\beta_2$ *improves the issuance price* $q_1$*, whereas the reverse occurs for* $z_1 > \hat{z}_1^\beta$*.*

**Proposition 2.** *There exists a cutoff level* $\hat{z}_1^\psi \in \mathbb{R}$ *such that when* $z_1 < \hat{z}_1^\psi$*, a decrease in* $\psi_2$ *improves the issuance price* $q_1$*, whereas the reverse occurs for* $z_1 > \hat{z}_1^\psi$*.*

Figure (3) illustrates these results with an example. We analyze the components of $q_1(z_1)$ more in detail and provide some intuition. The formal proofs of the propositions are in the appendix. We can rewrite $q_1$ as

$$q_1 = \theta + (1-\theta)\Phi\left[\frac{\mu_0 - \hat{s} + \overbrace{w_S}^{1}\, w_B(z_1-\mu_0)}{\sqrt{\underbrace{w_S^2}_{3}\underbrace{\left(\frac{1}{\gamma_1}+\frac{1}{\beta_2\psi_2}\right)}_{4}+\underbrace{\sigma_S^2}_{2}}}\right]. \tag{17}$$

We can decompose the effect of a change in $\beta_2$ and $\psi_2$ on $q_1$ into the four different channels we highlight in equation (17):

1. **second-period mean weight channel**: this is the same as described for $q_2$. In the context of the first-period price, it is multiplied by $w_B$, because that is the weight first-period traders give to $z_1$ when forecasting $z_2$.[21]

2. **second-period information precision channel**: this second effect is also what we described for $q_2$. It is now only one of the elements driving the denominator in equation (17).

3. **first-period variance weight channel**: as $\beta_2$ or $\psi_2$ increase, the first two channels make $q_2$ more responsive to $z_2$; however, $z_2$ is affected by noise agents as well as fundamentals, and this channel alone would decrease the first-period traders' ability to predict the second-period price through $z_1$. This effect is represented by $w_S^2$ in the denominator and would go in the direction of making the price *less* responsive to $z_1$.

4. **first-period guess precision channel**: closely related to the previous point, $\beta_2$ and $\psi_2$ affect the precision of the endogenous price signal in period two: in particular, as we see in equation (9), an increase in $\beta_2$ or $\psi_2$ implies that $z_2$ becomes more closely correlated with $s$ and thus $z_1$, while the importance of the noise agents is correspondingly diminished. This effect appears from the term $\left(\frac{1}{\gamma_1}+\frac{1}{\beta_2\psi_2}\right)$ in the denominator.[22]

[21] The presence of $w_B$ reflects the attenuation emphasized in Allen, Morris, and Shin [4]. In our comparative statics exercise, $w_B$ remains the same across the Yen and Euro economy, while $w_S$ changes.

[22] Combining effects (3) and (4) alone, we would get an ambiguous result. An increase in $\beta_2$ or $\psi_2$ increases

The proofs in the appendix show that the channels (1), (2), and (4) always dominate channel (3). Hence, when the realization of $z_1$ is low, the price $q_1$ is more resilient if second-period agents are less well informed (lower $\beta_2$ or $\psi_2$).[23]

According to our interpretation, lower values of $\beta_2$ and $\psi_2$ arise when debt is denominated in a currency over which the country has control, which allows recourse to inflation rather than outright default. In this case, second-period agents are workers setting their prices in the local currency. In contrast, when inflation is not an option and debt is subject to the risk of outright default, second-period agents correspond to a new cohort of well-informed bond traders. Propositions 1 and 2 then state that the price of debt will be more resilient to bad shocks in the former case. We view this result as particularly relevant for countries that start from a favorable prior: for them, there is limited upside from further confirming the creditors' belief that there is ample fiscal space, while there is substantial downside risk should they find out that fiscal constraints are tighter than they appeared. This is a good description of Eurozone countries in 2008, as well as other major developed economies, all of which paid very low interest rates before the onset of the crisis.

Our result also highlights a potentially opposite conclusion for a country that starts from an adverse prior. For such a country, issuing domestically-denominated debt may immediately lead workers to expect high inflation, and this pessimism will spill over to the traders who underwrite the debt, through the channels that we emphasize. When realized fiscal space is indeed limited, as will happen often if the prior is correct, there is not much that can be done to sustain the price of debt. However, in the event that fundamentals are more favorable, well-informed traders will be better placed to detect the situation, and debt will correspondingly fetch a higher price when issued in foreign currency. We view this as more relevant for countries such as those of Latin America and this may be another explanation for their past inclination to issue dollar-

the weight given to $z_2$, which can only be partially forecasted, but decreases the variance of such guess. As an example, on their own, these two channels would go in the direction opposite of Proposition 1 close to $\beta_2 = 0$: around that point, an increase in second-period precision decreases the predictability of $q_2$ given $z_1$.

[23] A bad realization of $z_1$ can be driven either by a low value of fiscal capacity $s$ or small demand from noise traders (low $\epsilon_1$). Both represent an adverse event for the government. When first-period traders are well informed, this realization will be mostly driven by fiscal capacity.

denominated debt.[24]

# 4 What if there is Recall of the Primary-Market Price?

In the previous section, we have examined the case where agents in the second period do not observe $q_1$. We now study what happens in the more likely scenario in which $q_1$ is known by second-period agents as well. Other than this, we retain the same structure as described in the previous section. In particular, we maintain the assumption that the default threshold is independent of the first-period price; in Section 5 we will show that the same results hold when the threshold is endogenous, as long as complementarities are not as strong as to generate equilibrium multiplicity.

## 4.1 Strategies and Equilibrium

The equilibrium structure of the modified game is largely identical to that of Section 3. We relegate the definition of an equilibrium to the appendix. Posterior beliefs over $s$ are still increasing in $x_{i,2}$ in the sense of first-order stochastic dominance,[25] and agents follow monotone strategies of the form

$$d(x_{i,2}, q_1, q_2) = \mathbb{1}[x_{i,2} \geq \hat{x}_2(q_1, q_2)].$$

Using the same steps as in Section 3, the market-clearing condition becomes

$$z_2 = s + \frac{\epsilon_2}{\sqrt{\beta_2}} = \hat{x}_2(q_1, q_2). \tag{18}$$

We focus once more on equilibria where conditioning beliefs on the prices $(q_1, q_2)$ is equivalent to conditioning them on the exogenous state state variables $(z_1, z_2)$ as defined in equations (12) and (9), and where conditioning beliefs on $q_1$ is equivalent to conditioning them on $z_1$. For these equilibria we obtain

$$s|x_{i,2}, z_2, z_1 \sim N\left(\frac{\alpha_0\mu_0 + \beta_1\psi_1 z_1 + \beta_2 x_{i,2} + \beta_2\psi_2 z_2}{\alpha_0 + \beta_1\psi_1 + \beta_2(1+\psi_2)}, \frac{1}{\alpha_0 + \beta_1\psi_1 + \beta_2(1+\psi_2)}\right) \tag{19}$$

[24]This reason is complementary to the time-inconsistency forces emphasized by Calvo [14], Bohn [12], Aguiar et al. [2], Engel and Park [24], and Ottonello and Perez [33].

[25]See Proposition 6 in the appendix.

and the marginal agent's indifference condition becomes

$$q_2(z_1, z_2) = \theta + (1-\theta)\Phi\left[\frac{(1-w_{1,S}-w_{2,S})\mu_0 + w_{1,S}z_1 + w_{2,S}z_2 - \hat{s}}{\sigma_S}\right], \tag{20}$$

where $w_{1,S} = \frac{\beta_1\psi_1}{\alpha_0+\beta_1\psi_1+\beta_2(1+\psi_2)}$, $w_{2,S} = \frac{\beta_2(1+\psi_2)}{\alpha_0+\beta_1\psi_1+\beta_2(1+\psi_2)}$ are the Bayesian weights given by the marginal second-period agent to first- and second-period information respectively, and, from (19), the standard deviation of conditional beliefs is

$$\sigma_S = \sqrt{\frac{1}{\alpha_0 + \beta_1\psi_1 + \beta_2(1+\psi_2)}}.$$

It is easy to see that $q_2(z_1, z_2)$ is unique and exists for all $(z_1, z_2) \in \mathbb{R}^2$. In Section 3, the prior was the only information element that was mutual common knowledge between period-1 and period-2 agents. Here, period-2 agents condition their demand on the first-period price $q_1$ as well, which creates a new source of common knowledge. This common information is the source of differences between the results of this section and the previous one.

Since the information set of first-period traders is the same of the previous section, their posterior beliefs on $z_2$ conditional on $x_{i,1}$ and $z_1$ are still given by (14). From the indifference condition of the marginal trader we can derive the equilibrium price function

$$\begin{aligned}
q_1(z_1) &= E[q_2(z_1, z_2)|x_{i,1} = z_1, z_1] \\
&= \theta + (1-\theta)\int \Phi\left[\frac{\mu_0(1-w_{1,S}-w_{2,S}) + w_{1,S}z_1 + w_{2,S}z_2 - \hat{s}}{\sigma_S}\right] \\
&\cdot \frac{1}{\sigma_{S|B}}\phi\left(\frac{z_2 - (1-w_B)\mu_0 - w_B z_1}{\sigma_{S|B}}\right) dz_2 \\
&= \theta + (1-\theta)\Phi\left[\frac{\mu_0\Big(1 - w_{1,S} - w_{2,S}w_B\Big) + z_1\Big(w_{1,S} + w_{2,S}w_B\Big) - \hat{s}}{\sqrt{w_{2,S}^2\left(\frac{1}{\gamma_1} + \frac{1}{\beta_2\psi_2}\right) + \sigma_S^2}}\right],
\end{aligned} \tag{21}$$

where $w_B$ and $\sigma^2_{S|B}$ continue to be defined as in (14) and (15).

Much of the intuition behind equation (21) follows that in (17). There we highlighted that a second-period agent's information set included prior information (common to first-period traders), and period-2 information (that first-period traders do not observe and must forecast using their information set). Here, the same dichotomy holds, with the difference that the intersection between primary and secondary agents' information sets now includes the first-period price, in addition to the prior. This is reflected in the weight given by first-period traders to state $z_1$ in the numerator, $w_{1,S} + w_{2,S}w_B$. $w_{1,S}$ represents the weight second-period agents give to $z_1$, a fact that is then taken into account by first-period traders. $w_{2,S}$ represents the weight second-period agents put on $z_2$, which traders predict using prior and first-period information with weight $1 - w_B$ and $w_B$ respectively.

## 4.2 Comparative Statics

We now prove results analogous to Propositions 1 and 2. While comparative statics for $\psi_2$ are the same as in Section 3, an increase in $\beta_2$ sharpens the sensitivity of the price to information for a smaller set of the parameter space, due to the more complex information structure of the current specification.

To build intuition, we rewrite $q_1$ as

$$q_1(z_1) = \theta + (1-\theta)\Phi\left[\frac{\mu_0 - \hat{s}}{\sqrt{w_{2,S}^2\sigma_{S|B}^2 + \sigma_S^2}} + K(z_1 - \mu_0)\right], \tag{22}$$

with

$$K := \frac{\overbrace{(w_{1,S} + w_{2,S}w_B)}^{1}}{\sqrt{\underbrace{w_{2,S}^2}_{3}\underbrace{\left(\frac{1}{\gamma_1} + \frac{1}{\beta_2\psi_2}\right)}_{4} + \underbrace{\sigma_S^2}_{2}}}. \tag{23}$$

The key difference between the case we analyze here and the one we considered in Section 3 is that now second-period agents form a posterior based on the first-period price as well as on their prior and their idiosyncratic signal. When $\beta_2$ or $\psi_2$ increase, they will rely less on the prior, which does not react to bad shocks, but also less on the first-period price. While second-period signals are aggregated through the second-period price, which first-period agents can only imperfectly forecast, the first-period price is effectively observable to them, as they are allowed to submit a conditional demand schedule. Hence, when the second-period posterior weight shifts from the first-period price to second-period signals, the correlation between the two prices will decrease and this may make first-period agents less responsive to their information. The further difference between the results for $\beta_2$ and $\psi_2$ stems from an asymmetry in the way these precisions enter in the problem of first- and second-period agents. Specifically, from the perspective of both, the product of $\beta_2$ and $\psi_2$ determines the precision of the second-period price as an aggregator of information. In addition to this, $\beta_2$ has a further role as the precision of the idiosyncratic signal observed by the marginal agent in the second period, which generates additional movement in the weight $w_{2,S}$ and the precision $\sigma_S$.[26]

From a mathematical perspective, the single-crossing condition illustrated in Figure 3 is driven by $K$, as defined in (23), which is the coefficient of $z_1$ in (22): when it is bigger, the first-period price becomes more responsive to the aggregate shock $z_1$. Comparing this expression with (17) from the previous section, the same four channels that we previously highlighted remain active. The first-period guess precision channel (channel 4) remains exactly as before, since the information set of first-period agents is unaffected. The second-period information precision and first-period variance weight channels (channels 2 and 3) also remain similar, although the new expressions for $w_{2,S}$ and $\sigma_S$ imply a weaker response to increases in $\beta_2$ and $\psi_2$ because the second-period agents now substitute away from the first-period price when their signal becomes more precise or the second-period price better aggregates information. The biggest difference emerges in the second-period mean weight channel (channel 1). When second-period agents receive more precise information on the fundamentals, the shift away from the unconditional prior continues to be a force increasing the impact of changes in fundamentals on the price; however, if the first-period price is sufficiently informative, a shift away from $z_1$ and towards $z_2$ would *decrease* the responsiveness of $q_2$ to fundamentals instead. Moreover, $z_1$ is known to first-period traders, whereas they can only predict $z_2$ with noise: hence, when $q_2$ responds less to $z_1$ directly and more to $z_2$, they respond themselves less aggressively.[27] While an increase in $\beta_2$ or $\psi_2$ continues to

[26] This asymmetry is discussed extensively in Albagli, Hellwig, and Tsyvinski [3].

[27] Mathematically, while the second-period agents' weight on $z_2$ ($w_{2,S}$) is multiplied by $w_B$, representing the imperfect ability of first-period traders to predict it, the weight second-period agents give on $z_1$ passes through to first-period traders without any dampening.

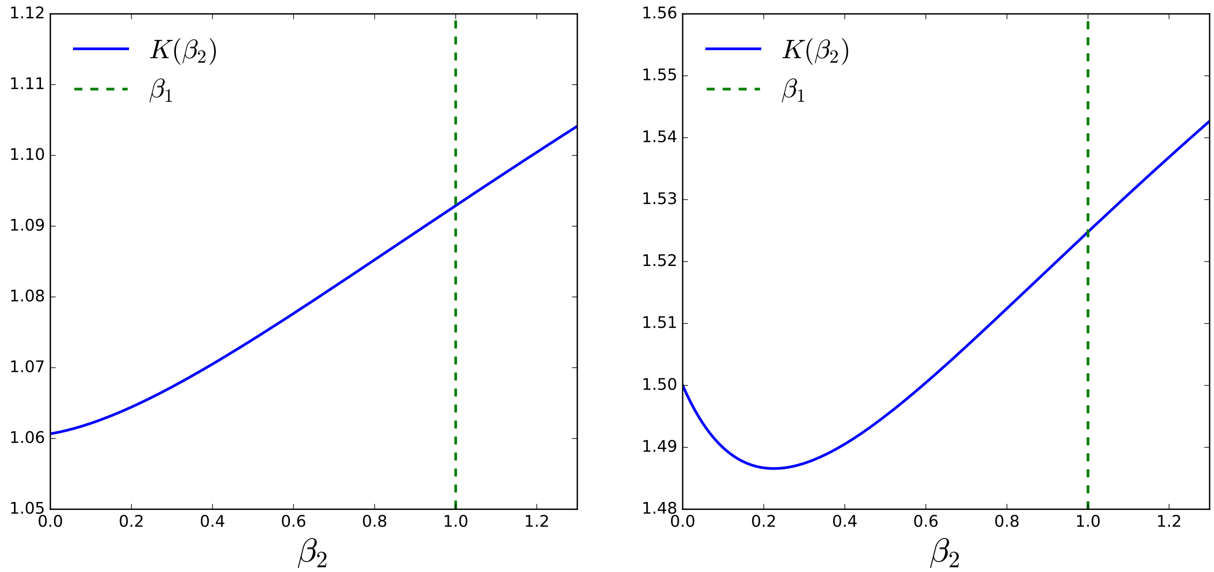


Figure 4: Examples of coefficient $K$ as a function of $\beta_2$.

increase responsiveness of the price through the second-period mean weight channel, this channel is now weakened, which matters when we combine all of the effects in equation (23).

In the case of Section 3, the coefficient of $z_1$ in (17) is globally increasing in both $\beta_2$ and $\psi_2$. Here, the analogous coefficient $K$ remains globally increasing in $\psi_2$, as we prove in Proposition 3, but it is not necessarily globally increasing in $\beta_2$. In the appendix, we prove that this coefficient is either monotonically increasing in $\beta_2$, or it has a single interior minimum, as illustrated in the two panels of Figure 4. In this latter case, it is possible that, starting from a situation in which second-period agents have no signal of their own, providing them with a very noisy signal would decrease the sensitivity of the first-period price to the aggregate shock $z_1$.

Our main case of interest is comparing the situation in which second-period agents are bond traders in the secondary market with the case in which they are less-informed price setters accepting local currency in exchange for goods. In this comparison, it would be natural to start from the case in which first and second-period bond traders are symmetric, in that they have a signal of equal precision. If anything, we would expect the second-period traders to receive more precise signals, as the passage of time could only reveal more information (in addition to the first-period price). Starting from such a situation, any move in the direction of lower second-period precision (whether it is a small local perturbation or a large deviation) decreases the sensitivity of the first-period price. This is illustrated in the right panel of Figure 4.

Formally, the following propositions apply:

**Proposition 3.** *There exists a cutoff level $\hat{z}_1^{\psi} \in \mathbb{R}$ such that when $z_1 < \hat{z}_1^{\psi}$, a decrease in $\psi_2$ improves the issuance price $q_1$, whereas the reverse occurs for $z_1 > \hat{z}_1^{\psi}$.*

**Proposition 4.** *Assume that $\psi_2 \geq \psi_1$ and $\beta_2^A \geq \beta_1$. Let $\beta_2^B < \beta_2^A$. Then there exists a cutoff level $\hat{z}_1^{\beta} \in \mathbb{R}$ such that when $z_1 < \hat{z}_1^{\beta}$, $q_1$ evaluated at $\beta_2^A$ is smaller than at $\beta_2^B$, whereas the reverse occurs for $z_1 > \hat{z}_1^{\beta}$, holding all other parameters fixed.*

We conclude that our main result is robust to the case in which the first-period price is observed by second-period agents: it remains the case that a government which starts from a good prior, but has a negative realization would fetch a better price for its debt when it is issued in local currency than when it is denominated in a currency over which it has no control.

# 5 Endogenous Default Threshold

In the previous section we maintained the assumption that the government's default cutoff is exogenous and independent of the primary-market price. We now relax this assumption as well and consider the case in which the default threshold is given by a function $\hat{s}(q_1)$. As an example, this happens if the debt auction follows the same structure as in Calvo [15]: the government requires a given debt auction revenue, which we normalize to unity, while its repayment obligations at the end of the second period depend on the interest rate and are given by $1/q_1$. A default occurs in this case if and only if $s < 1/q_1$, so in this case $\hat{s}(q_1) = 1/q_1$.

The introduction of an endogenous default threshold creates a new source of complementarity and could potentially generate multiple equilibria if information is sufficiently precise (Hellwig, Mukherji and Tsyvinski [27], Angeletos and Werning [11]). We study the case where a unique equilibrium is maintained, which happens when information is sufficiently dispersed.

The construction of an equilibrium is very similar to what we did in Section 4. All the steps up to equation (20) remain the same, where $\hat{s}$ is replaced by $\hat{s}(q_1)$. As of period 2, $\hat{s}(q_1)$ is a given, so that existence and uniqueness given $q_1$ are established as before. The main difference arises in equation (21), where now the endogenous threshold implies that $q_1(z_1)$ is only implicitly characterized by the solution to the following equation:

$$q_1 = \theta + (1-\theta)\Phi\left[\frac{\mu_0 - \hat{s}(q_1)}{\sqrt{w_{2,S}^2 \sigma_{S|B}^2 + \sigma_S^2}} + K(z_1 - \mu_0)\right], \qquad (24)$$

where $K$ is given by the same expression as in the case of an exogenous threshold, as defined in equation (23).

**Assumption 1.** *At any equilibrium price, the slope of the right-hand side of (24) with respect to $q_1$ is smaller than one.*

Assumption 1 is necessary and sufficient to guarantee the uniqueness of the equilibrium price function $q_1(z_1)$. As an example, for the Calvo threshold $\hat{s}(q_1) = 1/q_1$, a sufficient condition for Assumption 1 to hold is

$$\sqrt{w_{2,S}^2 \sigma_{S|B}^2 + \sigma_S^2} > \frac{1-\theta}{\theta^2}\frac{1}{\sqrt{2\pi}},$$

that is, the total amount of information in the economy should not be too high. In this specification, the price $q_1$ affects equilibrium equation (24) in two ways: it represents the cost of buying government bonds (left-hand side), and it affects the repayment probabilities via its impact on the default cutoff (right-hand side). The latter effect is amplified by information precision since it acts through posterior beliefs. When information precision is very high, locally it may happen that this default cutoff effect is strong enough to generate multiple equilibria. We instead consider the case in which there is enough noise that the curve describing how $q_1$ varies in response to $z_1$ does not bend backwards, so that $q_1$ remains a well-defined (and increasing) function of $z_1$.

In Section 4, we could establish results about the sensitivity of the price to $z_1$ by simply studying the properties of the coefficient $K$. Now, the analysis is complicated by the fact that $q_1$ appears on the right-hand side through its effect on the default threshold. We prove in the appendix that this does not change our results for the comparative statics when $\psi_2$ varies, so that Proposition 3 continues to hold.

Concerning $\beta_2$, in Section 4 we could always rely on the fact that two price functions drawn for different values would cross only once, with the direction dictated by the magnitude of $K$. We can no longer prove this here. However, even if single-crossing fails, prices will move in the same way as described in Proposition 4 following tail events. Formally:

**Proposition 5.** *Assume that $\psi_2 \geq \psi_1$ and $\beta_2^A \geq \beta_1$, and let Assumption 1 hold. Let $\beta_2^B < \beta_2^A$. Then there exist two cutoffs level $\hat{z}_1^L \leq \hat{z}_1^H \in \mathbb{R}$ such that when $z_1 < \hat{z}_1^L$, $q_1$ evaluated at $\beta_2^A$ is smaller than at $\beta_2^B$, whereas the reverse occurs for $z_1 > \hat{z}_1^H$, holding all other parameters fixed.*

The intuition behind Proposition 5 is that, for $z_1$ large in absolute value, the dominant force determining how the price moves with $\beta_2$ remains $K$, for which we already proved theorems in the previous section.

# 6 Conclusion

Inflation risk and default risk affect the real value of maturing government debt in a similar way. However, the general price level is driven by the interaction among a much larger fraction of the population than the restricted group of people who actively participate in the government debt market. To the extent that information about government finances is unevenly distributed within the population, we have shown that this asymmetry has important implications for the resilience of debt prices in the face of adverse shocks.

In this paper, we emphasized one reason why inflation reacts sluggishly to fundamentals. Our results would also apply in different contexts where other frictions force a slower adjustment in the prices of goods relative to asset prices, such as sticky-price models.

Our analysis opens a new dimension for the study of optimal debt management, in addition to the traditional channels of fiscal hedging and time consistency. The next step in this direction is to further develop a full theory of the optimal denomination of debt. Such a theory would take into account the insurance aspect that we have studied here, together with the effects of different structures of debt on the ex ante expected borrowing costs.[28]

[28] As emphasized in Albagli, Hellwig, and Tsyvinski [3], in the context of the model that we adopt, the rela-

# Appendix A Proofs

**Proposition 6** (Belief Stochastic Dominance)**.** *In each period, agents' posterior beliefs over s are increasing in their private signal in the sense of first-order stochastic dominance.*

**Proof of Proposition 6.** We prove this for the more complex case of Sections 4 and 5; the proof for the Section 3 economy is simpler and follows the same steps.

Denote with $F(s|x_{i,2}, q_1, q_2)$ the cumulative distribution function (cdf) of the posterior beliefs on $s$ for a second-period agent with private signal $x_{i,2}$, after observing primary-market price $q_1$ and when the equilibrium secondary-market price is $q_2$. Similarly, let $h(x|s, q_1, q_2)$ be the probability density function of the second-period idiosyncratic signal conditional on $(s, q_1, q_2)$, and $G(s|q_1, q_2)$ be the conditional cdf of $s$ given $q_1$ and $q_2$. By Bayes' rule,

$$F(s|x, q_1, q_2) = \frac{\int_{-\infty}^{s} h(x|y, q_1, q_2) dG(y|q_1, q_2)}{\int_{-\infty}^{+\infty} h(x|y, q_1, q_2) dG(y|q_1, q_2)}. \tag{25}$$

To prove first-order stochastic dominance, we show that, if $x_2 < \hat{x}_2$, then $\frac{F(s|x_2,q_1,q_2)}{F(s|\hat{x}_2,q_1,q_2)} > 1$ whenever the two cdf's are strictly between 0 and 1[29]. First, note that the ratio converges to 1 as $s \to +\infty$. We obtain

$$\frac{F(s|x_2, q_1, q_2)}{F(s|\hat{x}_2, q_1, q_2)} = \frac{\int_{-\infty}^{s} h(x_2|y, q_1, q_2) dG(y|q_1, q_2)}{\int_{-\infty}^{s} h(\hat{x}_2|y, q_1, q_2) dG(y|q_1, q_2)} \cdot \frac{\int_{-\infty}^{+\infty} h(\hat{x}_2|y, q_1, q_2) dG(y|q_1, q_2)}{\int_{-\infty}^{+\infty} h(x_2|y, q_1, q_2) dG(y|q_1, q_2)}.$$

The second fraction on the right-hand side is independent of $s$. $h(\cdot|s, q_1, q_2)$ is independent of $(q_1, q_2)$ and normally distributed, so that $\frac{h(x_2|y)}{h(\hat{x}_2|y)} > \frac{h(x_2|s)}{h(\hat{x}_2|s)}$ for all $y < s$. We next prove that $W(s) := \frac{\int_{-\infty}^{s} h(x_2|y,q_1,q_2)dG(y|q_1,q_2)}{\int_{-\infty}^{s} h(\hat{x}_2|y,q_1,q_2)dG(y|q_1,q_2)}$ is decreasing in $s$, and strictly so in regions of positive probability. This completes the proof, since we know that $\frac{F(s|x_2,q_1,q_2)}{F(s|\hat{x}_2,q_1,q_2)}$ converges to 1 in the limit. Let

tionship between the expected price of a security and its fundamental expected value ex ante is driven by the concavity or convexity of the payoff as a function of the underlying fundamental. In our case, the payoff of the first-period traders takes the shape of a normal cumulative distribution function, with both a convex and a concave piece that play against each other, so that we cannot establish a definite ranking.

[29]Since $h$ is a normal density (with unbounded support), equation (25) implies that $F(\cdot|x_2, q_1, q_2)$ and $F(\cdot|\hat{x}_2, q_1, q_2)$ are absolutely continuous with respect to each other, for any values $x_2$ and $\hat{x}_2$; hence, the sets on which they are 0 and 1 coincide.

$s_2 > s_1$ such that $G(s_1|q_1, q_2) > 0$,[30] then

$$W(s_2) - W(s_1) = \frac{\int_{y \le s_1} h(x_2|y)dG(y|q_1,q_2) + \int_{s_1}^{s_2} h(x_2|y)dG(y|q_1,q_2)}{\int_{y<s_1} h(\hat{x}_2|y)dG(y|q_1,q_2) + \int_{s_1}^{s_2} h(\hat{x}_2|y)dG(y|q_1,q_2)} - \frac{\int_{y \le s_1} h(x_2|y)dG(y|q_1,q_2)}{\int_{y<s_1} h(\hat{x}_2|y)dG(y|q_1,q_2)} =$$

$$\frac{\int_{s_1}^{s_2} h(x_2|y)dG(y|q_1,q_2) \int_{y \le s_1} h(\hat{x}_2|y)dG(y|q_1,q_2) - \int_{y \le s_1} h(x_2|y)dG(y|q_1,q_2) \int_{s_1}^{s_2} h(\hat{x}_2|y)dG(y|q_1,q_2)}{\int_{y \le s_2} h(\hat{x}_2|y)dG(y|q_1,q_2) \int_{y \le y_1} h(\hat{x}_2|y)dG(y|q_1,q_2)} \le$$

$$\frac{h(\hat{x}_2|y)}{h(x_2|y) \int_{y \le s_2} h(\hat{x}_2|y)dG(y|q_1,q_2) \int_{y \le y_1} h(\hat{x}_2|y)dG(y|q_1,q_2)} \cdot$$

$$\left[\int_{s_1}^{s_2} h(x_2|y)dG(y|q_1,q_2) \int_{y \le s_1} h(x_2|y)dG(y|q_1,q_2) - \int_{s_1}^{s_2} h(x_2|y)dG(y|q_1,q_2) \int_{y \le s_1} h(x_2|y)dG(y|q_1,q_2)\right] = 0,$$

where the inequality is strict if $G$ has positive mass on $(s_1, s_2]$. The posterior beliefs on $s$ of a first-period trader with private signal $x_{i,1}$ are given by $F(s|x_{i,1}, q_1)$. Proving these are increasing in $x_{i,1}$ in the sense of first-order stochastic dominance follows the same steps used above for second-period beliefs.

□

**Proposition 7** (Informational Equivalence of $z$ and $q$ in the case of no recall (Section 3))**.** *Assume that in equilibrium the price $q_1$ is a continuous function of $(s, \epsilon_1)$ and the second-period price $q_2$ is a continuous function of $(s, \epsilon_2)$. Let $\Sigma_1$ be the $\sigma$-algebra generated by the $\pi$-system $\{q \in \mathbb{R} : q_1 \le q\}$ and $\hat{\Sigma}_1$ by $\{z \in \mathbb{R} : z_1 \le z\}$, with $z_1$ as defined in (12). Similarly, let $\Sigma_2$ be the $\sigma$-algebra generated by the $\pi$-system $\{q \in \mathbb{R} : q_2 \le q\}$ and $\hat{\Sigma}_2$ by $\{z \in \mathbb{R} : z_2 \le z\}$, with $z_2$ as defined in (9). Then $\Sigma_1 = \hat{\Sigma}_1$ and $\Sigma_2 = \hat{\Sigma}_2$.*

**Proof of Proposition 7.** First, note that equation (9) follows directly from Proposition 6 and risk neutrality. Second, note that the function $\hat{x}_2(q_2)$ is defined via the indifference condition

$$\theta + (1-\theta)\mathrm{Prob}(s \ge \hat{s}|x_{i,2} = \hat{x}_2, q_2) = q_2. \tag{26}$$

Consider interior prices $q_2 \in (\theta, 1)$. Since conditional repayment probabilities are strictly increasing in the private signal $\hat{x}_2$, it follows that $\hat{x}_2(q_2)$ exists and is unique.[31] Then the market

[30] If $G(s_1|q_1, q_2) = 0$, then $F(s_1|x, q_1, q_2) = 0$ for all $x$.

[31] Existence follows because, when $q_2 \in (\theta, 1)$, the price does not reveal fully whether $s \ge \hat{s}$. Bayes' rule then implies that the left-hand side converges to $\theta$ as $\hat{x}_2 \to -\infty$ and to 1 as $\hat{x}_2 \to \infty$.

clearing condition (9) is a single-valued mapping from the price $q_2$ to the linear combination of shocks $z_2 := s + \epsilon_2/\sqrt{\beta_2} = \hat{x}_2(q_2)$.

Next, we use the property above to prove that corner prices cannot arise with positive probability in equilibria in which the price is continuous in $(s, \epsilon_2)$. Suppose by contradiction that a positive-probability set $H$ can be found for which $q_2$ is equal to $\theta$.[32] Since $H$ has positive probability, we can find two pairs $(s^A, \epsilon_2^A)$ and $(s^B, \epsilon_2^B)$ that correspond to two different values of $z_2$: $z_2^A$ and $z_2^B$. Next, consider the price as a function of $s$ moving along the two lines $s + \epsilon_2/\sqrt{\beta_2} = z_2^A$ and $s + \epsilon_2/\sqrt{\beta_2} = z_2^B$. As $s$ increases along the lines, the price will eventually have to increase, since a price of $\theta$ implies that $H$ must lie below $\hat{s}$ almost surely. Since $q_2$ is continuous, there must be two points $(\tilde{s}^A, \tilde{\epsilon}_2^A)$ and $(\tilde{s}^B, \tilde{\epsilon}_2^B)$ on the two lines where the price is interior and the same. This contradicts what we have proved, since we showed that, whenever the price is interior, $z_2 = \hat{x}_2(q_2)$, with $\hat{x}_2$ being single valued.

Having established that the price is almost surely interior, we return to market clearing and notice that $\hat{x}_2$ is continuous in $(s, \epsilon_2)$. Given that $q_2$ is also continuous in $(s, \epsilon_2)$ by assumption, $\hat{x}_2$ must be a measurable function of $q_2$ and thus it is measurable with respect to $\Sigma_2$ (i.e., $\hat{x}_2$ is known to somebody who knows the realization of $q_2$). This then implies that $z_2$ is also $\Sigma_2$-measurable.

We next prove that $q_2$ is $\hat{\Sigma}_2$-measurable. This proof follows the arguments of Pálvölgyi and Venter [34]. By contradiction, suppose that (on a set of positive measure) there are two vectors $(s^C, \epsilon_2^C) \neq (s^D, \epsilon_2^D)$ that lie on the same straight line indexed by $z_2$ but that correspond to different prices $q^C$ and $q^D$, i.e. such that

$$s^C + \epsilon_2^C/\sqrt{\beta_2} = z_2, \text{ and } q_2(s^C, \epsilon_2^C) = q^C$$
$$s^D + \epsilon_2^D/\sqrt{\beta_2} = z_2, \text{ and } q_2(s^D, \epsilon_2^D) = q^D$$

Since $q_2$ is continuous, the intermediate value theorem ensures that, for any curve that connects $(s^C, \epsilon_2^C)$ to $(s^D, \epsilon_2^D)$, there must be at least one point $(s, \epsilon_2)$ such that $q_2(s, \epsilon_2) = \frac{q^C + q^D}{2}$. First we apply the theorem to the curve represented by the straight line connecting $(s^C, \epsilon_2^C)$ to $(s^D, \epsilon_2^D)$, and denote with $(\hat{s}, \hat{\epsilon}_2)$ the point on such line such that $q_2(\hat{s}, \hat{\epsilon}_2) = (q^C + q^D)/2$. Along this

[32] The same logic applies to the case in which $q_2 = 1$.

line $z_2$ remains constant. Second, we apply the theorem to any other curve which intersects our straight line $z_2$ only at $(s^C, \epsilon_2^C)$ and $(s^D, \epsilon_2^D)$, again such that $(\tilde{s}, \tilde{\epsilon}_2)$ lies on the curve and $q_2(\tilde{s}, \tilde{\epsilon}_2) = (q^C + q^D)/2$. It follows that we have found two different points, $(\hat{s}, \hat{\epsilon}_2)$ and $(\tilde{s}, \tilde{\epsilon}_2)$, that correspond to the same price but are such that $\hat{s} + \hat{\epsilon}_2/\sqrt{\beta_2} \neq \tilde{s} + \tilde{\epsilon}_2/\sqrt{\beta_2}$. This contradicts the necessary market clearing condition (9).

The proof for the first period repeats the same steps as above. □

**Lemma 1.** *Let us denote a general version of the primary-market price as*

$$q_1(z_1) = \theta + (1-\theta)\Phi\left[\frac{\mu_0 - \hat{s}}{S} + K(z_1 - \mu_0)\right],$$

*where* $S := \sqrt{w_S^2 \sigma_{S|B} + \sigma_S^2}$ *and* $K := w_S w_B / S$ *for Section 3, while* $S := \sqrt{w_{2,S}^2 \sigma_{S|B}^2 + \sigma_S^2}$ *and* $K$ *is defined by (23) for Section 4. The partial derivatives of* $q_1(z_1)$ *with respect to* $\beta_2$ *and* $\psi_2$ *respectively are given by*

$$\begin{aligned} \frac{\partial q_1(z_1)}{\partial \beta_2} &= (1-\theta)\phi\left(\frac{\mu_0 - \hat{s}}{S} + K(z_1 - \mu_0)\right)\left[(z_1 - \mu_0)\frac{\partial K}{\partial \beta_2} - \left(\frac{\mu_0 - \hat{s}}{S^2}\right)\frac{\partial S}{\partial \beta_2}\right] \\ \frac{\partial q_1(z_1)}{\partial \psi_2} &= (1-\theta)\phi\left(\frac{\mu_0 - \hat{s}}{S} + K(z_1 - \mu_0)\right)\left[(z_1 - \mu_0)\frac{\partial K}{\partial \psi_2} - \left(\frac{\mu_0 - \hat{s}}{S^2}\right)\frac{\partial S}{\partial \psi_2}\right]. \end{aligned} \tag{27}$$

**Proof of Proposition 1.** Formally the proposition states that

$$\operatorname{sign}\left(\frac{\partial q_1(z_1)}{\partial \beta_2}\right) = \operatorname{sign}(z_1 - \hat{z}_1^\beta)$$

where $\hat{z}_1^\beta \in \mathbb{R}$ depends on all the parameters of the economy. From Lemma 1 and

$$\frac{\partial K}{\partial \beta_2} = \frac{\beta_1(1+\psi_1)(1+\psi_2)\left[2\alpha_0\psi_2 + \beta_2(1 + 3\psi_2 + 2\psi_2^2)\right]}{2\gamma_1\psi_2\sigma_S^{-4}\sqrt{\sigma_S^{-2}\beta_2(1+\psi_2)^2\sigma_{S|B}^2}} > 0$$

it follows that $\operatorname{sign}\left(\frac{\partial q_1(z_1)}{\partial \beta_2}\right) = \operatorname{sign}(z_1 - \hat{z}_1^\beta)$, where

$$\hat{z}_1^\beta = \mu_0 + \left(\frac{\mu_0 - \hat{s}}{S^2}\right)\left(\frac{\partial K}{\partial \beta_2}\right)^{-1}\frac{\partial S}{\partial \beta_2}.$$

□

**Proof of Proposition 2.** Formally the proposition states that

$$\operatorname{sign}\left(\frac{\partial q_1(z_1)}{\partial \psi_2}\right) = \operatorname{sign}(z_1 - \hat{z}_1^{\psi})$$

where $\hat{z}_1^{\psi} \in \mathbb{R}$ depends on all the parameters of the economy. From Lemma 1 and

$$\frac{\partial K}{\partial \psi_2} = \frac{\beta_1\beta_2(1+\psi_1)\left[2\psi_2\sigma_S^{-2} + \beta_2(2+2\psi_2)\right]}{2\psi_2\sqrt{\sigma_S^{-2} + \beta_2^2(1+\psi_2)^2\sigma_{S|B}^2}\Big(\psi_2[\alpha_0\gamma_1 + \beta_2^2(1+\psi_2)^2] + \gamma_1\beta_2(1+\psi_2)(1+2\psi_2)\Big)} > 0$$

it follows that $\operatorname{sign}\left(\frac{\partial q_1(z_1)}{\partial \psi_2}\right) = \operatorname{sign}(z_1 - \hat{z}_1^{\psi})$, where

$$\hat{z}_1^{\psi} = \mu_0 + \left(\frac{\mu_0 - \hat{s}}{S^2}\right)\left(\frac{\partial K}{\partial \psi_2}\right)^{-1}\frac{\partial S}{\partial \psi_2}. \tag{28}$$

□

**Definition 2** (Definition of an equilibrium when the first-period price is known in the second period)**.** A Perfect Bayesian Equilibrium consists of bidding strategies $d(x_{i,1}, q_1)$ and $d(x_{i,2}, q_1, q_2)$ for strategic players, price functions $q_1(s, \epsilon_1)$ and $q_2(s, \epsilon_1, \epsilon_2)$, and posterior beliefs $p_1(x_{i,1}, q_1)$ and $p_2(x_{i,2}, q_1, q_2)$ such that

(i) $d(x_{i,1}, q_1)$ and $d(x_{i,2}, q_1, q_2)$ are optimal given beliefs $p_1(x_{i,1}, q_1)$ and $p_2(x_{i,2}, q_1, q_2)$ respectively;

(ii) $q_1(s, \epsilon_1)$ and $q_2(s, \epsilon_1, \epsilon_2)$ clear the market for all $(s, \epsilon_1, \epsilon_2)$; and

(iii) $p_1(x_{i,1}, q_1)$ and $p_2(x_{i,2}, q_1, q_2)$ satisfy Bayes' Law for all market clearing prices $q_1$ and $q_2$.

**Proof of Proposition 3.** By the same arguments of the proof of Proposition 2, $q_1^A$ and $q_1^B$ satisfy the single-crossing condition and intersect at $\hat{z}_1^{\psi}$, function of all parameters of the Section 4 economy. Then $q_1^A$ crosses $q_1^B$ from below if and only if $\frac{\partial K}{\partial \psi_2} > 0$, which is always true.

We now prove that the same result also holds in the case of Section 5, where the government default threshold is endogenous. First, consider any of the (potentially multiple) intersections between $q_1^A$ and $q_1^B$, where $\psi_2^A > \psi_2^B$, and let us denote them with $(\hat{z}_1^{\psi}, \hat{q}_1^{\psi})$. The slope of the price function $q_1(z_1)$ at any of such points is given by

$$\left.\frac{\partial q_1(z_1)}{\partial z_1}\right|_{(\hat{z}_1^{\psi}, \hat{q}_1^{\psi})} = \frac{(1-\theta)\phi\left(\frac{\mu_0 - \hat{s}(\hat{q}_1^{\psi})}{S} + K(\hat{z}_1^{\psi} - \mu_0)\right)K}{1 + (1-\theta)\phi\left(\frac{\mu_0 - \hat{s}(\hat{q}_1^{\psi})}{S} + K(\hat{z}_1^{\psi} - \mu_0)\right)\frac{\hat{s}'(\hat{q}_1^{\psi})}{S}}.$$

Since $\hat{s}'(q) < 0$ and $K$ and $S$ are respectively increasing and decreasing in $\psi_2$, we can conclude that at all intersections $q_1^A$ crosses $q_1^B$ from below. This implies that (i) there can only exist one crossing point $(\hat{z}_1^\psi, \hat{q}_1^\psi)$, and (ii) the direction of the crossing is indeed as described in Proposition 3.

To explicitly characterize $\hat{z}_1^\psi$, let us rearrange equation (24) to get $S\Phi^{-1}\left(\frac{q_1-\theta}{1-\theta}\right) = \mu_0 - \hat{s}(q_1) + KS(z_1 - \mu_0)$: evaluated at $(\hat{z}_1^\psi, \hat{q}_1^\psi)$, this must hold when $\psi_2$ is equal to either $\psi_2^A$ or $\psi_2^B$. Subtracting and rearranging we can characterize the crossing further:

$$\hat{q}_1^\psi = \theta + (1-\theta)\Phi\left[\frac{K_A S_A - K_B S_B}{S_A - S_B}(\hat{z}_1^\psi - \mu_0)\right]. \tag{29}$$

where $K_A, S_A$ correspond to the case where $\psi_2 = \psi_2^A$, while $K_B, S_B$ correspond to the case where $\psi_2 = \psi_2^B$. It is then possible to plug equation (29) into (24) and solve for the value of $\hat{z}_1^\psi$.

□

**Proof of Proposition 4.** By the same arguments of the proof of Proposition 1, $q_1^A$ and $q_1^B$ satisfy the single-crossing condition and intersect at $\hat{z}_1^\beta$, function of all parameters of the Section 4 economy. Then $q_1^A$ crosses $q_1^B$ from below if and only if $K(\beta_2^A)$, the coefficient of $z_1$ evaluated at $\beta_2^A$, is larger than $K(\beta_2^B)$.

Note that condition $\frac{\partial K}{\partial \beta_2} > 0$ is equivalent to

$$\beta_2(1+\psi_2)(1+\psi_1+2\psi_2) - \beta_1\psi_1(1+\psi_1) + \alpha_0(\psi_1 - 2\psi_2) > 0 \tag{30}$$

which is linear and increasing in $\beta_2$, and equals zero at $\hat{\beta}_2 = \frac{\beta_1\psi_1(1+\psi_1)+\alpha_0(\psi_1-2\psi_2)}{(1+\psi_2)(1+\psi_1+2\psi_2)}$. The left panel of Figure 4 is an example of $\hat{\beta}_2 \le 0$, in which case $K(\beta_2^A) > K(\beta_2^B)$ for all $0 < \beta_2^B < \beta_2^A$. The right panel instead represents the scenario where $\hat{\beta}_2 > 0$ and $K(\beta_2)$ is not monotone increasing. To prove the Proposition it is sufficient to show that $K(\beta_2 = \beta_1) > K(\beta_2 \to 0)$, which is equivalent to

$$\frac{\beta_1[\gamma_1 + \beta_1(1+\psi_1)(\psi_1+\psi_2)]}{\sqrt{\gamma_1(\gamma_1+\beta_1\psi_2)\{\alpha_0\psi_2 + \beta_1[1+(3+\psi_1)\psi_2+\psi_2^2]\}/\psi_2}} > \frac{\beta_1\psi_1}{\sqrt{\alpha_0+\beta_1\psi_1}}.$$

When $\psi_2 \ge \psi_1$, this is always satisfied and concludes the proof. □

**Proof of Proposition 5.** Examine the argument of the cumulative distribution function on the right-hand side of (24). The second term is linear in $z_1$ with coefficient $K$, while the first term is a bounded function of $z_1$ since $\hat{s}(q_1) \in (\hat{s}(1), \hat{s}(\theta))$. It follows that when $z_1$ is sufficiently large in absolute value, the response of $q_1$ to changes in information precision will be driven solely by $\frac{\partial K}{\partial \beta_2}$ as defined in (23) and characterized in (30). $\square$

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