

## Bayesian Estimation of DSGE Models: identification using a diagnostic indicator

CFM-DP2018-25

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Many macroeconomists have expressed concern about the extent to which identification of structural, or DSGE, models may or may not have been achieved during estimation.

Reflecting the rapid progress of Bayesian estimation techniques, it is now common practice to estimate DSGE models rather than to simply calibrate them. The problem is, however, that if a parameter is not identified, this means that the data (and the prior) cannot pin down the value of this parameter, and if a parameter is only weakly identified, this means that a small change in, say, the sample variation causes a large change in the parameter estimate. Compared with standard linear identification problems in econometrics, DSGE models involve nonlinear estimation with many theoretical parameter restrictions and accordingly identification may be considerably more problematic.

As a result of these problems, two strands of diagnostic indicators have been developed. The first line of indicators sets an intermediate target and investigates the Jacobian of such a target with respect to the deep parameters of a model. This line of indicators has been pioneered by Iskrev (2010a), Iskrev and Ratto (2010) and Komunjer and Ng (2011). Typically, this intermediate target is a set of data moments. If the Jacobian of the data moments is column rank deficient, there are two possibilities; (i) one or more parameters do not affect any data moments at all; and (ii) a change in one parameter is totally offset by changes in other parameters and hence again may not affect any moments. The latter case, which is presumably more common than the former, is often referred to as partially identified or perfect collinearity among parameters. Iskrev (2010a) also proposes a check of the Jacobian of the reduced form parameters with respect to the deep parameters, so-called Iskrev's  $J_2$ .

The second line of indicators, such as Koop et al. (2013, KPS henceforth) and Iskrev (2010b), exploits the Information matrix, which is the expectation of the Hessian. This idea is very straightforward: if the likelihood function is flat along a particular direction at a likelihood mode, i.e. the Hessian is singular, the value of the likelihood (or posterior density) does not change along this direction and hence there are infinitely many combinations of parameters that achieve the maximum likelihood. The main difference between KPS and Iskrev (2010b) is that the former is

mainly interested in the identification by data, whereas Iskrev (2010b) checks the identification by both the prior and data. This point is very important and we will discuss this more deeply in our main analysis. One practical weakness of this second approach is that, as opposed to the Jacobian based methods, if the Hessian is singular it may be hard, if not impossible, to pin down the maximum point. This is because nearly all maximizing algorithms require a non-singular (i.e., strictly negative definite) Hessian; otherwise, the likelihood mode is not well defined. This Catch-22 problem seems to be common for most Hessian-based approaches.

The purpose of our paper is to investigate the KPS indicator. KPS suggest two separate methods for checking the presence and strength of identification of the parameters of DSGE models. Their first indicator is based on Bayesian theory. Suppose, for example, that it is not known if a parameter is identified or not. If it is unidentified, 'the marginal posterior of this parameter will equal the posterior expectation of the prior of this parameter conditional on the identified parameters'. The second method, relying on asymptotic theory, says that the precision of a parameter estimate will increase at the rate of the data size  $T$ , if it is identified. One merit of this second method lies in the simplicity of its implementation: in practice, it does not require any additional (time consuming) programming or simulations because it just examines the Hessian (or posterior variances) for (artificial) data sets with different sizes. As we shall explain, all a researcher then has to do, when estimating any model, is simply to check the speed at which the parameter precision increases. On the basis of our results, we will recommend using an Identification Ratio that compares the estimates with a sample of either 1,000 or 5,000 observations with those of 10,000.

The results are clear enough to allow us to make a number of observations. As many researchers use Smets and Wouters (SW), or its variants, as a testing ground for their identification methods, we are thus able to match our results in using a very simple indicator with theirs, see, for example, Iskrev (2010a) and Iskrev and Ratto (2010). Although we will need to investigate other key models, as well, to be conclusive, broadly speaking, because our findings on the SW model are consistent with other results, we should continue to be cautious about whether estimated parameters are indeed identified. The issues on identification in such a widely cited model, suggest that there continues to be a question mark about whether Bayesian estimation of DSGE models generates more heat than light. And so what we can suggest is that we should accordingly use a simple indicator to examine identification. In our view a little more clarity about identification when using DSGE procedures would aid and focus the debate on the development of models with more realistic economic structures. The regular use of KPS when estimating any DSGE model would allow the reader to make up their own mind on the question of whether model identification has been achieved of the estimates presented.