Macroeconomic Effects of Delayed Capital Liquidation*

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July 31, 2017

Abstract

This paper studies macroeconomic effects of financial shocks through the lens of delayed capital liquidation and reallocation among firms. I develop a model in which firms face idiosyncratic productivity risks, financial constraints, and random liquidation costs. Liquidation costs generate an option value of staying and a liquidation delay for unproductive firms. A novel feature arising from the delay is that unproductive firms have a higher debt-to-asset ratio than productive ones. I find that adverse financial shocks that tighten financial constraints can raise the option value. The financial shocks also have general equilibrium effects that further raise the option value and delay capital liquidation. Capital is thus persistently misallocated, leading to a long-lasting economic contraction. Using U.S. data from 1971-2015, I show that financial shocks can explain almost 67% of the variation in the capital liquidation-to-expenditures ratio and 72% of the variation in output.

Keywords: financial constraints and financial shocks; capital liquidation; option value of staying

Classification: E22; E32; E44; G11.

1 Introduction

The importance of financial shocks, i.e., perturbations that originate directly in the financial sector and that affect directly firms' debt, has recently gained increasing interest in the literature of business cycles. The total debt of U.S. non-financial firms was about 74% of total output from 1971 to 2015. Adverse financial shocks push firms to borrow less and cut

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*This paper is a significant revision of the earlier paper titled "Delayed Capital Reallocation". I am indebted to my advisors Markus Brunnermeier and Nobu Kiyotaki. Special thanks go to Ben Moll and Thomas Sargent who are in my thesis committee. I am also very grateful for the insightful discussions with Mark Aguiar, Marco Bassetto, Mariacristina De Nardi, Andrea Eisfeldt, Jonas Fisher, Mike Golosov, Francois Gourio, Antonio Guarino, Christian Hellwig, Ji Huang, Jakub Jurek, Leo Kaas, Adriano Rampini, Morten Ravn, Richard Rogerson, Esteban Rossi-Hansberg, Hyun Shin, Chris Sims, Gabriel Tenorio, and Mark Wright, as well as seminar participants at various institutes. All remaining errors are my own.

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investments, employment, and dividends (e.g., Jermann and Quadrini (2012) and Del Negro, Eggertsson, Ferrero, and Kiyotaki (2017)). This process reduces firms' net worth, which further tightens financial constraints and amplifies economic contractions.

This paper contributes to the literature by studying financial shocks through the liquidation and reallocation of capital among firms. Note that productivity differences across firms within narrow industries are well documented;[1] in general, the reallocation improves the productivity of capital.[2] Financial shocks may thus have a sizable impact on capital reallocation and hence on aggregate productivity. The main purpose of this paper is then to study together financial flow, capital liquidation, and output in the aggregate.

The U.S. market for reallocating firms' liquidated capital, including both full liquidation (i.e., acquisition) and partial liquidation (i.e., sales of properties, plants, and equipments), is sizable. In 2015, capital liquidation of publicly listed firms was $0.8 trillion, about 30% of total capital expenditures (new investment plus reallocation). I show that capital liquidation in recessions falls more than proportionally compared to capital expenditures - a procyclical liquidation-to-expenditures ratio. That is, both capital liquidation and capital expenditures are procyclical, but the degree of procyclicality (or volatility) of liquidation is higher.

To study the business cycle implications for capital liquidation and other macro aggregates, I propose an "option value of staying" theory of costly liquidation,[3] in which there is a rich interaction between capital liquidation and external financial conditions. This micro-level interaction implies that low productivity firms exhibit a higher debt-to-asset ratio than their productive pairs, supported by the empirical findings of İmrohoroğlu and Tüzel (2014). I find that negative financial shocks can delay the timing of when a firm is liquidated; this is crucial for procyclical liquidation-to-expenditures ratio and amplifies output dynamics.

As a first step, I present a simple firm model and illustrate the impact of financial constraints on the option value of staying. The model features a dividend-smoothing manager/owner, whose firm is subject to idiosyncratic productivity shocks and a borrowing constraint. The manager faces *random* liquidation costs if he/she chooses to liquidate the entire firm. When the firm is productive, the manager borrows (possibly hitting the borrowing limit) and invests in his firm technology. When the firm is unproductive, the manager decides whether to liquidate the firm or not.

Naturally, for a small enough liquidation cost, the manager is willing to sell his firm; for a large enough cost, he is not. There is thus an option value of staying, a value of waiting for the return of high productivity. If his firm is not liquidated, the firm has fewer resources because of the low productivity, and the manager needs to borrow to finance dividends.

[1] See, e.g., Hsieh and Klenow (2009) and İmrohoroğlu and Tüzel (2014).
[2] See, e.g., Maksimovic and Phillips (2001, 2002).
[3] For empirical evidence of costly liquidation, see, e.g., Ramey and Shapiro (2001).

When the productivity is low enough, the firm may become borrowing constrained.

For a given liquidation cost, an adverse financial shock that tightens the financial constraint has two effects. On the one hand, if the manager has a low productivity firm and is constrained in financing dividends, he has to cut dividends further. For a given liquidation cost, he is more willing to liquidate the firm. On the other hand, a tighter financial constraint - reducing borrowing today - will also reduce interest payments in future periods. The reduction of interest payments raises the attractiveness of the possible return of high productivity in the future, i.e., it increases the option value today.

As long as the firm's productivity is not too low, the second effect dominates. This means that a tightened financial constraint can increase the willingness to stay for a given liquidation cost. The *threshold liquidation cost*, above which the manager decides to stay, thus falls after the shock. In other words, it is more likely that the manager delays the liquidation. That is how an adverse financial shock can delay the liquidation decision.

I then embed the simple firm problem into a general-equilibrium framework, in order to study the dynamics of aggregate capital liquidation, total factor productivity (TFP), and aggregate output. The model features a continuum of firms which differ in their productivity and leverage. Each firm is similar to the one in the simple model. Besides full liquidation (still subject to liquidation costs), firms can now partially sell their capital (i.e., partial liquidation).

I calibrate and estimate the macro model in order to quantitatively examine the macro effects of financial shocks and exogenous aggregate productivity shocks. Note that financial shocks directly affect borrowing constraints and thus credit demand. To capture the effect of credit supply, the macro model has a representative household who consumes, saves, and works; the estimation then allows for shocks to the household intertemporal substitutions that affect their savings and thus credit supply.

As in the simple model, adverse financial shocks raise the option value of staying and delay full capital liquidation. I further find a general equilibrium effect from the shocks that raises the option value. Because productive firms cut capital expenditures and employment because of the tighter borrowing constraints, aggregate productivity and factor demand are lower. As a result, both the interest rate and the wage rate fall. Such general equilibrium effect generates a further delay on full liquidation. This is because both the borrowing cost and labor cost are lower for unproductive firms. The threshold liquidation cost thus falls further, and even more unproductive firms delay liquidation.

In contrast, adverse *exogenous* productivity shocks generate a "cleansing" effect, as lower aggregate productivity reduces the option value of staying. More managers with unproductive firms thus find it profitable to liquidate their firms in recessions. This result, together with the impact of financial shocks, can be used to back out underlying shocks that generate the data through the model. I find that financial shocks can significantly contribute to the observed procyclical debt flow and the procyclical capital liquidation-to-expenditures ratio. Overall, the financial shocks can explain around 72% of the variation in output, leaving 25% to aggregate productivity shocks and the rest to intertemporal shocks.

In sum, my contribution is to show that financial constraints interact with resale frictions. Notice that whether adverse financial shocks can generate the fall of capital liquidation in recessions is not obvious. Whether the shocks can produce a procyclical capital liquidation-to-expenditures ratio is even less clear. After all, if external financial conditions become worse in recessions, less productive firms always have an option to liquidate and reallocate their firm capital. Nevertheless, after adverse financial shocks, the option value of staying might well compensate for today's suffering. Some unproductive firms with low liquidation costs, which should have been liquidated without the adverse financial shocks, are thus kept running after the shocks.

As mentioned before, there is a unique feature arising from the interaction between financial constraints and resale frictions: compared to high productivity firms, low productivity firms exhibit a higher debt-to-asset ratio. The reason is that low productivity implies low profits for those firms and they have to exhaust the borrowing limit to finance dividend payments. We thus see a high level of debt relative to the level of their assets.

Why does one need the presence of resale frictions to generate a higher debt-to-asset ratio for unproductive firms? This is because if total liquidation is costless, there is no concern of option value. In that case, a firm borrows when it is productive; the firm will be liquidated by its manager (with the firm debt repaid) once it turns unproductive. As a result, if there are no resale frictions, we cannot see a higher debt-to-asset ratio when a firm becomes unproductive. Though the model shares similarities with financial frictions in Kiyotaki and Moore (1997), Bernanke, Gertler, and Gilchrist (1999), Mendoza (2010), Jermann and Quadrini (2012), and Brunnermeier and Sannikov (2014), the interaction between resale frictions and financial frictions generate more realistic leverage patterns as found by İmrohoroğlu and Tüzel (2014).

The technical aspect developed in this paper might be of some independent interest. I extend previous tractable firm problems with portfolio choices such as in Angeletos (2007), Moll (2014), and Liu and Wang (2014). I add random liquidation costs in a specific way, which leads to closed-form solutions of when a firm should exercise the liquidation option. The aggregation thus becomes straightforward. For general equilibrium responses to shocks, I can use standard perturbation methods to handle the option value with portfolio choices.[4]

The literature on macro effects of capital reallocation started around the seminal con-

[4] Cui and Kaas (2017) also apply this technique to firm default problems in general equilibrium.

tribution by Ramey and Shapiro (1998) and Eisfeldt and Rampini (2006).[5] In particular, Eisfeldt and Rampini (2006) show that aggregate capital reallocation is procyclical, while the potential benefits to reallocation (measured by the dispersion of Tobin's Q) are countercyclical.[6] Eisfeldt and Rampini (2008) and Fuchs, Green, and Papanikolaou (2016) use information frictions to endogenize liquidity and study procyclical reallocation. There is another line of research that uses search-and-matching to endogenize liquidity and generate procyclical reallocation, including Cao and Shi (2016) and Cui and Radde (2016).

The option value of staying is related to the option value of waiting to invest studied by Dixit and Pindyck (1994) and Bloom (2009). My contribution is to show how financial constraints directly affect the option value and liquidation strategies. In this respect, the paper is related to Caggese (2007), where financially constrained firms respond to idiosyncratic shocks by adjusting variable investment first and fixed capital later.[7]

This paper complements the previous work on capital reallocation. I document the procyclical capital liquidation-to-expenditures ratio. I also highlight that in recessions firms are less willing to use full liquidation when reallocating capital. That is why I propose the theory of option value of staying to directly link full liquidation and external financial frictions. Distinguishing full and partial liquidation follows the "bundled selling and disassembled selling" in Jovanovic and Rousseau (2002).

Additionally, compared to previous work, I use the general equilibrium model to simultaneously match real and financial aggregate variables in the data, by estimating the contribution of financial shocks and productivity shocks. I find that financial shocks are quantitatively important. The support for financial shocks can be found in business-cycle studies including Jermann and Quadrini (2012), Christiano, Motto, and Rostagno (2014), Ajello (2016), Del Negro, Eggertsson, Ferrero, and Kiyotaki (2017), and Cui and Kaas (2017), though capital liquidation is not the focus in these studies. Financial development across countries can also have a sizable impact on capital allocation across firms, aggregate productivity, and output (e.g., Buera, Kaboski, and Shin (2011) and Moll (2014)).

[5] Eberly and Wang (2009) look at the relationship between reallocation and growth in a two sector growth model. The interesting aspect is that it may not be optimal to reallocate capital from less productive to more productive firms during productivity transition periods because of the adjustment costs of capital.

[6] Hsieh and Klenow (2009) show that the gains after reallocating capital given empirically estimated level of misallocation is significant, implying the importance of research on capital reallocation. They find that removing the distortion can increase 30%-50% TFP in China and 40%-60% TFP in India. Lee (2016) uses advance identification strategies to show that a separate "reallocation" shock, which correlates well with investment-specific technology shocks, can explain more than 50% of the variation in capital reallocation.

[7] Lanteri (2014) focuses on the price of secondary market capital and the effect on the option value, by specifying an exogenous function that determines the substitutability between new and old capital.

2 Financial Cycles and Capital Liquidation Cycles

In this section, I briefly present three stylized facts about debt, capital liquidation, and output in the aggregate 1971-2015. The details about the data are in Appendix A.

The data of aggregate debt is from the U.S. Flow of Funds Accounts. Debt includes only liabilities that are directly related to credit market transactions or banking loans. I normalize the data in 2009 dollars by using the GDP implicit price deflator.

The data of aggregate capital liquidation or reallocation is obtained from the COMPUSTAT dataset (following Eisfeldt and Rampini (2006)). Aggregate capital liquidation, or aggregate capital reallocation, measures the aggregate changes of firm-level ownership of capital. It has two components. One is labeled as *full liquidation*, i.e., acquisition; the other is *partial liquidation*, i.e., sales of property, plants, and equipments (SPP&E). I also obtain capital expenditures from each firm. I aggregate the firm-level data and reach total full liquidation, total partial liquidation, and total capital expenditures for each year. Aggregate capital liquidation is thus the sum of the total full liquidation and total partial liquidation.

I use the liquidation-to-expenditures (L-E) ratio, i.e., aggregate capital liquidation divided by total capital expenditures, to analyze capital reallocation. The L-E ratio is about 0.298 in 2015.[8] The 1971-2015 sample mean is 0.219.

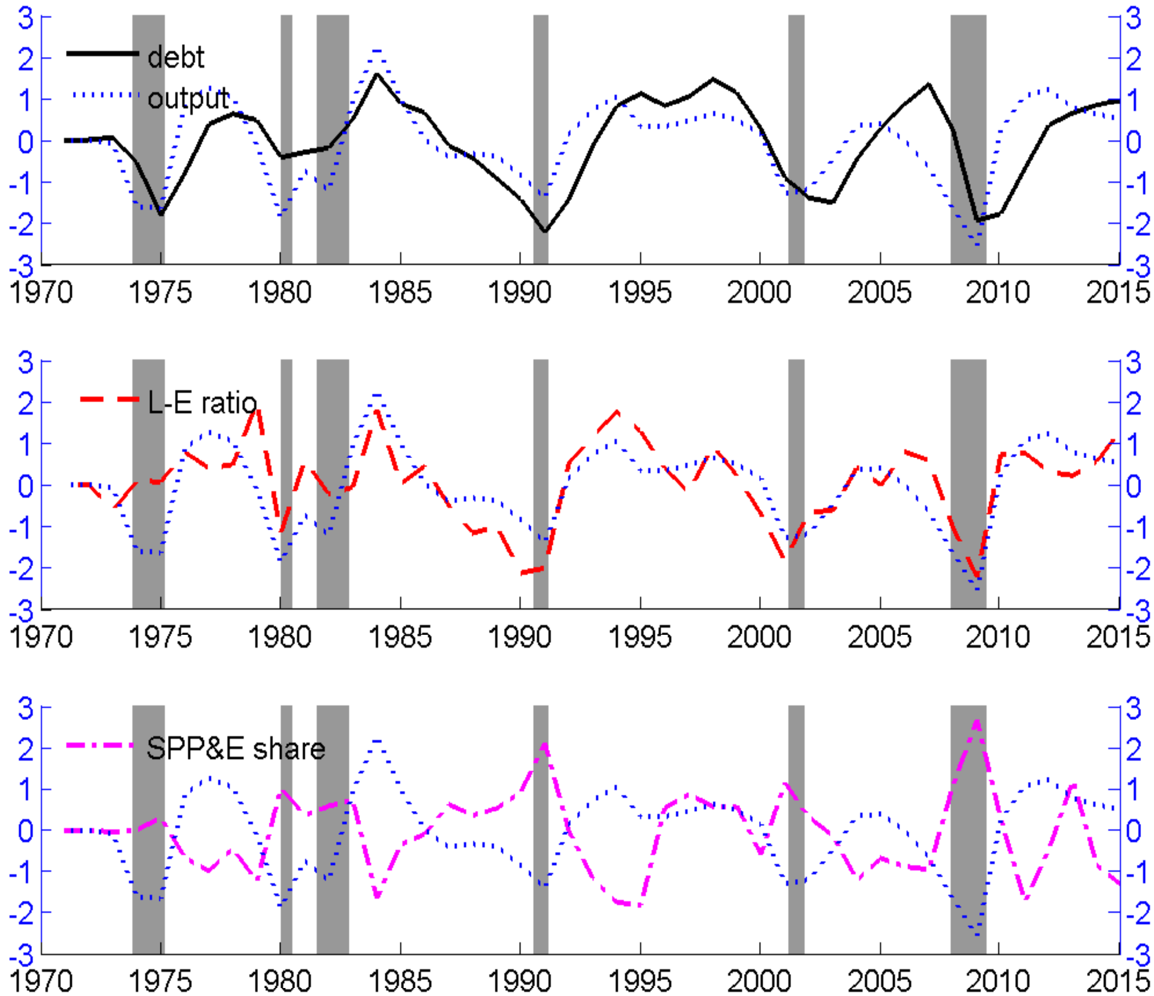
The data of corresponding aggregate output is *not* simply GDP in the National Income and Product Accounts (NIPA). Since capital liquidation does not include reallocating land, residential structure, and consumer durables, the capital corresponds to non-residential structures, plants, and equipments, while the investment corresponds to the non-residential investment. Output is then defined as the sum of the consumption and the investment. Consumption and investment in 2009 dollars are both obtained from NIPA. Using the output and the debt series, I find that the debt-to-output ratio is 82.4% in 2015. The 1971-2015 sample mean is 74% (as reported in the introduction).

The top two panels of Figure 1 show the cyclical components (one-sided HP filtered with a smoothing parameter 6.25) of debt, output, and the liquidation-to-expenditures ratio. Two stylized facts are clearly visible. First, debt falls in recessions and rises in booms: recessions push firms to restructure their financial positions by cutting debt. Second, less capital spending in recessions is in the form of used capital from liquidation, since the L-E ratio is procyclical. As it is well known that spending in new capital falls in recessions, we know that the spending cut on used capital is deeper than the cut in new capital.

What contributes to the falls of the liquidation-to-expenditures ratio in recessions? The

[8] In contrast to Eisfeldt and Rampini (2006) who use the level of capital reallocation, this measure tells us more about the importance of old capital relative to new capital, and could potentially attenuate the effect of variation in capital prices. Starting from 1984, this measure fluctuates around 0.30.

Figure 1: **Financial and Reallocation Cycles.** The series plotted are cyclical components of debt, the L-E ratio (aggregate capital liquidation divided by aggregate capital expenditures), the SPP&E share (total sales of properties, plants, and equipments divided by aggregate capital liquidation), and output, all normalized by their own standard deviations. The solid line: debt. The dashed line: L-E ratio. The dashed dotted line: SPP&E share. The three dotted lines: output. Shaded regions denote NBER recessions.



bottom panel of Figure 1 shows the cyclical components (with the same filtering) of the partial liquidation share in aggregate capital liquidation (i.e., the SPP&E share). The SP-P&E share is countercyclical. That is, in recessions, less reallocation is in the form of full liquidation than in booms.

Table 1 further confirms the three stylized facts. Debt and the liquidation-to-expenditures ratio are both procyclical, while the SPP&E share is countercyclical. Though debt is already 55% more volatile than output, the volatility of the L-E ratio and the SPP&E share are 5.59 and 7.52 times that of output, respectively. That is, the observed reallocation process is

Table 1: **Summary Statistics**

	Debt	L-E ratio	SPP&E share	Output
Debt	1	0.58	-0.46	0.60
L-E ratio	-	1	-0.79	0.68
SPP&E share	-	-	1	-0.64
Output	-	-	-	1
Relative Std	1.55	5.59	7.52	1

quite volatile and sensitive to business cycle conditions.

Reallocating capital is costly. But, in general, the reallocation process moves capital from less productive firms to more productive ones, documented for example in Maksimovic and Phillips (2001, 2002). The observed financial and reallocation cycles thus motivate me to study the impact of external financial conditions on capital liquidation, aggregate productivity, and output.

In the following, I start with a simple partial-equilibrium model, and later augment it into a general-equilibrium model. The general-equilibrium model is suitable for estimating potential shocks in order to replicate the stylized facts in Figure 1.

3 A Model of Financially-constrained Option Value

In this section, I present a simple entrepreneur's problem to illustrate how financial constraints can interact with liquidation decisions. Liquidation will become important for aggregate capital reallocation discussed in the macro model in Section 5.

3.1 An Entrepreneur's Problem

Time is discrete and infinite. Consider an entrepreneur who can save in risk-free bonds, borrow, and/or run a firm with a linear production technology in capital. The average return from production and the risk-free rate are constants and are exogenously given. On top of the average return, the entrepreneur faces productivity risks.

Preferences and Technology

The entrepreneur has a per-period utility from consumption (or the level of dividends) c represented by

$$u(c) = \log(c)$$

It may at first seem that the entrepreneur owns the firm, but the entrepreneur can also be thought of as the manager of a firm. Then, the curvature in $u(.)$ captures the preference of the manager for dividend smoothing. Lintner (1956) first showed that managers take into account dividend smoothing over time, a fact further confirmed by subsequent studies. Alternatively, putting curvature in $u(.)$ is a simple way of modeling the speed with which firms can vary the funding source when the financial condition changes.[9]

If the entrepreneur runs a firm with capital k at the beginning of t, his firm has a technology to produce $z\pi k$ units of consumption goods as profits. The productivity takes two values, i.e., $z \in \{z_h, z_l\}$ with $z_h > z_l > 0$, and z follows a Markov process with transition probabilities:

$$\text{Prob}(z_{it} = z_l \mid z_{it-1} = z_h) = p^{hl} \text{ and } \text{Prob}(z_{it} = z_h \mid z_{it} = z_l) = p^{lh}$$

where $0 < p^{hl} < 1$, $0 < p^{lh} < 1$, and $p^{hl} + p^{lh} < 1$. When the entrepreneur did not have a firm, he draws the higher productivity z_h also with a probability p^{lh}.[10]

Capital depreciates at a rate δ after production, and entrepreneurs can invest in new capital stock, buy, or sell existing capital stock. Capital stock thus evolves according to

$$k_{t+1} = (1 - \delta)k_t + i_t$$

where $i_t \geq -(1-\delta)k_t$ is investment. When $i_t < 0$, the entrepreneur sells capital. Importantly, adjusting capital is subject to resale frictions and financial frictions.

Resale frictions- Suppose an entrepreneur already owned a firm. When the entrepreneur liquidates capital (this may happen when his firm is not productive), i.e., $i_t < 0$, he has to sell the whole firm (note: this assumption will be relaxed in Section 5). At the same time, he incurs i.i.d. *stochastic liquidation cost* ζ with a cumulative distribution function $F(.)$. The time-varying cost ζ sometimes drives the entrepreneur to liquidate, and other times it forces them to stay in business.

For simplicity, ζ is modeled as a utility cost. But such simplification is not crucial. One can alternatively assume that ζ represents consumption goods. As long as it is proportional to the entrepreneur's net worth, we have identical analytical results.[11]

One interpretation of liquidation costs could be that the entrepreneur needs to spend time and resources in searching for and negotiating with potential acquirers, which costs

[9] Jermann and Quadrini (2012) also put a cost of adjusting dividend payout that controls the speed, supported by empirical evidence cited therein.

[10] This assumption simplifies the aggregation later on. One can assume a different probability, but this leads to a more complicated derivation.

[11] See Appendix B for the related discussion in the proof to Proposition 1.

him utility represented by ζ. Liquidation costs can arise for various reasons, such as search frictions (see, e.g., Cao and Shi (2016) and Cui and Radde (2016)) and asymmetric information (see, e.g., Eisfeldt and Rampini (2008) and Kurlat (2013)). Since the main goal is to examine the impact of financial shocks on the option value of staying, I abstract from the microfoundation of liquidation costs.

The entrepreneur's preference can now be represented by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \{\log(c_t) - \mathbf{1}_{\{\text{liquidating}\}}\zeta\}, \quad 0 < \beta < 1 \tag{1}$$

where $\mathbb{E}$ is a mathematical expectation operator, β is a subjective discount factor, and $\mathbf{1}$ is an indicator function. Readers will soon find out that such a modelling strategy admits closed-form solutions, even when both option values (choosing staying versus liquidation) and portfolio choices (choosing capital and bonds) are present, as in this paper.

Financial frictions- There is no insurance market for productivity risks. The only asset market is the credit market. The entrepreneur can save and borrow at a gross interest rate R; but if he decides to borrow, debt must be collateralized due to limited commitment issues. That is, the entrepreneur's external borrowing is bounded, because they face collateral constraints similar to those in Kiyotaki and Moore (1997) and Hart and Moore (1994). Assuming that in a case of default, the lender should be able to seize a $\theta \in [0,1]$ fraction of the residual capital after depreciation and reallocation. To ensure no "run away" default, the entrepreneur can thus borrow up to θ fraction of the residual capital at time $t+1$. Let b_t be the level of bonds held by the entrepreneur at the beginning of t. Then, the collateral constraint can be written as

$$Rb_{t+1} \geq -\theta(1-\delta)k_{t+1}$$

The Entrepreneur's Problem

Following the convention, I omit subscript t and use x' to denote x_{t+1}. Let v be the optimal value of the entrepreneur with state (k, b, z, ζ) at the beginning of t. I can now state the recursive problem as

$$v(k,b,z,\zeta) = \max_{\text{liquidate/stay}} \{v^0(k,b,z,\zeta), \quad v^1(k,b,z)\} \tag{2}$$

where v^0 and v^1 satisfy

$$v^0(k,b,z,\zeta) = \max_{c,b'} \{\log(c) - \zeta + \beta\mathbb{E}\left[v(0,b',z',\zeta')|z\right]\} \text{ s.t.}$$

$$c + b' = z\pi k + (1-\delta)k + Rb \tag{3}$$

$$v^1(k,b,z) = \max_{c,k',b'} \{\log(c) + \beta\mathbb{E}[v(k',b',z',\zeta')|z]\} \text{ s.t.}$$

$$c + k' - (1-\delta)k + b' = z\pi k + Rb \tag{4}$$

$$Rb' \geq -\theta(1-\delta)k' \tag{5}$$

$$k' - (1-\delta)k \geq 0 \tag{6}$$

Notice that when the entrepreneur did not have a firm (i.e., $k=0$), v^0 and v^1 correspond to values of continuing without a firm and starting running a new firm.

(3) and (4) are resource constraints. The entrepreneur uses profits $z\pi k$ and returns from bonds Rb to finance dividend payout c, investment $k' - (1-\delta)k$, and new bonds b'. When he decides to liquidate, $k' = 0$, and that is the difference between (3) and (4).

The entrepreneur's problem consists of two actions. Firstly, to liquidate the firm, to incur the liquidation cost, and to choose an optimal saving (in bonds), which yields v^0; in the next period, $k' = 0$ and the liquidation-cost draw ζ' will not matter. Secondly, to stay in business and to adjust capital, which yields v^1. When he chooses v^1, he also needs to choose the optimal consumption and a portfolio of capital stock and bonds. In so doing, the entrepreneur needs to respect both the financial constraint (5) and the resale constraint (6).

3.2 Characterizing the Problem

From here, I characterize the entrepreneur's decision rule. A key concept emerges: the option value of staying in business. Further, financial constraints can interact with the option value.

Before going into detail, I briefly explain why the resale constraint (6) may interact with the financial constraint (5). Suppose that the entrepreneur keeps the firm (i.e., he chooses v^1) but does not invest. This scenario can happen when the entrepreneur draws both low productivity z_l and a high liquidation cost ζ. That is, the entrepreneur wants to sell the existing unproductive firm. However, he keeps the firm if the liquidation cost is too large, and it is worthwhile to wait for the possibility of regaining productivity z_h tomorrow.

Then, the resale constraint $k' = (1-\delta)k$ binds and the resource constraint becomes

$$c + b' = z\pi k + Rb$$

where "cash on hand" $z\pi k + Rb$ is used to finance the dividend payout and savings in bonds. He can also borrow if that helps smooth dividends; but if he borrows more, the firm is closer to the borrowing limit imposed by (5). When the productivity $z = z_l$ is so low that b' has to reach the bound $-\theta(1-\delta)^2 k/R$, the entrepreneur is not able to raise more dividends.

One can view that waiting for a high productivity z_h tomorrow generates an option value, *the option value of staying*. Through affecting the option value, the tightness of financial constraint determines when (i.e., with what ζ) the entrepreneur should liquidate if the firm is unproductive.

The question is how to evaluate the option value. I first show what happens if the entrepreneur stays. Then, I characterize the option value of staying.

The Entrepreneur's Closed-form Policy Functions

The value function (2) has a form that is useful for solving the entrepreneur's decision.

Lemma 1:
$\forall \rho > 0$, *the value function satisfies*

$$v(\rho k, \rho b, z, \zeta) = v(k, b, z, \zeta) + \frac{\log \rho}{1-\beta} \tag{7}$$

Proof. See Appendix B. □

This Lemma implies a "scale-invariant" property. If the entrepreneur has a firm that is twice as large as the previous one (keeping the same productivity), the value of the firm in his view increases by a constant term. Notice that when a firm is scaled up, the *leverage ratio* remains. The leverage ratio is defined as capital stock over equity

$$\lambda \equiv \frac{k}{k+b}$$

Using the leverage ratio, I can derive the entrepreneur's decision rule in closed forms.

Proposition 1:
There exists a shadow value of capital $q = q(\lambda, z, \zeta)$ such that the entrepreneur's net worth n can be expressed as

$$n = n(k, b, z, \zeta) \equiv z\pi k + (1-\delta) q(\lambda, z, \zeta) k + Rb$$

The consumption function $c = c(k, b, z, \zeta)$, capital function $k' = k'(k, b, z, \zeta)$, and bond function $b' = b'(k, b, z, \zeta)$ can be expressed as

$$c = (1-\beta)n, \quad k' = \frac{\lambda'}{(q-1)\lambda' + 1}\beta n, \quad \textit{and} \quad b' = \frac{1-\lambda'}{(q-1)\lambda' + 1}\beta n$$

$\lambda' = \lambda'(\lambda, z, \zeta)$ and $q = q(\lambda, z, \zeta)$ are jointly determined. If the entrepreneur liquidates the

firm, $\lambda' = 0$ *by definition. Otherwise,* $\lambda' \in \left(0, \frac{1}{1-\theta(1-\delta)/R}\right]$ *satisfies*

$$\mathbb{E}_z \left[\frac{\frac{z'\pi+(1-\delta)q'}{q} - R}{\frac{q\lambda'}{(q-1)\lambda'+1}\left[\frac{z'\pi+(1-\delta)q'}{q} - R\right] + R} \right] \leq 0 \tag{8}$$

where "<" holds when the financial constraint (5) *binds. When the entrepreneur invests or liquidates the existing firm, or when the entrepreneur did not have a firm and continues without a firm, the resale constraint does not bind (i.e.,* $q = 1$*) . Otherwise, he has a firm and chooses to stay with a binding resale constraint with* q *solving*

$$\beta\left[z\pi + (1-\delta)q + R(\lambda^{-1} - 1)\right] = \left[q + 1/\lambda' - 1\right](1-\delta). \tag{9}$$

Proof. See Appendix B. □

That is, the entrepreneur consumes the $(1-\beta)$ fraction and saves the other β fraction of the net worth. The net worth is "economic net worth" that evaluates capital at a shadow price q related to resale constraint. If the resale constraint is not binding, $q = 1$. Otherwise, $q < 1$; the net worth is lower if q is lower, reflecting the tightness of resale constraint.

Notice that (8) implies a portfolio choice. Running a firm with a particular capital structure is similar to an investment in a portfolio. The portfolio consists of a $\tilde{\lambda}' = \frac{q\lambda'}{(q-1)\lambda'+1}$ fraction of risky capital stock and a $1 - \tilde{\lambda}'$ fraction of risk-free bonds, allowing short-selling of bonds to a limit (implied by the financial constraint) but not risky ones. The portfolio weight maximizes the expected log rate of return

$$\mathbb{E}_z \left[\log \left(\tilde{\lambda}' \left(\frac{z'\pi + (1-\delta)q'}{q} \right) + (1-\tilde{\lambda}')R' \right) \right]$$

which gives rise to (8), where $\frac{z'\pi'+(1-\delta)q'}{q}$ is the rate of return on capital and R' is the rate return on bonds.[12] Notice that when $q = 1$, $\tilde{\lambda}' = \lambda'$. One can thus interpret $\tilde{\lambda}'$ as the resale-constraint adjusted leverage ratio.

Next, I will show when the entrepreneur should exercise the liquidation option.

Liquidation and the Option Value of Staying

Depending on the profit rate π and the real interest rate R, two extreme cases might occur. One case is that the entrepreneur never liquidates. The other is that he always liquidates if he has a firm. Instead of looking at these two extreme scenarios, I focus on the interesting

[12] See, e.g., Campbell and Viceira (2002) for more detail discussion of portfolio choice.

equilibrium that features two properties.

First, z_h is large enough that the entrepreneur always invests when he draws z_h. Otherwise, eventually he will stop running a firm forever. Second, when the entrepreneur draws z_l, he liquidates the firm only if the liquidation cost is small enough, i.e., the entrepreneur follows a liquidation threshold strategy.

I thus restrict the parameters by $z_l\pi + 1 - \delta < R < z_h\pi + 1 - \delta$, or

$$z_l < \frac{R - (1-\delta)}{\pi} < z_h \tag{A1}$$

The assumption implies that the return from production is higher than the return from bonds when z_h is drawn, but lower when z_l is drawn. As a result, when the entrepreneur draws z_h, he always borrows and invests in his firm's technology. When the entrepreneur draws z_l, it is better to save in risk-free bonds, but the liquidation costs could prevent him from doing so.

Additionally, the gross interest rate should not be too large; otherwise, the entrepreneur may just save in risk-free bonds and entirely avoid the productivity risk

$$1 < R < \beta^{-1}(1-\delta) \tag{A2}$$

so that $\beta R < 1-\delta < 1$. In the following characterization, (A1) and (A2) are always assumed.

As discussed before, there exists a threshold liquidation strategy when he draws z_l, and one can explicitly express $\lambda' = \lambda'_l(\lambda)$ and $q = q_l(\lambda, \lambda')$, which jointly solve (8) and (9).

Lemma 2:
Given an end-of-period leverage λ', there exists a unique threshold $\tilde{\zeta}_l = \tilde{\zeta}_l(\lambda, \lambda')$ such that the entrepreneur liquidates the entire firm when he draws z_l and draws $\zeta < \tilde{\zeta}_l$. The shadow price $q(\lambda, z_l, \zeta)$ can be expressed as

$$q(\lambda, z_l, \zeta) = \begin{cases} 1 & \text{if } \zeta < \tilde{\zeta}_l(\lambda, \lambda'_l(\lambda)) \\ q_l(\lambda, \lambda'_l(\lambda)) & \text{if } \zeta \geq \tilde{\zeta}_l(\lambda, \lambda'_l(\lambda)) \end{cases}$$

where $\lambda'_l(\lambda)$ and $q_l(\lambda, \lambda')$ jointly solve (8) and (9).

Proof. See Appendix B. □

When $\zeta = \tilde{\zeta}_l(\lambda, \lambda')$, the net gain of liquidation is equal to the expected discounted cost of not doing so (i.e., an extra value of holding capital stock one more period). Holding on to capital is similar to gambling for z_h draws in the future. The gambling yields a risky return because of the uninsurable productivity risk. The gambling is not worthwhile when the cost of choosing the safer option (liquidation) is smaller than a threshold, i.e., $\zeta < \tilde{\zeta}_l(\lambda, \lambda')$. The threshold $\tilde{\zeta}_l(\lambda, \lambda')$ satisfies a simple forward-looking functional equation.

Proposition 2:

Given $q_l(\lambda, \lambda')$, let Δ_l be the difference in log net worth between two cases: the entrepreneur liquidates and the entrepreneur stays, i.e.,

$$\Delta_l = \Delta_l(\lambda, \lambda') \equiv \log\left(\frac{z_l\pi + (1-\delta) + R(\lambda^{-1} - 1)}{z_l\pi + (1-\delta)q_l(\lambda, \lambda') + R(\lambda^{-1} - 1)}\right) \tag{10}$$

Then, the cut-off value function $\tilde{\zeta}_l = \tilde{\zeta}_l(\lambda, \lambda')$ satisfies the following recursion

$$\frac{\Delta_l}{1-\beta} - \tilde{\zeta}_l = \frac{\beta\left[\log\tilde{\lambda}' + \sum_j p^{lj}\log\left(\frac{z_j\pi + (1-\delta)}{R} + \frac{1}{\lambda'} - 1\right)\right]}{1-\beta} + \beta p^{ll}\left[-\tilde{\zeta}'_l + \int_0^{\tilde{\zeta}'_l} F(\zeta)d\zeta\right] \tag{11}$$

where for notation simplicity $\tilde{\zeta}'_l = \tilde{\zeta}_l(\lambda', \lambda'_l(\lambda'))$ and $\tilde{\lambda}' = [q_l(\lambda, \lambda') + 1/\lambda' - 1]^{-1}$.

Proof. See Appendix B. □

I shall illustrate equation (11) when the entrepreneur draws both z_l and $\tilde{\zeta}_l$.

The left-hand side (LHS) of (11) is the *gain of liquidation* for the marginal case $\tilde{\zeta}_l$. When liquidating, the entrepreneur with one unit of capital and leverage ratio λ earns $\frac{\Delta_l}{1-\beta}$, from less distorted net worth. Recall from (10) that Δ_l is the gain in (log) net worth. This also means that consumption will go up if the entrepreneur liquidates. But he needs to pay the liquidation cost $\tilde{\zeta}_l(\lambda, \lambda')$. The net gain of liquidation thus amounts to $\frac{\Delta_l}{1-\beta} - \tilde{\zeta}_l$, which becomes the LHS.

The right-hand side (RHS) is the *option value of staying* in business. The value can be interpreted as the entrepreneur's net gain of *one-shot deviation*. This is the gain from staying relative to liquidating today, if the entrepreneur draws z_l today delays liquidation for just one period. This gain includes two terms. One is the capital return relative to the bond return (first term on the RHS), adjusted by the quantities of capital, i.e., $\log\tilde{\lambda}'$, "saved" from one unit of net worth today. The other is the discounted liquidation cost tomorrow, adjusted by the fact the entrepreneur will again follow the liquidation threshold strategy and will pay the liquidation cost if $\zeta' < \tilde{\zeta}'_l$.

For the marginal $\zeta_l = \tilde{\zeta}_l$, the entrepreneur is indifferent between staying and liquidating, i.e., the net gain of liquidation is the same as the option value of staying.

In sum, when the firm is unproductive, four unknowns $q_l(\lambda, \lambda'_l(\lambda))$, $\Delta_l(\lambda, \lambda'_l(\lambda))$, $\tilde{\zeta}_l(\lambda, \lambda'_l(\lambda))$, and $\lambda_l'(\lambda)$, jointly solve (8), (9), (10), and (11) to determine the liquidation strategy.

3.3 The "Dually Constrained" Outcome

For the ease of exposition, let $\lambda_\theta = 1/(1-\theta(1-\delta)/R)$ denote the highest leverage possible and $\lambda' \in [0, \lambda_\theta]$. The variation in θ is equivalent to the variation in λ_θ.

When the entrepreneur draws z_h, he may or may not be financially constrained. When z_l is low enough, he could be financially constrained when he draws z_l and a high liquidation cost, i.e, when he decides to stay with a low productivity. That is, $\lambda_l'(\lambda) = 1/(1-\theta(1-\delta)/R)$. This is because the resale constraint can also push the entrepreneur financially constrained, a "dually constrained" outcome.

Proposition 3:
Suppose the entrepreneur draws z_l and has a leverage λ that satisfies

$$\frac{\lambda_\theta R}{\beta R/(1-\delta) + \lambda_\theta[R - z_l\pi - (1-\delta)]} < \lambda$$

Both his financial constraint (5) and resale constraint (6) bind if he chooses to stay.

Proof. See Appendix B. □

To see why this "dually constrained" outcome can happen, we first consider the case in which the entrepreneur has unit capital stock and has the highest leverage λ_θ. Suppose he chooses to stay in business after drawing z_l this period. If he is not bounded by the resale constraint, then the shadow value of capital is one, and he saves a β fraction of the net worth $z_l\pi + 1 - \delta + R(\lambda_\theta^{-1} - 1)$.

Notice that λ' of the net worth is in the form of capital. If productivity z_l is so low that even if $\lambda' = \lambda_\theta$ we have

$$\lambda_\theta\beta\left[z_l\pi + 1 - \delta + R(\lambda_\theta^{-1} - 1)\right] < (1-\delta), \tag{12}$$

then the entrepreneur has to sell capital even if he borrows to the limit. However, this is not consistent with the fact that he chooses to stay in business and does not sell any capital. Therefore, his financial constraint and resale constraint bind at the same time.

Is (12) satisfied? Yes, since it is equivalent to $\lambda_\theta\left[z_l\pi + 1 - \delta - R\right] + R < \beta^{-1}(1-\delta)$, which is true under assumption (A1) and (A2). The entrepreneur's resale constraint thus strictly binds. Now, if the entrepreneur has a large debt burden today, i.e., λ is close to λ_θ as in Proposition 3, then he is so poor that he has to exhaust the credit limit to pay dividends.

According to Proposition 3, the lower the productivity z_l, the larger range of λ that makes the entrepreneur financially constrained. Intuitively, when the entrepreneur decides to stay, the financial constraint can bind more easily when the productivity z_l is lower, simply because the entrepreneur is poorer and has to rely more on borrowing. Notice that this feature has implication on the firm's debt-to-asset ratio that takes into account profits

$$\frac{-Rb}{z\pi k + (1-\delta)k} = \frac{R(1-\lambda^{-1})}{z\pi + 1 - \delta} \tag{13}$$

where $\lambda > 1$ as the entrepreneur borrows. Since the entrepreneur is likely to borrow more (i.e., λ reaches the upper bound λ_θ) and has a lower z when he is unproductive, the debt-to-asset ratio in (13) when he is unproductive has to be higher than the ratio when he is productive.

This new feature comes from the interaction between the financial constraint and the resale frictions. Without the liquidation costs, the entrepreneur liquidate the firm immediately after he draws the low productivity z_l and repay all the debt. One thus cannot see a higher debt-to-asset ratio when z_l is drawn.

The binding financial constraint simplifies the characterization of $q_l(\lambda, \lambda')$, $\Delta_l(\lambda, \lambda')$, and $\tilde{\zeta}_l(\lambda, \lambda')$, because now $\lambda' = \lambda_l'(\lambda) = \lambda_\theta$. To see this, first we know that $q_l = q_l(\lambda, \lambda_\theta)$ solves q in (9). Second, I can compute the difference in log net worth $\Delta_l(\lambda, \lambda_\theta)$ from (10). Finally, I solve the functional form $\tilde{\zeta}_l(\lambda, \lambda_\theta)$ from (11), which becomes

$$\begin{aligned} &\underbrace{-\tilde{\zeta}_l(\lambda, \lambda_\theta) + \beta p^{ll}\left[\tilde{\zeta}_l(\lambda_\theta, \lambda_\theta') - \int_0^{\tilde{\zeta}_l(\lambda_\theta, \lambda_\theta')} F(\zeta) d\zeta\right]}_{H^l(\tilde{\zeta}_l(\lambda, \lambda_\theta), \tilde{\zeta}_l(\lambda_\theta, \lambda_\theta'))} \\ = &\underbrace{\frac{\beta}{1-\beta}\left[\sum_j p^{lj}\log\left(\frac{z_j\pi + (1-\delta)}{R} + \frac{1}{\lambda_\theta} - 1\right) + \log\left(q_l(\lambda, \lambda_\theta) + \frac{1}{\lambda_\theta} - 1\right)^{-1}\right] - \frac{\Delta_l(\lambda, \lambda_\theta)}{1-\beta}}_{H^r(\lambda, \lambda_\theta)} \end{aligned} \tag{14}$$

Given any λ_θ, (14) specifies a condition that the function $\tilde{\zeta}_l(., \lambda_\theta)$ needs to satisfy. In (14), the LHS has cut-off values of liquidation costs today and tomorrow, while the RHS does not. For simplicity, let the LHS be $H^l(\tilde{\zeta}_l(\lambda, \lambda_\theta), \tilde{\zeta}_l(\lambda_\theta, \lambda_\theta'))$, and let the RHS be $H^r(\lambda, \lambda_\theta)$.

Remark: $H^r(\lambda, \lambda_\theta)$ is the *financially-constrained (net) option value* for an unproductive entrepreneur with leverage λ. It includes the gain of one-shot deviation (in the big bracket) arising from delaying liquidation for one period, net of the gain of liquidation, i.e., $\frac{\Delta_l(\lambda, \lambda_\theta)}{1-\beta}$. $H^r(\lambda, \lambda_\theta)$ crucially depends on the tightness of financial constraints λ_θ, and so does the liquidation threshold $\tilde{\zeta}_l(\lambda, \lambda_\theta)$.

4 How Do Shocks Affect Liquidations?

I use a thought experiment to illustrate the effect of financial shocks on the timing of when to exercise the liquidation option.

Suppose λ_θ is fixed at a certain level λ_{θ_h} until $t = s-1$. Unexpectedly, λ_θ falls permanently from λ_{θ_h} to λ_{θ_l}, from $t = s$ onwards as in the top panel of Figure 2. Now, consider a sample path in which the entrepreneur draws z_l every period from $t = s$ onwards.

Under some parameter conditions, the threshold liquidation value $\tilde{\zeta}_l(\lambda, \lambda')$ falls immediately at $t = s$, and from $t = s+1$ onwards it rises to a new level which is still below the initial level before $t = s$ (the bottom panel of Figure 2). Overall, if the entrepreneur draws z_l after the permanent financial shock, he is less likely to liquidate his firm because of the changes in $\tilde{\zeta}_l(\lambda, \lambda')$. To illustrate the adjustment process, I first look at periods $t \geq s+1$ and then come back to $t = s$.

4.1 From Time $t = s + 1$ Onwards

At the beginning of any t, I normalize the entrepreneur's capital stock to one unit and denote his leverage as λ. If he draws z_l today and decides to keep his firm, he is financially constrained because of the dually constrained outcome. Then, $k' = 1 - \delta$ and $\lambda' = \lambda_\theta$.

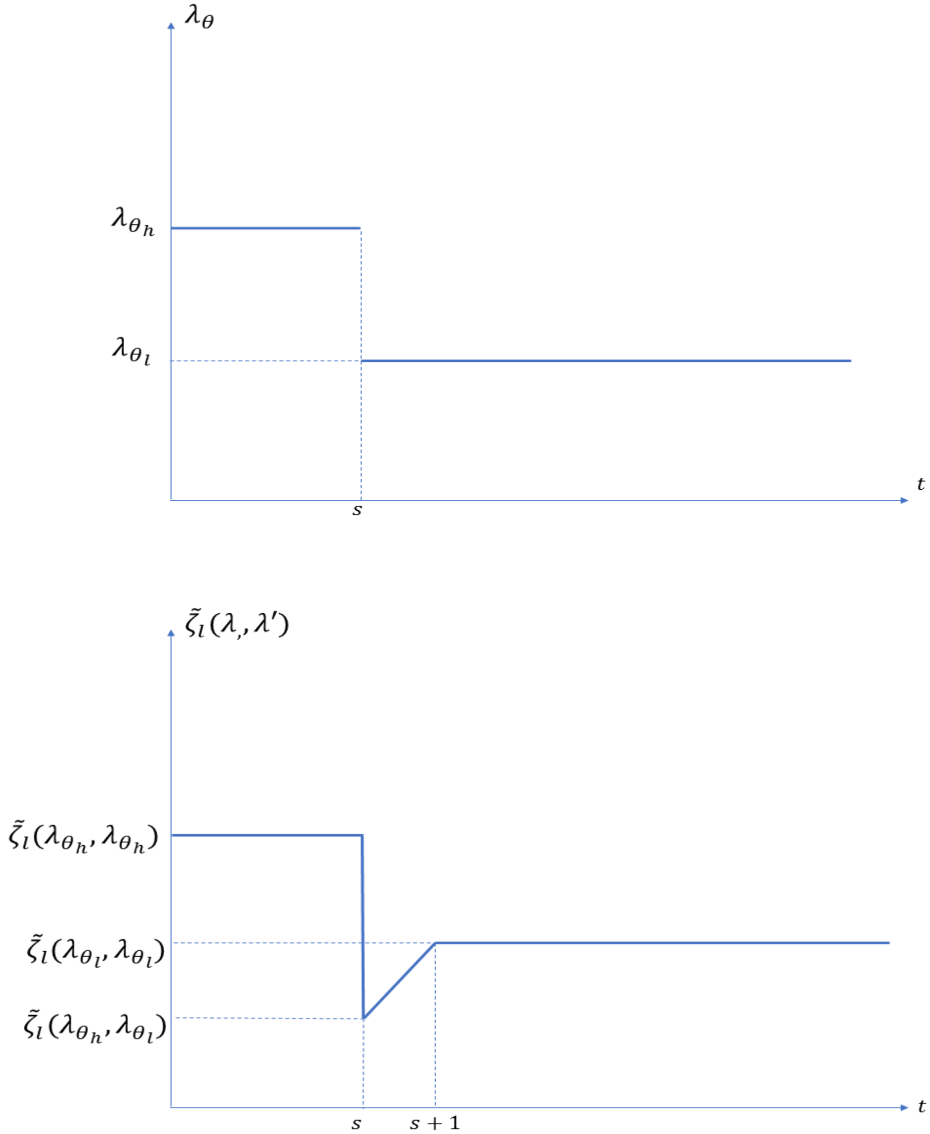
For any $t \geq s+1$, he comes to each period with λ_{θ_l} because he had z_l in the proceeding period $t-1$. If he chooses to stay after drawing z_l, he targets λ_{θ_l} again. Therefore, the threshold value is $\tilde{\zeta}_l(\lambda_{\theta_l}, \lambda_{\theta_l})$ for all $t \geq s+1$.

Notice that if λ_θ remains at λ_{θ_h} instead of falling to λ_{θ_l}, a similar argument applies and the threshold value becomes $\tilde{\zeta}_l(\lambda_{\theta_h}, \lambda_{\theta_h})$ for all $t \geq s+1$. One can see that the cut-off value is $\tilde{\zeta}_l(\lambda_\theta, \lambda_\theta)$ for any given λ_θ for all $t \geq s+1$. We should thus solve $\tilde{\zeta}_l(\lambda_\theta, \lambda_\theta)$ from (14), which becomes

$$
\begin{aligned}
& -\tilde{\zeta}_l(\lambda_\theta, \lambda_\theta) + \beta p^{ll}\left[\tilde{\zeta}_l(\lambda_\theta, \lambda_\theta) - \int_0^{\tilde{\zeta}_l(\lambda_\theta, \lambda_\theta)} F(\zeta)d\zeta\right] \qquad (15) \\
=& \frac{\beta}{1-\beta}\left[\underbrace{\underbrace{\sum_j p^{lj}\log\left(\frac{z_j\pi + (1-\delta)}{R} + \frac{1}{\lambda_\theta} - 1\right)}_{\text{relative return term}} \underbrace{+ \log\left(q_l(\lambda_\theta, \lambda_\theta) + \frac{1}{\lambda_\theta} - 1\right)^{-1}}_{\text{quantity term}}}_{\text{oneshot-deviation return term}}\right] - \frac{\Delta_l(\lambda_\theta, \lambda_\theta)}{1-\beta}
\end{aligned}
$$

For this reason, analyzing the effect of λ_θ on $\tilde{\zeta}_l(\lambda_\theta, \lambda_\theta)$ is essentially a comparative statics

Figure 2: **The Effect of a Permanently Tightened Financial Constraint**



analysis. To do so, I first show the effect of a falling λ_θ (from λ_{θ_h} to λ_{θ_l}) on the RHS (i.e., $H^r(\lambda_\theta, \lambda_\theta)$) and on the LHS (i.e., $H^l(\tilde{\zeta}_l(\lambda_\theta, \lambda_\theta), \tilde{\zeta}_l(\lambda_\theta, \lambda_\theta))$ later.

Given a z_h, if the low productivity z_l is not too low, the fall of λ_θ raises the RHS.

Proposition 4:
If $z_l > z_h - \frac{(1-\delta)(z_h\pi+1-\delta-R)}{\pi R}$*, then a fall of* λ_θ *increases* $H^r(\lambda_\theta, \lambda_\theta)$*, i.e.,* $\frac{\partial H^r(\lambda_\theta,\lambda_\theta)}{\partial \lambda_\theta} < 0$.

Proof. See Appendix B. □

Now, I explain the effect of adverse financial shocks on every terms on the RHS of (15).

$\Delta_l(\lambda_\theta, \lambda_\theta)$ in (15) represents the liquidation gain in net worth (in log) from less distorted consumption and saving decisions. A fall in λ_θ means that the entrepreneur can borrow less and the consumption distortion is more severe. However, a fall in λ_θ also implies that the entrepreneur starts every day with a lower leverage and less debt; if he chooses to stay, he is richer and the consumption distortion is smaller. In the proof, I show that $\Delta_l(\lambda_{\theta_l}, \lambda_{\theta_l}) < \Delta(\lambda_{\theta_h}, \lambda_{\theta_h})$, i.e., the second effect dominates. The liquidation gain in net worth is thus smaller when λ_θ falls, and such smaller liquidation gain raises the RHS (because of the "-" sign).

Now, I discuss the effect of a falling λ_θ on the "one-shot deviation return term", in (15). A falling λ_θ always pushes up the "relative return term", since the entrepreneur will begin with a lower leverage tomorrow; the effect of the falling λ_θ on the "quantity term" is, however, less clear. According to (9), the quantity term measures the remained capital stock (which is $1-\delta$) as a fraction of the economic net worth,

$$\left(q_l(\lambda_\theta, \lambda_\theta) + \frac{1}{\lambda_\theta} - 1\right)^{-1} = \frac{1-\delta}{\beta\left[z_l\pi + (1-\delta)q(\lambda_\theta, \lambda_\theta) + R(\lambda_\theta^{-1} - 1)\right]} \tag{16}$$

if one neglects the log transformation. On the one hand, the fall of λ_θ reduces debt payments (i.e., increases $1/\lambda_\theta$). It thus raises the economic net worth for each unit of capital and reduces the quantity term, i.e., the denominator in (16) rises . On the other hand, the fall of λ_θ also reduces the shadow value $q_l(\lambda_\theta, \lambda_\theta)$. It thus reduces the economic net worth and increases the quantity term, i.e., the denominator in (16) falls. As a result, the quantity term can either go up or down.

Proposition 4 implies that when z_l is high enough, the effect of a reduction in λ_θ on the net worth is rather limited. This is intuitive since the richer the entrepreneur is, the less concerned about the debt payment in the "quantity term" he is. The one-shot deviation return term thus rises with the fall in λ_θ.

In sum, we know that the RHS of (15) rises with the fall of λ_θ, i.e., the net gain of delaying liquidation goes up when the financial constraint becomes tightened. Now, I turn to the LHS of (15), and we can finally determine the variation of liquidation threshold $\tilde{\zeta}_l(\lambda_\theta, \lambda_\theta)$ in response to the drop in λ_θ.

Proposition 5:
$h^l(x) = H^l(x,x)$ is a strictly decreasing function of x (i.e., $\frac{dh^l(x)}{dx} < 0$) with $h^l(1) = 0$. Assuming an interior solution, we can express $\tilde{\zeta}_l(\lambda_\theta, \lambda_\theta) = \left(h^l\right)^{-1}(H^r(\lambda_\theta, \lambda_\theta))$, where $\left(h^l\right)^{-1}$ is the inverse function of h^l. Since $\frac{\partial H^r(\lambda_\theta, \lambda_\theta)}{\partial \lambda_\theta} < 0$, we know that $\frac{\partial \tilde{\zeta}_l(\lambda_\theta, \lambda_\theta)}{\partial \lambda_\theta} > 0$ and

therefore $\tilde{\zeta}_l(\lambda_{\theta_l}, \lambda_{\theta_l}) < \tilde{\zeta}_l(\lambda_{\theta_h}, \lambda_{\theta_h})$.

Proof. See Appendix B. $\square$

The implication is straightforward. When the financial constraint becomes permanently tightened from $t = s$, the option value of staying in business rises relative to the gains from liquidation from period $t = s + 1$ onwards. As a result, the threshold liquidation value $\tilde{\zeta}_l$ jumps from the old steady-state level $\tilde{\zeta}_l(\lambda_{\theta_h}, \lambda_{\theta_h})$ *down* to the new steady-state level $\tilde{\zeta}_l(\lambda_{\theta_l}, \lambda_{\theta_l})$. This liquidation strategy implies that the entrepreneur is less likely to liquidate his company, when he draws z_l in periods $t \geq s + 1$. The question now is what happens at time $t = s$.

4.2 At Time $t = s$

We have seen that from time $t = s+1$ onwards, the cut-off liquidation cost $\tilde{\zeta}_l$ takes the value $\tilde{\zeta}_l(\lambda_{\theta_l}, \lambda_{\theta_l})$, which is below $\tilde{\zeta}_l(\lambda_{\theta_h}, \lambda_{\theta_h})$. At time $t = s$, the beginning leverage λ_{θ_h} is higher than λ_{θ_l}, the new highest leverage allowed. We therefore know from Proposition 3 that the entrepreneur is financially constrained at time $t = s$.

According to (14), the cut-off $\tilde{\zeta}_l$ at $t = s$ takes the value $\tilde{\zeta}_l(\lambda_{\theta_h}, \lambda_{\theta_l})$, which solves

$$
\begin{aligned}
&- \tilde{\zeta}_l(\lambda_{\theta_h}, \lambda_{\theta_l}) + \beta p^{ll} \left[\tilde{\zeta}_l(\lambda_{\theta_l}, \lambda_{\theta_l}) - \int_0^{\tilde{\zeta}_l(\lambda_{\theta_l}, \lambda_{\theta_l})} F(\zeta) d\zeta \right] \qquad (17) \\
=& \frac{\beta}{1-\beta} \left[\sum_j p^{lj} \log \left(\frac{z_j \pi + (1-\delta)}{R} + \frac{1}{\lambda_{\theta_l}} - 1 \right) + \log \left(q_l(\lambda_{\theta_h}, \lambda_{\theta_l}) + \frac{1}{\lambda_{\theta_l}} - 1 \right)^{-1} \right] - \frac{\Delta_l(\lambda_{\theta_h}, \lambda_{\theta_l})}{1-\beta}
\end{aligned}
$$

given $\tilde{\zeta}_l(\lambda_{\theta_l}, \lambda_{\theta_l})$ known from the previous discussion. Now, setting $\lambda_\theta = \lambda_{\theta_l}$ in equation (15) and comparing it with (17), we immediately know that $\tilde{\zeta}_l(\lambda_{\theta_h}, \lambda_{\theta_l}) - \tilde{\zeta}_l(\lambda_{\theta_l}, \lambda_{\theta_l}) = H^r(\lambda_{\theta_l}, \lambda_{\theta_l}) - H^r(\lambda_{\theta_h}, \lambda_{\theta_l})$.

It turns out that $H^r(\lambda_{\theta_l}, \lambda_{\theta_l}) - H^r(\lambda_{\theta_h}, \lambda_{\theta_l}) < 0$ when the entrepreneur is sufficiently patient. That is, before reaching and staying at the new steady-state level $\tilde{\zeta}_l(\lambda_{\theta_l}, \lambda_{\theta_l})$ from time $t = s + 1$, the cut-off value overshoots below at time $t = s$.

Corollary 1:
When β is sufficiently close to 1, $H^r(\lambda_{\theta_l}, \lambda_{\theta_l}) < H^r(\lambda_{\theta_h}, \lambda_{\theta_l})$ and $\tilde{\zeta}_l(\lambda_{\theta_h}, \lambda_{\theta_l}) < \tilde{\zeta}_l(\lambda_{\theta_l}, \lambda_{\theta_l})$.

Proof. See the Appendix B. $\square$

The difference between $\tilde{\zeta}_l(\lambda_{\theta_h}, \lambda_{\theta_l})$ and $\tilde{\zeta}_l(\lambda_{\theta_l}, \lambda_{\theta_l})$ is whether the entrepreneur begins today with leverage λ_{θ_h} or λ_{θ_l}. Because the shock hits unexpectedly at time $t = s$, the entrepreneur begins with the high leverage λ_{θ_h}. This means that the entrepreneur needs to give up more consumption compared to the case in which he begins with the lower leverage λ_{θ_l}.

Yet, in these two cases, the target leverage λ_{θ_l} is the same. Because of this and because the entrepreneur is sufficiently patient, the loss of (log) net worth changes little whether $\lambda_\theta = \lambda_{\theta_h}$ or $\lambda_\theta = \lambda_{\theta_l}$. Instead, the dominating factor comes from the quantity term because

$$\log\left(q_l(\lambda_{\theta_h}, \lambda_{\theta_l}) + \frac{1}{\lambda_{\theta_l}} - 1\right)^{-1} > \log\left(q_l(\lambda_{\theta_l}, \lambda_{\theta_l}) + \frac{1}{\lambda_{\theta_l}} - 1\right)^{-1}$$

which means that the option value of staying is even higher when $\lambda_\theta = \lambda_{\theta_h}$ compared to $\lambda_\theta = \lambda_{\theta_l}$. The cut-off liquidation cost $\tilde{\zeta}_l(\lambda_{\theta_h}, \lambda_{\theta_l})$ thus needs to be below $\tilde{\zeta}_l(\lambda_{\theta_l}, \lambda_{\theta_l})$ to reflect the higher option value.

In summary, a tightened financial constraint increases the option value of staying. The increase is greater at the time of the shock $t = s$ than that in the future. As a result, the liquidation threshold falls on impact and increases to a new steady-state level, a level still below the old steady-state level shown in Figure 2. In all periods, the entrepreneur is less likely to liquidate his firm.

Before moving to the general equilibrium model, I briefly comment on the effect from varying the two factor prices, the average profit rate π and the real interest rate R. Similar to the previous experiment, suppose there is a permanent unexpected rise in the profit rate π or a permanent unexpected fall in the interest rate R from time $t = s$ onwards. Both of the changes increase the option value of staying, $H^r(\lambda_\theta, \lambda_\theta)$. The cut-off liquidation cost $\tilde{\zeta}_l(\lambda_\theta, \lambda_\theta)$ thus falls permanently.

Intuitively, either a rise in π or a fall in R gives the entrepreneur more resources when his firm turns unproductive. He is thus more willing to tolerate the loss in log net worth if he chooses to stay; he will be also richer in the future from a higher profit rate and a lower borrowing cost. As a result, the entrepreneur is less likely to exercise the liquidation option.

The next section incorporates general equilibrium effects from and on these two prices. Additionally, one can directly see the impact of financial shocks on aggregate liquidation, aggregate productivity, and output.

5 General Equilibrium with Aggregate Shocks

In this section, I extend the model into general equilibrium with aggregate shocks. This extension is useful, since one can later explore the dynamics of capital liquidation, total factor productivity (TFP), and aggregate output.

5.1 The Model

The economy is now populated by a representative household and a continuum of entrepreneurs with population measure one. For the purpose of writing recursive problems faced by agents in the economy, I use X as the aggregate state variable whose law of motion is taken as given by all agents. I will explain X after specifying those problems.

A Representative Household

A representative household, with a discount factor $\beta^h > \beta$ and intertemporal substitution γ (subject to shocks), comes to period t with bonds B_h accumulated before. The household chooses consumption, labor supply, and savings. The household's optimization can be represented by the following Bellman equation

$$v^h(B_H, \gamma; X) = \max_{C_H, L_H, B'_H} \left\{ u^h\left(C_H, L_H\right) + \gamma\beta^h \mathbb{E}\left[v^h(B'_H, \gamma'; X')|\gamma, X\right]\right\} \text{ s.t.}$$

$$C_H + B'_H = wL_H + RB_H \tag{18}$$

(18) is the resource constraint: the household uses labor income and return from savings to finance consumption and new savings. The wage rate $w = w(X)$ and the interest rate $R' = R'(X)$ are taken as given by the household. Shocks to γ affects the saving decisions of the household. The shocks are "credit-supply" shocks, different from financial shocks which mainly affect credit demand.

Entrepreneurs

If an entrepreneur i has a firm at the beginning of t, his firm has a constant-return-to-scale (CRS) production technology. This firm uses capital stock k_i (installed at $t-1$) and hires labor hours h_i at a competitive wage rate w to produce general consumption goods at the beginning of t.

$$y_i = (z_i k_i)^\alpha (Ah_i)^{1-\alpha}$$

where $\alpha \in (0,1)$ is the capital share in the production function, A is the level of aggregate labor augmenting productivity, and $z_i \in \{z_l, z_h\}$ follows the same Markov process as in Section 3. Aggregate and idiosyncratic productivities are common knowledge.

The labor market is frictionless, and we know entrepreneur i's profits after production is

$$\Pi(z_i, k_i; w) = (z_i k_i)^\alpha (Ah_i)^{1-\alpha} - wh_i$$

The profits are linear in k_i (see Appendix C for derivation), i.e., $\Pi(z_i, k_i; w) = z_i\pi(w)k_i$, where the associated labor demand and the aggregate profit rate are

$$h_i^* = \frac{z_i k_i}{A}\left[\frac{(1-\alpha)A}{w}\right]^{\frac{1}{\alpha}} \text{ and } \pi = \alpha\left(\frac{(1-\alpha)A}{w}\right)^{\frac{1-\alpha}{\alpha}}. \tag{19}$$

The total output produced by entrepreneur i can thus be written as $y_i = z_i \pi k_i/\alpha$, of which $z_i \pi k_i$ is the amount of resources owned by i. In contrast to Section 3, the profit rate π is endogenous. It increases with aggregate productivity while decreases with the wage rate.

Additionally, I allow entrepreneurs to partially sell their capital. That is, when entrepreneur i reallocates capital, he can liquidate the whole firm, or he can sell up to a fraction $\phi \in [0, 1]$ of his capital. Finally, entrepreneur i is subject to a similar collateral constraint as before.

Let v be the optimal value of an entrepreneur with individual state (k, b, z, ζ) at the beginning of t, given the aggregate state X. The value function $v(k, b, z, \zeta; X)$ satisfies the following Bellman equation:

$$v(k,b,z,\zeta;X) = \max_{\text{liquidate/stay}}\{v^0(k,b,z,\zeta;X), \quad v^1(k,b,z;X)\}$$

$$v^0(k,b,z,\zeta;X) = \max_{b'}\{\log\left(z\pi k + (1-\delta)k + Rb - b'\right) - \zeta + \beta\mathbb{E}[v(0,b',z',\zeta';X')|z,X]\}$$

$$v^1(k,b,z;X) = \max_{k',b'}\{\log(z\pi k + (1-\delta)k + Rb - k' - b') + \beta\mathbb{E}[v(k',b',z',\zeta';X')|z,X]\} \text{ s.t.}$$

$$Rb' \geq -\theta(1-\delta)k' \tag{20}$$

$$k' \geq (1-\phi)(1-\delta)k \tag{21}$$

where the profit rate $\pi = \pi(X)$ and interest rate $R' = R'(X)$ are taken as given. The financing constraint (20) remains the same as in the simple model. The resale constraint (21) takes into account the partial selling represented by ϕ.

One can now see that the aggregate state is $X = (\Gamma, R, S)$. $\Gamma \equiv \Gamma(k, b, z)$ is the distribution of individual entrepreneur's capital stock, bonds, and idiosyncratic productivity, R is the predetermined safe interest rate, $S \equiv (A, \gamma, \theta)$ is the set of aggregate productivity, intertemporal substitution, and tightness of financial constraints that are subject to shocks. The law of motion that concerns S is exogenous, which is denoted as G. The other law of motion that concerns updating Γ is denoted as H or $\Gamma' = H(\Gamma, R, S)$, which is endogenous. In summary, we have

$$S' = G(S), \ \Gamma' = H(\Gamma, R, S), \text{ and } X' = (H(\Gamma, R, S), R'(X), G(S)).$$

5.2 Recursive Competitive Equilibrium

As before, I focus on the type of equilibrium in which low productivity firms are both financially and resale constrained if they choose to stay in business. The productive firms may or may not be financially constrained.

The Household's and Entrepreneur's Decision Rules

Since $\beta^h > \beta$, households will be savers and the entrepreneurs are net debtors (though some of them are also creditors). The saving choice $B'_H = B'_H(B_H; X)$ and the labor supply decision $L_H = L_H(B_H; X)$ satisfy

$$\mathbb{E}_X \left[\frac{\beta^h u_c \left(C'_H, L'_H\right)}{u_c \left(C_H, L_H\right)} R' \right] = 1 \tag{22}$$

$$w = \mu L_H^{\nu} \tag{23}$$

Notice that the household's consumption $C_H = C_H(B_H; X)$ can be solved from the resource constraint (18).

Let λ_j ($j \in \{h, l\}$) be the time t beginning-of-period leverage of an entrepreneur who had z_j at $t-1$. For z_l entrepreneurs with leverage λ_j, I follow the notation in the simple model. Let $\tilde{\zeta}_{jl} = \tilde{\zeta}_l(\lambda_j, \lambda'_l; X)$ be the threshold liquidation cost, $q_{jl} = q_l(\lambda_j, z_l; X)$ be the shadow value of capital, and finally let $\Delta_{jl} = \Delta_l(\lambda_j, \lambda'_l; X)$ be the difference in log net worth between liquidating and staying

$$\Delta_{jl} = \Delta_l(\lambda_j, \lambda'_l; X) \equiv \log \left(\frac{z_l \pi + (1-\delta) + R(\lambda_j^{-1} - 1)}{z_l \pi + \phi(1-\delta) + (1-\phi)(1-\delta) q_{jl} + R(\lambda_j^{-1} - 1)} \right) \tag{24}$$

The following Corollary characterizes $\tilde{\zeta}_{jl}$, q_{jl}, and λ'_j, directly following the characterization in the simple model.

Corollary 2:
The cut-off value $\tilde{\zeta}_{jl} = \tilde{\zeta}_l(\lambda_j, \lambda'_l; X)$ and the shadow value $q_{jl} = q_l(\lambda_j, z_l; X)$ satisfy

$$\begin{aligned} & -\tilde{\zeta}_{jl} + \beta p^{ll} \mathbb{E}_X \left[\tilde{\zeta}'_{ll} - \int_0^{\tilde{\zeta}'_{ll}} F(\zeta') d\zeta' \right] \\ = & \frac{\beta}{1-\beta} \left[\mathbb{E}_X \sum_j p^{jl} \log \left(\frac{z_j \pi' + (1-\delta)}{R'} + \frac{1}{\lambda'_l} - 1 \right) - \log \left(q_{jl} + \frac{1}{\lambda'_l} - 1 \right) \right] - \frac{\Delta_{lj}}{1-\beta} \end{aligned} \tag{25}$$

$$\beta\left[z_l\pi+(1-\delta)\phi+(1-\delta)(1-\phi)q_{jl}+R(\lambda_j-1)\right]=\left(q_{jl}+\frac{1}{\lambda_l'}-1\right)(1-\phi)(1-\delta) \tag{26}$$

for $j\in\{h,l\}$. Finally, for an entrepreneur who draws z_j, the leverage $\lambda_j'=\lambda_j'(\lambda,z_j;X)$ solves

$$p^{jh}\mathbb{E}_X\left[\frac{z_h\pi'+1-\delta-R'}{\lambda_j'\left[z_h\pi'+1-\delta-R'\right]+R'}\right]+p^{jl}\mathbb{E}_X\left[1-F\left(\tilde{\zeta}_{jl}'\right)\right]\left[\frac{z_l\pi'+(1-\delta)\,q_{jl}'-R'}{\lambda_h'\left[z_l\pi'+(1-\delta)q_{jl}'-R'\right]+R'}\right]$$
$$+p^{jl}\mathbb{E}_X F\left(\tilde{\zeta}_{jl}'\right)\left[\frac{z_l\pi'+(1-\delta)-R'}{\lambda_h'\left[z_l\pi'+(1-\delta)-R'\right]+R'}\right]\le 0 \tag{27}$$

Proof. See Appendix B. □

In the type of equilibrium I focus on, the leverage λ_l' hits the borrowing limit

$$\lambda_l'=\frac{1}{1-\theta(1-\delta)/R'} \tag{28}$$

In quantitative analysis, I verify that the low productivity firms are financially constrained. For productive entrepreneurs, the leverage λ_h' may or may not hit the borrowing constraint. The unproductive firms thus have a higher debt-to-asset ratio than that of productive firms, because unproductive firms are more likely financially constrained. This new feature is again coming from the interaction between financial constraints and resale frictions.

Equilibrium

Here, I aggregate individual firms and define equilibrium.

Let K_h and K_l be the aggregate capital held by productive and unproductive entrepreneurs at the beginning of t. Let B be the corresponding aggregate bonds held by entrepreneurs who did not run firms at $t-1$. Because of the linearity of the policy function of entrepreneurs, they save a common β fraction of their net worth. Capital stock and bonds move proportionally with the mass of entrepreneurs. The wealth dynamics can thus be represented by the following (backward-looking) equations

$$K_h'=\lambda_h'\beta\sum_j\left[z_h\pi+1-\delta+R(\lambda_j^{-1}-1)\right]p^{jh}K_j+\lambda_h'\beta Rp^{lh}B \tag{29}$$

$$K_l'=(1-\delta)(1-\phi)\sum_j\left[1-F(\tilde{\zeta}_{jl})\right]p^{jl}K_j \tag{30}$$

$$B'=\beta\sum_j F(\tilde{\zeta}_{jl})\left[z_l\pi+(1-\delta)+R\left(\lambda_j^{-1}-1\right)\right]p^{jl}K_j+\beta Rp^{ll}B \tag{31}$$

(29) describes the capital transition in the hand of productive entrepreneurs. The LHS is the

end-of-period capital stock, while the RHS is the saving in capital stock from those shifting to the productive group. (30) describes the capital transition in the hand of entrepreneurs who become (or stay) unproductive and choose to stay in business. Since only ϕ fraction is saleable, those who were previously productive contribute $(1-\phi)(1-\delta)(1-F(\tilde{\zeta}_{hl}))p^{hl}K_h$, while those who were previously unproductive contribute $(1-\phi)(1-\delta)(1-F(\tilde{\zeta}_{ll}))p^{ll}K_l$. Finally, (31) represents savings in bonds from entrepreneurs who do not run firms. B' includes savings from entrepreneurs who draw z_l today and decide to liquidate, and savings from those who do not run firms before and draw z_l again today.

Let $\mathbb{L} \in \mathbb{R}^2_+$ be a compact set containing all leverage ratios $\{\lambda_h, \lambda_l\}$, and let $\mathbb{K} \subset \mathbb{R}^4_+$ be a compact set containing all possible values of (B_H, B, K_h, K_l). Then, $\mathbb{L} \times \mathbb{K}$ is the compact set that contains all possible values of endogenous state variables. Let $\mathbb{A} \subset \mathbb{R}^2_+ \times [0,1]^2$ be a compact set containing the pre-determined interest rate and all possible values of exogenous state variables (R, A, γ, θ). Then, $\mathbb{X} = \mathbb{L} \times \mathbb{K} \times \mathbb{A}$ is the compact set containing all possible values of all aggregate state variables.

A **recursive competitive equilibrium with capital liquidation** is a mapping $\mathbb{X} \to \mathbb{X}$ that consists of pricing functions (w, π, R'): $\mathbb{X} \to \mathbb{R}^3_+$, the household's policy functions (C_H, L_H, B'_H): $[0, +\infty) \times \mathbb{X} \to \mathbb{R}^3_+$, z_l entrepreneurs' liquidation strategies $(\tilde{\zeta}_{jl}, q_{jl}, \Delta_{jl})$: $\mathbb{L} \times z_l \times \mathbb{X} \to \mathbb{R}$, and leverage ratios λ'_j (for $j = h, l$): $\mathbb{L} \times \{z_j\} \times \mathbb{X} \to \mathbb{L}$, such that

(a) the household's choice $(C_H,\ L_H,\ B'_H)$ solves (18), (22), and (23);

(b) the entrepreneurs' liquidation strategy $\{\tilde{\zeta}_{jl}\}$ and portfolio choice $\{\lambda_{jl}\}$ follow (25), (27), and (28); the difference in log net worth Δ_{jl} is given by (24), and the shadow price $\{q_{lj}\}$ is given by (26);

(c) (B', K'_h, K'_l) satisfies the wealth dynamics in (29)-(31);

(d) Given that the wage rate w solves (19), R' and π are determined by the market clearing conditions for credit and labor[13]

$$\sum_{j=h,l} \left[\left(\lambda'_j\right)^{-1} - 1 \right] K'_j + B' + B'_h = 0 \tag{32}$$

$$\left(\frac{\pi}{\alpha}\right)^{\frac{1}{1-\alpha}} \sum_j \left(p^{jh} z^h + p^{jl} z^l\right) K_j = A L_h \tag{33}$$

[13]Since an individual entrepreneur's bond position is $\left[(\lambda')^{-1} - 1\right] k'$ with k' as the target capital level, we can aggregate the bond position as in (32). Additionally, from individual labor demand (19), we know the aggregate labor demand is $\left(\frac{\pi}{\alpha}\right)^{\frac{1}{1-\alpha}} \left[\sum \left(p^{jh} z^h + p^{jl} z^l\right) K_j\right] / A$ as in (33).

Some Macro Variables

To close the model, I show how to compute aggregate output, total factor productivity (TFP), capital liquidation, new investment, and capital expenditures.

First, since the output from an entrepreneur i is $y_i = \frac{\pi}{\alpha} z_i k_i$, aggregate output can be calculated as

$$Y = \frac{\pi}{\alpha}\left[\sum_j \left(p^{jh} z^h + p^{jl} z^l\right) K_j\right]$$

Given the production functional form, I define $TFP \equiv \frac{Y}{(\sum K_j)^\alpha L^{1-\alpha}}$. To calculate TFP, I first define $\rho_k \equiv K_h / K_l$ as the ratio of capital stock in the hand of productive entrepreneurs and unproductive entrepreneurs, $\bar{z}^h \equiv p^{hh} z_h + p^{hl} z_l$ as average productivity in this period for a previously productive entrepreneur, and similarly $\bar{z}^l \equiv p^{lh} z_h + p^{ll} z_l$ for a previously unproductive entrepreneur. Then,

$$TFP = A^{1-\alpha}\left(\frac{\left(p^{hh} z_h + p^{hl} z_l\right)\rho_k + p^{lh} z_h + p^{ll} z_l}{\rho_k + 1}\right)^\alpha = A^{1-\alpha}\left(\bar{z}^h - \frac{\bar{z}^h - \bar{z}^l}{\rho_k + 1}\right)^\alpha \tag{34}$$

If $\bar{z}^h > \bar{z}^l$, we know that TFP is higher when ρ_k is higher. That is, if previously productive entrepreneurs are on average more productive today than their unproductive pairs, the efficiency of the economy increases with capital in the hand of productive entrepreneurs.

Second, aggregate capital liquidation is

$$CL \equiv \underbrace{\phi(1-\delta)\left[\sum_j \left[1 - F\left(\tilde{\zeta}_{jl}\right)\right] p^{jl} K_j\right]}_{PL} + \underbrace{(1-\delta)\left[\sum_j F(\tilde{\zeta}_{jl}) p^{jl} K_j\right]}_{FL}$$

where the first part is the total partial liquidation (PL) and the second part is the total full liquidation (FL). The level of aggregate capital expenditures is equivalent to the capital spending of productive firms,

$$CX \equiv K_h' - (1-\delta)\sum p^{jh} K_j$$

Finally, since total capital expenditures come from new investment and capital liquidation, new investment is thus[14]

$$I = K_h' - (1-\delta)\sum p^{jh} K_j - CL$$

[14] Using the definition of aggregate investment, one can verify Walras' Law by deriving the goods market clearing condition, i.e., $C + I = Y$ where C is aggregate consumption.

6 Quantitative Analysis

In this section, I explore the quantitative properties of the macro model. In particular, I show different dynamics of capital liquidation and other macro aggregates, after shocks to aggregate productivity, the intertemporal substitution, and the external financial condition.

6.1 Parameterizations

The model parameters are calibrated to annual frequencies as the earliest capital liquidation recorded in COMPUSTAT data was for the year 1971 (recall Section 2).

For the productivity transition matrix and the size of idiosyncratic productivity shocks, I use the procedure developed by Tauchen (1986). That is, I set the transition probability as $p^{hh} = p^{ll}$ and the two levels of idiosyncratic productivity as

$$\log(\tilde{z}_h) = \sigma \text{ and } \log(\tilde{z}_l) = -\sigma$$

According to a micro-level study of plant level productivity by Ábrahám and White (2006), the idiosyncratic productivity fits an AR(1) process reasonably well. The yearly persistency is around 0.69, while the standard deviation of shocks is about 0.18.[15] Therefore, one can simply equate the variance and the serial correlation of the two-point Markov Chain in the model and the AR(1) process in the data, i.e.,

$$\sigma = \sqrt{\frac{0.18^2}{1 - 0.69^2}} = 0.2487 \text{ and } p^{hh} = \frac{1 + 0.69}{2} = 0.8450$$

Second, the household's discount factor β^h targets the (real) risk-free rate, set to 2% annually. To abstract the wealth effect on the household's labor supply, I assume that the household has a non-separable GHH utility function

$$U(C_H, L_H) = \frac{\left(C_H - \frac{\mu L_H^{1+\nu}}{1+\nu}\right)^{1-\varepsilon} - 1}{1 - \varepsilon}$$

I set $\varepsilon = 1$ to allow the same risk-aversion as entrepreneurs (with log utility). I set $\nu = 1$ so that the household's labor supply elasticity is 1, in line with a typical macro calibration. When one increases either ε or ν, the effect of financial shocks on capital liquidation and

[15] The standard deviation of shocks reported in İmrohoroğlu and Tüzel (2014) is close to 0.23 who look at COMPUSTAT data directly. If I calibrate to their target, the effect of financial shocks is even higher. This is because the dispersion of TFP is higher with a higher standard deviation, and the effect of financial shocks on capital reallocation will be stronger. I choose to be conservative because the model only features two states of idiosyncratic productivity.

output is even stronger. That is, when either ε or ν increases, the household saving and labor supply decisions are more sensitive to the changes of the interest rate and the wage rate. The effect of financial shocks on capital liquidation and output (which will be clear in the analysis of impulse responses) will be further amplified. Finally, $\mu = 4.6937$ is calibrated to match labor supply $L_h = 0.25$ in equilibrium (25% of total hours are used for work).

Third, the calibration matches the model steady state to several U.S. long-run aggregate statistics from 1971 to 2015 (Table 2). The capital share in the output α is set to a conventional value 0.33, so that labor share in the production is close to 2/3. Recall consumption and investment data are obtained from National Income and Product Accounts (NIPA) as in Section 2. Output is then defined as the sum of the consumption and the investment. This implies that the investment-to-output ratio is 0.185, and the corresponding capital-to-output ratio is about 2, which together calibrate the depreciation rate $\delta = 0.185/2 = 0.0925$. The leverage ratio is set to $\lambda_l = 1.58$ ($\theta = 0.4128$ as a result), so that the average debt-to-output ratio is 74% as in the sample period.

In order to obtain a tractable solution, the distribution function $F(.)$ is set to a uniform distribution with the lower bound zero and the upper bound $\bar{\zeta}$. To check robustness, I experiment with Pareto distribution, normal distribution, and log-normal distribution, and with different parameters for these distributions. All of them yield similar quantitative results.[16] This is due to the fact that the dominating force is the option value of staying, instead of the exact distribution of ζ.

Finally, I use the three targets, the average capital liquidation-to-expenditures ratio (0.3), the average share of SPP&E within capital reallocation (0.31),[17] and the capital-to-output ratio (2) to jointly calibrate the partial saleability ϕ, the upper bound $\bar{\zeta}$, the entrepreneurs' discount factor β (see Appendix C).

Notice that λ_h in equilibrium is the same as λ_l. One may classify both types of firms as constrained firms. However, they are constrained for different reasons. Productive firms borrow and invest because of the higher return of capital than the real interest rate. Unproductive firms, on the contrary, do not invest. They are constrained because the option value of keeping the firm is high, and they need to borrow to finance enough dividends.

Since productive firms have higher profits from production than their unproductive pairs, the debt-to-asset ratios (13) that take into account profits are 0.33 for the productive firms and 0.39 for the unproductive ones. That is, when productivity rises, the debt-to-asset ratio falls, consistent with the findings of İmrohoroğlu and Tüzel (2014).

[16] Available upon requests.

[17] The averages of the L-E ratio and the SPP&E share are the sample means of 1984-2015 data. Since 1984, these two ratios are clearly stationary. That is why I target the two steady-state ratios to the sample means of 1984-2015 data. To have the longest yearly data, I still filter the data from 1971.

Table 2: **Parameters (Steady State)**

Parameter	Value	Explanation/Target
β^h	0.9800	Annual risk-free rate 2%
ε	1.0000	Household risk aversion
ν	1.0000	Labor supply elasticity = 1
μ	4.0931	Hours worked 0.25
p^{hh}	0.8450	Productivity persistence 0.69; standard deviation 0.18
p^{ll}	0.8450	$p^{ll} = p^{hh}$
z_h	2.1247	Productivity persistence 0.69; standard deviation 0.18
z_l	0.4707	$\log(\tilde{z}_l) = -\log(\tilde{z}_h)$
δ	0.0925	investment / output = 0.185
α	0.33	capital share $\approx 1/3$
β	0.9049	capital / output = 2
ϕ	0.0435	Share of SPP&E = 0.31
$\bar{\zeta}$	29.8792	Liquidation-to-expenditures ratio = 0.3
θ	0.4128	Debt-to-output = 0.74

Now, I solve the dynamics around the deterministic steady state using log-linear approximations. I follow the tradition by specifying AR(1) processes for shocks to aggregate productivity A_t, the external financial condition θ_t, and the intertemporal substitution γ_t:

$$\begin{aligned}
\log A_t &= \rho_A \log A_{t-1} + \epsilon_t^A \\
\log \gamma_t &= \rho_\gamma \log \gamma_{t-1} + \epsilon_t^\gamma \\
\log \theta_t &= (1 - \rho_\theta) \log \theta + \rho_\theta \log \theta_{t-1} + \epsilon_t^\theta
\end{aligned}$$

where ρ_A, ρ_γ, and ρ_θ are persistent parameters, and $\epsilon_t^A \sim N(0, \sigma_A^2)$, $\epsilon_t^\gamma \sim N(0, \sigma_A^2)$, and $\epsilon_t^\theta \sim N(0, \sigma_\theta^2)$ are i.i.d. normal random variables. The model is then estimated by using the observed cyclical components of output, debt, capital liquidation-to-expenditures ratio, and the SPP&E share. Because only three structure shocks are considered so far, I add an "observation error" shock $\epsilon_t^m \sim N(0, \sigma_m^2)$ to the observed cyclical components of SPP&E share in percentage terms. The observation error shock measures how much the model *cannot* explain the SPP&E share. This is because the model abstracts many aspects of partial liquidation and full liquidation as well as the substitution between the two types of liquidation.

The estimates are reported in Table 3. The financial shocks seem to be mostly persistent. Aggregate productivity shocks are somewhat persistent, while one cannot reject that intertemporal shocks have no persistence at all. The estimated standard deviation of observation error is only 12.5% and is significant (i.e., the t statistic is larger than 2). Though not perfect for the dynamics of SPP&E share, the model can explain a large fraction of the substitution between partial and full liquidation. This will become clearer in the variance decomposition. Finally, the estimated standard deviation of all three structural shocks are also significant.

Table 3: **Estimation Results**

	Estimates	Standard Deviations	t statistics
ρ_A	0.2955	0.1396	2.1163
ρ_γ	-0.0330	0.1377	0.2396
ρ_θ	0.9279	0.0499	18.5797
σ_A	0.0134	0.0014	9.4442
σ_γ	0.0510	0.0054	9.4244
σ_θ	0.0581	0.0061	9.5439
σ_m	0.1247	0.0132	9.5439

6.2 Quantitative Results

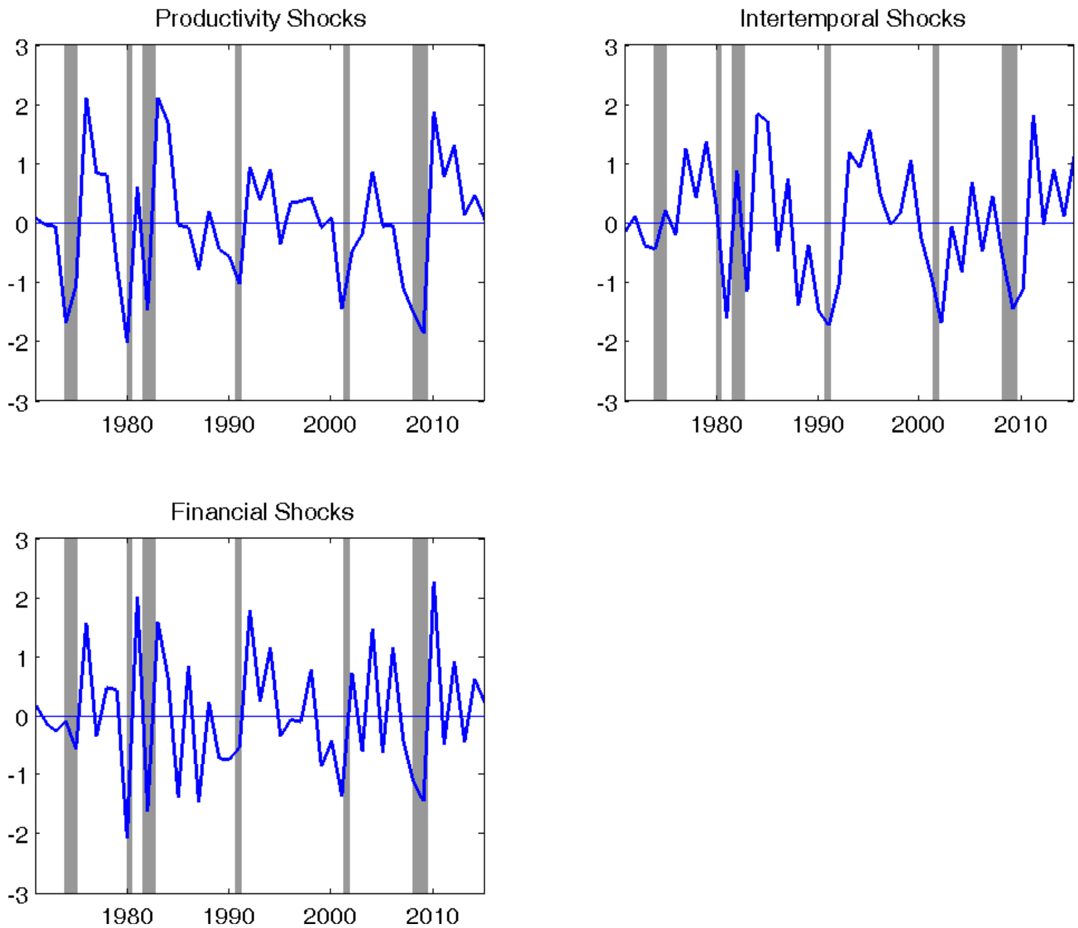
Three sets of quantitative results are shown in the following. First, I show the smoothed shocks from the maximum likelihood estimation. Second, I illustrate the impulse responses after one standard deviation innovations to each of the three exogenous states. Finally, I discuss the variance decomposition in detail.

Estimated Shocks

All three estimated structural shocks are plotted in Figure 3, normalized by their respective standard deviations. Through the lens of the model, the 2007-2009 recession (the "Great Recession") is indeed special compared to the previous ones. It has a combination of a sequence of adverse shocks to the external financial condition and aggregate productivity.

The Great Recession featured a large drop in liquidity and pledgeability of financial assets, which is captured in the model by the negative shocks to the collateral value θ. Note also the positive shocks to θ in the years prior to the Great Recession, which may reflect the real-estate boom and the surge of collateral assets in this period. After the recession, innovations to θ become positive but then stay almost at zero, possibly due to the asset-purchase programs implemented by the Federal Reserve in 2009-2010. As will become clear in impulse responses, financial shocks are quantitatively important for the business cycle, since they can move leverage significantly and thus affect capital reallocation, productivity, and aggregate activities.

Figure 3: **Estimated Innovations in Productivity, Intertemporal Substitutions, and Financial Conditions.** Note: All shocks are normalized by their respective standard deviations. Shaded areas are NBER dated recessions.



The negative shocks to intertemporal substitution in the 1991-1992, 2001-2002, and 2007-2009 recessions reflect the increase of savings from the household sector. In reality, some households borrow to buy housing properties or to buy durable goods. Although the model does not feature household debt, we can interpret this result as a consequence of household deleveraging.

Impulse Responses

I now use the model as a laboratory to examine the effects of one standard-deviation shock to aggregate productivity, external financial conditions, and intertemporal substitutions, respectively. All three shocks hit at time 0, and one can see the impact of shocks on the $A/\gamma/\theta$ panel in Figure 4.

As expected, a negative productivity shock generates a persistent fall in consumption, investment, and therefore output (dashed lines). Notice that the size of initial productivity shock is about 1.34%, and output falls by 1.4% on impact. Though the model features a financial accelerator through the borrowing constraint, there is not much amplification. Why is the financial accelerator not powerful? The answer is that the fall of aggregate productivity generates a "cleansing" effect, and such an effect improves resource allocation after the initial shock.

To see this, notice that with a lower profit rate (see the dynamics of π_t), productive firms have less net worth and can borrow less to invest, in both new and old capital. If z_l firms liquidate the same amount of capital as in the steady state, then z_h firms will shrink capital expenditures entirely through reducing (new) investment. Nevertheless, with a lower profit rate, z_l firms have less incentive to produce and stay in business; fewer z_l entrepreneurs remain to operate their firms, creating a cleansing effect. Full liquidation thus increases. As the capital expenditures from productive firms fall at the same time, the liquidation-to-expenditures ratio rises on impact and stays persistently above the steady-state level.

Another view of the cleansing effect against the financial accelerator is by looking at the adjusted TFP, i.e., the TFP after one eliminates the impact of A_t:

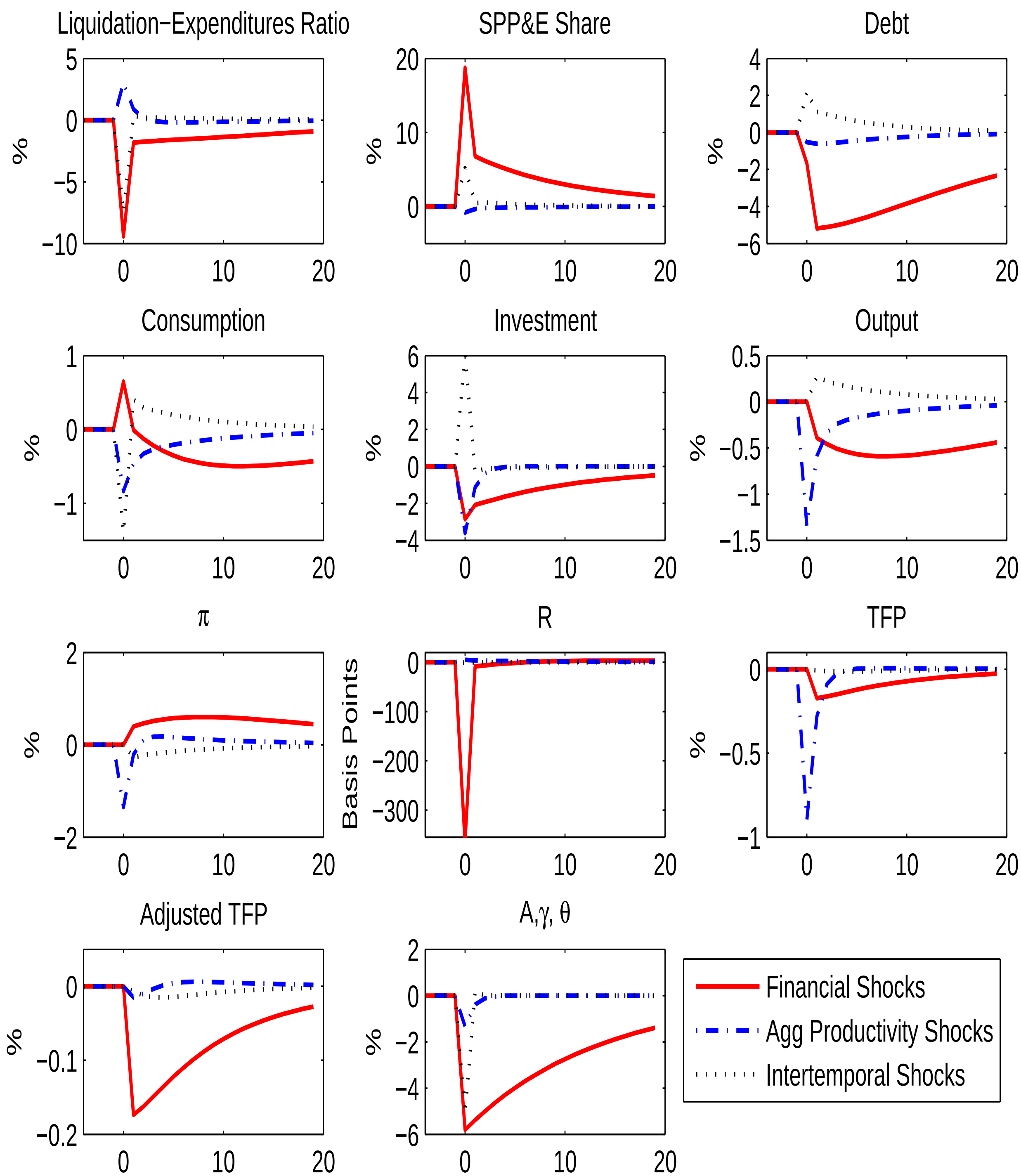
$$TFP_t^{adj} = \frac{Y_t}{K_t^{\alpha}(A_t L_t)^{1-\alpha}}$$

When A_t falls, the productive firms have less net worth and can thus borrow less (see the debt panel in Figure 4). Capital is worse allocated, and TFP_t^{adj} falls in period 0 and period 1. However, from period 2 onwards, the adjusted TFP is already above the steady-state level; it stays persistently above that level because of more liquidation of unproductive firms. In other words, the cleansing effect of adverse productivity shocks push unproductive firms to liquidate and transfer resources to productive ones. Unproductive firms liquidate more capital, save more in bonds, and lend more to productive firms. The cleansing effect prevents debt from falling dramatically, which facilitates the productive firms to reallocate capital from unproductive ones.

Financial shocks, on the other hand, do *not* generate the cleansing effect (solid lines). The performance of the model in response to financial shocks crucially depends on the impact of these shocks on capital liquidation. As shown in the first and second panel of Figure 4, the capital liquidation-to-expenditures ratio falls by about 10% on impact and stays persistently below the steady-state level, while the SPP&E share increases by about 18% on impact and also stays persistently above its steady-state level.

When financial constraints become tighter, productive firms can borrow less and invest

Figure 4: **Impulse Responses to Shocks** Note: All series plotted are changes from their steady-state (ss) levels. % in the figure means % of steady-state levels.



less both in new or old capital, an effect similar to Khan and Thomas (2013). However, the tightened financial constraints also increase the option value of staying of entrepreneurs who have unproductive firms, the key new effect in this paper as shown in the simple model. Reallocation of used capital is thus persistently delayed, and the magnitude is *not* mainly caused by the lower demand from productive firms. To understand this claim, recall that the capital liquidation-to-expenditures ratio is

$$\text{L-E ratio} = \frac{CL}{CL + I} = \frac{FL + PL}{FL + PL + I}$$

where FL is full liquidation, PL is partial liquidation, and I is new investment. To generate a falling L-E ratio, capital liquidation (CL) must fall more than the fall of investments (I). That is, the lower supply of liquidated capital is the main driving force behind the fall of reallocation. If instead the demand from productive firms is the main factor, CL and I should fall at approximately the same rate, leaving L-E ratio almost unchanged.

If full liquidation falls, then partial liquidation rises, which is reflected by the rise of SPP&E share in Figure 4. The magnitude of the falling full liquidation (FL) needs to be large enough to dominate the rise of partial liquidation (PL).

Importantly, besides the above partial-equilibrium effect of adverse financial shocks, the macro model showcases two general equilibrium effects that further delay full liquidation.

First, adverse financial shocks largely reduce investment and labor hiring from productive firms. Less competition in the labor market then reduces wages and pushes up π_t. Second, the shocks bring down the real interest rate and further increase the option value of staying. To see the second effect, notice again that productive firms are more constrained after the financial shocks. Aggregate TFP thus declines, followed by the reduction of real interest rate (on impact about 3.5%). The real interest rate stays below the steady-state level for almost five years before it returns to only slightly above that level.

The two general equilibrium effects further raise the option value of staying of unproductive firms. With the persistently high profit rate, the benefits of staying in business become higher; with the persistent low risk-free rate, the benefit of liquidating firms and saving the proceeds in risk-free bonds become smaller. For unproductive firms, holding on to capital is thus even more attractive. As a comparison, the real interest rate barely moves after aggregate productivity shocks.

Output does not fall on impact. But it falls one period after the hit of initial financial shock. Why? The initial fall of θ only changes capital liquidation and the efficiency of using capital today. It does not change production today, but it will reduce output tomorrow. The same reason implies that consumption rises initially with the fall of investment so that output remains at its steady-state level.

Though the reduction in θ is persistent, financial shocks generate much more persistently lower consumption, investment, and output. For example, output will be 0.5% below the steady-state level even after 19 years. This is because the debt level is still 3% lower than the steady-state level after 19 years, while one sees a much milder response in debt after aggregate productivity shocks. Part of the persistence is from the financial accelerator channel, and another part of the persistence comes from lower full liquidation and worsened capital misallocation.

What are the effects (represented by dotted lines) of intertemporal shocks? After a negative shock to γ, the household consumes less and saves more, which channels resources to the business sector. That is why on impact firm debt increases by 2.5% and investment increases by almost 5%. The rise of investment means that capital expenditures are less in the form of reallocation, so that we see a fall of liquidation-to-expenditures ratio. With the fall of reallocation, TFP falls at the same time. That is, a sudden increase of credit to the firm sector has a negative impact on efficiency, despite the fact that it raises investment and output.

Remark: adverse financial shocks raise the option value of staying of entrepreneurs with unproductive firms; the general equilibrium effects on profit rate and interest rate further push up the option value. Therefore, unproductive firms delay full liquidation after adverse financial shocks. The delays lead to persistent falls in reallocation, TFP, and output. Since the declines of aggregate TFP are endogenous, we do not need large and persistent variation in exogenous productivity A_t (recall the estimates of productivity process in Table 3).

Variance Decomposition

The impulse response exercises suggest that financial shocks are necessary to generate declined capital liquidation in recessions with the falling output. The dynamics of capital liquidation thus could provide some useful information of the source(s) of business cycles. The following variance-decomposition exercise shows how much the variation of the estimated model can be separately explained by financial shocks, aggregate productivity shocks, and intertemporal shocks.

Evidently, from Table 4, the dynamics of debt (96.51), L-E ratio (66.81), and SPP&E share (77.25) are mainly explained by financial shocks (shocks to collateral in the table). It follows from the previous impulse analysis that aggregate productivity shocks only mildly affect debt and capital liquidation. In fact, compared to financial shocks, aggregate productivity shocks move capital liquidation in the opposite direction. The intertemporal shocks affect the total supply of credit and investments, and therefore the shocks can explain 28.03% of the variance of liquidation-to-expenditures ratio.

The "error" shocks capture 19.16% of the variation in SPP&E share that cannot be explained by the other three structural shocks. Given that financial shocks are the most important shocks to explain SPP&E, one can say that financial shocks capture reasonably well the substitution between total liquidation and partial liquidation.

Table 4: **Variance Decomposition in Percent**

	Exogenous Shocks to			
	Collateral (1)	Productivity (2)	Savings (3)	"Errors"
$\frac{\text{Total Liquidation}}{\text{Capital Expenditures}}$	66.81	5.16	28.03	0
$\frac{\text{Partial Liquidation}}{\text{Total Liquidation}}$	77.25	0.12	3.47	19.16
Debt	96.51	0.72	2.77	0
Output	71.75	25.40	2.85	0
Consumption	59.50	14.45	26.05	0
Investment	51.70	16.33	31.97	0
TFP	16.36	83.49	0.15	0

Not surprisingly, most variations in consumption and investment are caused by financial and productivity shocks. At the same time, 26.05% of consumption variation and 31.97% of investment variation are attributed to intertemporal shocks. This is because such shocks affect consumption-saving decisions of the representative household. However, investment is different from the household savings, and that is why the intertemporal shocks explain a larger fraction of investment variation than that of consumption variation.

Regarding output, financial shocks can explain 71.75% of the variation, while productivity shocks can explain about 25.40%. The other two shocks barely move output. Compared to Jermann and Quadrini (2012), financial shocks can explain more of the output variation (about 25 percentage points more). One might suspect the difference is mainly due the fact that TFP is endogenous and financial shocks significantly affect TFP.

Nevertheless, there are two channels through which credit and liquidation impact the dynamics of output. On one hand, the financial flow affects the capital allocation among productive and unproductive firms. This is the *productivity effect* of financial flow. On the other hand, the flow also affects the firms' aggregate demand for capital and labor, and therefore aggregate production. This is the *factor effect* of the financial flow. The two effects clearly interact with each other.

To shed light on these two effects, the last row of Table 4 shows how much the variation of TFP in the model can be explained by each shock. Endogenous fluctuation of TFP due to the credit allocation is only 16.36%, which is much less important than the exogenous fluctuations in TFP due to direct productivity shocks (83.49%).

The impact of reallocation on TFP can be seen after log-linearizing TFP in (34)

$$\hat{TFP} = (1-\alpha)\hat{A} + \frac{\alpha\left(\bar{z}^h - \bar{z}^l\right)}{\bar{z}^h\bar{\rho}_k + \bar{z}^l}\hat{\rho}_k$$

where $\hat{TFP}$, $\hat{A}$, and $\hat{\rho}_k$ are log-deviation from their steady-state levels. In our calibration, $\alpha = 0.33$, $\bar{z}^h = 1.87$, $\bar{z}^l = 0.73$, and $\bar{\rho}_k = 2.70$. This means that the coefficient in front of $\hat{\rho}_k$ is about 0.065, and 1% variation in the ratio ρ_k can only lead to 0.065% variation in TFP. Such small variation in TFP comes from the fact that the steady-state $\bar{\rho}_k$ is already high. That is, the calibrated economy is already quite efficient, which is not that surprising given the effective functioning of US financial markets.[18]

Though financial flow does not generate large endogenous variation in TFP, the induced capital flow (from and to the productive firms) strongly affects the net worth of those productive firms, thereby changing capital accumulation and labor demand in the future. Notice that Jermann and Quadrini (2012) emphasize the variation in employment and production *directly* from financial shocks. The transmission mechanism in this paper is the variation in employment, investment, and output in the future arising *indirectly* from financial shocks. In this respect, the quantitative studies here complement Jermann and Quadrini (2012).

7 Final Remarks

This paper highlights how financial constraints affect the option value of staying when an unproductive firm is about to liquidate.

Due to liquidation costs, unproductive firms have an option value to stay, hoping to regain productivity in the future. These unproductive firms may not liquidate due to high liquidation costs. Meanwhile, because their profits are low, they might have to borrow to the credit limit to finance dividends. Unproductive firms may thus have higher debt-to-asset ratios than their productive pairs, a unique feature of the model.

When a recession is generated by adverse financial shocks, the option value of staying can increase. This reduces the unproductive firms' possibility of exercising the liquidation option. The falling wage rate and interest rate following the shocks further raise the option value, worsening the allocation of capital and deepening the recession.

The model presented has rich implications for interest rate and (capital) tax policies. For example, a lower interest rate facilitates productive firms to borrow more and reallocate

[18] To illustrate, I keep the technology but reduce $\bar{\rho}_k$ to 0.1. The coefficient in front of $\hat{\rho}_k$ then becomes 0.41, almost 6.31 times the previous calculation. In that case, the variation in reallocation due to financial shocks will have a much larger impact on output and thus TFP.

more capital; but it also reduces the incentive for unproductive firms to reallocate capital, as the benefit of liquidation is lower due to the falling interest rate.

In addition, a higher capital tax rate reduces the profit rate of productive firms, which distorts investment decisions in new and old capital. But for unproductive firms, it can also encourage more liquidation and reallocation, since a lower profit rate means a lower opportunity cost of exercising the liquidation option. All these explorations are beyond the scope of a single paper and are left for future research.

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Appendix

A Data

Financial data is from the Flow of Funds Accounts (Z1 report of Federal Reserve Board). I use the "Coded Table" released on December 8, 2016. New editions of the Coded Tables may use different coding, so one should take into account the changes for future uses.

Total business (corporate and non-corporate) debt in nominal terms is the sum of Debt Securities (Table F.102, item 30) and Loans (Table F.102, item 34). I use the GDP implicit price deflator, Table 1.1.9 in the National Income and Product Account (NIPA) to deflate total business debt into real terms (in 2009 dollars)

For aggregate consumption and investment in 2009 dollars, I use Table 1.1.3 in the NIPA account. I exclude residential investment, consumer durables, government expenditures, and net exports, because the model abstracts from these.

For firm-level capital reallocation, the COMPUSTAT (North America) contains useful information for ownership changes of productive assets started from 1971.

I first exclude all observations with Canadian dollars (because I focus on the U.S. market). Then, I follow Eisfeldt and Rampini (2006) and measure capital liquidation or capital reallocation of each firm by sales of property, plant and equipment (SPPE, data item 107 with combined data code entries excluded), plus acquisitions (AQC, data item 129 with combined data code entries excluded) from 1971 to 2015. The measure captures transactions after which the capital is used by a new firm (and new productivity is thus applied).

I also use capital spending (CAPX, data item 128). Since capital spending in COMPUSTAT excludes acquisitions, the level of capital expenditures of each firm is calculated as the sum of AQC and CAPX.

B Proofs

In the following, I prove results when the resale constraint is written as $k' \geq (1-\phi)(1-\delta)k$ and when there are aggregate risks (i.e., the value function is written as $v(k, b, z, \zeta; X)$). For the simple model, one can just let $\phi = 0$ and omit the aggregate state variable X.

Proof to Lemma 1

Given the aggregate state variable X and its evolution, I define the Bellman operator $\mathcal{T}$ as

$$\mathcal{T}v(k, b, z, \zeta; X) = \max\left\{v^0(k, b, z, \zeta; X), v^1(k, b, z; X)\right\}$$

$$v^0(k, b, z, \zeta; X) = \max_{b'}\left\{\log\left(z\pi k + Rb + (1-\delta)k - b'\right) - \zeta + \beta\mathbb{E}\left[v(0, b', z', \zeta'; X')|z, X\right]\right\}$$

$$v^1(k, b, z; X) = \max_{k', b'}\left\{\log(z\pi k + Rb + (1-\delta)k - k' - b') + \beta\mathbb{E}[v(k', b', z', \zeta'; X')|z, X]\right\}$$

$$\text{s.t. } R'b' \geq -\theta(1-\delta)k' \tag{35}$$

$$k' \geq (1-\phi)(1-\delta)k \tag{36}$$

The value function of an entrepreneur is the fixed point of the contraction mapping in some closed function space $\mathcal{V}_1$ (Stokey, Lucas, and Prescott, 1989). The value function is increasing in k, b, and z. To prove that the value function satisfies (7), I will prove the contraction mapping $\mathcal{T}$ preserves the same property if any arbitrary function v satisfies (7). Then, the unique fixed point $v = v^*$ satisfies (7).

Consider an entrepreneur with state (k, b, z, ζ) who chooses (k', b'). Now, consider another entrepreneur with $(\rho k, \rho b, z, \zeta)$, where $\rho > 0$. Notice that the policy $(\rho k', \rho b')$ is feasible, i.e., it satisfies the resource, the borrowing, and the resale constraints. Then, we know that $\mathcal{T}v$ can be either v^1 or v^0, and

$$\begin{aligned}\mathcal{T}v(\rho k,\rho b,z,\zeta;X) &\geq \log\rho(z\pi k+Rb+(1-\delta)k-k'-b')+\beta\mathbb{E}[v(k',b',z',\zeta';X')|z,X]+\frac{\beta\log\rho}{1-\beta}\\ &=\log(z\pi k+Rb+(1-\delta)k-k'-b')+\beta\mathbb{E}[v(k',b',z',\zeta';X')|z,X]+\frac{\log\rho}{1-\beta}\end{aligned}$$

$$\begin{aligned}\mathcal{T}v(\rho k,\rho b,z,\zeta;X) &\geq \log\rho(z\pi k+Rb+(1-\delta)k-b')-\zeta+\beta\mathbb{E}[v(0,b',z',\zeta';X')|z,X]+\frac{\beta\log\rho}{1-\beta}\\ &=\log(z\pi k+Rb+(1-\delta)k-b')-\zeta+\beta\mathbb{E}[v(0,b',z',\zeta';X')|z,X]+\frac{\log\rho}{1-\beta}\end{aligned}$$

That is,

$$\mathcal{T}v(\rho k,\rho b,z,\zeta;X)\geq v(k,b,z,\zeta;X)+\frac{\log\rho}{1-\beta}$$

Conversely, starting at $(\rho k,\rho b,z,\zeta)$, scaling by $1/\rho$, and following similar procedure, one has

$$\mathcal{T}v(k,b,z,\zeta;X)\geq\mathcal{T}v(\rho k,\rho b,z,\zeta;X)-\frac{\log\rho}{1-\beta}$$

Combining the two equations above, we have

$$\mathcal{T}v(\rho k,\rho b,z,\zeta;X)=\mathcal{T}v(k,b,z,\zeta;X)+\frac{\log\rho}{1-\beta}$$

Therefore, the mapping $\mathcal{T}$ preserves the same property. Because v is the unique fixed point, I conclude that $v(\rho k,\rho b,z,\zeta;X)=v(k,b,z,\zeta;X)+\frac{\log\rho}{1-\beta}$.

Proof to Proposition 1

To save notation, I write π and R' instead of $\pi(X)$ and $R'(X)$. I focus on the case in which an entrepreneur already has an existing firm, i.e., $k>0$. If the entrepreneur does not have a firm, the proof is almost identical except that there is no well-defined shadow value of capital.

If an entrepreneur decides to liquidate the existing firm

The value of doing so is $v(k,b,z,\zeta;X)=v^0(k,b,z,\zeta;X)$, where

$$\begin{aligned}v^0(k,b,z,\zeta;X)&=\max_{b'}\left\{\log(z\pi k+(1-\delta)k+Rb-b')-\zeta+\beta\mathbb{E}\left[v(0,b',z',\zeta';X')|z,X\right]\right\}\\ &=\max_{b'}\left\{\log(z\pi k+(1-\delta)k+Rb-b')-\zeta+\beta\mathbb{E}\left[v(0,1,z',\zeta';X')|z,X\right]+\frac{\beta\log(b')}{1-\beta}\right\}\end{aligned}\tag{37}$$

One can maximize out b' with the optimal solution $b'=\beta\left[z\pi k+(1-\delta)k+Rb\right]$, which implies that the consumption function is

$$c=(1-\beta)(z\pi k+(1-\delta)k+Rb)$$

We thus know that $v^0(k,b,z,\zeta;X)$ in (37) can be rewritten as

$$v^0(k,b,z,\zeta;X)=\log(1-\beta)+\frac{\beta\log\beta}{1-\beta}+\frac{\log(z\pi k+(1-\delta)k+Rb)}{1-\beta}-\zeta+\beta\mathbb{E}\left[v(0,1,z',\zeta';X')|z,X\right]$$

Since $\mathbb{E}\left[v(0,1,z',\zeta';X')|z,X\right]$ only depends on z and X, $v^0(k,b,z,\zeta;X)$ can be further simplified to

$$v^0(k,b,z,\zeta;X)=J^0(z;X)-\zeta+\frac{\log(z\pi k+(1-\delta)k+Rb)}{1-\beta}\tag{38}$$

where $J^0(z;X) = \log(1-\beta) + \frac{\beta\log\beta}{1-\beta} + \beta\mathbb{E}\left[v(0,1,z',\zeta';X')|z,X\right]$. In this case, the shadow value satisfies $q(\lambda,z,\zeta;X)=1$. I thus verify that $n(k,b,z,\zeta;X) = z\pi k+\phi(1-\delta)+(1-\phi)(1-\delta)qk+Rb = z\pi k+(1-\delta)k+Rb$, and the policy functions satisfy

$$c=(1-\beta)n, \quad k' = \frac{\lambda'}{(q-1)\lambda'+1}\beta n = \lambda'\beta n, \quad \text{and} \quad b' = \frac{1-\lambda'}{(q-1)\lambda'+1}\beta n = (1-\lambda')\beta n.$$

with λ' to be determined.

If an entrepreneur decides to keep the existing firm

The value of doing so is $v(k,b,z,\zeta;X) = v^1(k,b,z;X)$, where

$$v^1(k,b,z;X) = \max_{k',b'}\left\{\log(z\pi k+Rb+(1-\delta)k-k'-b') + \beta\mathbb{E}[v(k',b',z',\zeta';X')|z,X]\right\}$$

subject to (35) and (36). According to Lemma 1, we know that

$$v^1(k,b,z;X) = \max_{\lambda',k'}\left\{\log\left(z\pi k+(1-\delta)k+Rb-\frac{k'}{\lambda'}\right) + \frac{\beta\log(k'/\lambda')}{1-\beta} + \beta\mathbb{E}\left[v\left(\lambda',1-\lambda',z',\zeta';X'\right)|X\right]\right\} \tag{39}$$
$$\text{s.t. } 0<\lambda'\le\lambda_\theta \text{ and } (36)$$

$\lambda'=0$ is ignored because the resale constraint prevents λ' hitting 0. As in the main text $\lambda_\theta = \frac{1}{1-\theta(1-\delta)/R'}$ denote the upper bound of leverage ratio.

Given any λ', the first-order condition for k' is

$$\frac{-1/\lambda'}{z\pi k+(1-\delta)k+Rb-k'/\lambda'} + \frac{\beta}{(1-\beta)\,k'} + \mu_\phi = 0$$

where $\mu_\phi\ge 0$ is the Lagrangian multiplier attached to the resale constraint $k'\ge(1-\phi)(1-\delta)k$. For notation simplicity, I let $k'=(1-\varphi)(1-\delta)k$ where $\varphi\le\phi$. That is, the entrepreneur chooses to sell a $\varphi\le\phi$ fraction of residual capital $(1-\delta)k$. Then,

$$\frac{-1/\lambda'}{z\pi k+(1-\delta)k+Rb-(1-\varphi)(1-\delta)k/\lambda'} + \frac{\beta}{(1-\beta)\,(1-\varphi)(1-\delta)k} + \mu_\phi = 0$$

or,

$$(1-\varphi)(1-\delta)\frac{k}{\lambda'} = \frac{\beta+\mu_\phi(1-\varphi)(1-\delta)k(1-\beta)}{1+\mu_\phi(1-\varphi)(1-\delta)k(1-\beta)}\left[z\pi k+(1-\delta)k+Rb\right]$$

which together with the resources constraint $c=z\pi k+(1-\delta)k+Rb-k'/\lambda'$ implies that consumption is

$$c = \frac{(1-\beta)}{1+\mu_\phi(1-\varphi)(1-\delta)k(1-\beta)}\left[z\pi k+(1-\delta)k+Rb\right]$$

Notice that when the resale constraint is slack, $\mu_\phi=0$ and $(1-\varphi)(1-\delta)k/\lambda' = \beta\left[z\pi k+(1-\delta)k+Rb\right]$. Consumption function is thus similar to the case in which the entrepreneur liquidates the existing firm.

$$c=(1-\beta)\left[z\pi k+(1-\delta)k+Rb\right]$$

To replace the Lagrangian multiplier μ_ϕ, I define the shadow value $q\equiv q(\lambda,z,\zeta;X)$ that satisfies

$$\frac{z\pi+\phi(1-\delta)+(1-\phi)(1-\delta)q+R(\lambda^{-1}-1)}{z\pi+(1-\delta)+R(\lambda^{-1}-1)} = \frac{1}{1+\mu_\phi(1-\phi)(1-\delta)k(1-\beta)}$$

From this definition, when $\mu_\phi>0$ (i.e., the resale constraint is binding), $q=q(\lambda,z,\zeta;X)<1$. In addition, when $\mu_\phi > 0$, we know that $k' = (1-\phi)(1-\delta)k$ and

$$\mu_\phi(1-\phi)(1-\delta)k = \frac{1/\lambda'}{\left[z\pi + (1-\delta) + R(\lambda^{-1}-1)\right](1-\phi)^{-1}(1-\delta)^{-1} - 1/\lambda'} - \frac{\beta}{(1-\beta)}$$

Since λ' is a function of leverage λ and productivity z, $\mu_\phi(1-\phi)(1-\delta)k$ thus only depends on λ and z. This verifies that the shadow value q can be written as $q = q(\lambda, z, \zeta; X)$. When $\mu_\phi = 0$, we know that $q = 1$, consistent with the form $q = q(\lambda, z, \zeta; X)$.

To summarize, consumption and savings in capital and debt can be written as

$$c = (1-\beta)\left[z\pi + (1-\delta)\left[\phi + (1-\phi)q(\lambda, z, \zeta; X)\right] + R(\lambda^{-1}-1)\right]k \tag{40}$$

$$q(\lambda, z, \zeta; X)k' + b' = \beta\left[z\pi + (1-\delta)\left[\phi + (1-\phi)q(\lambda, z, \zeta; X)\right] + R(\lambda^{-1}-1)\right]k$$

no matter whether the resale constraint is binding or not. When the resale constraint binds, $q < 1$ solves

$$k' + b' = k'/\lambda' = \beta\left[z\pi + (1-\delta)q + R(\lambda^{-1}-1)\right]k/\left[(q-1)\lambda' + 1\right]$$

which proves (9). Now, $v^1(k, b, z; X)$ in (39) can be rewritten as

$$\begin{aligned} v^1(k, b, z; X) = {} & \log(1-\beta) + \frac{\beta\log\beta}{1-\beta} + \frac{\log(z\pi k + \phi(1-\delta)k + (1-\phi)(1-\delta)qk + Rb)}{1-\beta} \\ & + \beta\max_{\lambda' \le \lambda_\theta}\left\{-\frac{\log\left(1 + (q-1)\lambda'\right)}{1-\beta} + \mathbb{E}\left[v\left(\lambda', 1-\lambda', z', \zeta'; X'\right)|z, X\right]\right\} \end{aligned}$$

Since $\max_{\lambda' \le \lambda_\theta}\left\{-\frac{\log\left(1+(q-1)\lambda'\right)}{1-\beta} + \mathbb{E}\left[v\left(\lambda', 1-\lambda', z', \zeta'; X'\right)|X\right]\right\}$ only depends on $(\lambda, z; X)$, we know that $v^1(k, b, z; X)$ has the form

$$v^1(k, b, z, \zeta; X) = J^1(\lambda, z; X) + \frac{\log(z\pi k + \phi(1-\delta) + (1-\phi)(1-\delta)qk + Rb)}{1-\beta} \tag{41}$$

where $J^1(\lambda, z; X) = \log(1-\beta) + \frac{\beta\log\beta}{1-\beta} + \beta\max_{0<\lambda' \le \lambda_\theta}\left\{-\frac{\log\left(1+(q-1)\lambda'\right)}{1-\beta} + \mathbb{E}\left[v\left(\lambda', 1-\lambda', z', \zeta'; X'\right)|X\right]\right\}$. I thus again verify $n(k, b, z, \zeta; X) = z\pi k + \phi(1-\delta) + (1-\phi)(1-\delta)qk + Rb$ and the policy functions

$$c = (1-\beta)n, \quad k' = \frac{\lambda'}{(q-1)\lambda' + 1}\beta n, \quad \text{and} \quad b' = \frac{1-\lambda'}{(q-1)\lambda' + 1}\beta n.$$

with λ' to be determined.

Remark: When an entrepreneur is investing in capital, $q = 1$ and $J^1(z, \lambda; X)$ is even independent of λ.

Remark: The consumption function has the algebraic form in (40), regardless of whether the entrepreneur chooses to invest, partially sell capital, or liquidate the entire firm. According to (38) and (41), I can define net worth as $n \equiv n(k, b, z, \zeta; X) = z\pi k + \phi(1-\delta) + (1-\phi)(1-\delta)qk + Rb$ and state the value function as

$$v(k, b, z, \zeta; X) = J(\lambda, z, \zeta; X) + \frac{\log n(k, b, z, \zeta; X)}{1-\beta} \tag{42}$$

Solving portfolio weights

The remaining task is to solve

$$\max_{\lambda' \le \lambda_\theta}\left\{-\frac{\log\left(1 + (q-1)\lambda'\right)}{1-\beta} + \mathbb{E}\left[v\left(\lambda', 1-\lambda', z', \zeta'; X'\right)|z, X\right]\right\}$$

From the envelop condition,[19] we know that $v_k(k,b,z,\zeta;X) = \frac{z\pi+(1-\delta)q}{(1-\beta)[z\pi k+(1-\delta)qk+Rb]}$ and $v_b(k,b,z,\zeta;X) = \frac{R}{(1-\beta)[z\pi k+(1-\delta)qk+Rb]}$. This implies that the first-order condition becomes

$$\mathbb{E}\left[\frac{z'\pi'+\phi(1-\delta)+(1-\phi)(1-\delta)q'-R'}{[z'\pi'+\phi(1-\delta)+(1-\phi)(1-\delta)q']\lambda'+R'(1-\lambda')}|z,X,\right]+\mu_\lambda = \frac{q-1}{1+(q-1)\lambda'}$$

where $\mu_\lambda \geq 0$ is the Lagrangian multiplier attached to the borrowing constraint $\lambda' \leq \lambda_\theta$. After dividing both sides by $[1+(q-1)\lambda']^{-1}$, we have

$$\mathbb{E}\left[\frac{z'\pi'+\phi(1-\delta)+(1-\phi)(1-\delta)q'-R'}{\frac{q\lambda'}{(q-1)\lambda'+1}\left[\frac{z'\pi'+\phi(1-\delta)+(1-\phi)(1-\delta)q'}{q}-R'\right]+R'}|z,X\right]+[1+(q-1)\lambda']^{-1}\mu_\lambda = q-1$$

Now, I subtract $q-1$ on both sides and obtain

$$\mathbb{E}\left[\frac{\frac{z'\pi'+\phi(1-\delta)+(1-\phi)(1-\delta)q'}{q}-R'}{\frac{q\lambda'}{(q-1)\lambda'+1}\left[\frac{z'\pi'+\phi(1-\delta)+(1-\phi)(1-\delta)q'}{q}-R'\right]+R'}|z,X\right]+\frac{q}{1+(q-1)\lambda'}\mu_\lambda = 0$$

Since $\mu_\lambda \geq 0$, we thus prove (8).

A Discussion

First, if $k=0$, i.e., the entrepreneur does not have a firm, the proof is almost identical except that there is no well-defined shadow value of capital.

Second, if we model the liquidation cost in terms of consumption goods, the same results apply as long as the liquidation cost is proportional to the net worth of the entrepreneur. To see this, suppose the liquidation cost is $\hat{\zeta}\,(z\pi k+(1-\delta)k+Rb)$ units of consumption goods. One can redefine the net worth net of liquidation cost as

$$n(k,b,z,\zeta;X) = (1-\hat{\zeta})\,[z\pi k+(1-\delta)k+Rb]$$

Then, the only change is that (38) becomes $v^0(k,b,z,\zeta;X) = J^0(z;X) - \frac{\log(1-\hat{\zeta})}{1-\beta} + \frac{\log(z\pi k+(1-\delta)k+Rb)}{1-\beta}$. Proposition 1 still holds, if we set $\zeta = \log(1-\hat{\zeta})/(1-\beta)$.

Proof to Lemma 2

First, when an entrepreneur draws z_h, the entrepreneur's problem is interesting only if he invests. We know that he does not sell and the resale cost ζ does not matter; further, $q(\lambda,z,\zeta;X)=1$ so that current leverage λ does not matter either. Therefore, for some function $J_h(X)$, we know

$$J(\lambda,z_h,\zeta;X) = J_h(X)$$

Second, when the entrepreneur draws z_l, the entrepreneur can choose between staying and liquidating. When he decides to liquidate, the leverage λ does not affect $J(\lambda,z_l,\zeta;X)$. Therefore, $q(\lambda,z_l,\zeta;X)=1$ and

$$J(\lambda,z_l,\zeta;X) = J_0(X) - \zeta$$

[19] The envelope condition requires differentiability of $v(k,b,z,\zeta;X)$. When $k' > (1-\phi)(1-\delta)k$ and $k' < (1-\phi)(1-\delta)k$ are standard, which relies on the differentiability of standard dynamic programming problem as proved by Benveniste and Scheinkman (1979) (or see again Stokey, Lucas, and Prescott (1989)). When $k' = (1-\phi)(1-\delta)k$, I follow methods from Clausen and Strub (2012) in Banach space (the space of k and b) and adjust to the dynamic programming problem of my model. The general idea is that the value function is the upper envelop of value function of buying, inactive, and selling. It is therefore super-differentiable. At the same time, it has potential downward kink (sub-differentiable). Therefore, the value function will be both super-differentiable and sub-differentiable, and thus differentiable. The detail derivation is long and tedious but available upon request.

for some function $J_0(X)$. When he decides to stay, ζ does not affect $J(\lambda, z_l, \zeta; X)$. Therefore, $q(\lambda, z_l, \zeta; X) = q_l(\lambda, \lambda'; X)$ for some $\lambda' = \lambda_l'(\lambda; X)$ chosen by the entrepreneur, and we know that for some function $J_l(\lambda; X)$

$$J(\lambda, z_l, \zeta; X) = J_l(\lambda; X)$$

Now, I need to show that there exists the liquidation threshold $\tilde{\zeta}_l = \tilde{\zeta}_l(\lambda, \lambda'; X)$ that gives rise to $q_l(\lambda, \lambda'; X)$. Whether to liquidate the existing firm depends on two values given by the two choices according to (42). Those z^l entrepreneurs liquidate the entire firm when ζ is smaller than $\tilde{\zeta}_l$ that satisfies

$$J_l(\lambda; X) + \frac{\log(z_l \pi k + \phi(1-\delta)k + (1-\phi)(1-\delta)q_l(\lambda, \lambda'; X)k + Rb)}{1-\beta} \geq J_0(X) + \frac{\log(z_l \pi k + (1-\delta)k + Rb)}{1-\beta} - \tilde{\zeta}_l$$

where $\tilde{\zeta}_l = 0$ (a boundary case) if " $>$". Further notice that after subtracting $\log k$ on both sides, we immediately see that the threshold level $\tilde{\zeta}_l$ solves

$$\tilde{\zeta}_l = J_0(X) - J_l(\lambda; X) + \Delta_l(\lambda, \lambda'; X) \equiv \tilde{\zeta}_l(\lambda, \lambda'; X) \tag{43}$$

where $\Delta_l(\lambda, \lambda'; X) = \log\left(\frac{z_l\pi + 1 - \delta + R(\lambda^{-1} - 1)}{z_l\pi + \phi(1-\delta) + (1-\phi)(1-\delta)q_l(\lambda, \lambda'; X) + R(\lambda^{-1} - 1)}\right)$. Therefore, $\tilde{\zeta}_l = \tilde{\zeta}_l(\lambda, \lambda'; X)$ which completes the proof.

Proof to Proposition 2

I normalize an entrepreneur's net worth to be $n = 1$. Out of the net worth, he consumes $(1-\beta)$ fraction and saves β fraction. Denote $\lambda' = k'/(k' + b')$ as the resulting leverage chosen by the entrepreneur, then $\frac{q}{q - 1 + 1/\lambda'}$ is the share of net worth spent on capital (and the rest is spent on bonds).

For an entrepreneur who draws z_l today and does not liquidate the whole firm, the value function $v(k, b, z, \zeta; X) = J(\lambda, z_l, \zeta; X) + \frac{\log n}{1-\beta}$ is equivalent to (when $n = 1$)

$$\begin{aligned}
J_l(\lambda; X) = \log(1-\beta) &+ p^{lh}\beta\mathbb{E}_X\left[J_h(X') + \frac{\log\beta\left(\tilde{\lambda}'\left(z_h\pi' + 1 - \delta - R'\right) + R'\right)}{1-\beta}\right] \\
&+ p^{ll}\beta\mathbb{E}_X\int_{\tilde{\zeta}_l'}^{+\infty}\left[J_l(\lambda'; X') + \frac{\log\beta\left(\tilde{\lambda}'\left(z_l\pi' + \phi\left(1-\delta\right) + (1-\phi)(1-\delta)q_l' - R'\right) + R'\right)}{1-\beta}\right]dF(\zeta') \\
&+ p^{ll}\beta\mathbb{E}_X\int_0^{\tilde{\zeta}_l'}\left[J_0(X') - \zeta' + \frac{\log\beta\left(\tilde{\lambda}'\left(z_l\pi' + (1-\delta) - R'\right) + R'\right)}{1-\beta}\right]dF(\zeta')
\end{aligned} \tag{44}$$

where the right-hand side includes the utility from consumption $\log(1-\beta)$ and three possible continuation values next period. To save notation, I denote $\lambda' = \lambda_l'(\lambda; X)$, $\tilde{\lambda}' = \frac{q_l(\lambda, \lambda'; X)}{q_l(\lambda, \lambda'; X) - 1 + 1/\lambda'}$, $\tilde{\zeta}_l' = \tilde{\zeta}_l(\lambda', \lambda_l'(\lambda'); X')$, and $q_l' = q_l(\lambda', \lambda_l'(\lambda'; X'); X')$.

For an entrepreneur who draws z_l today and liquidates the whole firm, the value function $v(k, b, z, \zeta; X) = J(\lambda, z_l, \zeta; X) + \frac{\log n}{1-\beta}$ is equivalent to

$$J_0(X) - \zeta = \log(1-\beta) - \zeta + \frac{\beta\log(\beta R')}{1-\beta} + \beta p^{lh}\mathbb{E}_X\left[J_h(X')\right] + p^{ll}\beta\mathbb{E}_X\left[J_0(X')\right] \tag{45}$$

where the right-hand side includes the utility from consumption $\log(1-\beta)$, the liquidation cost ζ, and two possible continuation values next period.

For an entrepreneur who draws z_h, one can obtain a similar recursion for $J_h(X)$. This recursion, together with (44) and (45), solve $J_h(X)$, $J_l(\lambda; X)$, and $J_0(X)$ jointly. To save space, I do not write the recursion.

Finally, I subtract (45) from (44), and I use (43) to obtain the forward looking equation for $\tilde{\zeta}_l$ and complete the proof:

$$\frac{\Delta_l(\lambda,\lambda';X)}{1-\beta}-\tilde{\zeta}_l(\lambda,\lambda';X) \tag{46}$$
$$=\frac{\beta\mathbb{E}_X\left(\log\tilde{\lambda}'+\sum_j p^{lj}\log\left(\frac{z_j\pi'+(1-\delta)}{R'}+\frac{1}{\lambda'}-1\right)\right)}{1-\beta}+p^{ll}\beta\mathbb{E}_X\left[-\tilde{\zeta}(\lambda',\lambda_l'(\lambda');X')+\int_0^{\tilde{\zeta}_l'}F(\zeta')d\zeta'\right]$$

Proof to Proposition 3

Suppose the entrepreneur's resale constraint does not bind. We know that $q=1$. He thus consumes $(1-\beta)\left[z_l\pi+1-\delta+R(\lambda^{-1}-1)\right]$ and saves $\beta\left[z_l\pi+1-\delta+R(\lambda^{-1}-1)\right]$. If

$$\lambda_\theta\beta\left[z_l\pi+1-\delta+R(\lambda^{-1}-1)\right]<1-\delta \tag{47}$$

then even if the entrepreneur borrows to the limit, i.e., $\lambda'=\lambda_\theta$, he has to sell capital to finance dividends which violates the resale constraint. Therefore, the resale constraint must bind together with financial constraint, and we have $\lambda_\theta\beta\left[z_l\pi+(1-\delta)\,q+R(\lambda^{-1}-1)\right]=(1-\delta)\left[(q-1)\lambda_\theta+1\right]$. According to (47), we thus prove that when λ is sufficiently large, i.e.,

$$\lambda>\frac{\lambda_\theta R}{\beta R/(1-\delta)+\lambda_\theta\left[R-z_l\pi-(1-\delta)\right]}$$

the two constraints bind together. Notice that $\frac{\lambda_\theta R}{\beta R/(1-\delta)+\lambda_\theta(R-z_l\pi-(1-\delta))}<\lambda_\theta$, because it is equivalent to $\lambda_\theta\left[z_l\pi+1-\delta-R\right]+R<\beta^{-1}(1-\delta)$, which is satisfied under assumptions (A1) and (A2).

Proof to Proposition 4

I omit aggregate state variable X for notation simplicity. Let me write again the definition of $H^r(\lambda,\lambda_\theta)$

$$H^r(\lambda,\lambda_\theta)=\beta\frac{p^{lj}\sum_j\log\left(\frac{z_j\pi'+(1-\delta)}{R'}+\frac{1}{\lambda_\theta}-1\right)-\log\left(q_l(\lambda,\lambda_\theta)-1+\frac{1}{\lambda_\theta}\right)}{1-\beta}-\frac{\Delta_l(\lambda_\theta,\lambda_\theta)}{1-\beta}$$

First, we can use the definition of $\Delta_l(\lambda,\lambda_\theta)$ to reach

$$\begin{aligned}(1-\beta)H^r(\lambda,\lambda_\theta)=&-\beta\log\left(q_l(\lambda,\lambda_\theta)+\lambda_\theta^{-1}-1\right)-\beta\log R\\&+\beta p^{lh}\log\left(z_h\pi'+(1-\delta)+R'(\lambda_\theta^{-1}-1)\right)+\beta p^{ll}\log\left(z_l\pi'+(1-\delta)+R'(\lambda_\theta^{-1}-1)\right)\\&+\log\left(z_l\pi+(1-\delta)q_l(\lambda,\lambda_\theta)+R(\lambda^{-1}-1)\right)-\log\left(z_l\pi+(1-\delta)+R(\lambda^{-1}-1)\right)\end{aligned}$$

To replace $q_l(\lambda,\lambda_\theta)$ in $(1-\beta)H^r(\lambda,\lambda_\theta)$, I use (9), $\beta\left[z_l\pi+(1-\delta)q_l(\lambda,\lambda_\theta)+R(\lambda^{-1}-1)\right]=(1-\delta)\left[q_l(\lambda,\lambda_\theta)+\lambda_\theta^{-1}-1\right]$ to reach

$$q_l(\lambda,\lambda_\theta)+\lambda_\theta^{-1}-1=\frac{\beta}{(1-\beta)(1-\delta)}\left[z_l\pi+1-\delta+R(\lambda^{-1}-1)-(1-\delta)\lambda_\theta^{-1}\right]$$

Now, $(1-\beta)H^r(\lambda,\lambda_\theta)$ can be stated as

$$\begin{aligned}(1-\beta)H^r(\lambda,\lambda_\theta)=&\,C(\beta,\delta,R)-\beta\log\left(z_l\pi+1-\delta+R(\lambda^{-1}-1)-(1-\delta)\lambda_\theta^{-1}\right)\\&+\beta p^{lh}\log\left(z_h\pi'+(1-\delta)+R'(\lambda_\theta^{-1}-1)\right)+\beta p^{ll}\log\left(z_l\pi'+(1-\delta)+R'(\lambda_\theta^{-1}-1)\right)\\&+\log\left(z_l\pi+1-\delta+R(\lambda^{-1}-1)-(1-\delta)\lambda_\theta^{-1}\right)-\log\left(z_l\pi+(1-\delta)+R(\lambda^{-1}-1)\right)\end{aligned} \tag{48}$$

where $C(\beta,\delta,R)$ is collection of terms that only depend on β, δ, and R.

When there are no shocks to factor prices, $\pi=\pi'$ and $R=R'$. Now, replacing λ by λ_θ and taking the derivative w.r.t λ_θ^{-1}, I obtain

$$\frac{\partial(1-\beta)H^r(\lambda_\theta,\lambda_\theta)}{\partial\lambda_\theta^{-1}} = \beta\left[-\frac{[R-(1-\delta)]}{z_l\pi+(1-\delta)+R(\lambda_\theta^{-1}-1)-(1-\delta)\lambda_\theta^{-1}} + \sum_j \frac{p^{lj}R}{z_j\pi+(1-\delta)+R(\lambda_\theta^{-1}-1)}\right] + \frac{[R-(1-\delta)]}{z_l\pi+(1-\delta)+R(\lambda_\theta^{-1}-1)-(1-\delta)\lambda_\theta^{-1}} - \frac{R}{z_l\pi+(1-\delta)+R(\lambda_\theta^{-1}-1)} \tag{49}$$

Notice first that the term in the big bracket of (49) is bounded below by

$$\frac{R}{z_h\pi+(1-\delta)+R(\lambda_\theta^{-1}-1)} - \frac{R-(1-\delta)}{z_l\pi+(1-\delta)+R(\lambda_\theta^{-1}-1)-(1-\delta)\lambda_\theta^{-1}}$$

which is non-negative if and only if

$$\frac{R\pi}{1-\delta}(z_h-z_l) \le z_h\pi+1-\delta-R$$

This proves the condition stated in the beginning of the proposition. Second, the term outside of the big bracket in (49) is strictly positive because

$$\frac{[R-(1-\delta)]}{z_l\pi+(1-\delta)+R(\lambda_\theta^{-1}-1)-(1-\delta)\lambda_\theta^{-1}} - \frac{R}{z_l\pi+(1-\delta)(1-d)+R(\lambda_\theta^{-1}-1)} > 0$$

is equivalent to $z_l\pi+1-\delta<R$, which has to be true in equilibrium. In summary, I prove that $H^r(\lambda_\theta,\lambda_\theta)$ decreases with λ_θ because

$$\frac{\partial H^r(\lambda_\theta,\lambda_\theta)}{\partial\lambda_\theta} = \underbrace{\frac{\partial H^r(\lambda_\theta,\lambda_\theta)}{\partial\lambda_\theta^{-1}}}_{>0}\underbrace{\frac{\partial\lambda_\theta^{-1}}{\partial\lambda_\theta}}_{<0} < 0$$

Proof to Proposition 5

Let x represents $\zeta(\lambda_\theta,\lambda_\theta)$ and $h^l(x)=H^l(x,x)$. Then, we know that $h^l(1)=0$. Taking derivative w.r.t. x, one has

$$dh^l(x)/dx = -(1-\beta p^{ll}) - \beta p^{ll}F(x) = -1+\beta p^{ll}(1-F(x)) < 0$$

since β, p^{ll}, and $F(x)$ are all in $(0,1)$. h^l is thus a decreasing function, and this implies that $\tilde{\zeta}_l(\lambda_\theta,\lambda_\theta) = \left(h^l\right)^{-1}(H^r(\lambda_\theta,\lambda_\theta))$. Since $H^r(\lambda_\theta,\lambda_\theta)$ is also a decreasing function of λ_θ, $\tilde{\zeta}_l(\lambda_\theta,\lambda_\theta)$ is thus an increasing function of λ_θ, which completes the proof.

Proof to Corollary 1

The goal is to prove that when β is sufficiently large, $H^r(\lambda_{\theta_l},\lambda_{\theta_l})-H^r(\lambda_{\theta_h},\lambda_{\theta_l})<0$. For exposition purposes, I prove $(1-\beta)\left[H^r(\lambda_{\theta_l},\lambda_{\theta_l})-H^r(\lambda_{\theta_h},\lambda_{\theta_l})\right]<0$. Using the identity in (48), I have

$$\begin{aligned}
&(1-\beta)\left[H^r(\lambda_{\theta_l},\lambda_{\theta_l})-H^r(\lambda_{\theta_h},\lambda_{\theta_l})\right]\\
=&(1-\beta)\log\left(\frac{z_l\pi+(1-\delta)+R(\lambda_{\theta_l}^{-1}-1)-(1-\delta)\lambda_{\theta_l}^{-1}}{z_l\pi+(1-\delta)+R(\lambda_{\theta_h}^{-1}-1)-(1-\delta)\lambda_{\theta_l}^{-1}}\right)+\log\left(\frac{z_l\pi+(1-\delta)+R(\lambda_{\theta_h}^{-1}-1)}{z_l\pi+(1-\delta)+R(\lambda_{\theta_l}^{-1}-1)}\right)\\
=&(1-\beta)\log\left(\frac{x_l-(1-\delta)\lambda_{\theta_l}^{-1}}{x_h-(1-\delta)\lambda_{\theta_l}^{-1}}\right)+\log\left(\frac{x_h}{x_l}\right)
\end{aligned}$$

where I have used $x_l=z_l\pi+(1-\delta)+R(\lambda_{\theta_l}^{-1}-1)$ and $x_h=z_l\pi+(1-\delta)+R(\lambda_{\theta_h}^{-1}-1)$ to simplify notation. Since $\lambda_{\theta_l}<\lambda_{\theta_h}$, it must be true that $x_l>x_h$.

Now, notice that $(1-\beta)\left[H^r(\lambda_{\theta_l}, \lambda_{\theta_l}) - H^r(\lambda_{\theta_h}, \lambda_{\theta_l})\right] < 0$ is equivalent to

$$\left(\frac{x_l - (1-\delta)\lambda_{\theta_l}^{-1}}{x_h - (1-\delta)\lambda_{\theta_l}^{-1}}\right)^{1-\beta} < \frac{x_l}{x_h} \tag{50}$$

Suppose $\beta > \xi$ where $\xi \in [0,1]$, then a sufficient condition for (50) is $\frac{x_l-(1-\delta)\lambda_{\theta_l}^{-1}}{x_h-(1-\delta)\lambda_{\theta_l}^{-1}} < \left(\frac{x_l}{x_h}\right)^{\frac{1}{1-\xi}}$. Because $\frac{x_l-(1-\delta)\lambda_{\theta_l}^{-1}}{x_h-(1-\delta)\lambda_{\theta_l}^{-1}} > \frac{x_l}{x_h} > 1$, the sufficient condition is satisfied when $\xi \to 1$. In other words, when β is sufficiently large (i.e., close to one), (50) is satisfied and $(1-\beta)\left[H^r(\lambda_{\theta_l}, \lambda_{\theta_l}) - H^r(\lambda_{\theta_h}, \lambda_{\theta_l})\right] < 0$ is true. According to (50), the threshold value of β is $\beta^* = 1 - \log\left(\frac{x_l}{x_h}\right) - \log\left(\frac{x_l-(1-\delta)\lambda_{\theta_l}^{-1}}{x_h-(1-\delta)\lambda_{\theta_l}^{-1}}\right)$.

Proof to Corollary 2

The proof is represented by the proof to Proposition 1 and the proof to Proposition 2.

C The Macro Model

In this part, I provide some omitted details in Section 5.

C.1 Entrepreneurs' Profits

From the main text, the profits are $\Pi(z_{it}, k_{it}; w_t) = \max_{l_{it}}\{(z_{it}k_{it})^\alpha (A_t h_{it})^{1-\alpha} - w_t h_{it}\}$. The first-order condition for labor hours is $A_t^{1-\alpha}(z_{it}k_{it})^\alpha(1-\alpha)h_{it}^{-\alpha} = w_t$, so that the optimal labor demand is $h_{it}^* = z_{it}k_{it}\left[\frac{(1-\alpha)A_t^{1-\alpha}}{w_t}\right]^{1/\alpha}$. Profits are thus

$$\begin{aligned}
\Pi(z_{it}, k_{it}; w_t) = (z_{it}k_{it})^\alpha (A_t h_{it})^{1-\alpha} - w_t h_{it} &= A_t^{1-\alpha} z_{it}k_{it}\left[\left[\frac{(1-\alpha)\,A_t^{1-\alpha}}{w_t}\right]^{\frac{1-\alpha}{\alpha}} - w_t\left[\frac{(1-\alpha)\,A_t^{1-\alpha}}{w_t}\right]^{\frac{1}{\alpha}}\right] \\
&= A_t^{\frac{1-\alpha}{\alpha}} z_{it}k_{it}\left[\frac{(1-\alpha)}{w_t}\right]^{1/\alpha}\left(\frac{w_t}{1-\alpha} - w_t\right) = z_{it}\pi_t k_{it}.
\end{aligned}$$

where $\pi_t = \alpha\left[\frac{(1-\alpha)A_t}{w_t}\right]^{\frac{1-\alpha}{\alpha}}$.

C.2 A Collection of Equilibrium Conditions

From the main text, we know that there are six endogenous state variables B_H, λ_h, λ_l, K_h, K_l, B and three exogenous variables. All other equilibrium objects are functions of these state variables.

Instead of writing down six equations of evolution for the six endogenous states, which involves various substitutions of variables, I choose to solve 17 unknowns C_H, L_H, B'_H, λ'_h, λ'_l, K'_h, K'_l, B', π, R, w, q_{hl}, q_{ll}, Δ_{hl}, Δ_{ll}, $\tilde{\zeta}_{hl}$, $\tilde{\zeta}_{ll}$ together from 17 equilibrium conditions, given exogenous stochastic processes (A, γ, θ). These equilibrium conditions are

(1). Households' decisions:

$$\mu L_H^\nu = w$$

$$\mathbb{E}_X\left[\frac{\gamma'\beta^h u'\left(C'_H - \frac{\mu(L'_H)^{1+\nu}}{1+\nu}\right)}{\gamma u'\left(C_H - \frac{\mu(L_H)^{1+\nu}}{1+\nu}\right)}R'\right] = 1$$

$$C_H + B'_H = wL_H + RB_H$$

(2). Entrepreneurs portfolio choices and liquidation decisions:

$$\lambda'_l = \frac{1}{1-\theta(1-\delta)/R'}$$

$$p^{hh}\mathbb{E}_X\left[\frac{z_h\pi' + 1 - \delta - R'}{\lambda'_h\left[z_h\pi' + 1 - \delta - R'\right] + R'}\right] + p^{hl}\mathbb{E}_X\left[1 - F(\tilde{\zeta}'_{hl})\right]\left[\frac{z_l\pi' + (1-\delta)\,q_{hl} - R'}{\lambda'_h\left[z_l\pi' + (1-\delta)q_{hl} - R'\right] + R'}\right]$$
$$+ p^{hl}\mathbb{E}_X F(\tilde{\zeta}'_{hl})\left[\frac{z_l\pi' + (1-\delta) - R'}{\lambda'_h\left[z_l\pi' + (1-\delta) - R'\right] + R'}\right] \leq 0$$

$$\beta\left[z^l\pi + \phi(1-\delta) + (1-\phi)(1-\delta)q_{lj} + R(\lambda_j - 1)\right] = (1-\phi)(1-\delta)\left(q_{lj} - 1 + 1/\lambda'_l\right)$$

$$\Delta_{lj} = \log\left(\frac{z^l\pi + (1-\delta) + R_s(\lambda_l^{-1} - 1)}{z^l\pi + \phi(1-\delta) + (1-\phi)(1-\delta)q_{lj} + R(\lambda_l^{-1} - 1)}\right)$$

$$-\tilde{\zeta}_{jl} + \beta p^{ll}\mathbb{E}_X\left[\tilde{\zeta}_{ll} - \int_0^{\tilde{\zeta}'_{ll}} F(\zeta')d\zeta'\right] = \beta\mathbb{E}_X\frac{\sum_j p^{lj}\log\left(\frac{z_j\pi' + (1-\delta)}{R'} + \frac{1}{\lambda'_l} - 1\right)}{1-\beta} - \beta\frac{\log\left(q_{jl} + \frac{1}{\lambda'_l} - 1\right)}{1-\beta} - \frac{\Delta_{jl}}{1-\beta}$$

with $\tilde{\zeta}_{jl} = 0$ if there is no $\tilde{\zeta}_{jl}$ that satisfies the above equation.

(3). Wealth evolution conditions:

$$K'_h = \lambda'_h\beta\sum_j\left[z_h\pi + 1 - \delta + R(\lambda_j^{-1} - 1)\right]p^{jh}K_j + \lambda'_h\beta R p^{lh}B$$

$$K'_l = (1-\phi)(1-\delta)\sum_j\left[1 - F(\tilde{\zeta}_{lj})\right]p^{jl}K_j$$

$$B' = \beta\sum_j F(\tilde{\zeta}_{jl})\left[z_l\pi + (1-\delta) + R\left(\lambda_j^{-1} - 1\right)\right]p^{jl}K_j + \beta R p^{ll}B$$

(4). Market clearing conditions:

$$\pi = \alpha\left(\frac{(1-\alpha)A}{w}\right)$$

$$\sum_j\left[(\lambda_j)^{-1} - 1\right]K_j + B + B_h = 0$$

$$\left(\frac{\pi}{\alpha}\right)^{\frac{1}{1-\alpha}}\left[\left(p^{lh}z_h + p^{ll}z_l\right)K_l + \left(p^{hh}z_h + p^{hl}z_l\right)K_h\right] = AL_h$$

C.3 Calibration

All parameters to be calibrated

- Parameters related to the representative household: β^h, ε, ν, and μ
- Parameters related to entrepreneurs and production technology: p^{hh}, p^{ll}, z_h, z_l, α, β, δ, θ, ϕ, and $\bar{\zeta}$

The calibration of household parameters, p^{hh}, p^{ll}, z_h, z_l, and α are detailed in the main text. I use five data targets to calibrate β, δ, ϕ, θ, and $\bar{\zeta}$. The five targets are partial liquidation / total liquidation (shares of SPP&E) ρ_1, capital liquidation-expenditure ratio ρ_2, investment / output ratio ρ_3, capital / output ratio ρ_4, and leverage constraint λ_θ (coming from sample debt-to-output ratio). Notice that the model implied ρ_1, ρ_2, ρ_3, and ρ_4 are

$$\rho_1 = \frac{\left[(1-F_h)p^{hl}\rho_k + (1-F_l)p^{ll}\right]\phi}{F_h p^{hl}\rho_{K_h} + F_l p^{ll} + \left[\left[(1-F_h)\right]p^{hl}\rho_k + \left[(1-F_l)\right]p^{ll}\right]\phi} \tag{51}$$

$$\rho_2 = \frac{\left(F_h p^{hl}\rho_k + F_l p^{ll}\right)(1-\delta)(1+\rho_1)}{\rho_k - \left(p^{hh}\rho_k + p^{lh}\right)(1-\delta)} \tag{52}$$

$$\rho_3 = \frac{\rho_k - \left(p^{hh}\rho_k + p^{lh}\right)(1-\delta) - \left(F_h p^{hl}\rho_k + F_l p^{ll}\right)(1-\delta)(1+\rho_1)}{\frac{\pi}{\alpha}\left[p^{lh}z^h + p^{ll}z^l + \rho_k\left(p^{hh}z^h + p^{hl}z^l\right)\right]} \tag{53}$$

$$\rho_4 = \frac{K_h + K_l}{Y} = \frac{\rho_k + 1}{Y/K_l} = \frac{\rho_k + 1}{\frac{\pi}{\alpha}\left[p^{lh}z^h + p^{ll}z^l + \rho_k\left(p^{hh}z^h + p^{hl}z^l\right)\right]} \tag{54}$$

where $\rho_k = K_h/K_l$, $F_h = F(\tilde{\zeta}_{hl})$, and $F_l = F(\tilde{\zeta}_{ll})$.

First, I show how to calibrate δ, ϕ, and θ. Using (53) and (54), we can substitute out π/α, which yields

$$\frac{\delta(\rho_{K_h}+1)}{\rho_3} = \frac{\rho_{K_h}+1}{\rho_4} \to \delta = \rho_3/\rho_4 \tag{55}$$

From (51) again, we know that

$$F_h p^{hl}\rho_k + F_l p^{ll} = (1-\rho_1)\left[\left(p^{hl}\rho_k + p^{ll}\right) - \frac{1}{1-\delta}\right] \tag{56}$$

Using (56), I substitute out $F_h p^{hl}\rho_k + F_l p^{ll}$ in (52) so that

$$\frac{\rho_2}{1+\rho_2} = \frac{\left[p^{hl}\rho_k + p^{ll}\right](1-\delta) - 1}{\delta(\rho_k+1)} \to \rho_k = \frac{p^{ll}(1-\delta) - 1 - \frac{\rho_2\delta}{1+\rho_2}}{\frac{\rho_2\delta}{1+\rho_2} - p^{hl}(1-\delta)} \tag{57}$$

Once having ρ_k, we can back out $F_h p^{hl}\rho_k + F_l p^{ll}$ and π from (56) and (54). ϕ is then calibrated from (30) which becomes

$$1 = \left[(1-F_h)p^{hl}\rho_k + (1-F_l)p^{ll}\right](1-\phi)(1-\delta)$$

Since I target $\lambda_l = \lambda_\theta$, one can compute θ from $\lambda_l = \frac{1}{1-\theta(1-\delta)/R}$.

Next, I show how to calibrate β and $\bar{\zeta}$. Since λ_h, q_h, and q_l can be calculated as

$$\left(q_{hl} - 1 + \lambda_l^{-1}\right)(1-\phi)(1-\delta) = \beta\left[z_l\pi + \phi(1-\delta) + (1-\phi)(1-\delta)q_{hl} + R\left(\lambda_h^{-1} - 1\right)\right]$$

$$\left(q_{ll} - 1 + \lambda_l^{-1}\right)(1-\phi)(1-\delta) = \beta\left[z_l\pi + \phi(1-\delta) + (1-\phi)(1-\delta)q_{ll} + R\left(\lambda_l^{-1} - 1\right)\right]$$

$$\lambda_h = \min\left\{-\frac{R(p^{lh}\left[z_h\pi + (1-\delta)\right] + p^{ll}\left[z_l\pi + \phi(1-\delta) + (1-\phi)(1-\delta)q_{hl}\right] - R)}{\left(z_h\pi + (1-\delta) - R\right)\left(z_l\pi + \phi(1-\delta) + (1-\phi)(1-\delta)q_{hl} - R\right)}, \lambda_\theta\right\}$$

I find that $\lambda_h = \lambda_\theta$ with the previous calibration, and therefore we know that $F_h = F_l$. To calibrate β, I solve $\frac{B}{K_l}$ and β together from the two conditions for the evolution of wealth (29) and (31)

$$\frac{B}{K_l} = \frac{1}{p^{lh}\beta R}\left\{\frac{\rho_k}{\lambda_h} - \beta\left[z_h\pi + (1-\delta) + R(\lambda_h^{-1} - 1)\right]p^{hh}\rho_k - \beta\left[z_h\pi + (1-\delta) + R(\lambda_l^{-1} - 1)\right]p^{lh}\right\}$$

$$\frac{B}{K_l} = F_h\beta\left[z_l\pi + (1-\delta) + R(\lambda_h^{-1} - 1)\right]p^{ll}\rho_k + F_l\beta\left[z_l\pi + (1-\delta) + R(\lambda_l^{-1} - 1)\right]p^{ll} + p^{ll}\beta R\frac{B}{K_l}$$

Finally, we calibrate the upper bound $\bar{\zeta}$ of the uniform distribution from (25)

$$-\bar{\zeta}F_l + p^{ll}\beta\mathbb{E}_X\left[\bar{\zeta}F_l - \frac{\bar{\zeta}F_l^2}{2}\right] = \frac{\beta}{1-\beta}\mathbb{E}_X\sum_j p^{lj}\log\left(\frac{z_j\pi + (1-\delta)}{R} + \frac{1}{\lambda_l'} - 1\right) - \frac{\beta\log\left(q_{jl} + \frac{1}{\lambda_l'} - 1\right)}{1-\beta} - \frac{\Delta_{jl}}{1-\beta}$$