Agnostic Structural Disturbances (ASDs): Detecting and Reducing Misspecification in Empirical Macroeconomic Models

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Abstract

Exogenous random structural disturbances are the main driving force behind fluctuations in most business cycle models and typically a wide variety is used. This paper documents that a minor misspecification regarding structural disturbances can lead to large distortions for parameter estimates and implied model properties, such as impulse response functions with a wrong shape and even an incorrect sign. We propose a novel concept, namely an agnostic structural disturbance (ASD), that can be used to both detect and correct for misspecification of the structural disturbances. In contrast to regular disturbances and wedges, ASDs do not impose additional restrictions on policy functions. When applied to the Smets-Wouters (SW) model, we find that its risk-premium disturbance and its investment-specific productivity disturbance are rejected in favor of our ASDs. While agnostic in nature, studying the estimated associated coefficients and the impulse response functions of these ASDs allows us to interpret them economically as a risk-premium/preference and an investment-specific productivity *type* disturbance as in SW, but our results indicate that they enter the model quite differently than the original SW disturbances. Our procedure also selects an additional wage mark-up disturbance that is associated with increased capital efficiency.

Key Words: DSGE, full-information model estimation, structural disturbances
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1 Introduction

Exogenous random shocks are the lifeblood of modern macroeconomic business cycle models. These shocks enter the model as innovations to structural disturbances that affect key aspects of the model. Whereas the prototype real business cycle (RBC) model features total factor productivity (TFP) as the only structural disturbance, recent generations of business cycle models typically include a multitude of structural disturbances. To avoid singularities when estimating a model, one needs at least as many unobserved random disturbances as observables. These random shocks can take the form of measurement error or structural disturbances. The larger the role for measurement error, the smaller the role of the theoretical model.[1] Thus, if researchers want the theoretical model to explain an important part of the data and they do not want to set aside information contained in additional observables, then they need to come up with a sufficiently large set of structural disturbances.

Incorporating structural disturbances correctly is nontrivial and it is not enough to have the right set. Structural disturbances impose (cross-equation) restrictions on model equations and, thus, on the model's solutions. Thus, each structural disturbance has to enter *each* model equation correctly. This is a real concern, since we often do not have independent evidence on how structural disturbances should affect the system. For example, should a risk-premium disturbance affect all Euler equations or only those of a specific type of investment? Is it correct to assume that structural disturbances are uncorrelated as is commonly done? Chari, Kehoe, and McGrattan (2007) propose "wedges" as alternatives to standard structural disturbances. However, it is important to realize that wedges also impose restrictions. For example, suppose one adds a "labor wedge" to the labor first-order condition. The assumption that this wedge does not enter any other equation implies restrictions on how this disturbance affects policy rules.

The contributions of this paper are threefold. First, based on a series of econometric estimation exercises using data generated from a known economic model, we document that a *minor* misspecification of the empirical model regarding structural disturbances can easily lead to *large* distortions for parameter estimates and model properties, such as business cycle statistics and impulse response functions (IRFs). Specifically, we consider the case where the empirical model wrongly excludes one of the structural disturbances and wrongly includes another. Everything else is correctly specified, including functional forms. Even though we adjust parameter values to ensure that this is a relatively minor type of misspecification, the results can be very drastic. For example, standard deviations as implied by the misspecified model are frequently multiples of their true values and correlation coefficients and IRFs can flip sign. These results are due solely to misspecification, since we use large samples and a consistent estimator.

[1] There is a fundamental difference between measurement error and structural random disturbances. The latter are part of the economic model and their shocks affect the system through time according to the equations of the model. Measurement error does not.

Second, we propose agnostic structural disturbances (ASDs) as an alternative *structural* disturbance. In contrast to regular structural disturbances, ASDs impose *no* additional restrictions on policy rules. Nevertheless, they are very different from measurement error, because they are *structural* disturbances and propagate through the system like regular structural disturbances. Our ASD procedure can be used in two ways. First, it can be used to test whether regular structural disturbances are correctly specified. Second, an empirical specification can be enriched by adding ASDs as additional structural disturbances. Using Monte Carlo experiments, we document that the ASD procedure is capable of detecting and correcting for misspecification in samples of typical size.

The third contribution of our paper is to test whether the structural disturbances of the model in Smets and Wouters (2007) (SW) are correctly specified using the same US postwar data set. We find that the risk-premium and the investment-specific productivity disturbance are not correctly specified. We use our procedure to improve on the SW empirical specification. Specifically, our preferred specification (based on marginal likelihood considerations) has three ASDs and excludes the SW risk-premium and the SW investment-specific disturbance.

A nice feature of our procedure is that its outcomes provide insights into the nature of the agnostic disturbances. That is, although the ASD procedure itself does not rely on any theory, the estimation results – both the associated coefficients and their IRFs – may reveal a lot about the type of structural disturbance the data has identified.

One of the ASDs in our adjusted empirical specification of the SW model has a strong impact on the investment Euler equation and plays an important role for the fluctuations in investment. While the same is true for the standard investment-specific disturbance used in SW, our ASD enters the capital accumulation with a different sign than the investment-specific disturbance and also has a direct positive effect on capacity utilization. This ASD could capture an "investment-modernization" disturbance that positively affects the return on new investment, but goes together with an increased depreciation of *existing* capital. The latter would imply that this disturbance affects the capital accumulation with the opposite sign as a standard investment disturbance, consistent with our empirical results. The direct effect on utilization could compensate for this scrapping of existing vintages of capital.

The second ASD shares similarities with the SW risk-premium disturbance. Specifically, it plays a key role in the bond Euler equation. However, the way it enters the capital valuation equation indicates it is a preference disturbance, not a risk-premium disturbance. Interestingly, Smets and Wouters (2007) prefer the risk-premium disturbance over the preference disturbance of Smets and Wouters (2003) because it generates a positive comovement of the main economic aggregates, whereas a preference disturbance does not. Our ASD generates a typical business cycle even though it affects the capital valuation equation like a preference disturbance. The reason is that it also has an important impact on the investment Euler equation. Another noteworthy feature of this ASD is that it directly affects the policy rate. This indicates that the central bank responds differently to economic developments when these are due to changes in investors' required rates of return.

The third ASD has an important impact on the wage mark-up. Whereas the SW wage mark-up disturbance only affects one equation, our ASD also has an important effect on the capital value, the utilization, and the capital accumulation equation. Specifically, the data indicate that increased upward wage pressure goes together with more efficient use of capital. This disturbance has a very temporary impact. In contrast to the first two ASDs, this ASD does not replace a SW disturbance. Leaving the SW wage mark-up disturbance out of our preferred empirical specification reduces the marginal data density substantially. However, including this ASD does substantially lower the value of the MA coefficient in the ARMA representation of the SW wage mark-up disturbance.

In our application, we could give a sensible interpretation to each of the three ASDs like one often can do for wedges. Similar to wedges, there may not be a unique interpretation.[2] However, with wedges the researcher has to take a stand on where the wedges enter the model. The whole idea about our procedure is that it starts by being agnostic and it lets the data decide where and how ASDs should enter each model equation.

In the next section, we discuss the outcomes of our misspecification experiments, in which we generate data using the SW model as the data generating process (*dgp*) and then estimate parameters with slightly misspecified empirical models. We use large samples, so the results are only due to misspecification and not to sampling variation. Section 3 provides a general discussion and motivation of our proposed misspecification detection and correction procedure. Section 4 describes how to use ASDs in practice. Section 5 documents the ability of ASDs to detect and correct for misspecification using Monte Carlo experiments for a typical application. We use again the SW model to generate data and the same type of misspecification of the empirical model as in section 2. But now we use a sample length of typical size. Section 6 discusses the results when our procedure is applied to the SW model on US data.

2 Large sample consequences of misspecification

In this section, we consider the consequences of estimating a misspecified empirical model. Specifically, we generate data with a known structural business cycle model and then estimate parameters using a misspecified empirical specification. We focus on large sample properties and use a Maximum Likelihood (ML) estimator, which is consistent in this environment. Thus, the results presented are not due to sampling variation. We document that even a minor misspecification can lead to substantial distortions in parameter estimates. These distortions matter in the sense that they imply model properties that are quite different from the true ones. In fact, even

[2]For example, regarding our third ASD, it is possible that it captures a higher wage mark-up that induces a more efficient use of capital. But it is also possible that it captures a desire to use capital more efficiently and that the higher wage mark-up is the price firms have to pay to obtain this efficiency increase.

implied model moments for variables that are used as observables in the estimation can deviate substantially from their data counterparts (which represent the truth given that we focus on large sample properties).

2.1 The true underlying model for our experiment

We use the New Keynesian model of Smets and Wouters (2007), the workhorse model of empirical business cycle analysis, as the basis of our analysis. Parameter values of the true data generating process are set equal to those of the SW posterior mode.[3] The list of parameters estimated and their interpretation is given in table 1.[4]

2.2 The specification of the empirical model

The original SW model has seven exogenous random variables. Those are a TFP disturbance, ε_t^a, a risk-premium disturbance, ε_t^b, a government spending disturbance, ε_t^g, an investment-specific disturbance, ε_t^i, a monetary policy disturbance, ε_t^r, a price mark-up disturbance, ε_t^p, and a wage mark-up disturbance, ε_t^w. We leave out one of these seven disturbances when generating data for our misspecification experiments. The empirical specification also leaves out one disturbance, *but* not the right one. This means we have $7 \times 6 = 42$ experiments. Everything else is always correctly specified, including functional forms, specification of the processes for the exogenous random variables, and the values of the parameters that are not estimated. The observables used in SW consists of employment, the federal funds rate, the inflation rate, GDP, consumption, investment, and the real wage rate. We exclude the real wage rate so we have the same number of observables as structural disturbances which is consistent with the empirical exercise in SW.

Is this a likely misspecification? We believe that this type of misspecification is likely to be important in practice even if one includes a large set of structural disturbances. The first reason is that having a large set does not necessarily imply one includes all the true disturbances. Moreover, one does not only need to include all

[3]The only exception is the parameter ρ_{ga}, which captures the impact of the TFP structural disturbance on the government expenditures structural disturbance. We set this coefficient equal to zero in both the true *dgp* and in the empirical model. This implies that all structural disturbances are uncorrelated. This is a typical assumption and makes our misspecification experiment more transparent. As discussed below, the misspecification considered is related to the specification of the set of structural disturbances. If $\rho_{ga} \neq 0$, then we would have to make additional choices whenever the misspecification involves either the TFP or the government spending shock. We explored some alternative cases in which $\rho_{ga} \neq 0$ and found similar results.

[4]We follow SW and do not estimate the depreciation rate, δ, the steady-state wage mark-up, $\overline{\mu}$, the steady-state level of government expenditures, $\overline{g}$, the curvature in the Kimball goods-market aggregator, ε_p, and the curvature in the Kimball labor-market aggregator, ε_w. Since we use demeaned data, we also fix the trend growth rate, $\overline{\gamma}$, the parameter controlling steady state hours, $\overline{l}$, the parameter controlling steady state inflation, $\overline{\pi}$, and the discount factor, β.

Table 1: Parameter explanations

α	Capital share
σ_c	Inverse IES of consumption
Φ	Fixed cost in production
ϕ	Elasticity of adjustment cost function
λ	Degree of consumption habits
ξ_w	Degree of wage rigidity
σ_ℓ	Inverse IES of leisure
ξ_p	Degree of price rigidity
ι_w	Degree of indexation for wages
ι_p	Degree of indexation for prices
ψ	Elasticity of capital utilization adj. cost function
r_π	Taylor rule coefficient on inflation
ρ	Degree of interest rate smoothing in Taylor rule
r_y	Taylor rule coefficient on output gap
$r_{\Delta y}$	Taylor rule coefficient on change in output gap
ρ_j	Persistence of exogenous disturbance j
μ_j	MA coefficient of exogenous disturbance j
σ_j	Standard deviation of exogenous disturbance j
	$j \in \{a, b, g, I, r, p, w\}$

Notes. The table reports the parameters of the SW model that are estimated and their interpretation. The list of exogenous disturbances is given in the text.

true disturbances, *each* disturbance has to enter *each* model equation correctly. For example, a TFP disturbance is typically modeled as a labor-augmenting productivity shock, but productivity changes could affect the production function differently. Moreover, TFP increases may also affect other aspects of the production process such as the depreciation rate.[5]

Is this a "minor" misspecification? When generating the data, we adjust the standard deviation of the disturbance that is incorrectly excluded from the empirical specification to ensure that it is responsible for at most 10% of the volatility for *any* of the six observables used in the estimation. By doing this we reduce the quantitative importance of the misspecification.

One could argue that a misspecification is only minor if one would not detect it in a typical data set using some model selection criterion such as the marginal likelihood. This is a very strict requirement. Comparing a misspecified model with the correct one requires that researchers are aware of the correct specification and test their empirical model against it. Since structural disturbances can enter models in many different ways, researchers may not consider the correct one even if they consider several alternatives.[6] In this section, we use very large samples and misspecified models would be rejected against the truth. In section 2.6, we select two of the forty-two experiments and data sets of typical length and document using a computer intensive Monte Carlo analysis that a test comparing the misspecified model with the correct model would often *not* lead to a rejection of the misspecified model. This supports our claim that the misspecification considered here is indeed minor.

Estimation procedure. DSGE models are typically estimated with Bayesian techniques, which means that the estimation outcome is a weighted combination of the prior and the empirical likelihood. Misspecification of the empirical model affects the latter. With a tight prior, observed data – and thus misspecification of the likelihood – matter less for posterior estimates. Then, the quality of those estimates will depend on the quality of the prior. This paper focuses on the question how misspecification affects what the observed data imply for parameter estimates and implied model properties. Thus, we focus on the likelihood and use Maximum Likelihood estimation.[7]

[5]Similarly, Cúrdia and Reis (2012) argue that assumptions about the correlation of structural disturbances are important and that one can question the standard assumption that structural disturbances in macroeconomic models are not correlated.

[6]Indeed, although the SW empirical specification is a very carefully constructed model that incorporates insights of many previous empirical studies, it is still rejected against some minor modifications, as is shown in 6.

[7]Our optimization problem is relatively well defined. It helps, of course, that our experiments rely on very large samples and on empirical models that are only misspecified in terms of the driving processes. Moreover, we use the true parameter values as the initial conditions for the optimization routine and we specify bounds for the parameter values. These choices decrease computing time and also give a misspecified model the best possible chance to deliver estimates that are close to the truth. The innovation standard deviations of the disturbances are restricted to be in the interval $[0, 10]$

In practice, there could be interesting interactions between the misspecification of the empirical model and small sample properties of the estimator. We abstract from small sampling variation by using a large enough sample. In particular, our experiments are based on a sample of 10,000 observations. Our estimator is consistent and estimates are very close to the truth when the empirical model is correctly specified. Section 5 studies the small sample properties in detail for two out of the forty-two misspecification experiments.

Priors on the standard deviation of structural disturbances typically do not allow for point mass at zero. Ferroni, Grassi, and León-Ledesma (2015) point out that this biases the results towards a positive role of all structural disturbances.[8] This is not an issue for us, since we use ML estimation. In fact, estimated standard deviations of disturbances that are part of the empirical model but *not* part of the true *dgp* turn out to be often close to zero.[9]

Identification. In appendix B, we document that the estimated parameter values are identified using a strong version of the identification test of Komunjer and Ng (2011). This is true according to the correct and the misspecified empirical model. Thus, none of the results should be driven by non-identification rather than misspecification. Further justification for this claim is given in section 2.5.

2.3 Misspecification: Consequences for parameter values

Table 2 reports some key percentiles (across experiments) to characterize the range of the estimated parameter values. We only consider parameters that are in both the true and empirical specification.[10] All parameters are affected by misspecification to some extent. Moreover, the minor misspecifications considered in these forty-two experiments lead to massive distortions for several parameter estimates.

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and the coefficients of their time series process in the interval $[0, 99]$. Given our focus on misspecified disturbances, we want these intervals to be large. For the structural parameters we set the lower bound and the upper bound to the first and ninety-ninth percentile according to the SW prior, centered at the parameter values of the true *dgp*.

[8]There are several differences between their and our setup. They only consider one specific misspecified empirical model whereas we consider forty-two. Although they consider a limited Monte Carlo experiment (with 100 replications), the main discussion focuses on particular sample of 200 observations. In this section, we abstract from small sample issues by focusing on one very long sample. Most importantly, their main focus is on the consequences of using an inverse gamma prior for parameters that could well be zero. Our focus is on the misspecification of the empirical model, not the specification of the prior.

[9]In those cases, the role of the structural disturbance that is wrongly excluded from the empirical specification is "taken over" by some of the correctly included disturbances, not the one that is wrongly included.

[10]Specifically, for the parameters of the exogenous random processes, the experiments in which the disturbance is part of the empirical model – but not part of the true *dgp* – are excluded from the calculations of the percentiles.

Table 2: Parameter values: Point estimates across misspecification experiments

	Truth	Imposed Min	Min	10%	25%	Median	75%	90%	Max	Imposed Max
α	0.19	0.07	0.07	0.11	0.17	0.19	0.20	0.23	0.31	0.31
σ_c	1.39	0.53	0.53	0.78	1.14	1.35	1.60	1.82	2.25	2.25
Φ	1.61	1.33	1.33	1.33	1.53	1.77	1.89	1.89	1.89	1.89
ϕ	5.48	1.99	2.71	3.59	5.47	7.38	8.97	8.97	8.97	8.97
λ	0.71	0.45	0.45	0.59	0.71	0.74	0.84	0.89	0.90	0.90
ξ_w	0.73	0.47	0.50	0.67	0.73	0.75	0.82	0.87	0.91	0.92
σ_ℓ	1.92	0.18	0.18	0.18	0.52	1.87	2.71	3.66	3.66	3.66
ξ_p	0.65	0.40	0.53	0.60	0.65	0.78	0.86	0.86	0.86	0.86
ι_w	0.59	0.24	0.24	0.27	0.38	0.58	0.61	0.80	0.89	0.89
ι_p	0.22	0.01	0.01	0.01	0.10	0.22	0.32	0.48	0.63	0.65
ψ	0.54	0.20	0.20	0.20	0.42	0.54	0.68	0.86	0.86	0.86
r_π	2.03	1.45	1.45	1.45	1.71	2.07	2.39	2.61	2.61	2.61
ρ	0.81	0.53	0.62	0.73	0.79	0.81	0.85	0.88	0.92	0.97
r_y	0.08	-0.04	-0.04	0.01	0.05	0.09	0.16	0.20	0.20	0.20
$r_{\Delta y}$	0.22	0.10	0.10	0.10	0.10	0.20	0.24	0.34	0.34	0.34
ρ_a	0.95	0.00	0.50	0.82	0.92	0.96	0.98	0.99	0.99	0.99
ρ_b	0.18	0.00	0.04	0.09	0.13	0.17	0.26	0.36	0.80	0.99
ρ_g	0.97	0.00	0.94	0.96	0.97	0.97	0.99	0.99	0.99	0.99
ρ_I	0.71	0.00	0.57	0.60	0.68	0.71	0.78	0.84	0.95	0.99
ρ_r	0.12	0.00	0.01	0.06	0.11	0.13	0.18	0.33	0.50	0.99
ρ_p	0.90	0.00	0.70	0.77	0.84	0.89	0.93	0.96	0.98	0.99
ρ_w	0.97	0.00	0.93	0.95	0.97	0.97	0.98	0.99	0.99	0.99
μ_p	0.74	0.00	0.08	0.22	0.43	0.73	0.82	0.91	0.95	0.99
μ_w	0.88	0.00	0.00	0.00	0.87	0.89	0.92	0.96	0.98	0.99
σ_a	0.45	0.00	0.42	0.47	0.67	0.92	1.49	2.57	3.20	10
σ_b	0.24	0.00	0.07	0.20	0.23	0.24	0.26	0.27	0.29	10
σ_g	0.52	0.00	0.52	0.52	0.52	0.53	0.55	0.56	0.57	10
σ_I	0.45	0.00	0.14	0.25	0.39	0.44	0.46	0.48	0.54	10
σ_r	0.24	0.00	0.22	0.23	0.23	0.24	0.26	0.28	0.31	10
σ_p	0.14	0.00	0.04	0.09	0.12	0.14	0.15	0.16	0.17	10
σ_w	0.24	0.00	0.18	0.20	0.21	0.24	0.25	0.29	0.31	10

Notes. This table gives information about the parameter estimates across the forty-two misspecification experiments. For the parameters of the laws of motion of the disturbances, we exclude an experiment from the calculations of the percentiles when the disturbance is part of the empirical model, but not part of the true *dgp*. The table also reports the bounds imposed on parameter estimates. See table 1 for the definitions of the parameters.

The median parameter estimates (across experiments) are relatively close to the true parameter values. Thus, our choice of experiments does not favor bias in a particular direction. There is one exception. The median value of the estimated standard deviation of the productivity disturbance innovation, σ_a, is equal to 0.92 compared to a true value of 0.45. The reason is that this disturbance often "absorbs" the variation of the disturbance that is not included in the empirical specification. Thus, the disturbance that is wrongly included in the empirical specification does not necessarily fulfill this role.

Even if we exclude cases for which the estimates fall in the bottom or top 10%, then we find that estimates are substantially different from their true value for many parameters. For example, for the labor supply elasticity with respect to the real wage, σ_l, the 10$^{\text{th}}$ percentile is equal to 0.18 and the 90$^{\text{th}}$ percentile is equal to 3.66, compared with a true value of 1.92. For the parameter capturing the indexation of wages ι_w, the same two percentiles are 0.27 and 0.8, compared with a true value of 0.59. For the parameter capturing the indexation of prices, ι_p, the two numbers are 0.01 and 0.48, compared with a true value of 0.22. When the two 10% tails are not excluded and the full range of estimates is considered, then the range substantially increases. Specifically, the largest values are 0.89 and 0.63 for the indexation of wages and prices, respectively.[11] Recall that these distortions are solely due to misspecification, not to small-sample variation.

For several parameters, the results remain bad when we narrow the range of outcomes considered. For example, when we exclude the bottom and the top 25%, then the values for σ_l, vary between 0.52 and 2.71 compared with a true value of 1.92. The results are also quite bad for ϕ, the elasticity in the capital adjustment cost function, for which the 25$^{\text{th}}$ percentile is equal to 5.47 and the 75$^{\text{th}}$ percentile is equal to 8.97.

2.4 Missspecification: Consequences for model properties

The previous section documents that misspecification can lead to large distortions in parameter values. Parameter estimates are often of interest in themselves. At least as important are the properties of the estimated structural model. It could be that different parameter configurations lead to similar model properties. In this section, we address this by looking at implied moments and IRFs.

2.4.1 Implied model moments

We begin by documenting the consequences of model misspecification for implied model moments using the misspecification setup described above. Table 3 reports the range of values for typical business cycle properties as implied by the estimated parameter values of the forty-two experiments considered. Specifically, it reports standard deviations and correlation coefficients relative to their true values. Thus, a value equal

[11] Parameter estimates are constrained to be in a range, and the largest estimate of the wage indexation parameter is constrained by the imposed upper bound.

to 1 means that there is no distortion. The column labeled "true value" reports the range of values the corresponding moment has according to the true *dgp*.[12]

Table 3: Moments: Ratio of implied value to truth across experiments with misspecification

	True value (across experiments)	Min	10%	25%	Median	75%	90%	Max
		(estimates, scaled by true value)						
Std(y_t)	[3.48 , 5.12]	0.51	0.78	0.92	1.03	1.64	4.46	6.03
Std(c_t)	[3.30 , 5.58]	0.45	0.76	0.92	1.03	1.81	4.12	6.62
Std(i_t)	[9.73 , 12.94]	0.70	0.87	0.99	1.11	1.71	3.81	6.47
Std(r_t)	[0.52 , 0.61]	0.76	0.90	0.94	1.00	1.36	2.28	2.78
Std(π_t)	[0.37 , 0.54]	0.64	0.72	0.94	1.01	1.25	2.21	2.98
Std(w_t)	[2.13 , 2.70]	0.73	0.83	0.92	1.08	2.28	5.57	10.87
Corr(y_t, c_t)	[0.65 , 0.94]	0.28	0.68	0.93	0.99	1.07	1.15	1.52
Corr(y_t, i_t)	[0.74 , 0.87]	0.69	0.83	0.95	1.00	1.10	1.16	1.29
Corr(c_t, i_t)	[0.63 , 0.89]	-0.68	0.60	0.92	1.00	1.19	1.34	1.57
Corr(c_t, r_t)	[-0.65 , -0.35]	-0.71	0.54	0.86	0.99	1.11	1.52	2.13
Corr(i_t, w_t)	[0.29 , 0.69]	-1.52	0.10	0.64	1.07	1.49	1.99	3.28
Corr(i_t, π_w)	[0.51 , 0.80]	0.36	0.84	0.97	1.02	1.17	1.34	1.75

Notes. This table reports the outcomes across experiments for the indicated moment as implied by parameter estimates relative to its true value. Thus a value equal to 1 indicates that there is no distortion due to misspecification. Each row reports percentiles across our forty-two experiments. It also reports the range of values of the true moments across the experiments. All moments considered are related to variables that are used in the estimation as observables.

Misspecification implies an upward bias for volatility in our experiments.[13] This upward bias could be specific to our particular type of misspecification. However, the observed upward bias is consistent with the simple analytical example discussed in appendix A.[14] The results are solely due to misspecification, since we use very large samples and our ML estimator is consistent when the empirical model is correctly specified.

The overestimation of volatility is enormous in some cases. Even if we exclude the top 25%, then standard deviations can be multiples of the true standard deviation. For example, the 75^{th} percentile for the standard deviation of wages is 2.28 times its true value. This ratio increases to 5.57 when we only exclude the top 10%. The 90^{th} percentiles for the consumption and output standard deviation ratios are 4.12

[12]Moments are not the same across experiments, since we adjust the standard deviations of the structural disturbances to ensure that the wrongly omitted disturbance does not play an important role.

[13]Section 2.3 documents an upward bias for σ_a, the standard deviation of the TFP disturbance. Since one disturbance is missing from the empirical model, it is not surprising that there is a shift towards some of the other disturbances. By contrast, here we find an upward bias for *total* variability.

[14]In appendix A, we discuss a simple example which documents analytically how maximum likelihood estimation of a misspecified model can lead to an *arbitrarily* large upward bias in the implied variance of an observable.

and 4.46, which also indicates massive over-prediction. The 90$^{\text{th}}$ percentile number for investment is equal to 3.81 and in the worst experiment the implied standard deviation is 6.47 times as big as the true value. By contrast, the values in the lower tail are less drastic. Excluding the bottom 10%, we find that the largest distortions are found for inflation for which the 10$^{\text{th}}$ percentile is 0.72, that is, implied volatility is 28% below its true value. If we consider all experiments, then the smallest ratio is equal to 0.45, which is found for the implied standard deviation of consumption.

Misspecification also has large quantitative implications for correlation coefficients. In fact, the sign of the correlation coefficient as implied by parameter estimates turns out to be different from its sample analogue in several cases. This would not be a big deal if the two correlation coefficients are both close to zero. But there are also cases in which the implied correlation coefficient according to the estimated empirical model and the true correlation coefficient are both large in absolute value and differ in sign.[15,16]

2.4.2 Impulse response functions (IRFs)

To conclude the discussion on the consequences of misspecification, we document that misspecification can also have a large impact on impulse response functions. There are many IRFs to consider. Figure 1 plots for three IRFs the outcomes across the experiments and documents that the distortions can be large. We exclude the cases when the disturbance of interest is in the empirical specification, but not part of the true *dgp*. It would not be surprising if these are different.[17] Thus, the disturbance of interest is part of the true *dgp* as well as the empirical model for all three cases considered.

Figure 1a plots the response of output to a TFP disturbance. This is obviously a key characteristic of the model. The black line plots the true IRF and the grey lines plot the IRFs as implied by the empirical model for the different experiments. All IRFs are based the same size shock.[18] If the grey lines are close to the black line, then misspecification of the empirical model has only minor consequences for the IRF considered. The sign of the IRF is virtually always correct and TFP disturbances always have a noticeable positive impact on aggregate output.[19] Nevertheless, the figure documents that there are large differences in terms of initial impact, overall

[15]A striking example is the experiment in which the government disturbance is not present in the true *dgp* and the empirical model excludes the risk-premium disturbance instead. The true correlation between consumption and investment is equal to 0.67 whereas the one implied by the estimated model is equal to -0.41.

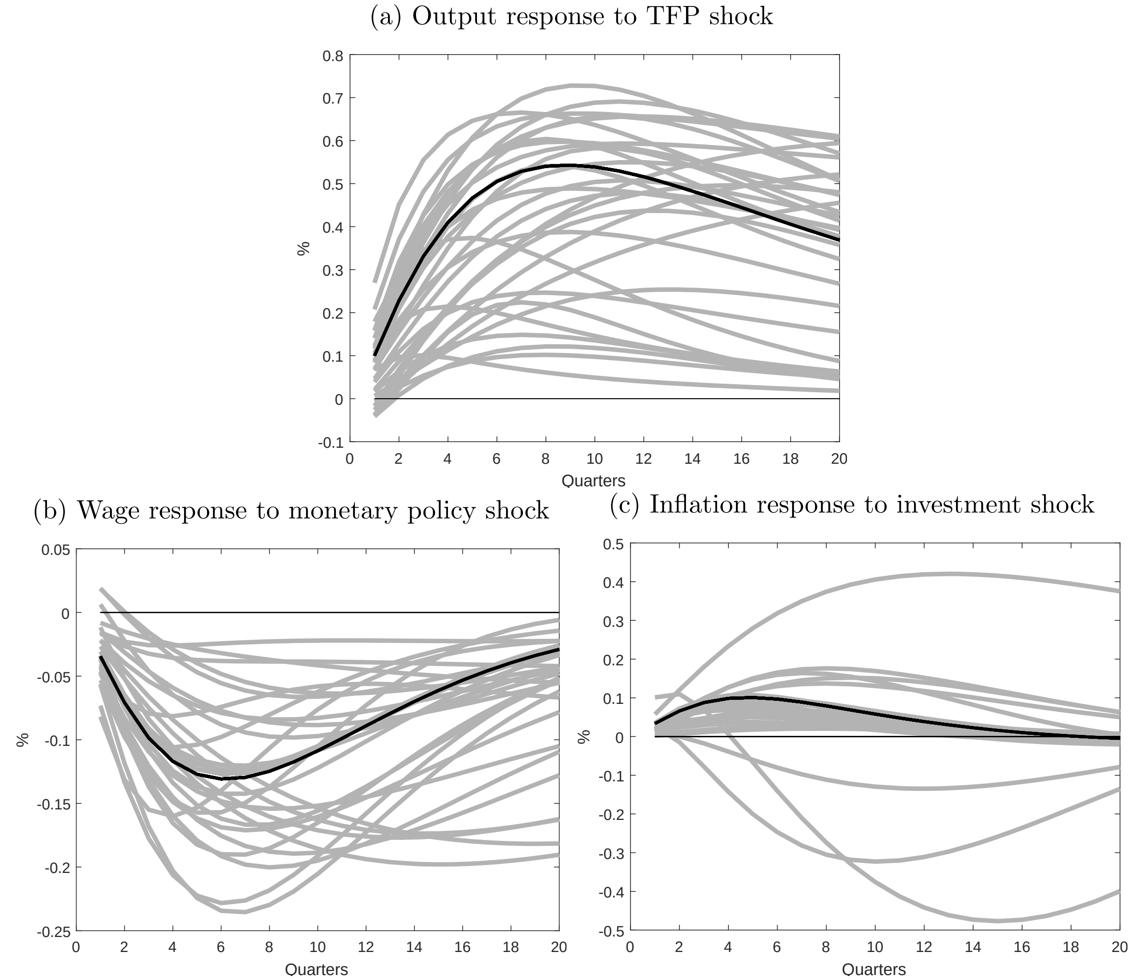
[16]The smallest correlation coefficient (in absolute value) according to the true model is 0.29, so any sign change implies a nontrivial change in the correlation coefficient.

[17]Also, we cannot calculate IRFs for a particular disturbance if that disturbance is not part of the empirical specification. This means that each figure plots IRFs for thirty-two cases.

[18]That is, one standard deviation according to the original SW model. Differences across IRFs are bigger if we use the estimated standard deviations for the different experiments.

[19]In some experiments, the initial response is negative. However, its value is then very small.

Figure 1: IRFs according to true (black) and misspecified (grey) empirical models



Notes. The figure plots the true IRF (black)and the IRFs implied by the misspecified (grey) empirical models considered. The results are based on a very large sample, so results are not due to small sample variation. These IRFs are for shocks that are correctly included in the model. Also, we do not use estimated standard deviations, but use the same size shock for all IRFs.

magnitude, shape, and persistence.

Figure 1b plots the response of the real wage to a monetary policy shock. This is clearly the kind of model property one would want to get right when analyzing monetary policy. The figure shows again a wide variety of responses across the different empirical specifications. Whereas the true response is substantial, there are several empirical specifications that predict a very small change. There are also a few specifications that give a much larger response. We want to reemphasize that the plotted IRFs are for a disturbance that is correctly included in the empirical model.

Figure 1c reports the results for the inflation IRF of an investment-specific shock. For most experiments the IRFs display a similar pattern, but there are important differences in terms of magnitude. For three experiments, however, the IRFs are completely at odds with the true IRF. Whereas the true IRF is positive and has reverted back to zero after twenty periods, the IRFs implied by these three misspecified empirical models are negative and indicate larger volatility and more persistence. Again, relatively small changes in parameter values can change these IRFs such that they are much closer to the true IRF.[20]

2.5 Is weak identification the cause?

In appendix B, we demonstrate that all parameters are identified in all models considered.[21] Moreover, we use a very large sample to estimate the parameters so the large range of values for parameter estimates cannot be caused by samples being too short to be informative. Also, the finding that the different parameter values are associated with quite different model properties indicates that the results discussed in this section are not due to parameters not being identified. As a final check, we compare the values of the likelihood according to the misspecified model at the estimated values and the true values. When using the true values, we do re-estimate the parameters of the exogenous random variables.[22] The smallest difference between the two log likelihood values is equal to 14.5 and there are only four experiments for which the difference is less than 100. The mean (median) difference is equal to $10,371$ (5501).[23]

2.6 Is the misspecification really minor?

The misspecification experiments considered above involve the inclusion of one wrong and the exclusion of one correct structural disturbance. Everything else is correctly specified. So the misspecification affects only a small part of all the model features researchers have to specify when writing down a complete empirical model.

Nevertheless, one could argue, that this misspecification is not that likely for the analysis in Smets and Wouters (2007), since SW was preceded by years of empirical analysis by many authors. However, in section 6, we document that we clearly reject

[20]Specifically, if σ_c, the parameter controlling curvature in the utility function and λ, the parameter indicating the habit component in the utility function, are set equal to their true values, then these three IRFs have a shape that is similar to the true IRF, that is, also predict a hump-shaped positive response. The responses still differ somewhat from the truth in having a more delayed response and a more persistent effect. The estimated values for σ_c in the three experiments are 0.65, 0.53, and 0.53, whereas the true value if equal to 1.39. The estimated values for λ are equal to 0.86, 0.87, and 0.85, whereas the true value is equal to 0.71.

[21]All true specifications have one structural disturbance less than the original SW model. This turns out not to matter for identification. In fact, estimated parameters remain identified when we do the identification test for specifications with five disturbances that exclude the disturbance that is not part of the true *dgp* as well as the one that is erroneously omitted from the empirical specification.

[22]This is a conservative choice, since differences in the likelihoods would be larger if these parameters are not re-estimated.

[23]It is not surprising that across experiments, there are some for which the misspecification is smaller than for others resulting in smaller differences between the two likelihood values. After all, our experiments are not designed to find large misspecification. Our set is constructed using a simple variation in the set of the original structural disturbances.

the null that two of the included structural disturbances are correctly specified against several alternatives. It is important to recall that correct specification of a structural disturbance is not only getting the nature of the disturbance right, but also that it enters each model equation correctly. In section 6, we will argue that this is not the case for two of the seven SW structural disturbances.

Furthermore, it could be argued that a misspecification is only minor if the misspecified model is not rejected when its fit is compared with the fit of the correct model. This is, of course, a test that one could never implement in practice, since it requires knowing the truth. The large differences in likelihood discussed in section 2.5 indicate that the misspecified model would be easily rejected. However, those likelihood values correspond to tests using unrealistically large samples. The appropriate question is whether one would reject the misspecified model with a sample of typical length and typical estimation procedure.

To address this question, we do a Monte Carlo experiment in which the model is estimated as in SW. That is, the data set has the same number of observations, the parameters are estimated with the same Bayesian methodology, and the priors are also the same. We assess model fit using the marginal data density (MDD). These are expensive Monte Carlo experiments.[24] Therefore, we consider only two of the possible forty-two misspecification experiments of section 2. They were chosen as follows. We ranked all experiments by the likelihood value of the misspecified specification relative to the likelihood of the correct specification. The idea is that misspecification is less severe if the likelihood values are close to each other. The first experiment chosen is the one corresponding to the sixty-sixth percentile and the second is the one corresponding to the thirty-third percentile.[25] Thus, our experiments are neither the least nor the most problematic in terms of misspecification. In section 5, we return to these two examples and we will document that consequences of misspecification are severe for both cases.

For the experiment at the thirty-third percentile we find that the misspecified model has a higher marginal data density in 17.8% of the Monte Carlo replications. Thus, one would prefer the wrong empirical model over the correct one in about four out of five cases in cases *if* one is so lucky to be able to do the test against the true model specification.

For the experiment at the sixty-sixth percentile, this number decreases to 52%. That is, the correct and the misspecified model have roughly an equal chance of having the best fit when realistic samples are used.[26]

[24]The reason is that they involve an optimization problem containing many parameter values. In contrast to the exercise in section 2, the optimization here is a bit more difficult, since it is affected by small-sample sampling variation. Moreover, it has to be repeated for every Monte Carlo replication.

[25]The first (second) Monte Carlo experiment corresponds to the case when the true *dgp* does not include a monetary policy (TFP) disturbance, but the empirical model leaves out the investment disturbance instead.

[26]This Monte Carlo experiment does indicate an interesting aspect of sampling variation. The large sample analysis indicates that the empirical model considered in this second Monte Carlo experiment

3 Agnostic Structural Disturbances

In this section, we develop and motivate our "structural agnostic disturbance" (ASD) procedure to detect and correct for misspecification. ASDs can be added to a structural model and they can be used to test whether a regular structural disturbance is correctly specified.

3.1 Underlying theoretical model

Consider the following linearized model

$$0_{n\times 1} = \mathbb{E}_t\left[\Lambda_2\left(\Psi\right)s_{t+1} + \Lambda_1\left(\Psi\right)s_t + \Lambda_0\left(\Psi\right)s_{t-1} + \Gamma\left(\Psi\right)\varepsilon_{t+1} + \Upsilon\left(\Psi\right)\varepsilon_t\right], \tag{1a}$$

$$\varepsilon_t = G\varepsilon_{t-1} + H\eta_t, \tag{1b}$$

$$\mathbb{E}_t\left[\eta_{t+1}\right] = 0, \tag{1c}$$

$$\mathbb{E}_t\left[\eta_{t+1}\eta_{t+1}'\right] = I_{m\times m}, \tag{1d}$$

where Ψ is the vector containing the structural parameters, s_t is the $n \times 1$ vector of endogenous variables, and ε_t is the $m \times 1$ vector of exogenous random variables. All variables are defined relative to their steady state values. Most linearized DSGE models can be represented with such a system of equations.[27]

Type of misspecification considered. As in section 2, the misspecification focuses on the modeling of the structural disturbances. That is, whether the included disturbances are the right ones and whether the restrictions they impose on the model equations correct.

3.2 The ASD procedure

There are two ways to describe and implement the ASD procedure. The first formulation is discussed in section 3.2.1. This formulation highlights that our procedure is more general than the procedure that adds wedges to particular model equations. We provide the second formulation in section 3.2.3 after discussing some background information in section 3.2.2. This second formulation makes clear that our procedure is more efficient than the misspecification procedures that combine a DSGE model with a reduced-form empirical model as in Ireland (2004) and Del Negro, Schorfheide, Smets, and Wouters (2007). This efficiency advantage is made possible by focusing on one particular type of misspecification, namely exogenous disturbances not being the right

is more misspecified than the one of the first Monte Carlo, since there was a large difference in the marginal likelihoods of the correct and misspecified model. In terms of marginal likelihoods, this ranking is reversed in small samples.

[27] Linearization leads to accurate solutions for many business cycle models. When this is not the case, then this is an additional source of misspecification.

ones or not being modeled correctly. As explained below, this allows us to use some of the structure of the model.

We will show that these two formulations are not different procedures, but different ways to implement this procedure.[28] Which procedure is more convenient in practice will depend on the application.

3.2.1 ASD procedure: First formulation based on model equations

Consider the model given in equation (1). To simplify the exposition, we start with the case for which s_t includes only state variables and all n state variables are observables. Suppose that the researcher is only sure about m_1 structural disturbances. These are part of the vector, $\varepsilon_{1,t}$. If $m_1 < n$ and there are no other disturbances, then there is a singularity problem. One option would be to add measurement error. But structural disturbances and measurement errors are very different. Structural disturbances affect economic variables and propagate through the system according to the economic mechanisms of the model. Measurement error disturbances do not.[29] Another option is to make a best guess and to add a vector $\varepsilon_{2,t}$ with m_2 additional structural disturbances with $m_2 \geq n - m_1$. Equation (1) can then be written as

$$\begin{aligned} 0_{n\times 1} &= \mathbb{E}_t\left[\Lambda_2(\Psi)\, s_{t+1} + \Lambda_1(\Psi)\, s_t + \Lambda_0(\Psi)\, s_{t-1} + \Gamma(\Psi)\,\varepsilon_{t+1} + \Upsilon(\Psi)\,\varepsilon_t\right] \\ &= \mathbb{E}_t\left[\begin{array}{c} \Lambda_2(\Psi)\, s_{t+1} + \Lambda_1(\Psi)\, s_t + \Lambda_0(\Psi)\, s_{t-1} \\ +\left[\Gamma_1(\Psi)\ \ \Gamma_2(\Psi)\right]\left[\begin{array}{c}\varepsilon_{1,t+1}\\ \varepsilon_{2,t+1}\end{array}\right] + \left[\Upsilon_1(\Psi)\ \ \Upsilon_2(\Psi)\right]\left[\begin{array}{c}\varepsilon_{1,t}\\ \varepsilon_{2,t}\end{array}\right]\end{array}\right], \end{aligned} \tag{2a}$$

$$\left[\begin{array}{c}\varepsilon_{1,t}\\ \varepsilon_{2,t}\end{array}\right] = \left[\begin{array}{cc} G_{11} & G_{12}\\ G_{21} & G_{22}\end{array}\right]\left[\begin{array}{c}\varepsilon_{1,t-1}\\ \varepsilon_{2,t-1}\end{array}\right] + \left[\begin{array}{cc} H_{11} & H_{12}\\ H_{21} & H_{22}\end{array}\right]\left[\begin{array}{c}\eta_{1,t}\\ \eta_{2,t}\end{array}\right], \tag{2b}$$

$$\mathbb{E}_t\left[\begin{array}{c}\eta_{1,t+1}\\ \eta_{2,t+1}\end{array}\right] = 0, \tag{2c}$$

$$\mathbb{E}_t\left[\left[\begin{array}{c}\eta_{1,t+1}\\ \eta_{2,t+1}\end{array}\right]\left[\begin{array}{cc}\eta_{1,t+1} & \eta_{2,t+1}\end{array}\right]\right] = I_{m\times m}. \tag{2d}$$

The column vectors $\Gamma_2(\Psi)$ and $\Upsilon_2(\Psi)$ capture the restrictions imposed by the m_2 additional structural disturbances. In the remainder of this section, we document that no such restrictions are imposed when agnostic structural disturbances are added.

Adding ASDs to model equations. If one adds agnostic structural disturbances instead of regular structural disturbances, then the system of equations is modified as

[28] See section 3.2.5.

[29] See section 3.2.2 for an explanation. Moreover, most researchers would find it undesirable if "measurement" error explains a large part of the data.

follows:

$$0_{n\times 1}=\mathbb{E}_t\left[\begin{array}{c}\Lambda_2\left(\Psi\right)s_{t+1}+\Lambda_1\left(\Psi\right)s_t+\Lambda_0\left(\Psi\right)s_{t-1}\\ +\left[\Gamma_1\left(\Psi\right)\ \mathbf{\Gamma}_2\right]\left[\begin{array}{c}\varepsilon_{1,t+1}\\ \varepsilon_{2,t+1}\end{array}\right]+\left[\Upsilon_1\left(\Psi\right)\ \mathbf{\Upsilon}_2\right]\left[\begin{array}{c}\varepsilon_{1,t}\\ \varepsilon_{2,t}\end{array}\right]\end{array}\right]. \tag{3}$$

The key aspect of our procedure is that $\mathbf{\Gamma}_2$ and $\mathbf{\Upsilon}_2$ are reduced-form coefficients that do *not* contain any restrictions on Ψ. Moreover, when $G_{21}=0$, which is typically the case as structural disturbances are usually modeled to be uncorrelated, then

$$\mathbb{E}_t\left[\mathbf{\Gamma}_2\varepsilon_{2,t+1}+\mathbf{\Upsilon}_2\varepsilon_{2,t}\right]=\left(\mathbf{\Gamma}_2G_{22}+\mathbf{\Upsilon}_2\right)\varepsilon_{2,t}. \tag{4}$$

Using this insight, we can write the system as

$$0_{n\times 1}=\mathbb{E}_t\left[\begin{array}{c}\Lambda_2\left(\Psi\right)s_{t+1}+\Lambda_1\left(\Psi\right)s_t+\Lambda_0\left(\Psi\right)s_{t-1}\\ +\Gamma_1\left(\Psi\right)\varepsilon_{1,t+1}+\Upsilon_1\left(\Psi\right)\varepsilon_{1,t}+\widehat{\mathbf{\Upsilon}}_2\varepsilon_{2,t}\end{array}\right], \tag{5}$$

where $\widehat{\mathbf{\Upsilon}}_2=G_{22}\mathbf{\Gamma}_2+\mathbf{\Upsilon}_2$. All that matters for the model is $\widehat{\mathbf{\Upsilon}}_2$, which means that adding an agnostic disturbance introduces one additional parameter for each model equation.[30,31] Replacing regular structural disturbances with agnostic structural disturbances may make it harder to identify Ψ, the structural parameters of the model. As discussed in appendix B, this turned out to be not an issue for the experiments discussed in this paper. Identification of $\widehat{\mathbf{\Upsilon}}_2$ will be discussed in section 3.2.3.

3.2.2 Useful proposition for second ASD formulation

In this section, we will proof a proposition that will be helpful with the second formulation of the ASD procedure. Consider again the model given in equation (2), which divides the vector with exogenous disturbances, ε_t, into two parts, the $m_1\times 1$ vector, $\varepsilon_{1,t}$, and the $m_2\times 1$ vector, $\varepsilon_{2,t}$. A recursive solution to equation (2) has the following form:

$$\begin{aligned}s_t&=A\left(\Psi\right)s_{t-1}+B(\Psi)\varepsilon_t\\&=A\left(\Psi\right)s_{t-1}+\left[\begin{array}{cc}B_1\left(\Psi\right)&B_2\left(\Psi\right)\end{array}\right]\left[\begin{array}{c}\varepsilon_{1,t}\\ \varepsilon_{2,t}\end{array}\right].\end{aligned} \tag{6}$$

The following proposition states that the properties of $\varepsilon_{2,t}$ do *not* affect the coefficients of the policy rule related to s_{t-1} and $\varepsilon_{1,t}$, that is, they do not affect $A(\Psi)$ and $B_1(\Psi)$. Thus, it does not matter whether $\varepsilon_{2,t}$ is a regular or an agnostic structural disturbances and the time series properties of $\varepsilon_{2,t}$ do not matter either. The only assumption needed is that the elements of G_{21} are equal to zero, which corresponds to

[30] Without loss of generality one can set the standard deviations of the innovation of the ASDs equal to 1, which in this case is a normalization of the diagonal elements of $H_{2,2}$. As with regular structural disturbances, one would need to estimate the parameters of the time series specification contained in G.

[31] As discussed later, one could choose to leave the agnostic disturbance out of some equations.

the case when $\varepsilon_{1,t}$ has no effect on *future* values of $\varepsilon_{2,t}$. This is not very restrictive given that the literature usually sets all elements of G_{21} equal to zero (and also all elements of G_{12}, $H_{1,2}$, and $H_{2,2}$ as well as the off-diagonal elements of G_{11}, G_{22}, $H_{1,1}$ and $H_{2,2}$).

Proposition 1 *If the model is given by equation* (2) *and all elements of* G_{21} *are equal to zero, then* (*i*) $A(\Psi)$ *and* $B_1(\Psi)$ *do not depend on* $\Gamma_2(\Psi)$ *and* $\Upsilon_2(\Psi)$*, which characterize the nature of the additional disturbances, and* (*ii*) $A(\Psi)$ *and* $B_1(\Psi)$ *do not depend on* G_{22}*,* H_{21}*, and* H_{22}*, which characterize the time series properties of* $\varepsilon_{2,t}$*.*

Proof. Substitution of the policy rule as given in equation (6) into the system of equations (2) gives,

$$\begin{aligned}
0_{n\times 1} &= \left(\Lambda_2 A^2 + \Lambda_1 A + \Lambda_0\right) s_{t-1} + \left(\Lambda_2 AB + \Lambda_2 BG + \Lambda_1 B + \Gamma G + \Upsilon\right)\varepsilon_t, && (7)\\
\varepsilon_t &= \left[\begin{array}{cc} \varepsilon_{1,t} & \varepsilon_{2,t} \end{array}\right]', && (8)\\
B &= \left[\begin{array}{cc} B_1 & B_2 \end{array}\right], && (9)\\
G &= \left[\begin{array}{cc} G_{11} & G_{12} \\ G_{21} & G_{22} \end{array}\right], && (10)
\end{aligned}$$

where we have suppressed the dependence of coefficients on Ψ. The first equation has to hold for all values of s_{t-1} and ε_t. This implies that a solution must satisfy

$$\Lambda_2 A^2 + \Lambda_1 A + \Lambda_0 = 0_{n\times n} \tag{11}$$

and

$$\Lambda_2 AB + \Lambda_2 BG + \Lambda_1 B + \Gamma G + \Upsilon = 0_{n\times(m_1+m_2)}. \tag{12}$$

A does not depend on the time series properties of $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$, since B, G, and H do not appear in equation (11). Equation (12) can be written as follows

$$\overline{\Lambda}\left[\begin{array}{cc} B_1 & B_2 \end{array}\right] + \Lambda_2\left[\begin{array}{cc} B_1 & B_2 \end{array}\right]\left[\begin{array}{cc} G_{11} & G_{12} \\ G_{21} & G_{22} \end{array}\right] + \Gamma\left[\begin{array}{cc} G_{11} & G_{12} \\ G_{21} & G_{22} \end{array}\right] + \Upsilon = 0_{n\times(m_1+m_2)}), \tag{13}$$

where $\overline{\Lambda} = \Lambda_2 A + \Lambda_1$. This is a system of $n \times (m_1 + m_2)$ equations to solve for the elements of B. It can be split into the following two sets of systems:

$$\begin{aligned}
\overline{\Lambda} B_1 + \Lambda_2 B_1 G_{11} + \Lambda_2 B_2 G_{21} + \Gamma_1\left[\begin{array}{c} G_{11} \\ G_{21} \end{array}\right] + \Upsilon_1 &= 0_{n\times m_1}, && (14)\\
\overline{\Lambda} B_2 + \Lambda_2 B_1 G_{12} + \Lambda_2 B_2 G_{22} + \Gamma_2\left[\begin{array}{c} G_{12} \\ G_{22} \end{array}\right] + \Upsilon_2 &= 0_{n\times m_2}. && (15)
\end{aligned}$$

If $G_{21} = 0$, then equation (14) contains $n \times m_1$ equations to solve for all the elements of B_1. The solution cannot depend on G_{22} or H_2 since these matrices do not appear in this equation.■

It is intuitive that the elements of G_{21} have to be equal to zero, that is, $\varepsilon_{1,t}$ should not affect future values of $\varepsilon_{2,t}$. If current values of $\varepsilon_{1,t}$ do affect future values of $\varepsilon_{2,t}$ and therefore future values of s_t, then one has to know how $\varepsilon_{2,t}$ affects model outcomes to determine how $\varepsilon_{1,t}$ affects current outcomes for s_t.

3.2.3 ASD procedure: Second formulation based on policy functions

The second formulation highlights the differences with alternative procedures that deal with misspecification by combining a DSGE model and a VAR. This alternative formulation is also useful in terms of understanding whether adding agnostic disturbances leads to identification issues.

An alternative way of writing the solution to the model is the following:

$$s_t = \sum_{i=1}^{m} s_t^{[i]}, \tag{16}$$

$$s_t^{[i]} = A(\Psi)\, s_{t-1}^{[i]} + B_{\cdot,i}(\Psi)\, \varepsilon_{i,t}, \tag{17}$$

where $s_t^{[i]}$ represents the outcome of the state variable if the *only* disturbance in the economy is the i^{th}-disturbance, $\varepsilon_{i,t}$, and $B_{\cdot,i}$ is the i^{th} column of B. Thus, one can think of the s_t variables as the sum of the outcomes in "one-disturbance" economies. The linearity of the model is important for this additive property. According to proposition 1, the coefficients on the lagged state variable, $A(\Psi)$, do not depend on the particular disturbance considered. That is, whereas $B_{\cdot,i}(\Psi)$ is indexed by i because it depends on what kind of disturbance is the driving force of the economy, $A(\Psi)$ does not. This property greatly increases the efficiency of our procedure.

Our proposed procedure consists of including m_2 agnostic structural disturbances. This results in the following time series representation of the policy functions:[32]

$$s_t = \sum_{i=1}^{m} s_t^{[i]}, \tag{18a}$$

$$s_t^{[i]} = A(\Psi)\, s_{t-1}^{[i]} + B_{\cdot,i}(\Psi)\, \varepsilon_{i,t} \quad \text{for } i \leq m_1, \tag{18b}$$

$$s_t^{[i]} = A(\Psi)\, s_{t-1}^{[i]} + \boldsymbol{B}_{\cdot,i}\varepsilon_{i,t} \qquad \text{for } m_1 + 1 \leq i \leq m_1 + m_2 = m. \tag{18c}$$

In terms of notation, $B_{\cdot,i}(\Psi)$ contains coefficients associated with a regular structural disturbance which are a function of Ψ and bold font $\boldsymbol{B}_{\cdot,i}$ contains reduced-form coefficients associated with a structural agnostic disturbance. The *only* difference between this specification and the standard DSGE specification with only regular structural disturbances is that the $\boldsymbol{B}_{\cdot,i}$ coefficients are unrestricted reduced-form coefficients. Since our agnostic disturbances are structural disturbances, their impact propagates through

[32]According to proposition 1, this specification is valid as long as the elements of G_{12} are equal to zero, which is usually the case.

the system exactly as regular structural disturbances do, that is, as described by $A(\Psi)$. The property of linear models that $A(\Psi)$ does not depend at all on what is the nature of the structural disturbances nor on their time series properties makes it possible to efficiently add structural disturbances to the specification without having to be specific on what they are.

The dimension of $\boldsymbol{B}_{\cdot,i}$ is equal to n, the number of state variables. This means that adding an agnostic disturbance means estimating an additional n parameters. The number of additional parameters to be estimated is limited because structural disturbances differ in their initial impact, but their propagation through time is the same for all disturbances and controlled by $A(\Psi)$. Moreover, an increase in the standard deviation of an agnostic structural disturbance affects the model variables in exactly the same way as an identical proportional increase of the elements of $\boldsymbol{B}_{\cdot,i}$. Consequently, the standard deviation of an agnostic disturbance can be normalized to equal 1.[33] If there are observables that are not state variables, then one also needs an additional equation for these y_t variables, which for our set-up is given by

$$
\begin{aligned}
y_t &= \sum_{i=1}^{m} y_t^{[i]}, && (19a) \\
y_t^{[i]} &= C(\Psi)\, s_{t-1}^{[i]} + D_{\cdot,i}(\Psi)\, \varepsilon_{i,t} \quad \text{for } i \leq m_1, && (19b) \\
y_t^{[i]} &= C(\Psi)\, s_{t-1}^{[i]} + \boldsymbol{D}_{\cdot,i} \varepsilon_{i,t} \quad \text{for } m_1 + 1 \leq i \leq m_1 + m_2 = m, && (19c)
\end{aligned}
$$

where y_t is the $(\overline{n} \times 1)$ vector with observables that are not state variables. Each additional observable used in the estimation will introduce one more coefficient related to the agnostic structural disturbances.

3.2.4 Identification

Replacing $B_{\cdot,i}(\Psi)$ with $\boldsymbol{B}_{\cdot,i}$ reduces the number of restrictions on structural parameters, which could affect the identification of Ψ. We have verified that the structural parameters, Ψ, continue to satisfy the local identification conditions as specified in Komunjer and Ng (2011) when we replace regular structural disturbances by ASDs. The coefficients of $\boldsymbol{B}_{\cdot,i}$ are also identified locally since they directly enter the policy functions. However, there is no global identification of the $\boldsymbol{B}_{\cdot,i}$ coefficients when $m_2 > 1$, since the agnostic disturbances are interchangeable, that is, there is no difference between say the first and the second agnostic disturbance in how they affect model equations. This is a consequence of being agnostic.

The first formulation of our procedure adds agnostic disturbances to the model equations. Under what conditions are the associated coefficients, i.e., the elements of

[33] If the time series processes of the two disturbances have the same number of parameters, then replacing a regular structural disturbance by an agnostic disturbance typically means estimating an additional $n-1$ parameters. The number would be less if some structural parameters are associated only with the regular structural disturbance that is replaced.

$\widehat{\boldsymbol{\Upsilon}}_2$ identified? What matters for identification are the policy functions, that is, the $\boldsymbol{B}_{\cdot,i}$ and the $\boldsymbol{D}_{\cdot,i}$ coefficients. These coefficients are a function of the $\widehat{\boldsymbol{\Upsilon}}_2$ coefficients. Since the total number of coefficients in $\boldsymbol{B}_{\cdot,i}$ and $\boldsymbol{D}_{\cdot,i}$ is equal to the number of state variables plus the number of observables that are not state variables, $n+\overline{n}$, one can add the agnostic disturbance to at most $n+\overline{n}$ model equations for each of the elements of $\widehat{\boldsymbol{\Upsilon}}_2$ to be identified. This is a necessary, not a sufficient condition.[34]

Identification of the elements of $\widehat{\boldsymbol{\Upsilon}}_2$ only becomes important if one wants to give an economic interpretation of the agnostic disturbance. As discussed in section 6, however, this can be a useful exercise.

3.2.5 Equivalence of first and second formulation

The easiest case to consider is the one in which the model consists of $n+\overline{n}$ equations and the observables are the n state variable plus $\overline{n}$ other observables, where $\overline{n}$ could be zero. If the agnostic disturbance is added to all $n+\overline{n}$ model equations, then the two different ways to implement the procedure are identical.

The first formulation, which adds agnostic disturbances to model equations, is more flexible. The reason is that it allows us to add the agnostic disturbances to only a subset of the $n+\overline{n}$ equations. By excluding the agnostic disturbance from some equations one does impose restrictions on the agnostic disturbance and this implies that the first and the second implementation will lead to different policy functions and different estimation results. Imposing such restrictions moves us away from being fully agnostic, but there may be cases where this flexibility of the first formulation is very useful. In section 6, we document how model selection procedures can be used to impose restrictions leading to more concise formulations that make it possible to interpret the agnostic disturbances.

Now consider the case when the model has more than $n+\overline{n}$ equations, that is, some model variables are not state variables or observables, and the agnostic disturbance is added to more than $n+\overline{n}$ model equations. From the discussion above, we know that not all the elements of $\widehat{\boldsymbol{\Upsilon}}_2$ can be identified. That is, different combinations of the coefficients in $\widehat{\boldsymbol{\Upsilon}}_2$ lead to the same values for the $n+\overline{n}$ coefficients in $\boldsymbol{B}_{\cdot,i}$ and $\boldsymbol{D}_{\cdot,i}$. As long as the agnostic disturbance remains agnostic and there is no need to interpret the $\widehat{\boldsymbol{\Upsilon}}_2$ coefficients, then this is not a problem. Specifically, it does not affect the identification of the structural parameters Ψ.[35]

[34] To understand why this is not a sufficient condition consider a system that consists of two equations containing the model's two state variables, $s_{1,t}$ and $s_{2,t}$, and no other variables. Also, y_t satisfies the equation $y_t = 2s_{1,t} + s_{2,t}$. If y_t is not an observable, then one could not add the agnostic disturbance to this equation, because its associated coefficient would, of course, not be identified.

[35] In practice, a good optimization routine should still be able to find the true optimized value of the objective function and associated values for Ψ even though it may take some time before it realizes that several variations in the elements of $\widehat{\boldsymbol{\Upsilon}}_2$ do not lead to improvements in the target.

3.2.6 Comparison with alternative procedures

In this section, we discuss how our procedure compares with alternatives proposed in the literature. A detailed description of these alternative approaches can be found in appendix E.2.

Agnostic structural disturbances versus wedges. Equations (2) and (5) point out the difference between adding an agnostic structural disturbance and adding a regular structural disturbance. Adding a regular structural disturbance requires specifying in which equation the disturbance appears and how the associated elements of $\Gamma_2(\Psi)$ and $\Upsilon_2(\Psi)$ depend on the structural parameters Ψ. Adding an agnostic disturbance does not impose such restrictions. Wedges are similar to *regular* structural parameters in that they only appear in a subset of equations. Sometimes only one equation. Wedges may or may not impose restrictions on the structural parameters, Ψ. For example, one of the wedges considered in Chari, Kehoe, and McGrattan (2007) is a productivity disturbance. This disturbance appears in the budget constraint and the first-order condition for capital and imposes cross-equation parameter restrictions. By contrast, when a "labor wedge" is added to the labor-supply first-order condition, then this does not impose restrictions on the structural parameters, since it does not appear in any other equation. Relative to an agnostic disturbance, however, it is restrictive because it is not allowed to appear in other model equations.[36]

Agnostic structural disturbances versus measurement error. ASDs differ from measurement error in that the latter is not a *structural* disturbance. Consequently, its impact on the different elements of *model* variables does not propagate through the system as structural disturbances do. To understand this difference consider the following system of equations:

$$s_t = As_{t-1} + B\varepsilon_t, \tag{20}$$
$$y_t = Cs_t + D\varepsilon_t, \tag{21}$$
$$\varepsilon_t = G\varepsilon_{t-1} + H\eta_t. \tag{22}$$

The first equation represents a very simple structural model that governs the law of motion of the state variable, s_t. The second equation specifies the relationship between the observable, y_t and the state variable. ε_t is a scalar exogenous random variable. A value of C equal to 1 means that the state variable is the observable. If ε_t is measurement error, then $D \neq 0$ *and* $B = 0$. That is, measurement error affects the difference between data and model variables, but does not affect how model variables

[36] Inoue, Kuo, and Rossi (2015) provide a formal analysis for using wedges to detect and identify misspecification. Using a New Keynesian model, they introduce a labor wedge into the cost minimization problem of the intermediate good producing firm, and a final good wedge and a bond demand wedge into the household budget constraint. Similar to the productivity disturbance, such wedges only appear in a limited set of equations and do impose parameter restrictions.

behave. By contrast, if ε_t is a structural disturbance, then $D = 0$ and $B \neq 0$. Now ε_t does affect model variables and propagates through the system according to the structural model, that is, according to equation (20).

The idea of the agnostic procedure is to add not one but two different ASDs to equations (20) and (21). This procedure would allow for several possibilities discussed above *and* combinations, namely one or two structural disturbances with no measurement error, one structural disturbances that is correlated with measurement error, one structural disturbance and uncorrelated measurement error, or just measurement error.

Agnostic disturbances versus a DSGE-VAR. Ireland (2004) and Del Negro, Schorfheide, Smets, and Wouters (2007) combine a DSGE model with a reduced-form VAR that contains the observables. Specifically, they start with a fully specified DSGE model as represented by equations (18a), (18b), (19a), and (19b). Since they have no agnostic structural disturbances, the value of m_2 is equal to zero.

There are two key differences between these two approaches and ours. First, our approach focuses on a particular type of misspecification, which allows it to use aspects of the model that are not affected by this misspecification, namely $A(\Psi)$ and $C(\Psi)$. Second, introducing a VAR into the estimation means that the number of disturbances necessarily increases by a number equal to the number of variables in the VAR. Moreover, adding a VAR introduces many more parameters unless the number of observables is small. Our procedure allows for a more parsimonious approach and could consist of adding just one new disturbance or replacing one regular structural disturbance with an agnostic structural disturbance.

Both differences imply that our approach is more efficient in terms of the number of parameters that it has to be estimate.[37] The price of parsimony is that our procedure is not designed to detect misspecification unrelated to structural disturbances, that is, misspecification associated with restrictions imposed by $B(\Psi)$ and $D(\Psi)$. Although, it is not designed to do so, ASDs might very well pick up other types of misspecification such as wrong functional forms and time variation in structural parameters. The DSGE-VAR approach explicitly allows misspecification in $A(\Psi)$ and $C(\Psi)$. However, Chari, Kehoe, and McGrattan (2008) point out that the VAR with a finite number of lags that does not contain *all* the model's state variables is likely to be misspecified. This means that the DSGE-VAR approach cannot deal with all possible misspecifications either.

Another difference emerges as the sample size goes to infinity. With the DSGE-VAR approach one has two "competing" empirical specifications, a DSGE model and a VAR. Since every DSGE suffers from at least some minor misspecification, one can expect the VAR to fully take over as the sample size goes to infinity. If that happens, then one is left with a reduced-form model that can no longer be used for policy analysis. This will

[37] For example, for the popular DSGE model of Smets and Wouters (2007) with 7 observables, a VAR with 4 lags would mean estimating 204 additional coefficients. As discussed in section 6, the implementation of our procedure for this model means estimating twelve more parameters.

never happen with our approach, since the propagation of state variables will always be determined by $A(\Psi)$ and the relationship between state variables and observables by $C(\Psi)$. If the number of regular structural disturbances in the true data generating process is less than or equal to the number of agnostic structural disturbances, then one can expect the role of regular structural disturbances to be driven to zero as the sample size goes to infinity.[38] The restrictions imposed by $B(\Psi)$ and $D(\Psi)$ would then no longer play a role.

4 What to do in practice?

In this section, we first discuss how agnostic structural disturbances can be used as a test for misspecification. Next, we discuss how agnostic structural disturbances can be applied to reduce misspecification.

4.1 ASDs to test for misspecification

Agnostic structural disturbances differ from regular structural disturbances in that their *initial* impact on the economy is not restricted and, thus, imposes no restrictions on model parameters. As indicated in equation (5), regular structural disturbances, $\varepsilon_{1,t}$, enter model equations as $\Gamma_1(\Psi)\varepsilon_{1,t+1} + \Upsilon_1(\Psi)\varepsilon_{1,t}$, whereas agnostic structural disturbances, $\varepsilon_{2,t}$, enter models equations as $\widehat{\boldsymbol{\Upsilon}}_2\varepsilon_{2,t}$, where $\widehat{\boldsymbol{\Upsilon}}_2$ is a vector of reduced-form coefficients that does not impose restrictions on Ψ. Since these are two competing models, and the former is a restricted version of the latter, standard model selection statistics can be used to test whether the restrictions imposed by structural disturbances are correct.

Specifically, a simple and transparent way to proceed is to carry out a model selection test, such as a likelihood-ratio test, for each of the regular structural disturbance considered separately. For example, if the disturbance in question is a wage mark-up disturbance, then one first estimates the model with a wage mark-up disturbance and then re-estimates the model with the wage mark-up disturbance replaced by an ASD. Let $\mathcal{L}(\Psi)$ be the log likelihood of the model with the wage mark-up disturbance and let $\mathcal{L}(\Psi, \widehat{\boldsymbol{\Upsilon}}_2)$, be the log likelihood of the model with the wage mark-up disturbance replaced by an ASD.[39] To test the restrictions imposed by the wage mark-up disturbance one checks whether $\mathcal{L}(\Psi, \widehat{\boldsymbol{\Upsilon}}_2) - \mathcal{L}(\Psi)$ exceeds the critical value of a $\chi^2(q)$ distribution with q degrees of freedom, where q is the difference in the number of parameters between the two models. One could compare marginal data densities if one prefers a Bayesian methodology.

[38] Assuming that there are enough ASDs to avoid any singularity issues.

[39] These expressions are based on the notation of the first formulation of our ASD procedure. In terms of the notation of the second formulation, the log likelihood of the model with the agnostic disturbance would be denoted by $\mathcal{L}(\Psi, \boldsymbol{B})$ when all observables are state variables or by $\mathcal{L}(\Psi, \boldsymbol{B}, \boldsymbol{D})$ when some observables are not state variables.

One can also assess whether a particular regular structural disturbance is restrictive by looking at changes in parameter estimates and model properties after the regular structural disturbance has been replaced by an agnostic structural disturbance.

If the restrictions imposed by the wage mark-up disturbance are rejected, then one has two options. First, one could modify how the wage mark-up affects the model. In section 6, we show how the estimated model with agnostic disturbances provides useful insights for such modifications. Second, one could simply use the estimated model with the agnostic disturbance.

4.2 ASDs to reduce misspecification

To estimate models one would like to use all available observables. When estimating DSGE models one needs at least as many disturbances as observables to avoid a singularity problem. As the number of observables increases, it becomes more difficult to come up with sensible structural disturbances. Recall that it is not just a question of conjecturing a particular type of structural disturbance. The structural disturbance has to enter each and every model equation correctly. An alternative is to add ASDs.

Adding agnostic disturbances does not complicate the estimation in practice. For example, to add an agnostic disturbance, $\varepsilon_{2,t}$, to a model estimated with Dynare one would add $\widehat{\mathbf{\Upsilon}}_{i,2}\varepsilon_{2,t}$ to the i^{th} model equation, where $\widehat{\mathbf{\Upsilon}}_{i,2}$ is the i^{th} element of $\widehat{\mathbf{\Upsilon}}_{2}$. Under our second formulation, adding an ASD simply means adding an extra column to the policy rules with the ASD and its reduced-form coefficients.

5 ASDs and Misspecification: Small-Sample Monte Carlo experiments

In section 2, we considered the large-sample consequences of using a (slightly) misspecified empirical model which wrongly excluded one structural disturbance and included one that was not part of the true model. In that experiment, an empirical model that includes an ASD instead of the wrongly included regular structural disturbance, would uncover the true parameter values. The reason is that this ASD-augmented empirical model is correctly specified, the ML estimator is consistent, a large sample is used, and the structural parameters remain identified.[40]

In this section, we consider the same misspecification experiment, but now consider *small-sample* Monte Carlo experiments. This will allow us two answer two questions. First, is the ASD procedure effective in detecting misspecification when we compare the ASD-augmented empirical model with the misspecified empirical model? Second, what is the efficiency loss if one replaces a regular structural disturbance that is part of the true underlying model with an ASD? The empirical model remains correctly specified if one does so, but one looses efficiency because one estimates additional reduced-form parameters and imposes less true restrictions.

[40]In fact, parameters remain identified, if two regular structural disturbances are replaced by ASDs.

5.1 Experiments and empirical specifications.

As in section 2, the *dgp* is the SW model with six of the seven structural disturbances, but now we use a sample of typical length, namely 156, which is the same as the number of observations used to estimate the model in Smets and Wouters (2007). The number of Monte Carlo replications is equal to 1,000. In each Monte Carlo replication, we estimate model parameters using the SW empirical model that is identical to the true *dgp* and two additional specifications. The first is an empirical model, that is – as in section 2 – misspecified, because it excludes one of the structural disturbances of the true *dgp* and erroneously includes another. The second excludes the same structural disturbance, but now includes an ASD. This last specification is also correct. That is, there are values of the reduced-form coefficients such that the specification is identical to the original SW one.

These are computationally expensive Monte Carlo experiments. Therefore, we only consider two of the possible forty-two combinations to misspecify and those are the same two as those considered in section 2.6.

5.2 ASD misspecification test when alternative is misspecified

To evaluate whether the ASD procedure can detect misspecification, we first use a Likelihood Ratio (LR) test that compares the likelihood of the empirical specification with the agnostic disturbance to the likelihood of the misspecified empirical model. The number of degrees of freedom is equal to ten, since the agnostic specification has ten more parameters.[41] With this procedure, the ASD procedure rejects the misspecified model in all Monte Carlo replications in both experiments. The procedure is, thus, quite powerful in detecting misspecification. As discussed in section 2.6, however, if we use a Bayesian model comparison procedure based on SW priors, then the ASD procedure rejects the misspecified specification in 82.2% and 52% of the generated samples for the first and the second Monte Carlo experiment, respectively. It is not surprising that the power reduces with a Bayesian approach. The reason is that the posteriors of the misspecified and the agnostic specification are more similar than their likelihood functions since the posteriors share the same prior.

5.3 ASD misspecification test when alternative is correct

Next, we do the same ASD test for misspecification when the alternative model is correctly specified. For the first Monte Carlo experiment, we find that the rejection rate is 21.5% at the 10%-level and 12% at the 5%-level. For the second experiment, these two numbers are 20.9% and 12.6%. Thus, the small-sample results do not coincide precisely with the theoretical predictions based on large-sample theory. However,

[41] We use the second formulation of our procedure. This formulation introduces the smallest possible number of additional parameters.

the distortions are not that unreasonable. In appendix C, we document that the histogram of estimated χ^2 statistics is reasonably close to the theoretical (large-sample) χ^2 distribution, but has a slightly fatter upper tail.

5.4 Correcting for misspecification

The discussion above made clear that the ASD procedure does very well in terms of detecting misspecified models and reasonably well in not rejecting correctly specified models in small samples. In this subsection, we document that the estimates of the structural parameters obtained with the agnostic procedure are much closer to the true values than those obtained with the misspecified empirical model. In fact, they are very similar to those obtained with the correctly specified fully-structural empirical model.

Table 4 reports the average absolute error of the parameter estimates relative to the true value for the three different empirical models across Monte Carlo replications. Consistent with the large-sample results discussed in section 2, parameter estimates obtained with the misspecified structural model are substantially worse than those obtained with the correctly specified model. The average of the errors for the misspecified model is more than twice as large as the one for the correctly specified model for several parameters.[42] Average errors for the misspecified model are typically better for the second experiment. However, that is not true for all parameters. For example, the average error for σ_c is substantially higher in the second experiment, whereas there is only a modest increase for the correctly specified model.

For the first Monte Carlo experiment, the average error outcomes for the agnostic setup and the correct specification are very similar. Although only slightly, the average error is actually lower for the agnostic specification for ten of the twenty-seven parameters. For the second Monte Carlo experiment, the fully specified SW specification comes with some noticeable efficiency advantages for several parameter estimates. Nevertheless, the estimates obtained using the agnostic procedure are still much better than the one obtained with the misspecified model.

Figures 2 and 3 plot histograms characterizing the distribution of the parameter estimates across Monte Carlo replications for a selected set of parameters.[43] Each panel reports the results for the correctly specified model (dark line and dots), the agnostic procedure (white bars), and the misspecified model (blue/dark bars).

[42] Particular problematic is the standard deviation of the TFP disturbance in the first Monte Carlo experiment for which the average error is almost nine time as large as the one for the correct empirical model. Consistent with the results of section 2, this disturbance often takes over the role of the wrongly excluded structural disturbance.

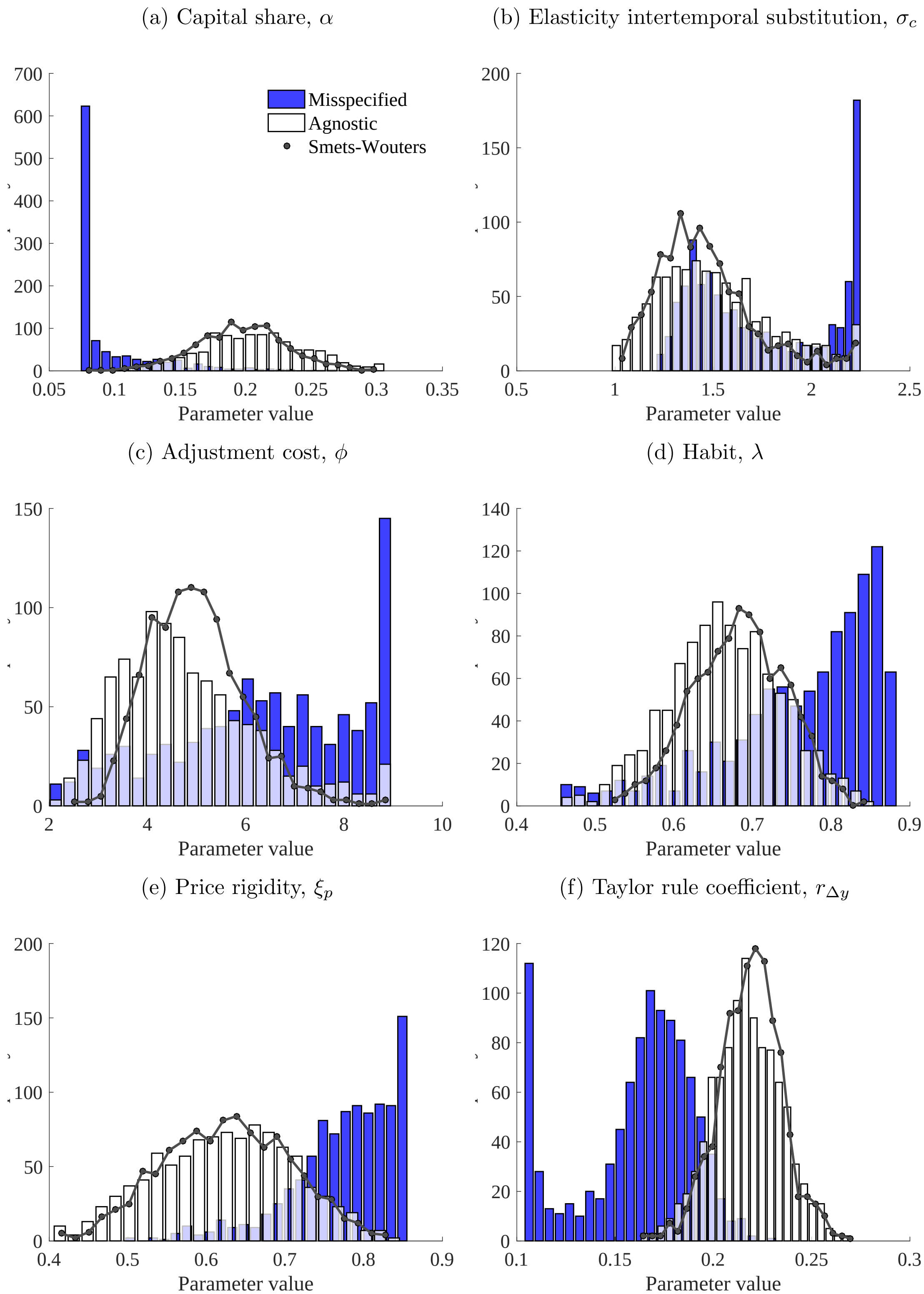
[43] A full set of results for all parameters is given in appendix C.

Table 4: Average absolute errors across Monte Carlo experiments

	true value	**average error first MC** misspecified	agnostic	SW	**average error second MC** misspecified	agnostic	SW
α	0.19	0.098	0.035	0.028	0.056	0.048	0.037
σ_c	1.39	0.384	0.246	0.191	0.540	0.288	0.226
Φ	1.61	0.217	0.212	0.191	0.192	0.212	0.164
ϕ	5.48	1.793	1.326	0.899	1.429	1.269	0.896
h	0.71	0.096	0.069	0.052	0.083	0.077	0.057
ξ_w	0.73	0.082	0.090	0.076	0.092	0.095	0.081
σ_ℓ	1.92	1.652	0.640	0.532	1.506	0.939	0.831
ξ_p	0.65	0.130	0.074	0.068	0.090	0.080	0.070
ι_w	0.59	0.205	0.165	0.159	0.190	0.168	0.160
ι_p	0.22	0.142	0.109	0.101	0.128	0.112	0.100
ψ	0.54	0.182	0.128	0.109	0.150	0.134	0.118
r_π	2.03	0.295	0.277	0.241	0.347	0.380	0.333
ρ	0.81	0.031	0.025	0.022	0.034	0.038	0.030
r_y	0.08	0.051	0.025	0.021	0.055	0.034	0.029
$r_{\Delta y}$	0.22	0.058	0.014	0.012	0.057	0.039	0.033
ρ_a	0.95	0.071	0.028	0.020	-	-	-
ρ_b	0.18	0.161	0.078	0.073	0.133	0.079	0.071
ρ_g	0.97	0.020	0.016	0.013	0.018	0.016	0.014
ρ_I	0.71	-	-	-	-	-	
ρ_r	0.12	-	-	-	0.089	0.072	0.067
ρ_p	0.90	0.181	0.090	0.067	0.188	0.070	0.053
ρ_w	0.97	0.031	0.030	0.019	0.022	0.029	0.021
μ_p	0.74	0.246	0.188	0.161	0.250	0.173	0.139
μ_w	0.88	0.071	0.072	0.056	0.069	0.071	0.057
σ_a	0.45	0.441	0.061	0.052	-	-	-
σ_b	0.24	0.050	0.021	0.021	0.040	0.023	0.021
σ_g	0.52	0.035	0.027	0.026	0.026	0.027	0.025
σ_I	0.45	-	-	-	-	-	
σ_r	0.24	-	-	-	0.013	0.015	0.014
σ_p	0.14	0.022	0.017	0.015	0.019	0.017	0.015
σ_w	0.24	0.026	0.021	0.020	0.022	0.023	0.021

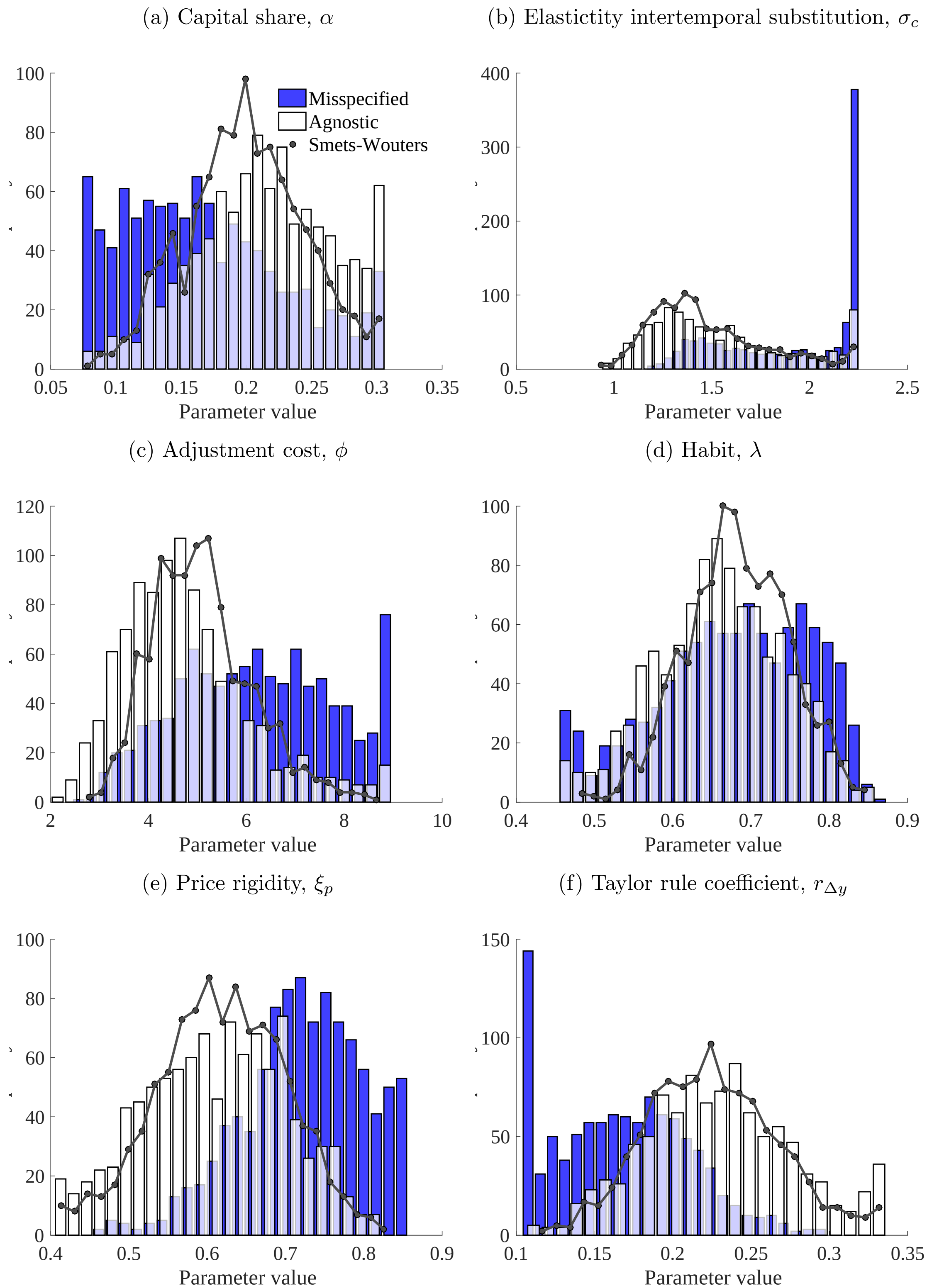
Notes. This table reports the average absolute error across Monte Carlo replications for the indicated parameter and empirical specification. See table 1 for the definitions of the parameters. The first (second) Monte Carlo experiment corresponds to the case when the true *dgp* does not include the monetary policy (TFP) disturbance, but the empirical model leaves out the investment disturbance instead.

Figure 2: Histograms for parameter estimates: First Monte Carlo experiment



Notes. The panels plot the distribution of the indicated parameter across the Monte Carlo replications. The color of the histograms for the misspecified case changes in a lighter shade when they overlap with the histogram for the agnostic specification. In this experiment, the true *dgp* does not include the monetary policy disturbance, but the empirical model leaves out the investment disturbance instead.

Figure 3: Histograms for parameter estimates: Second Monte Carlo experiment



Notes. The panels plot the distribution of the indicated parameter across the Monte Carlo replications. The color of the histograms for the misspecified case changes in a lighter shade when they overlap with the histogram for the agnostic specification. In this experiment, the true *dgp* does not include the TFP disturbance, but the empirical model leaves out the investment disturbance instead.

The figures document that the distributions of estimates obtained with the correct specification and the agnostic procedure are both qualitatively and quantitatively very similar. By contrast, the distribution of estimates obtained with the misspecified empirical model can be vastly different. For example, panel a of figure 2 documents that the distribution of estimates of the capital share parameter, α, displays a strong downward bias when the misspecified empirical model is used. The associated mean is equal to 0.09, whereas the true value is equal to 0.19. The figure also documents that a large number of estimates are clustered at the imposed lower bound. That is, by imposing bounds we limited the distortions due to misspecification. For α, the leftward shift is so large, that there is little overlap between the distribution of the estimates based on the misspecified model and the other two empirical models. Bunching at the lower or upper bound is more pervasive for the first experiment, but also observed for the second.

For the parameters considered in these figures, the distribution of estimates for the agnostic and the fully-specified SW specification are almost always centered around the true parameter value. In principle, there could be a small sample bias, since this is a complex nonlinear estimation problem. The full set of results, discussed in appendix C, do indeed indicate that there is a bias for some parameters. In those cases, the bias is similar for the estimator based on the fully-specified specification and the agnostic one. An example of a parameter that is estimated with bias is the labor supply elasticity with respect to the real wage, σ_l. Its true value is equal to 1.92. In the first experiment, the average estimate across the Monte Carlo replications is equal to 1.84 for the SW and 1.71 for the agnostic specification. By contrast, the associated average estimate is equal to 0.27 for the misspecified model, which indicates a large bias.

6 Are the SW disturbances the right ones for US data?

The Monte Carlo experiment of section 5 documents that the ASD procedure is a powerful tool to detect and correct for misspecification when the SW model is used as the true *dgp*. In this section, we use the ASD procedure on actual data. Specifically, we first use the ASD procedure to test the restrictions imposed by structural disturbances in the SW model using the same US postwar data as in the original SW paper. We will document that the restrictions imposed by the risk premium and the investment-specific technology disturbance are rejected by the ASD procedure. That is, replacement of these regular structural disturbances by an agnostic structural disturbance leads to an increase in the marginal data density. The restrictions of the other five disturbances are not rejected. Next, we use model selection procedures to determine the number of ASDs to include and to construct a more concise specification that excludes the agnostic disturbances from *some* model equations. The best specification obtained from these selection procedures is one with three ASDs. To conclude, we interpret the nature of these three agnostic structural disturbances by examining the sign and magnitude of the associated coefficients in model equations and the IRFs of the agnostic disturbances.

6.1 Introducing ASDs into the Smets-Wouters model

The ASD procedure can be used in frequentist and Bayesian settings. Since SW use a Bayesian estimation procedure, we will do the same. To estimate the model with agnostic disturbances, we use the formulation of the procedure as described in section 3.2.1, which entails adding the agnostic disturbance to model equations without restricting its impact.[44] This only requires a minor modification of the Dynare program that estimates the model for the original SW specification. We do not include agnostic disturbances in equations that define observables.[45] This means that there are thirteen coefficients to measure the impact of an agnostic disturbance on the system. We can normalize the standard deviation of the agnostic disturbances to one, since its coefficients are of a reduced-form nature. Thus, the difference in the number of parameters between the most general agnostic specification considered and the original SW specification is equal to twelve times the number of ASDs that are introduced. Details are given in appendix D.1.

The priors for the structural parameters are identical to the ones used by SW. The prior for each agnostic coefficient is a Normal with a mean equal to what the coefficient would be according to the SW restrictions and the SW prior means.[46] By centering the priors of the agnostic coefficients around the SW restrictions, we favor the SW specification. However, the means of these priors hardly matter and our results are robust to setting the prior mean equal to zero for all coefficients.[47]

The standard deviation of the prior distribution is set equal to 0.5. This implies a very uninformed prior, since the model is linear in log variables. As a robustness check we also consider a standard deviation equal to 0.1 and we find very similar results.

[44] The SW model has an output gap measure that depends on the outcomes of a hypothetical parallel economy with flexible prices. If an equation in the sticky-price part of the model has an associated equation in the flexible-price part of the model, then we assume that the agnostic disturbance enters the two equations with the same reduced-form coefficient.

[45] For example, the SW specification uses consumption growth as an observable and has an equation that defines consumption growth. Allowing an agnostic disturbance to affect this equation would capture measurement error (which would be correlated with structural disturbances if this ASD also appears in other model equations with a non-zero coefficient). We do not explore this possibility to keep the analysis parsimonious and to stay close the SW approach, which does not allow for measurement error.

[46] For example, suppose we use the ASD procedure to test the restrictions of the risk-premium disturbance by replacing it with an ASD. The risk-premium disturbance appears in two equations, namely the consumption/bond Euler equation and the capital-valuation equation. The prior means of the reduced-form agnostic coefficients for these two equations are set equal to the values according to the SW restrictions with structural parameters evaluated at their prior means. The reduced-form coefficients associated with the other equations have a prior mean equal to zero.

[47] Having a non-zero prior has a practical advantage. The signs of the coefficients of an agnostic disturbance are not identified. That is, one can switch the signs of the coefficients of an ASD as long as one does it for all coefficients. A necessary consequence of its agnostic nature is that the sign of an ASD disturbance has no a priori meaning. If the prior means of all ASD coefficients are zero, then the ASD coefficients can flip sign for different runs of the MCMC procedure.

6.2 Smets-Wouters structural disturbances: Specification tests

A specification that replaces a regular structural disturbance with an agnostic one encompasses the original specification which gives it an advantage in terms of achieving a better fit. The additional parameters, however, act as a penalty term in the marginal data density. Table 5 reports the marginal data densities for the different specifications. The first row reports the marginal data density for the original SW specification with its seven regular structural disturbances. The seven subsequent rows give the results when the indicated regular structural disturbances is replaced by an ASD.

Table 5: Misspecification tests for the original Smets-Wouters empirical model

structural SW disturbance excluded	SAD added	marginal data density
None (original SW)	no	-922.40
TFP, ε_t^a	yes	-931.21
Risk premium, ε_t^b	yes	**-908.79**
Government expenditure, ε_t^g	yes	-934.14
Investment-specific, ε_t^i	yes	**-919.81**
Monetary policy,ε_t^r	yes	-926.88
Price mark-up, ε_t^p	yes	-938.85
Wage mark-up, ε_t^w	yes	-947.31

Notes. The table reports the marginal data density for different empirical specifications. The first row reports the value for the original SW specification. The specifications considered in subsequent rows replace the indicated structural disturbance with an agnostic structural disturbance. The bold numbers indicate the cases for which the MDD is higher when the indicated structural disturbance is replaced by an agnostic disturbance.

Overall, these results are quite supportive of the original SW specification as the SW restrictions are preferred for five of the seven structural disturbances.[48] But the results for the risk-premium and the investment specific disturbance indicate that improvement is possible.[49]

6.3 Which regular and agnostic disturbances to include?

The results do not necessarily imply that we should exclude the structural risk-premium and investment disturbance. After all, it is possible that a model that includes agnostic disturbances *as well as* these two SW structural disturbances has an even higher marginal data density. To investigate this issue, we compare a set of models that do or do not include the risk-premium disturbance, that do or do not include the

[48]However, it is possible that the SW specification would be rejected against more concise agnostic disturbances, that is specifications that exclude the agnostic disturbance from some equations.

[49]When we narrow the prior of the agnostic coefficients by reducing the standard deviation to 0.1, then the restrictions of the monetary policy disturbance are also rejected. But the increase in the marginal data density is relatively small, namely from -922.40 to -920.82.

investment disturbance, and that include one, two, or three ASDs.[50]

Table 6: Model selection procedure for SW model: Step 1

regular structural		agnostic			marginal
ε_t^b	ε_t^i	$\widetilde{\varepsilon}_t^A$	$\widetilde{\varepsilon}_t^B$	$\widetilde{\varepsilon}_t^C$	data density
no	no	yes	yes	no	**-906.85**
no	no	yes	yes	yes	-925.55
no	yes	yes	no	no	-908.79
no	yes	yes	yes	no	-907.46
no	yes	yes	yes	yes	-922.94
yes	no	no	yes	no	-919.81
yes	no	yes	yes	no	-907.32
yes	no	yes	yes	yes	-921.71
yes	yes	no	no	no	-922.40
yes	yes	yes	no	no	-909.35
yes	yes	no	yes	no	-920.26
yes	yes	yes	yes	no	-908.09
yes	yes	yes	yes	yes	-922.82

Notes. The table reports the marginal data density for different empirical specifications regarding three agnostic disturbances and the two disturbances that are misspecified, that is, the risk-premium disturbance, ε_t^b, and the investment disturbance, ε_t^i. The number in bold indicates the highest outcome.

Table 6 reports the results. It shows that the model with the highest marginal data density is one with two agnostic disturbances and without the SW risk-premium as well as the SW investment-specific disturbance. Another indication that there is no need for these two SW structural disturbances is that their role in terms of explaining variation in the data is very small when agnostic disturbances are included. According to the (unconditional) variance decomposition of the estimated SW model, the risk-premium disturbance is especially important for the price of capital, consumption growth, and output growth explaining 45.4%, 61.2%, and 22.1% of total variability, respectively. It only plays a minor role for other variables. When agnostic disturbances are added, then these three numbers drop to 3.88%, 3.88%, and 2.05%, respectively.[51] The reduction in the role of the investment disturbance is even stronger. In the SW model, the investment disturbance plays a quantitatively important role for many variables. For investment growth it even explains 82.1% of the volatility. With agnostic disturbances added, its role becomes minuscule. Even for investment growth it only explains 0.31%.

[50] To estimate the model with all seven observables, an empirical specification with only one ASD would need either the risk-premium or the investment disturbance to avoid a singularity.

[51] These numbers are based on the specification with two ASDs and all seven SW structural disturbances using posterior mode estimates.

6.4 Finding the best agnostic empirical specification

To interpret ASDs, we could use the best specification found so far. However, interpretation of an ASD is easier when the specification is more concise. To determine whether an agnostic disturbance should be excluded from some equations, we implement model selection procedures using the marginal data density as the criterion of fit. This statistic increases when fit improves, but also penalizes additional parameters.

We consider both a specific-to-general procedure and a general-to-specific procedure and we apply the procedure for the specifications with two and three ASDs.[52,53] The specific-to-general procedure with three ASDs leads to the highest MDD and the selected outcome is our preferred empirical model. The specific-to-general procedure with two ASDs and the general-to-specific procedure with two ASDs lead to slightly lower MDDs.[54] Moreover, the models selected by these three procedures are very similar. Specifically, the additional ASD in the specification with three ASDs only plays a minor role. The zero restrictions imposed for the other two ASDs are not exactly the same, but the differences are due to coefficients that turn out to be small. As documented in appendix D.3, the estimates of the parameters are similar and the estimates obtained with these three empirical specifications imply similar model properties. The general-to-specific procedure with three ASDs leads to a specification that has a much lower MDD.[55]

In our preferred specification, the first agnostic disturbance enters eight of the thirteen equations, the second in three, and the third in five. By contrast, the original SW risk-premium and the investment-specific disturbance appear in only two. In the remainder of this section, we discuss the estimation results for our preferred specification and give an interpretation to the three ASDs.

[52] See appendix D.2 for details.

[53] An informal alternative selection procedure would be the following. One starts at the same point as the general-to-specific procedure, that is, with ASDs included in every equation. The marginal posteriors of the agnostic coefficients provides information on the lack of importance of different agnostic coefficients and may provide the researcher promising combinations of zero restrictions to impose. In fact, the posteriors for the coefficients with the fully unrestricted ASD specifications are very predictive of the equations selected by the specific-to-general procedures for this application. Of course, there are good reasons why this informal procedure is not a generally accepted model selection procedure and we cannot expect this to always work well.

[54] The specific-to-general procedure generates an MDD equal to -892.92 with two ASDs and -890.76 with three. The general-to-specific with two ASDs results in an MDD of -894.94.

[55] Namely, -909.48. The general-to-specific procedure already stops after two steps. That is, the procedure does not detect that imposing *multiple* restrictions *simultaneously* does lead to substantial improvements. One has to impose some structure on any model selection procedure, because it would be impossible to consider all possible combinations. That is, one has to give instructions on what paths to follow and which ones to ignore. But this means that the model selection procedure may not find the best model. This motivates our use of different model selection criteria.

6.5 Impact on parameter estimates and model properties

As documented in appendix D.3, table 11, there are several differences between the estimated values of the structural parameters obtained with the fully structural SW specification and our preferred agnostic specification with three ASDs. For example, the inflation coefficient in the Taylor rule is equal to 2.05 in the SW specification and 1.77 in ours.[56] The SW estimate is right at the upper bound of our 90% highest posterior density (HPD) interval. The SW mean estimate for the parameter characterizing the share of fixed cost in production is equal to 1.61 which is quite a bit higher than our mean estimate of 1.47 and outside our 90% HPD interval. Also, the mean posterior value of the MA coefficient of the wage mark-up disturbance is equal to 0.85 according to the SW specification and 0.59 according to ours. Our mean estimate for the standard deviation of this disturbance is roughly a third of the SW estimate.

Although there are some nontrivial differences, they are relatively small and the IRFs of the five regular structural disturbances that are included in both specifications are very similar for the two empirical models. The same is true when we consider the role of these five disturbances for the variance decomposition. Details are given in appendix D.3, tables 12 and 13. One nontrivial change is the role of the productivity disturbance for output growth, which is 16.1% according to SW and 22.2% according to ours. Although the differences seem minor if we consider the five structural disturbances in isolation, the combined role changes quite a bit for some variables. For example, the combined role of these five structural disturbances for investment (amount of capital used) is equal to 55.5% (74.1%) for the SW specification and 68.7% (92.6%) for our preferred specification.

6.6 Giving the ASDs an economic interpretation

ASDs are agnostic by nature. The model selection procedure also does not use any economic reasoning. Here we will show how the estimation results, such as parameter estimates of ASD coefficients and IRFs, can be used to give a meaningful interpretation to the ASDs. We will argue that one ASD can be interpreted as an investment-specific disturbance, but with some quite striking differences from the regular one used in the literature and in SW. We will refer to this ASD as the agnostic "investment-modernization disturbance." The second ASD has features in common with the SW risk-premium disturbance, although it is closer to a preference disturbance. Moreover, like the first ASD it displays some striking differences with its original SW counterpart. We will refer to this ASD as the agnostic "Euler disturbance". The role of the third ASD is quantitatively less important than the other two. It mainly affects wage growth and is associated with a more efficient use of capital. We will refer to this ASD as the "capital-efficiency wage mark-up disturbance." By assigning names to agnostic disturbances, we may open ourselves to criticism. Our main reason for assigning these

[56] We report posterior mean estimates unless indicated otherwise.

labels is that we want to make clear that agnostic disturbances are in principle theory-free, *and yet* allow the researcher to go one step further, towards giving an economic interpretation to them.

Table 7: Role of structural disturbances for variance

	risk/preference		investment		
	SW ε_t^b	agnostic ε_t^A	SW ε_t^i	agnostic ε_t^B	agnostic ε_t^C
output	1.53	1.14	7.34	2.17	0.28
flex. price output	0	2.08	5.39	1.02	0.36
consumption	2.18	1.51	2.83	0.49	0.25
investment	0.22	1.06	44.2	29.3	1.00
hours	2.52	1.29	8.15	4.97	2.03
capital	0.04	0.12	32.5	2.37	9.75
utilization	0.86	4.14	35.4	9.46	14.7
price of capital	45.4	18.6	36.0	31.6	7.21
marginal cost	0.87	15.2	3.11	2.61	5.13
policy rate	7.40	17.2	18.3	12.5	0.65
inflation	0.58	0.68	3.18	3.96	0.91
output growth	22.1	21.3	15.8	8.04	1.82
consumption growth	61.2	61.7	0.95	2.03	0.10
investment growth	2.46	12.6	82.1	70.0	0.81

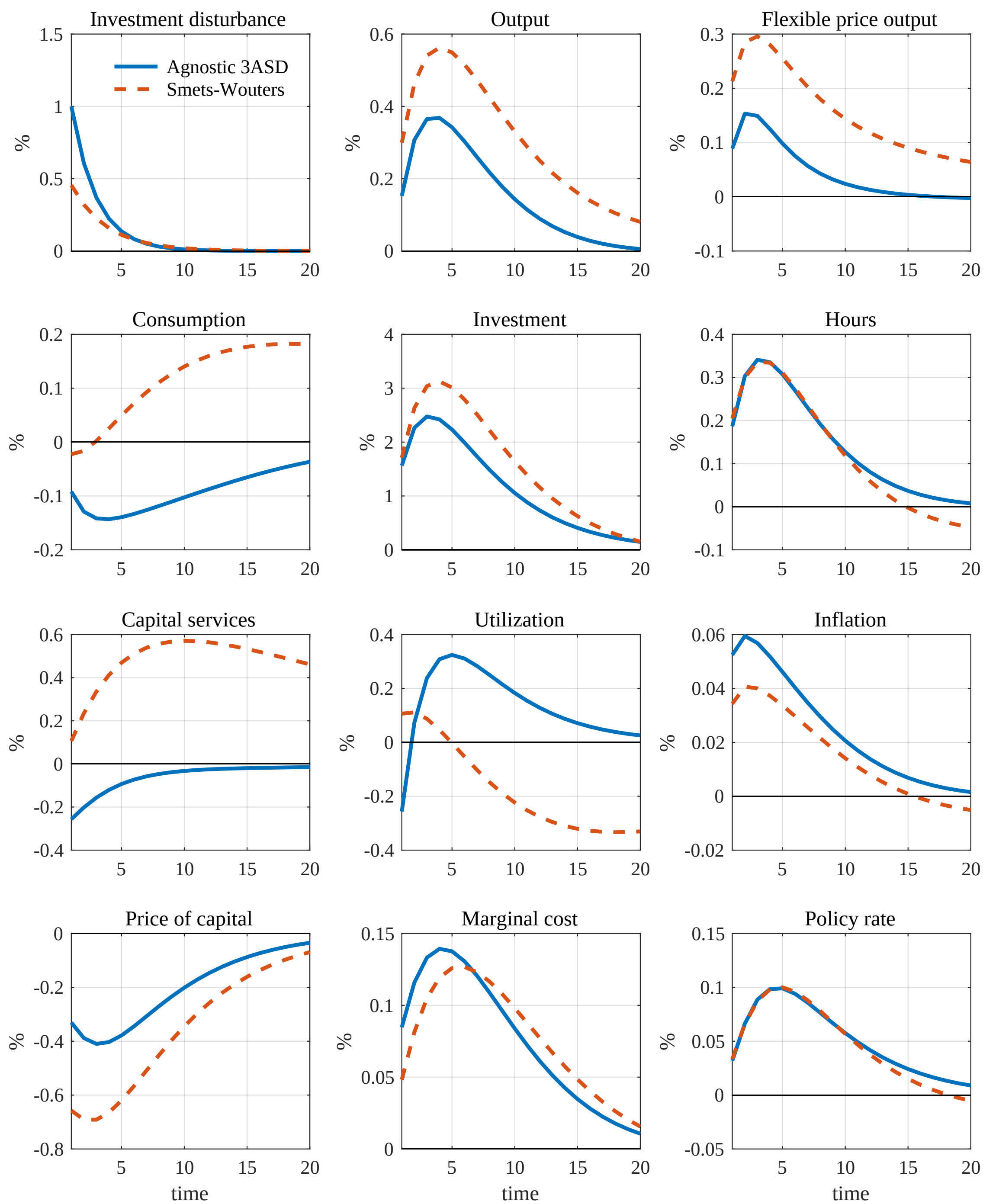
Notes. The table reports the percentage of total variability explained by the SW and the agnostic risk-premium disturbance and the SW and the agnostic investment disturbance. The numbers for the SW disturbance are from estimation of the original SW model. The numbers for the agnostic disturbance are from our preferred empirical model with three agnostic disturbances.

6.6.1 The agnostic investment-modernization disturbance, $\widetilde{\varepsilon}_t^B$

In the SW model, the investment-specific technology disturbance shows up in the investment Euler equation and in the capital accumulation equation. One of our agnostic disturbances, $\widetilde{\varepsilon}_t^B$, also shows up in these two equations.[57] The only other equation in which $\widetilde{\varepsilon}_t^B$ appears is the utilization equation that relates capacity utilization to the rental rate of capital. These findings indicate that $\widetilde{\varepsilon}_t^B$ could be interpreted as an investment-specific productivity disturbance. Furthermore, as documented in table 7, $\widetilde{\varepsilon}_t^B$, plays an important role for the volatility of investment. Specifically, it explains 70% of the volatility of investment growth compared to 82.1% for the investment-specific disturbance in the SW model. Interestingly, $\widetilde{\varepsilon}_t^B$ is not important for the volatility of capital. Specifically it only explains 2.37% of the volatility of the capital stock, whereas the SW investment disturbance explains 32.5%. Thus, if $\widetilde{\varepsilon}_t^B$ is an investment-specific disturbance, then it is not a typical one.

[57] In our computer programs, the ASDs are referred to as agnA, agnB, and agnC. The economic story we are going to tell works best if we start with agnB. Labels for agnostic disturbances are arbitrary and we could relabel this disturbance as $\widetilde{\varepsilon}_t^A$, which may seem more logical given that it is discussed first. We chose not to do so, because it would create an inconsistency with our computer programs.

Figure 4: IRFs of the SW investment and the agnostic investment-modernization disturbance



Notes. These figures plot the IRFs of the SW investment-specific productivity disturbance and the agnostic disturbance $\tilde{\varepsilon}_t^B$ that we interpret as an investment-modernization disturbance.

Figure 4 plots the IRFs of our agnostic disturbance and the SW investment-specific disturbance. This graph documents that there are some remarkable differences. The SW investment disturbance generates a typical business cycle with key aggregates moving in the same direction. A positive agnostic investment disturbance also leads to a strong comovement between output and investment. However, a positive shock leads to a *reduction* in consumption and capital.[58] Also, whereas capacity utilization decreases in the SW model, our specification indicates an increase.

To understand these differences and to explain why we still think that $\widetilde{\varepsilon}_t^B$ is an investment-specific disturbance, we have to take a closer look at the relevant equations and how $\widetilde{\varepsilon}_t^B$ affects these equations differently than the SW investment specific disturbance, ε_t^i. The three relevant equations are the following:[59,60]

Smets-Wouters investment-specific disturbance, $\boldsymbol{\varepsilon_t^i}$

$$\begin{aligned} \text{Investment Euler:} \quad & i_t = i_1(\Psi)\, i_{t-1} + (1 - i_1(\Psi))\, \mathbb{E}_t[i_{t+1}] + \boldsymbol{\varepsilon_t^i}, & (23) \\ \text{Utilization:} \quad & z_t = z_1(\Psi)\, r_t^k, & (24) \\ \text{Capital:} \quad & k_t = k_1(\Psi)\, k_{t-1} + (1 - k_1(\Psi))\, i_t + \boldsymbol{k_2(\Psi)\, \varepsilon_t^i}, \quad \boldsymbol{k_2(\Psi) > 0}. & (25) \end{aligned}$$

Agnostic investment-modernization disturbance, $\boldsymbol{\widetilde{\varepsilon}_t^B}$

$$\begin{aligned} \text{Investment Euler:} \quad & i_t = i_1(\Psi)\, i_{t-1} + (1 - i_1(\Psi))\, \mathbb{E}_t[i_{t+1}] + \boldsymbol{d_3^B \widetilde{\varepsilon}_t^B}, \boldsymbol{d_3^B > 0}, & (26) \\ \text{Utilization:} \quad & z_t = z_1(\Psi)\, r_t^k + \boldsymbol{d_7^B \widetilde{\varepsilon}_t^B}, \quad \boldsymbol{d_7^B < 0}, & (27) \\ \text{Capital:} \quad & k_t = k_1(\Psi)\, k_{t-1} + (1 - k_1(\Psi))\, i_t + \boldsymbol{d_8^B \widetilde{\varepsilon}_t^B}, \quad \boldsymbol{d_8^B < 0}. & (28) \end{aligned}$$

The reason for the striking differences between the IRFs of our ASD and the SW investment disturbance is that our unrestricted approach lets the agnostic investment specific disturbance appear in the capital accumulation equation without restrictions. That is, the sign of the coefficient of $\widetilde{\varepsilon}_t^B$, d_8^B, is unrestricted, but the coefficient of ε_t^i in the SW specification, $k_2(\Psi)$ is restricted by the values of the structural parameters, Ψ.

[58] Justiano, Primiceri, and Tambalotti (2010) also report a negative consumption response to an investment disturbance, but only for the first five periods. As discussed in Ascari, Phaneuf, and Sims (2016), most models would predict a countercyclical consumption response to an investment disturbance. The SW model overturns this property due to a sufficiently high degree of price and wage stickiness. Our agnostic approach implies similar estimates for price and wage stickiness, but nevertheless indicates that the data actually prefer a countercyclical consumption response.

[59] These are equations (3), (7), and (8) in the original SW paper, respectively. Ψ is the vector with structural coefficients and these restrict the coefficients in the model equations. See Smets and Wouters (2007) for the definitions of the coefficient functions. The subscripts of the coefficients of the agnostic disturbance refer to the SW equation number. For example, $d_3^B \widetilde{\varepsilon}_t^B$ is the term added to equation (3) of SW. i_t is the investment level, r_t^k the rental rate of capital, z_t the utilization rate, ε_t^i the SW investment-specific investment disturbance, $\widetilde{\varepsilon}_t^B$ the agnostic disturbance, and Ψ is the vector with structural parameters. Variables are defined relative to their steady-state values.

[60] The other two ASDs also enter some of these equations. We leave these terms out for transparency reasons and because there are no interactions in a linear framework.

The outcome is that the posterior mean of d_8^B has the *opposite* sign relative to $k_2(\Psi)$.[61]

This means that a reduction in the cost of transforming *current* investment into capital goes together with increased depreciation of the *existing* capital stock in our specification. In the SW model, an investment specific disturbance does not affect the economic viability of the existing capital stock. Our agnostic approach questions this assumption and suggests that the investment-specific productivity disturbance goes together with scrapping of older vintages. This is the reason why we refer to it as an agnostic investment-modernization disturbance.

In the SW model, capacity utilization is proportional to the rental rate and there are no shocks that can affect this relationship. An accelerated depreciation of the capital stock increases the rental rate, which in turn would induce an increase in the utilization rate. In our agnostic specification, this relationship is dampened somewhat, since a positive agnostic disturbance has a *direct* negative impact on capacity utilization since it enters the capacity utilization with a negative coefficient. The overall effect is still an increase in capacity utilization. It seems plausible that scraping of old vintages goes together with higher utilization of the remaining capital stock.

6.6.2 The agnostic Euler disturbance, $\widetilde{\varepsilon}_t^A$

The agnostic disturbance $\widetilde{\varepsilon}_t^A$ appears in eight equations. This leaves open many possible interpretations. The key equation, however, is the Euler equation for bonds, because excluding the disturbance from this equation leads to by far the largest drop in the marginal data density. This suggests that it could have key characteristics in common with a preference or a risk-premium disturbance. This view is also supported by table 7 which documents that $\widetilde{\varepsilon}_t^A$ is important for the same variables as the SW risk-premium disturbance. However, this agnostic disturbance also has some quite different characteristics from both. Therefore, we will adopt an alternative name and refer to it as the agnostic Euler disturbance. For the interpretation of $\widetilde{\varepsilon}_t^A$, it is important to understand the differences in impact of a regular preference and a regular (bond) risk-premium disturbance.

Difference between a preference and (bond) risk-premium disturbance. A preference disturbance affects current utility. This means it affects the marginal rate of substitution and, thus, *all* Euler equations. Such a preference disturbance is used in Smets and Wouters (2003). By contrast, Smets and Wouters (2007) include instead a (bond) risk premium that introduces a wedge between the policy rate and the required rate of return on bonds without affecting other Euler equations. Both disturbances have a strong impact on current consumption. However, a positive preference disturbance makes current consumption more desirable and reduces the attractiveness of *all* types of saving. A positive risk-premium disturbance only makes savings in bonds less

[61] Moreover, the 90% HPD does not include 0. Although it does not make much of a difference, we give the SW outcome the best possible chance by setting the prior means of the coefficients of $\widetilde{\varepsilon}_t^B$ to what they would be under the SW specification using SW prior means.

attractive. That is, it induces a desire to substitute out of bonds and into investment, in addition to an increase in consumption. Thus, a preference disturbance leads to a negative comovement of consumption and investment, whereas a (bond) risk-premium disturbance leads to a positive comovement. Smets and Wouters (2007) mention this as the reason for using a risk-premium instead of a preference disturbance.

There is another key difference between these two disturbance. A preference disturbance affects output in both the flexible-price and the sticky-price part of the model. By contrast, a risk-premium disturbance has no affect on key aggregates such as consumption and output in the flexible price part of the SW model.[62]

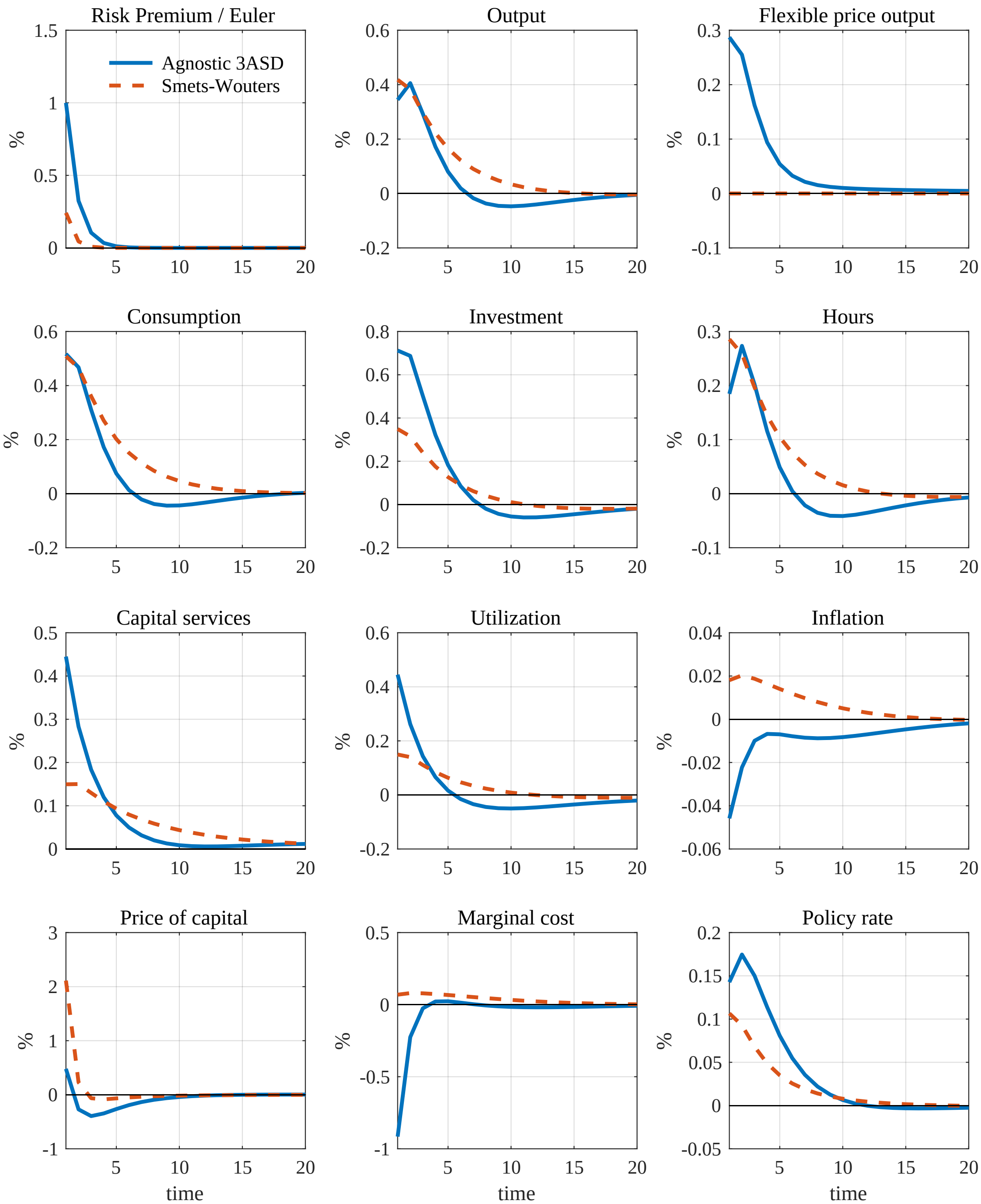
Is $\widetilde{\varepsilon}_t^A$ a preference, a risk-premium, or another type of disturbance? Figure 5 plots the IRFs of the SW risk-premium and our agnostic disturbance. The figure documents that both generate a regular business cycle with positive comovement for output, consumption, investment, and hours. The positive comovement suggest that the agnostic disturbance is a bond risk-premium disturbance as in Smets and Wouters (2007) and not a preference disturbance as in Smets and Wouters (2003). However, the agnostic disturbance has a strong impact on flexible-price output which is inconsistent with it being a (bond) risk-premium disturbance and consistent with it being a preference disturbance. These differences are big enough for us to come up with a new label and we choose Euler disturbance.

To better understand the nature of the agnostic Euler disturbance, we take a closer look at the equations in which $\widetilde{\varepsilon}_t^A$ enters. It appears in the aggregate budget constraint, the bond Euler equation, the investment Euler equation, the capital value equation, the utilization rate equation, the price mark-up equation, the rental rate of capital equation, and the Taylor rule.

Although our agnostic disturbance does have some effect on quite a few different aspects of the model, the interpretation is eased by the fact that the role of the agnostic disturbance is minor in most of the eight equations in the sense that allowing it to enter these equations only has a minor quantitative impact on the behavior of model variables or only affects the qualitative behavior of one or two variables without affecting the behavior of the key macroeconomic variables.

[62] The reason is the following. In the flexible price part of the model, the nominal policy rate, r_t, the expected inflation rate, $\mathbb{E}_t[\pi_{t+1}]$, and the risk-premium disturbance, ε_t^b, only appear in the combination $r_t - \mathbb{E}_t[\pi_{t+1}] + \varepsilon_t^b$. Consequently, a change in ε_t^b is simply absorbed by the real rate. This is not the case in the sticky-price economy, because it would be inconsistent with the Taylor rule.

Figure 5: IRFs of the SW risk-premium and the agnostic Euler disturbance



Notes. These figures plot the IRFs of the SW risk-premium disturbance and the agnostic disturbance $\tilde{\varepsilon}_t^A$ that we interpret as an Euler-equation disturbance.

Specifically, to understand the role of $\widetilde{\varepsilon}_t^A$ on key macroeconomic aggregates we can restrict ourselves to the Taylor rule and the three model equations that are relevant for the savings/investment decisions, which are the bond Euler equation, the investment Euler equation, and the capital value equation. The following set of equations documents how the SW risk-premium disturbance and our agnostic Euler enter these equations:[63]

Smets-Wouters risk premium, $\boldsymbol{\varepsilon_t^b}$

$$
\begin{aligned}
\text{Bond Euler:} \quad & c_t = c_1(\Psi)\, c_{t-1} + (1 - c_1(\Psi))\, \mathbb{E}_t[c_{t+1}] \\
& + c_2(\Psi)\,(l_t - \mathbb{E}_t[l_{t+1}]) \\
& - c_3(\Psi)\, \boldsymbol{(r_t - \mathbb{E}_t[\pi_{t+1}] + \varepsilon_t^b)},\ c_3(\Psi) > 0, && (29) \\
\text{Inv. Euler:} \quad & i_t = i_1(\Psi)\, i_{t-1} \\
& + (1 - i_1(\Psi))\, \mathbb{E}_t[i_{t+1}] + \varepsilon_t^i, && (30) \\
\text{Capital value:} \quad & q_t = q_1 \mathbb{E}_t[q_{t+1}] + (1 - q_1)\mathbb{E}_t\left[r_{t+1}^k\right] \\
& \boldsymbol{-(r_t - \mathbb{E}_t[\pi_{t+1}] + \varepsilon_t^b)}, && (31) \\
\text{Policy rate:} \quad & r_t = \rho r_{t-1} + (1 - \rho)\{r_\pi + r_Y(y_t - y_t^p)\} \\
& + r_{\Delta y}[(y_t - y_t^p) - (y_{t-1} - y_{t-1}^p)] + \varepsilon_t^r. && (32)
\end{aligned}
$$

Agnostic Euler disturbance, $\boldsymbol{\widetilde{\varepsilon}_t^A}$

$$
\begin{aligned}
\text{Bond Euler:} \quad & c_t = c_1(\Psi)\, c_{t-1} + (1 - c_1(\Psi))\, \mathbb{E}_t[c_{t+1}] \\
& + c_2(\Psi)\,(l_t - \mathbb{E}_t[l_{t+1}]) \\
& - c_3(\Psi)\, \boldsymbol{(r_t - \mathbb{E}_t[\pi_{t+1}]) - d_2^A \widetilde{\varepsilon}_t^A,\ d_2^A > 0}, && (33) \\
\text{Inv. Euler:} \quad & i_t = i_1(\Psi)\, i_{t-1} \\
& + (1 - i_1(\Psi))\, \mathbb{E}_t[i_{t+1}] + \varepsilon_t^i \boldsymbol{- d_3^A \widetilde{\varepsilon}_t^A,\ d_3^A > 0}, && (34) \\
\text{Capital value:} \quad & q_t = q_1 \mathbb{E}_t[q_{t+1}] + (1 - q_1)\mathbb{E}_t\left[r_{t+1}^k\right] \\
& \boldsymbol{-(r_t - \mathbb{E}_t[\pi_{t+1}]) - d_4^A \widetilde{\varepsilon}_t^A,\ d_4^A > 0}, && (35) \\
\text{Policy rate:} \quad & r_t = \rho r_{t-1} + (1 - \rho)\{r_\pi + r_Y(y_t - y_t^p)\} \\
& + r_{\Delta y}[(y_t - y_t^p) - (y_{t-1} - y_{t-1}^p)] + \varepsilon_t^r \boldsymbol{+ d_{14}^A \widetilde{\varepsilon}_t^A,\ d_{14}^A > 0}. && (36)
\end{aligned}
$$

As in SW, we use the bond Euler equation to substitute the marginal rate of substitution out of the capital valuation equation. While the SW bond risk-premium disturbance, ε_t^b, does *not* appear in the original capital valuation equation, it does show up *after* this substitution has taken place. Moreover, it appears in these two equations with the exact same coefficient as the nominal interest rate for bonds, r_t. By contrast, *after* substituting out the marginal rate of substitution in the capital value equation,

[63] In these equations, c_t is consumption, l_t is hours worked, r_t is the nominal policy rate, π_t is the inflation rate, q_t is the price of capital, y_t is output, and y_t^p is output in the flexible-price economy. Also see information given in footnote 59.

a preference disturbance would *no longer* appear in the capital valuation equation.[64]

Our ASD appears in the bond Euler equation and the capital valuation equation and it shows up with the same sign as the SW risk-premium disturbance. This supports the view that our ASD is similar to a risk-premium disturbance. Nevertheless, one could argue that the ASD is a preference and not a bond risk-premium disturbance for the following reasons. Although d_4^A has the right sign for a risk-premium coefficient, its magnitude, evaluated using the posterior mean, is too small.[65] The 90% HPD interval of the coefficient of $\widetilde{\varepsilon}_t^A$ in the capital valuation equation, d_4^A, includes zero and setting the coefficient equal to zero has very little impact on model properties and virtually none on the marginal data density.

As pointed out in Smets and Wouters (2003), a preference disturbance generates consumption and investment responses that move in opposite directions. Our ASD predicts responses in the same direction even if we impose that the ASD does not enter the capital valuation equation (after substituting out the MRS). The reason for the positive comovement is that our ASD also enters the investment Euler equation. The investment Euler equation is a dynamic equation, but its dynamic aspects are due solely to investment adjustment costs.[66] Our agnostic approach indicates that the structural disturbance that plays a key role in the bond Euler equation should also appear in the investment Euler equation. In fact, it is the first equation chosen in our specific-to-general model selection procedure.

So what could this agnostic disturbance represent? The simplest – and our preferred explanation – is that it is a preference disturbance that is correlated with an investment-specific disturbance.[67] A more structural interpretation would be the following. A preference disturbance would also affect the (linearized) investment Euler equation if investment does not only lead to expenses in the current, but also in subse-

[64]In the SW specification, the impact of the risk-premium disturbance is normalized to be equal to 1 in one of the equations. The actual impact of this disturbance on this equation is then determined by the estimated standard deviation. Instead, we normalize the standard deviation of the ASDs. We do not want to impose the SW normalization, since it would imply that the agnostic disturbance must affect the equation in which the coefficient is normalized unless the estimated standard deviation is equal to zero, which would mean that it would not have an effect on any other equation either.

[65]If our ASD is a risk-premium disturbance, then d_4^A/d_2^A should be equal to $1/c_3(\Psi)$, but using posterior means, we find that $d_4^A/d_2^A = 3.3$, whereas $1/c_3(\Psi) = 7.27$, substantially higher. Here, c_3 is a function of the habit, the elasticity of inter-temporal substitution, and the trend growth rate parameter. c_3 is calculated using posterior means of our preferred specification.

[66]Adjustment costs are zero in the steady state, which implies that neither a preference disturbance nor a risk-premium disturbance appear in a *linearized* investment Euler equation. A preference disturbance would appear in the original *nonlinear* equation. The main intertemporal aspect of the investment decision, which is also present without adjustment costs, is captured by the capital valuation equation.

[67]As discussed above, our agnostic structural investment disturbance, $\widetilde{\varepsilon}_t^B$, enters the capital accumulation equation with a sign that is the opposite of the regular investment disturbance, which we interpreted as scrapping of older vintages. $\widetilde{\varepsilon}_t^A$ does *not* enter the capital accumulation equation. This would indicate that this investment disturbance which goes together with an upswing in agents' mood is between a regular investment disturbance and our agnostic investment disturbance in terms of what it implies for the viability or depreciation of the existing capital stock.

quent periods. For example, investment may lead to additional expenses when capital becomes productive. A positive preference disturbance would lower the value of such future liabilities.

This disturbance appears directly in the Taylor rule with a negative coefficient. This means that the central bank responds more aggressively to business cycle fluctuations induced by this Euler disturbance. Without this effect on the Taylor rule this disturbance would have a stronger impact on economic aggregates and inflation would no longer be procyclical.[68]

6.6.3 The agnostic capital-efficiency wage mark-up disturbance, $\widetilde{\varepsilon}_t^C$

The third agnostic disturbance chosen by our model selection criterion increases the total number of structural disturbances to eight, that is, one more than the number in the SW specification. Thus, this agnostic disturbance cannot be interpreted as a replacement of a SW disturbance. Figure 6 plots its IRFs.

This third agnostic disturbance, $\widetilde{\varepsilon}_t^C$, appears in five equations. The first equation into which it is selected is the wage-adjustment equation. It also shows up into three equations related to capital, namely the capital accumulation equation, the capital utilization equation, and the capital-valuation equation. Finally, it appears in the economy-wide budget constraint, although the impact on the latter is minor.

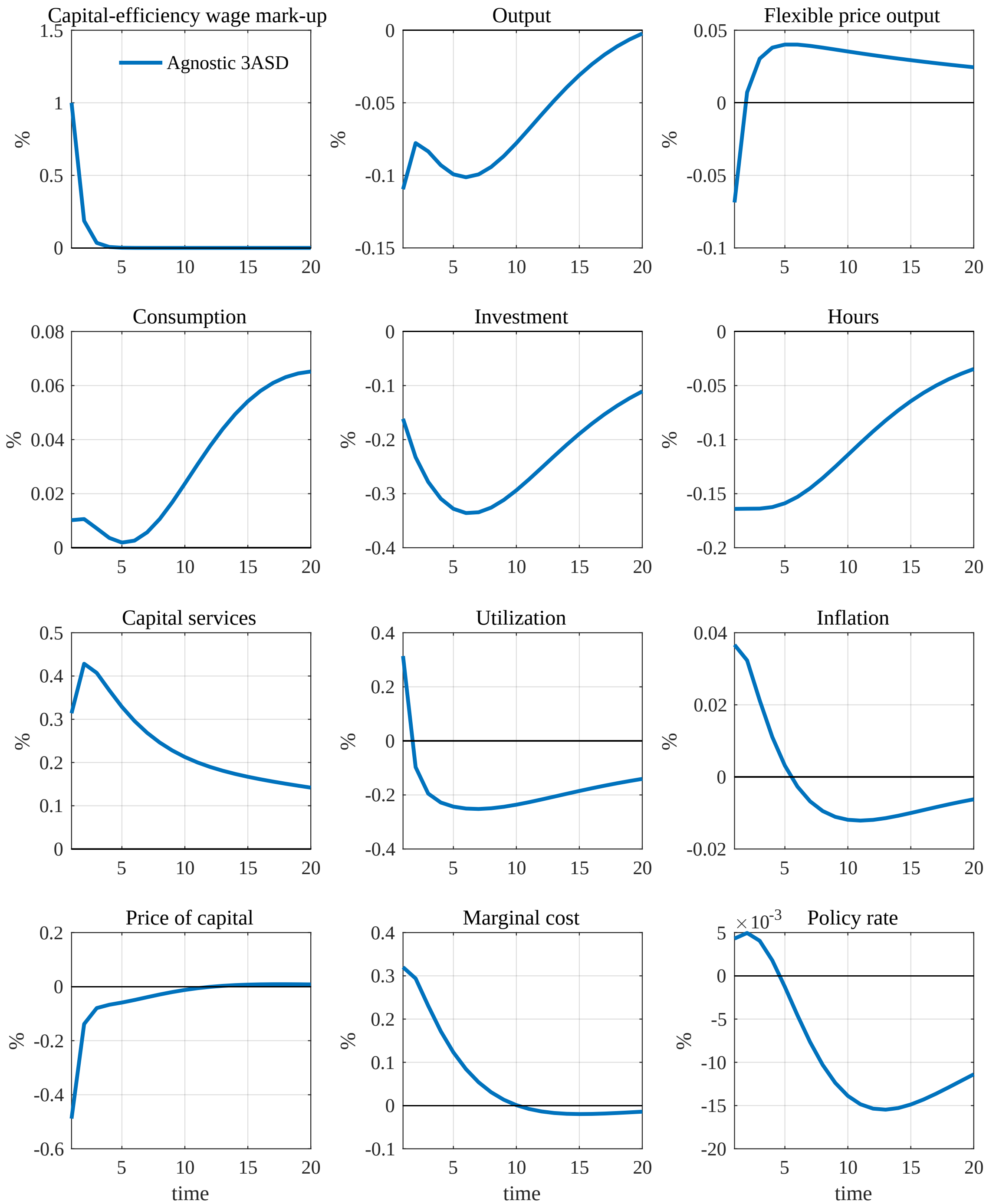
The SW wage mark-up disturbance, ε_t^w also shows up in the wage-adjustment equation. The differences with $\widetilde{\varepsilon}_t^C$ are the following. First, ε_t^w *only* shows up in the wage-adjustment equation, whereas $\widetilde{\varepsilon}_t^C$ has a direct impact on key equations related to capital. This is an important difference that results in quite different IRFs. A positive shock to ε_t^w induces a regular economic downturn with all key macroeconomic aggregates moving in the same direction, except for the price of capital which increases initially. A positive shock to $\widetilde{\varepsilon}_t^C$ also induces a recession with a reduction in output, investment, and employment. However, it leads to an increase in potential output, installed capital, and initially also an increase in capacity utilization. In contrast to the SW ε_t^w shock it leads to a decrease in the price of capital.

The second difference between our agnostic $\widetilde{\varepsilon}_t^C$ and the SW ε_t^w disturbance is that a shock to $\widetilde{\varepsilon}_t^C$ is very temporary. $\widetilde{\varepsilon}_t^C$ is an AR(1) process, and the posterior mean of the auto-regressive coefficient is equal to 0.19. The SW ε_t^w disturbance is a very persistent ARMA(1,1) process. The presence of $\widetilde{\varepsilon}_t^C$ in the empirical model strongly reduces the coefficient of the MA component of ε_t^w, but has little impact on the AR component.[69]

[68] See appendix D.4.

[69] Specifically, with $\widetilde{\varepsilon}_t^C$ included in the empirical specification the posterior means of the AR and the MA coefficients of ε_t^w are equal to 0.97 and 0.59, respectively. Estimates with the SW specification for these two numbers are 0.97 and 0.85.

Figure 6: IRFs of the agnostic capital-efficiency wage mark-up disturbance



Notes. These figures plot the IRFs of the agnostic disturbance $\widetilde{\varepsilon}_t^C$ that we interpret as a capital-efficiency wage mark-up disturbance.

Including $\widetilde{\varepsilon}_t^C$ in the empirical specification does not reduce the role of ε_t^w for fluctuations of key variables. ε_t^w remains the most important disturbance for key economic aggregates. The only exception is the wage *growth* rate. In the SW specification ε_t^w explains 61.6% of the volatility of wage growth, whereas it only explains 13.3% in our preferred specification. This role is clearly taken over by $\widetilde{\varepsilon}_t^C$ which explains 53.5% of wage growth volatility. $\widetilde{\varepsilon}_t^C$ also plays a nontrivial role for fluctuations in the capital stock, capacity utilization, and the rental rate of capital, explaining 9.8%, 14.7%, and 13.1%, of total variability respectively.

The following equations document how $\widetilde{\varepsilon}_t^C$ enters the model:[70]

Agnostic capital-efficiency wage mark-up disturbance, $\boldsymbol{\widetilde{\varepsilon}_t^C}$

$$
\begin{aligned}
\text{Capital value: } q_t &= q_1\mathbb{E}_t\left[q_{t+1}\right] + (1-q_1)\mathbb{E}_t\left[r_{t+1}^k\right] \\
&- \boldsymbol{\left(r_t - \mathbb{E}_t\left[\pi_{t+1}\right]\right) - d_4^C\widetilde{\varepsilon}_t^C, d_4^C < 0}, && (37)\\
\text{Utilization: } z_t &= z_1\left(\Psi\right) r_t^k + \boldsymbol{d_7^C\widetilde{\varepsilon}_t^C,\quad d_7^C > 0}, && (38)\\
\text{Capital: } k_t &= k_1\left(\Psi\right) k_{t-1} + \left(1 - k_1\left(\Psi\right)\right) i_t + \boldsymbol{d_8^C\widetilde{\varepsilon}_t^C,\quad d_8^C > 0}, && (39)\\
\text{Wage mark-up: } w_t &= w_1 w_{t-1} + (1-w_1)(\mathbb{E}_t[w_{t+1} + \pi_{t+1}] && (40)\\
&- w_2\pi_t + w_3\pi_{t-1} - w_4\mu_t^w + \boldsymbol{d_{13}^C\widetilde{\varepsilon}_t^C,\quad d_{13}^C > 0}. && (41)
\end{aligned}
$$

The equations indicate that this agnostic disturbance increases the wage mark-up and is associated with increased efficiency of the capital stock, both in terms of a lower depreciation rate and increased utilization. It also goes together with a reduction in the value of existing capital. Thus, this ASD could capture an increase in the wage rate, for example, because of increased bargaining power of workers, in response to which firms use capital more efficiently. An alternative is that its origin lies in changes in the ability or need to use capital more efficiently, but that a more efficient use of capital comes at the cost of higher wage rates. That is, to adopt this more efficient use of capital, firms have to pay a higher wage rate, perhaps in terms of an overtime premium.

[70] We leave out the overall budget constraint since the role of the disturbance in this equation is minor, but its impact in this equation is like a contractionary fiscal expenditure shock. Details are given in appendix D.5. w_t is the real wage rate and μ_t^w is the real wage mark-up, i.e., the difference between the wage rate and the marginal rate of substitution between consumption and leisure. Also see footnote 59 for additional information.

A Consequences of misspecification: An analytical example

In this section, we give a *very* simple example to indicate that misspecification can have large distortive effects in the sense that *implied* properties of the model using the parameter estimates can be at odds with the *actual* corresponding properties of the data that are used to estimate the parameters. The model is linear, and all variables have a Normal distribution. Throughout this section, parameter estimates are based on population moments. Thus, the results are not due to small sample variation. The estimation procedure is Maximum Likelihood (ML).

More specifically, this example demonstrates that there can be massive differences between the variances of observables as *implied* by the model using estimated parameter values and the actual variances in the data set. This result is surprising since the ML estimator of the variance of a given time series is the sample variance when the variable has a Normal distribution. We will show that this is not necessarily true for implied variances when the empirical model is misspecified.[71]

True model. The true model is given by the following set of equations:

$$y_t = \begin{bmatrix} y_{1,t} \\ y_{2,t} \end{bmatrix} = \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix} = \Lambda \varepsilon_t, \tag{42}$$

$$\mathbb{E}\left[\varepsilon_t \varepsilon_t'\right] = \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{bmatrix}, \tag{43}$$

and we make the following assumption about the distribution of the error terms:

$$\varepsilon_{1,t} \sim N(0, \sigma_1^2) \text{ and } \varepsilon_{2,t} \sim N(0, \sigma_2^2). \tag{44}$$

Misspecification. The objective is to estimate the standard deviations of the structural disturbances, σ_1^2 and σ_2^2. The researcher takes the value of Λ as given. The empirical model is misspecified, because $\overline{\Lambda} \neq \Lambda$ is used instead of the true value.

[71] As a byproduct of this paper, we learned that there also can be large gaps between *actual* properties of the data used and the corresponding *implied* properties according to the Maximum Likelihood estimates of the model parameters when the DSGE model is correctly specified, but a data sample with finite length is used. Since the objective of Maximum Likelihood is not to match moments, there is no reason why there should be a close match, but we were surprised by the large magnitudes of the differences. For example, using a sample of 1,000 observations generated by the SW model with seven disturbances and the correct empirical specification, it is not unusual to find implied standard deviations for the observables that are three to five times their data counterpart. Such differences will disappear as the sample size increases, since the estimator is consistent, but such asymptotic results do not provide much assurance if there is a small sample bias even at a relatively large sample size of 1,000 observations.

Empirical specifications. We consider the following two empirical specifications:

Case 1: Empirical model given by

$$y_t = \begin{bmatrix} y_{1,t} \\ y_{2,t} \end{bmatrix} = \overline{\Lambda}\varepsilon_t, \quad \mathbb{E}\left[\varepsilon_t \varepsilon_t'\right] = \begin{bmatrix} \overline{\sigma}_1^2 & \overline{\sigma}_{12} \\ \overline{\sigma}_{12} & \overline{\sigma}_2^2 \end{bmatrix}. \tag{45}$$

Case 2: Empirical model given by

$$y_t = \begin{bmatrix} y_{1,t} \\ y_{2,t} \end{bmatrix} = \overline{\Lambda}\varepsilon_t, \quad \mathbb{E}\left[\varepsilon_t \varepsilon_t'\right] = \begin{bmatrix} \overline{\sigma}_1^2 & 0 \\ 0 & \overline{\sigma}_2^2 \end{bmatrix}. \tag{46}$$

Both empirical models are misspecified, because they use the wrong value of Λ. In the first case, the empirical model allows the correlation between the two innovations to be non-zero even though it is equal to zero according to the true data generating process. In the second case, the empirical model imposes that the correlation is equal to zero, just as it is in the true model.

Case 1: Wrong Λ and allow for wrong σ_{12}. Since the model is linear and the shocks have a normal distribution, the ML estimator of the variance-covariance matrix $\mathbb{E}\left[\varepsilon_t \varepsilon_t'\right]$, $\widehat{\Sigma}_\varepsilon$, is given by

$$\widehat{\Sigma}_\varepsilon \;\; = \;\; \overline{\Lambda}^{-1}\widehat{\Sigma}_y'\overline{\Lambda}^{-1\prime}. \tag{47}$$

As mentioned above, we abstract from sampling variation and $\widehat{\Sigma}_y'$ is estimated using population moments. This means that the ML estimator of $\widehat{\Sigma}_\varepsilon'$ is given by

$$\begin{aligned} \widehat{\Sigma}_\varepsilon &= \overline{\Lambda}^{-1}\mathbb{E}\left[y_t y_t'\right]\overline{\Lambda}^{-1\prime} && (48) \\ &= \overline{\Lambda}^{-1}\Lambda\Lambda'\overline{\Lambda}^{-1\prime}. && (49) \end{aligned}$$

True versus implied variance. The purpose of this section is to document the consequences of misspecification for the implied variance of the observable y_t according to the estimated model. The *true* variance-covariance matrix is given by:

$$\Sigma_y^{\text{true}} = \mathbb{E}\left[y_t y_t'\right] = \Lambda\Lambda'. \tag{50}$$

The *implied* variance of y_t according the researcher's (misspecified) model, $\widehat{\Sigma}_y$, is given by

$$\begin{aligned} \widehat{\Sigma}_y &= \overline{\Lambda}\widehat{\Sigma}_\varepsilon\overline{\Lambda}' && (51) \\ &= \overline{\Lambda}\overline{\Lambda}^{-1}\Lambda\Lambda'\overline{\Lambda}^{-1\prime}\overline{\Lambda}' && (52) \\ &= \Lambda\Lambda' = \Sigma_y^{\text{true}}. && (53) \end{aligned}$$

Thus, the procedure actually generates the correct answer even though an incorrect empirical specification is used. In this case, the estimated empirical model is misspecified for two reasons, namely it has the wrong Λ and the estimated value of σ_{12} is not equal to its true value. These have exactly offsetting effects in terms of their impact on the implied variance. Another way to look at this result is the following. By allowing for a more flexible specification, i.e., a non-zero value for σ_{12}, the researcher would get a better answer for the implied variance of y_t even though the flexibility implies that the estimated model is wrong in more dimensions.

Case 2: Wrong Λ and correct σ_{12}. Obtaining the estimate for $\widehat{\Sigma}_\varepsilon$ is just as easy as in the previous case. Given $\widehat{\Lambda}$ and data for y_t, one can calculate the values for ε_t and use these to calculate the variance of ε_t and the implied variance of y_t. The following is a complicated, but useful way to express the outcome:

$$\widehat{\Sigma}_\varepsilon = \begin{array}{c} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \overline{\Lambda}^{-1} \Lambda\Lambda' \overline{\Lambda}^{-1\prime} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \\ + \\ \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \overline{\Lambda}^{-1} \Lambda\Lambda' \overline{\Lambda}^{-1\prime} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \end{array}. \tag{54}$$

True versus implied variance. The implied variance of y_t is equal to

$$\widehat{\Sigma}_y = \left(\begin{array}{c} \overline{\Lambda} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \overline{\Lambda}^{-1} \Lambda\Lambda' \overline{\Lambda}^{-1\prime} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \overline{\Lambda}' \\ + \\ \overline{\Lambda} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \overline{\Lambda}^{-1} \Lambda\Lambda' \overline{\Lambda}^{-1\prime} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \overline{\Lambda}' \end{array} \right) \neq \Lambda\Lambda' = \Sigma_y^{\text{true}} \tag{55}$$

The reason $\widehat{\Sigma}_y \neq \Sigma_y^{\text{true}}$ is that the $\overline{\Lambda}$ terms do not cancel out. In our Monte Carlo experiments with misspecified models, we find that there often are large gaps between the variances of the observables used in the estimation and the corresponding variances as implied by the model using the estimated parameters. Moreover, there is a bias. That is, the implied variance is typically larger than the actual variance. Our Monte Carlo experiments are a lot more complicated than this example, but this example may shed light on the coincidence of high implied variances. Specifically, because the $\overline{\Lambda}$s do not cancel out, the expression for $\widehat{\Sigma}_y$ contains terms like the following:

$$\overline{\Lambda} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \overline{\Lambda}^{-1} = \frac{1}{\overline{\lambda}_{11}\overline{\lambda}_{22} - \overline{\lambda}_{12}\overline{\lambda}_{21}} \begin{bmatrix} \overline{\lambda}_{11}\overline{\lambda}_{22} & -\overline{\lambda}_{11}\overline{\lambda}_{12} \\ \overline{\lambda}_{21}\overline{\lambda}_{22} & -\overline{\lambda}_{12}\overline{\lambda}_{21} \end{bmatrix}. \tag{56}$$

This equation documents that the ratio of the implied variance relative to the true variance could be arbitrarily large if the term in the denominator goes to zero.[72] For a

[72] The opposite is less likely, since it would require values for the λ_{ij} coefficients such that the com-

correctly specified model this would not matter, since the small term in the denominator would then be offset by an equally small term in the numerator. But this is not necessarily the case for an incorrectly specified model.

B Identification of structural parameters

We use the test proposed in Komunjer and Ng (2011) to check whether the parameters of the empirical specifications used in our experiments are identified. We will refer to this test as the KN test. This test provides both necessary and sufficient conditions for local identification under a set of weak conditions.[73] It focuses on the state-space representation of the model and – in contrast to earlier identification tests – does not require the user to specify a set of particular autocovariances.[74]

Identification of original Smets-Wouters estimation exercise. SW fix the values of five parameters: depreciation, δ, steady-state wage mark-up, $\overline{\mu}$, steady-state exogenous spending, $\overline{g}$, curvature in the Kimball goods-market aggregator, ε_p, and curvature in the Kimball labor-market aggregator, ε_w. Komunjer and Ng (2011) consider the identification of the SW model, but their empirical specification is slightly different from the one of SW in that all variables are demeaned. By contrast, the data in the SW estimation exercise does contain information about the level, since the inflation rate and the nominal interest rate are in levels. Komunjer and Ng (2011) show that several subsets of the five parameter restrictions mentioned above are sufficient to obtain identification *if* the parameter controlling steady state hours, $\bar{l}$, and the parameter controlling steady state inflation, $\overline{\pi}$, are fixed as well. It makes sense that identification requires more restrictions when information about the levels is not used in the estimation.

Identification of our specifications. The empirical and true specifications used in our Monte Carlo experiments have six structural disturbances, whereas the original SW empirical model has seven. This may imply that less parameters are identified. It is important that the parameters that we try to estimate are identified. If parameters are not identified, then different parameter combinations lead to the same criterion of fit used in the estimation, so it would not be surprising if parameter estimates are different for slightly different specifications.

Consequently, we adopt the following conservative strategy to ensure identification. The KN test checks rank conditions of matrices and to see whether there is a singularity one needs to choose a tolerance criterion. We set the criterion at a level that is more strict than the one chosen by KN.[75] We follow SW and fix the values of the five

binations appearing in square brackets are small, but the particular combination in the denominator is not. For example, one cannot accomplish this by simply choosing small values for the λ_{ij} terms.

[73]These are a stability condition and regularity conditions on the innovations.

[74]An example of such an earlier test is Iskrev (2010).

[75]We set "Tol" equal to 1e-2 instead of 1e-3 (a higher number is more strict).

Table 8: Komunjer and NG identification test.

required number		41	225	36	302	-
	n	$\overline{\Delta}_{\Lambda}^{S}$	$\overline{\Delta}_{T}^{S}$	$\overline{\Delta}_{U}^{S}$	$\overline{\Delta}^{S}$	pass?
ε_t^a excluded	12	41	225	36	302	yes
ε_t^b excluded	12	41	225	36	302	yes
ε_t^g excluded	12	41	225	36	302	yes
ε_t^i excluded	12	41	225	36	302	yes
ε_t^r excluded	12	41	225	36	302	yes
ε_t^w excluded	13	41	225	36	302	yes
ε_t^p excluded	13	41	225	36	302	yes

Notes. Here, n is the number of restrictions, which includes the number of coefficients fixed in all experiments and the number of coefficients in the law of motion of the excluded exogenous random variable that are all set to zero. $\overline{\Delta}_{\Lambda}^{S}$ is a matrix that contains the derivatives of all the vectorized elements in the state-space representation of the model (the A, B, C, D matrices and the variance-covariance matrices) evaluated at the true parameter values. It is intuitive that this matrix needs to have full rank for identification. But it is not sufficient. $\overline{\Delta}_{T}^{S}$ and $\overline{\Delta}_{U}^{S}$ are matrices with particular elements related to the state-space representation. The matrix $\overline{\Delta}^{S} = [\overline{\Delta}_{\Lambda}^{S}\ \overline{\Delta}_{T}^{S}\ \overline{\Delta}_{U}^{S}]$ needs to have full rank to pass the KN test.

parameters mentioned above. In addition, we fix all parameters that have a direct effect on the means of variables, since we use demeaned variables in the estimation. The associated parameters are the trend growth rate, $\overline{\gamma}$, the parameter controlling steady state hours, $\bar{l}$, the parameter controlling steady state inflation, $\overline{\pi}$, and the discount factor, β.[76] Finally, as discussed in section 2.1, we fix the spillover from the productivity disturbance to exogenous spending and set it equal to zero.

The results of the KN test are reported in table 8 and it indicates that the identification test is passed in all cases. That is, lack of identification is not driving the results in section 2.

C Additional results for Monte Carlo experiments

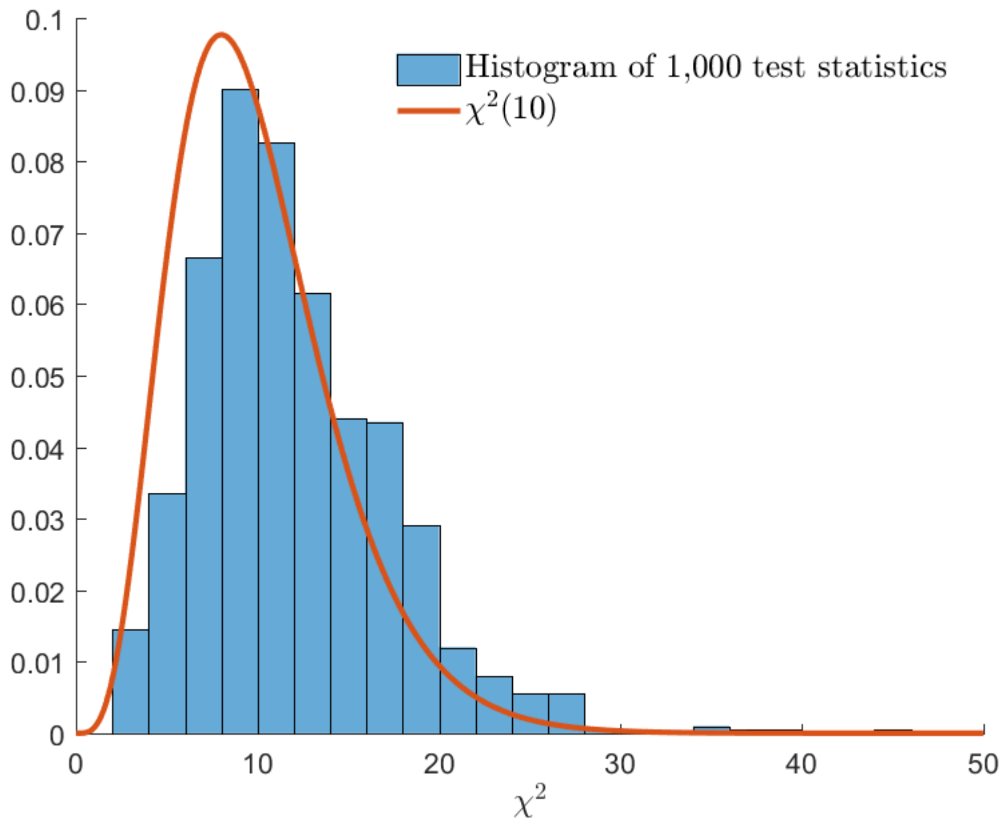
In this appendix, we report additional results for the analysis of section 5 in which we compared estimation outcomes using the ASD specification, the SW model with only regular structural disturbances, and an incorrect empirical model.

Figures 7 and 8 plot the histograms of the estimated χ^2 statistics across Monte Carlo replications for the two experiments of section 5 together with the theoretical (large-sample) χ^2 distribution. The number of degrees of freedom is equal to 10.

Tables 9 and 10 document detailed information on the distribution of parameter estimates for the two Monte Carlo experiments.

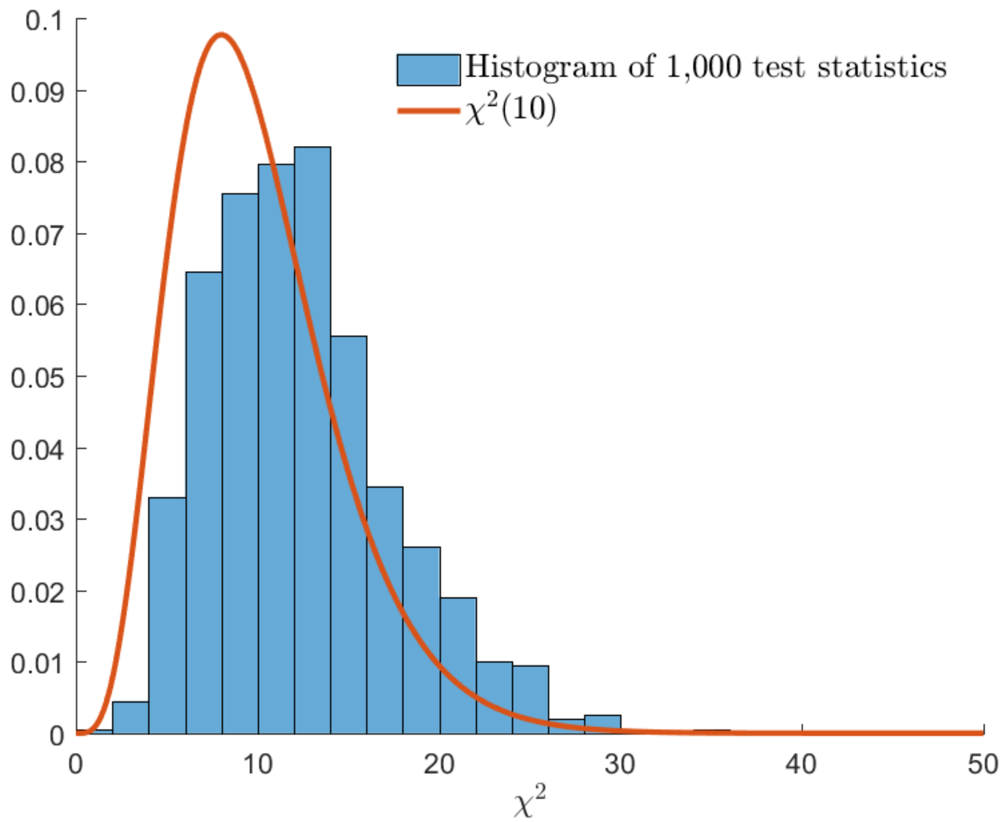
[76] It is a conservative choice to fix all four, since identification only requires that two parameters are fixed according to the test of Komunjer and Ng (2011).

Figure 7: Likelihood ratio test agnostic versus fully specified model: First experiment



Notes. The figure plots the distribution of χ^2 statistics of the first Monte Carlo experiment and the theoretical distribution according to large sample theory. This Monte Carlo experiment corresponds to the case when the true *dgp* does not include a monetary policy disturbance, but the empirical model leaves out the investment disturbance instead.

Figure 8: Likelihood ratio test agnostic versus fully specified model: Second experiment



Notes. The figure plots the distribution of χ^2 statistics of the first Monte Carlo experiment and the theoretical distribution according to large sample theory. This Monte Carlo experiment corresponds to the case when the true *dgp* does not include a TFP disturbance, but the empirical model leaves out the investment disturbance instead.

Table 9: Parameter estimates across Monte Carlo replications: First experiment

				misspecified estimation					**ASD procedure**					**SW specification**				
	Truth	**LB**	**UB**	**10%**	**25%**	**50%**	**75%**	**90%**	**10%**	**25%**	**50%**	**75%**	**90%**	**10%**	**25%**	**50%**	**75%**	**90%**
α	0.19	0.07	0.31	0.07	0.07	0.08	0.10	0.14	0.15	0.18	0.21	0.23	0.26	0.15	0.17	0.20	0.22	0.24
σ_c	1.39	0.53	2.25	1.35	1.44	1.66	2.16	2.24	1.16	1.29	1.47	1.69	1.95	1.19	1.29	1.42	1.58	1.81
Φ	1.61	1.33	1.89	1.46	1.69	1.86	1.89	1.89	1.33	1.34	1.57	1.85	1.89	1.33	1.38	1.60	1.83	1.89
ϕ	5.48	1.99	8.97	3.42	5.03	6.48	8.05	8.87	3.18	3.78	4.59	5.63	6.74	3.80	4.27	4.91	5.55	6.24
λ	0.71	0.45	0.90	0.62	0.71	0.79	0.84	0.86	0.57	0.62	0.66	0.71	0.76	0.61	0.64	0.68	0.72	0.76
ξ_w	0.73	0.47	0.92	0.58	0.65	0.71	0.78	0.86	0.57	0.62	0.68	0.75	0.82	0.58	0.63	0.69	0.75	0.81
σ_ℓ	1.92	0.18	3.66	0.18	0.18	0.18	0.19	0.54	0.89	1.16	1.55	2.17	2.77	1.07	1.36	1.76	2.24	2.71
ξ_p	0.65	0.40	0.86	0.69	0.74	0.79	0.83	0.85	0.50	0.56	0.63	0.69	0.73	0.52	0.57	0.63	0.68	0.73
ι_w	0.59	0.24	0.89	0.24	0.28	0.38	0.53	0.68	0.32	0.44	0.60	0.76	0.88	0.34	0.47	0.61	0.76	0.88
ι_p	0.22	0.01	0.65	0.04	0.13	0.25	0.38	0.51	0.02	0.08	0.16	0.25	0.34	0.03	0.08	0.16	0.24	0.31
ψ	0.54	0.20	0.86	0.26	0.38	0.59	0.75	0.84	0.38	0.47	0.57	0.69	0.81	0.41	0.48	0.57	0.66	0.77
r_π	2.03	1.45	2.61	1.58	1.78	2.05	2.33	2.55	1.71	1.88	2.10	2.41	2.60	1.76	1.91	2.09	2.35	2.57
ρ	0.81	0.53	0.97	0.74	0.77	0.80	0.82	0.83	0.78	0.80	0.82	0.84	0.85	0.78	0.80	0.82	0.84	0.85
r_y	0.08	-0.04	0.20	0.05	0.07	0.11	0.17	0.20	0.05	0.07	0.08	0.11	0.13	0.06	0.07	0.08	0.10	0.12
$r_{\Delta y}$	0.22	0.10	0.34	0.11	0.15	0.17	0.18	0.19	0.19	0.21	0.22	0.23	0.24	0.20	0.21	0.22	0.23	0.24
ρ_a	0.95	0.00	0.99	0.60	0.93	0.95	0.96	0.96	0.88	0.92	0.94	0.95	0.96	0.90	0.93	0.94	0.96	0.96
ρ_b	0.18	0.00	0.99	0.03	0.08	0.16	0.27	0.75	0.03	0.08	0.14	0.20	0.26	0.04	0.09	0.15	0.21	0.26
ρ_g	0.97	0.00	0.99	0.99	0.99	0.99	0.99	0.99	0.94	0.96	0.97	0.98	0.98	0.94	0.96	0.97	0.98	0.98
ρ_p	0.90	0.00	0.99	0.50	0.66	0.78	0.91	0.97	0.68	0.80	0.87	0.92	0.95	0.74	0.82	0.88	0.92	0.94
ρ_w	0.97	0.00	0.99	0.93	0.97	0.99	0.99	0.99	0.93	0.95	0.97	0.98	0.99	0.94	0.96	0.97	0.98	0.99
μ_p	0.74	0.00	0.99	0.21	0.38	0.64	0.86	0.94	0.31	0.48	0.62	0.73	0.80	0.38	0.51	0.64	0.73	0.80
μ_w	0.88	0.00	0.99	0.72	0.81	0.87	0.91	0.94	0.73	0.80	0.85	0.89	0.92	0.76	0.82	0.86	0.89	0.92
σ_a	0.45	0.00	10.00	0.62	0.70	0.85	1.05	1.22	0.35	0.38	0.42	0.48	0.53	0.37	0.39	0.44	0.48	0.53
σ_b	0.24	0.00	10.00	0.06	0.20	0.24	0.26	0.28	0.21	0.22	0.24	0.26	0.28	0.21	0.23	0.24	0.26	0.28
σ_g	0.52	0.00	10.00	0.51	0.53	0.55	0.57	0.59	0.48	0.49	0.52	0.54	0.56	0.48	0.50	0.52	0.54	0.56
σ_p	0.14	0.00	10.00	0.12	0.14	0.15	0.17	0.18	0.11	0.12	0.14	0.15	0.17	0.11	0.12	0.14	0.15	0.16
σ_w	0.24	0.00	10.00	0.19	0.20	0.22	0.24	0.25	0.21	0.23	0.24	0.26	0.28	0.21	0.23	0.25	0.26	0.28

Notes. The table provides information on the distribution of the indicated parameter across the Monte Carlo replications. See table 1 for the definitions of the parameters. This Monte Carlo experiment corresponds to the case when the true *dgp* does not include a monetary policy disturbance, but the empirical model leaves out the investment disturbance instead.

Table 10: Parameter estimates across Monte Carlo replications: Second experiment

				misspecified estimation					ASD procedure					SW specification				
	Truth	**LB**	**UB**	**10%**	**25%**	**50%**	**75%**	**90%**	**10%**	**25%**	**50%**	**75%**	**90%**	**10%**	**25%**	**50%**	**75%**	**90%**
α	0.19	0.07	0.31	0.09	0.12	0.16	0.21	0.26	0.14	0.18	0.21	0.25	0.29	0.14	0.17	0.20	0.23	0.26
σ_c	1.39	0.53	2.25	1.39	1.57	2.07	2.24	2.25	1.14	1.27	1.45	1.75	2.14	1.16	1.26	1.41	1.63	1.93
Φ	1.61	1.33	1.89	1.39	1.58	1.79	1.87	1.89	1.33	1.35	1.58	1.85	1.89	1.34	1.43	1.61	1.78	1.87
ϕ	5.48	1.99	8.97	4.09	4.99	6.23	7.43	8.48	3.29	3.85	4.57	5.43	6.54	3.85	4.34	4.98	5.64	6.44
λ	0.71	0.45	0.90	0.54	0.61	0.69	0.76	0.80	0.55	0.61	0.66	0.72	0.77	0.60	0.64	0.68	0.73	0.77
ξ_w	0.73	0.47	0.92	0.55	0.61	0.67	0.72	0.78	0.55	0.60	0.67	0.74	0.81	0.58	0.62	0.69	0.75	0.80
σ_ℓ	1.92	0.18	3.66	0.18	0.18	0.20	0.55	1.03	0.46	0.89	1.56	2.58	3.54	0.67	1.07	1.78	2.64	3.38
ξ_p	0.65	0.40	0.86	0.62	0.68	0.73	0.78	0.83	0.49	0.54	0.62	0.68	0.73	0.51	0.56	0.62	0.68	0.72
ι_w	0.59	0.24	0.89	0.24	0.30	0.42	0.57	0.69	0.32	0.46	0.61	0.78	0.89	0.34	0.47	0.62	0.77	0.88
ι_p	0.22	0.01	0.65	0.04	0.11	0.24	0.34	0.46	0.01	0.07	0.15	0.24	0.32	0.03	0.08	0.16	0.24	0.32
ψ	0.54	0.20	0.86	0.30	0.40	0.53	0.67	0.80	0.37	0.45	0.57	0.70	0.81	0.40	0.48	0.58	0.68	0.79
r_π	2.03	1.45	2.61	1.74	2.04	2.32	2.54	2.60	1.59	1.86	2.28	2.59	2.61	1.68	1.89	2.22	2.52	2.60
ρ	0.81	0.53	0.97	0.79	0.81	0.84	0.86	0.87	0.73	0.77	0.81	0.84	0.86	0.76	0.78	0.81	0.84	0.85
r_y	0.08	-0.04	0.20	0.08	0.10	0.13	0.16	0.19	0.04	0.06	0.10	0.12	0.14	0.05	0.07	0.09	0.12	0.13
$r_{\Delta y}$	0.22	0.10	0.34	0.11	0.13	0.17	0.20	0.22	0.17	0.19	0.23	0.26	0.29	0.17	0.20	0.22	0.25	0.28
ρ_b	0.18	0.00	0.99	0.07	0.14	0.22	0.33	0.53	0.02	0.08	0.14	0.21	0.26	0.04	0.09	0.15	0.21	0.26
ρ_g	0.97	0.00	0.99	0.98	0.99	0.99	0.99	0.99	0.94	0.96	0.97	0.98	0.98	0.94	0.96	0.97	0.98	0.98
ρ_r	0.12	0.00	0.99	0.00	0.00	0.02	0.06	0.10	0.00	0.04	0.11	0.18	0.23	0.01	0.05	0.11	0.17	0.22
ρ_p	0.90	0.00	0.99	0.48	0.65	0.78	0.91	0.97	0.74	0.82	0.89	0.92	0.95	0.78	0.84	0.89	0.92	0.95
ρ_w	0.97	0.00	0.99	0.94	0.96	0.97	0.98	0.99	0.93	0.95	0.97	0.98	0.99	0.94	0.96	0.97	0.98	0.98
μ_p	0.74	0.00	0.99	0.17	0.37	0.61	0.84	0.92	0.36	0.50	0.62	0.72	0.81	0.44	0.55	0.65	0.73	0.80
μ_w	0.88	0.00	0.99	0.73	0.79	0.84	0.89	0.91	0.73	0.79	0.84	0.89	0.91	0.76	0.81	0.85	0.89	0.92
σ_b	0.24	0.00	10.00	0.14	0.19	0.22	0.24	0.26	0.20	0.22	0.24	0.26	0.28	0.21	0.23	0.24	0.26	0.28
σ_g	0.52	0.00	10.00	0.49	0.51	0.53	0.55	0.57	0.47	0.49	0.51	0.54	0.56	0.48	0.50	0.52	0.54	0.56
σ_r	0.24	0.00	10.00	0.22	0.23	0.24	0.25	0.26	0.22	0.23	0.24	0.25	0.26	0.22	0.23	0.24	0.25	0.26
σ_p	0.14	0.00	10.00	0.12	0.14	0.15	0.16	0.18	0.11	0.12	0.14	0.15	0.17	0.11	0.12	0.14	0.15	0.16
σ_w	0.24	0.00	10.00	0.20	0.21	0.23	0.24	0.26	0.21	0.23	0.25	0.27	0.28	0.22	0.23	0.25	0.27	0.28

Notes. The table provides information on the distribution of the indicated parameter across the Monte Carlo replications. See table 1 for the definitions of the parameters. This Monte Carlo experiment corresponds to the case when the true *dgp* does not include a TFP disturbance, but the empirical model leaves out the investment disturbance instead.

D ASD procedure for the Smets-Wouters model

In this appendix, we provide further details on how the ASD procedure is implemented in section 6. We also provide additional results.

D.1 Including ASDs in SW equations

To apply the ASD procedure to the SW model, we adapt the Dynare program provided by the authors.[77] Adapting a Dynare program to add an agnostic disturbance is easy. Specifically, for the first ASD, $\widetilde{\varepsilon}_t^{\,A}$, we do the following.

1. In the model block, we add $\boldsymbol{d}_i^A \widetilde{\varepsilon}_t^A$ to the i^{th} equation, where $\widetilde{\varepsilon}_t^A$ is the agnostic disturbance and $\boldsymbol{d}_i^A$ the coefficient associated with the agnostic disturbance in the i^{th} equation. Details are given below.[78]

2. We add an equation to the model block that describes the law of motion for $\widetilde{\varepsilon}_t^A$. If the agnostic disturbance replaces a regular structural disturbance, then this disturbance should be taken out of the program.

3. Declare $\widetilde{\varepsilon}_t^A$ as a variable and declare the elements of $\boldsymbol{d}_i^A$ and the coefficients of the law of motion for $\widetilde{\varepsilon}_t^A$ as parameters.

4. Specify a prior for the elements of $\boldsymbol{d}_i^A$.

We do not add the agnostic disturbance to equations (6) and (12) of the SW model, because these equations just contain definitions for capacity utilization and the wage mark-up, respectively.[79] The set of equations for the SW model consists of two parts. The first part models the flexible price economy and the second part models the actual economy with sticky prices. One needs to model the flexible-price economy, because the flexible-price output level is used to define the output gap, which is one of the arguments in the monetary policy rule. In principle, one could let the agnostic disturbance enter the equations of the sticky-price economy and the associated equations in the flexible-price economy with a different coefficient.[80] In most economic models, however, structural disturbances would enter the associated pair of equations in the same way. Therefore, we also restrict the agnostic disturbance to enter the associated equations in the same way. The exception is SW equation (13) because it captures

[77] The program is available at https://www.aeaweb.org/articles?id=10.1257/aer.97.3.586 under the "Download Data Set" link.

[78] The other two ASDs are added using the same procedure. The $\boldsymbol{d}^A$ coefficients correspond to the $\widehat{\boldsymbol{\Gamma}}_{\mathbf{2}}$ coefficients in section 3.2.1. We adapt the notation, since SW also use lower case Roman letters for coefficients.

[79] Equation numbers refer to those in Smets and Wouters (2007). We do allow the agnostic disturbances to affect the utilization rate and the wage mark-up directly by including it in the model equations that specify their relationship with other model variables.

[80] The sticky-price block contains some equations, such as the monetary policy rule, that do not have a counterpart in the flexible-price economy.

both potential stickiness in wages and the relationship between the wage rate and its mark-up.

Specifically, we add the agnostic disturbance to equations (1), (2), (3), (4), (5), (7), (8), (9), and (11) of the SW model and the associated equations of the flexible-price economy. We also add it to equation (13) in both the flexible and the sticky-price part of the model, but here we allow coefficients to differ. In addition, we add the agnostic disturbance to equations (10) and (14) which do not have a counterpart in the flexible-price economy. This means that $\boldsymbol{d}^A$ has thirteen elements. The last coefficient associated with the agnostic disturbance is the autoregressive coefficient of its law of motion. The standard deviation of the agnostic disturbance is normalized to be equal to 1.

D.2 Model selection procedures

The general-to-specific model selection procedure starts with the specification in which the agnostic disturbances are allowed to enter each model equation. It then calculates the marginal data densities for all possible specifications in which the ASD is *not* allowed to enter *one* of the model equations. Thus, we estimate a set of models, each having one less coefficient. If none of the specifications lead to a better fit, then the procedure stops. If improvements are found, then the procedure is repeated using the specification that led to the biggest improvement as the benchmark.

The specific-to-general procedure starts with the specifications in which each of the two ASDs are allowed to enter only one model equation. To avoid a singularity, one cannot start with a more parsimonious model.[81] In the next step, we estimate a set of models in which one of the ASDs is added to one equation and, thus, one additional parameter is estimated. The procedure stops if none of the specifications leads to an improvement. If there is an improvement, then the specification with the largest improvement becomes the next benchmark and the procedure is repeated.

Why not consider even more general specifications? Although our model selection procedures consider a rich set of models, they are not the most general. Unfortunately, there are practical limitations to what is feasible. Five SW disturbance are always included in our specifications. The most ideal setup would be flexible in this dimension as well and not safeguard any of the seven SW regular disturbances and allow for the possibility of including seven ASDs (or more). With such a setup all SW disturbances could be replaced by an ASD. The first problem one would have to deal with is that identification of structural parameters is likely to limit the number of regular structural disturbances one can replace with ASDs. Let us consider a simple setup in which there are seven equations for seven state variables and all state variables

[81] The posteriors of the ASD coefficients in the fully agnostic model provide clear evidence that one of the ASDs is very important for the bond Euler equation and one for the investment Euler equation. So these are natural choices.

are observables. Moreover, each equation has one regular structural disturbance. A general-to-specific procedure would be complicated since the first-stage model would have a large number of coefficients to estimate. Specifically, if all seven ASDs appear in all equations, then one needs to estimate forty-nine reduced-form coefficients. One may need a rich data set to identify all of them. In our application, the number of coefficients would be equal to ninety-one, since we have thirteen equations. The specific-to-general procedure faces the problem that each specification needs at least seven disturbances to avoid singularities. This means that there are a large number of different models one can start with. For the simple setup with seven equations described above, this would mean that there are already $2^7 = 128$ different models to consider in the first round alone.

D.3 Additional results

Specifications with and without restrictions on ASDs. Table 11 compares structural parameter estimates of models chosen by our model selection procedures with those that contain the same number of ASDs, but allow ASDs to enter all equations. The latter are fully agnostic. The parameter estimates are fairly similar. IRFs for the included regular structural disturbances are also quite similar. That is not always the case for the IRFs of the agnostic disturbances themselves. The IRFs for some variables do differ between the concise and the fully unrestricted ASD specification. Given the misspecification results of section 2, it is not surprising that different empirical specifications lead to different results. Another issue with the fully unrestricted ASD specification is that it estimates a large number of coefficients which complicates generating an accurate posterior with Monte Carlo Markov Chain algorithms. Especially, for the 3-ASD fully unrestricted specification, the Brooks-Gelman statistics did not look particularly good for some of the coefficients associated with the agnostic disturbances.

Specifications with two and three ASDs. Tables 12 and 13 provide the role of the regular and agnostic disturbances for the fluctuations of a wide range of variables. In addition to the results of the SW specification, it also shows the results for the two-ASD and three-ASD specification chosen by our specific-to-general procedure. It shows that the results are very similar for the two chosen ASD specifications. The same conclusion can be drawn from figures 9 and 10 that plot the IRFs for two agnostic disturbances.

Table 11: Posterior Means

Parameter	Original SW	Agnostic: 2 ASDs		Agnostic: 3 ASDs	
		concise	unrestricted	concise	unrestricted
α	0.1903	0.2044	0.1878	0.1877	0.2089
σ_c	1.3889	1.4657	1.4535	1.4618	1.4772
Φ	1.6083	1.5211	1.5242	1.4741	1.4762
ϕ	5.7405	5.3843	4.4031	5.3425	4.6933
λ	0.7136	0.6544	0.7055	0.6679	0.6930
ξ_w	0.7066	0.6660	0.6706	0.7268	0.6453
σ_ℓ	1.8458	1.9094	1.7733	2.0770	1.5916
ξ_p	0.6541	0.6566	0.6981	0.6412	0.6902
ι_w	0.5783	0.5556	0.5432	0.5077	0.5557
ι_p	0.2389	0.2010	0.1997	0.1871	0.1891
ψ	0.5426	0.5345	0.5049	0.5283	0.3176
r_π	2.0469	1.7676	1.7797	1.7746	1.7438
ρ	0.8105	0.7933	0.8082	0.8018	0.8032
r_y	0.0887	0.0725	0.0860	0.0787	0.0819
$r_{\Delta y}$	0.2237	0.1903	0.1703	0.1941	0.1608
ρ_a	0.9572	0.9555	0.9483	0.9532	0.9510
ρ_g	0.9764	0.9719	0.9710	0.9702	0.9018
ρ_r	0.1464	0.1376	0.1219	0.1286	0.1227
ρ_p	0.8893	0.8975	0.8899	0.9262	0.9080
ρ_w	0.9680	0.9751	0.9790	0.9747	0.9822
ρ_b / ρ_A	0.2165	0.3344	0.6386	0.3239	0.4527
ρ_i / ρ_B	0.7116	0.6087	0.1660	0.6069	0.7232
ρ_C	-	-	-	0.1865	0.1577
μ_p	0.6977	0.6764	0.6923	0.7166	0.7172
μ_w	0.8466	0.8241	0.8368	0.5945	0.8168
ρ_{ga}	0.5184	0.6438	0.6525	0.6709	0.5448
σ_a	0.4586	0.4436	0.4421	0.4524	0.4411
σ_g	0.5299	0.4702	0.4689	0.4428	0.2285
σ_r	0.2449	0.2180	0.2171	0.2171	0.2114
σ_p	0.1403	0.1346	0.1299	0.1308	0.1311
σ_w	0.2427	0.2384	0.2361	0.0763	0.2249
σ_b	0.2398	-	-	-	-
σ_i	0.4525	-	-	-	-
$100(\beta^{-1}-1)$	0.1648	0.1685	0.1826	0.1656	0.2038
$\bar{\gamma}$	0.4316	0.4349	0.4386	0.4367	0.4352
$\bar{\pi}$	0.7845	0.7483	0.7443	0.7391	0.7534
$\bar{\ell}$	0.5617	0.1263	0.5216	0.1303	1.0360
MDD	-922.40	-892.92	-906.85	-890.73	-925.50

Notes. MDD stands for marginal data density. The "concise" ASD specifications are the ones chosen by the specific-to-general model selection procedure. The "unrestricted" ASD specifications are the fully agnostic with no zero restrictions. See table 1 for the definitions of the parameters.

Table 12: Variance decomposition for observables across model specifications

		ε^a	ε^g	ε^r	ε^p	ε^w	$\varepsilon^b/\widetilde{\varepsilon}^A$	$\varepsilon^i/\widetilde{\varepsilon}^B$	$\widetilde{\varepsilon}^C$
Δy	Original SW	16.10	28.88	6.17	4.55	6.39	22.12	15.79	-
	Agnostic: 2 ASDs	20.29	27.01	7.15	6.04	8.12	20.53	10.85	-
	Agnostic: 3 ASDs	22.21	24.60	7.04	4.66	10.30	21.33	8.04	1.82
Δc	Original SW	5.29	2.10	11.56	4.40	14.54	61.17	0.95	-
	Agnostic: 2 ASDs	3.26	1.62	11.29	4.56	15.33	62.34	1.61	-
	Agnostic: 3 ASDs	2.95	1.28	10.69	3.37	17.90	61.67	2.03	0.1
Δi	Original SW	6.01	0.84	2.47	3.80	2.37	2.46	82.05	-
	Agnostic: 2 ASDs	4.86	0.91	2.19	4.24	2.76	12.25	72.80	-
	Agnostic: 3 ASDs	5.49	1.02	2.38	3.80	3.94	12.55	70.01	0.81
ℓ	Original SW	1.94	10.34	3.15	6.23	67.66	2.52	8.15	-
	Agnostic: 2 ASDs	1.29	6.84	2.47	6.04	71.23	1.56	10.57	-
	Agnostic: 3 ASDs	1.08	4.33	2.15	4.44	79.70	1.29	4.97	2.03
Δw	Original SW	4.53	0.09	1.48	29.47	61.61	0.79	2.03	-
	Agnostic: 2 ASDs	3.82	0.22	2.43	30.84	54.34	3.02	5.34	-
	Agnostic: 3 ASDs	4.09	0.11	1.25	25.18	13.32	2.23	0.38	53.45
π	Original SW	3.92	1.00	4.25	27.64	59.43	0.58	3.18	-
	Agnostic: 2 ASDs	3.16	1.28	4.43	24.91	61.96	0.79	3.46	-
	Agnostic: 3 ASDs	2.95	0.90	3.28	16.87	70.46	0.68	3.96	0.91
r	Original SW	10.09	3.90	14.67	7.17	38.42	7.40	18.34	-
	Agnostic: 2 ASDs	6.50	3.49	9.77	5.79	38.96	21.49	14.02	-
	Agnostic: 3 ASDs	5.70	2.77	8.18	4.33	48.61	17.29	12.47	0.65

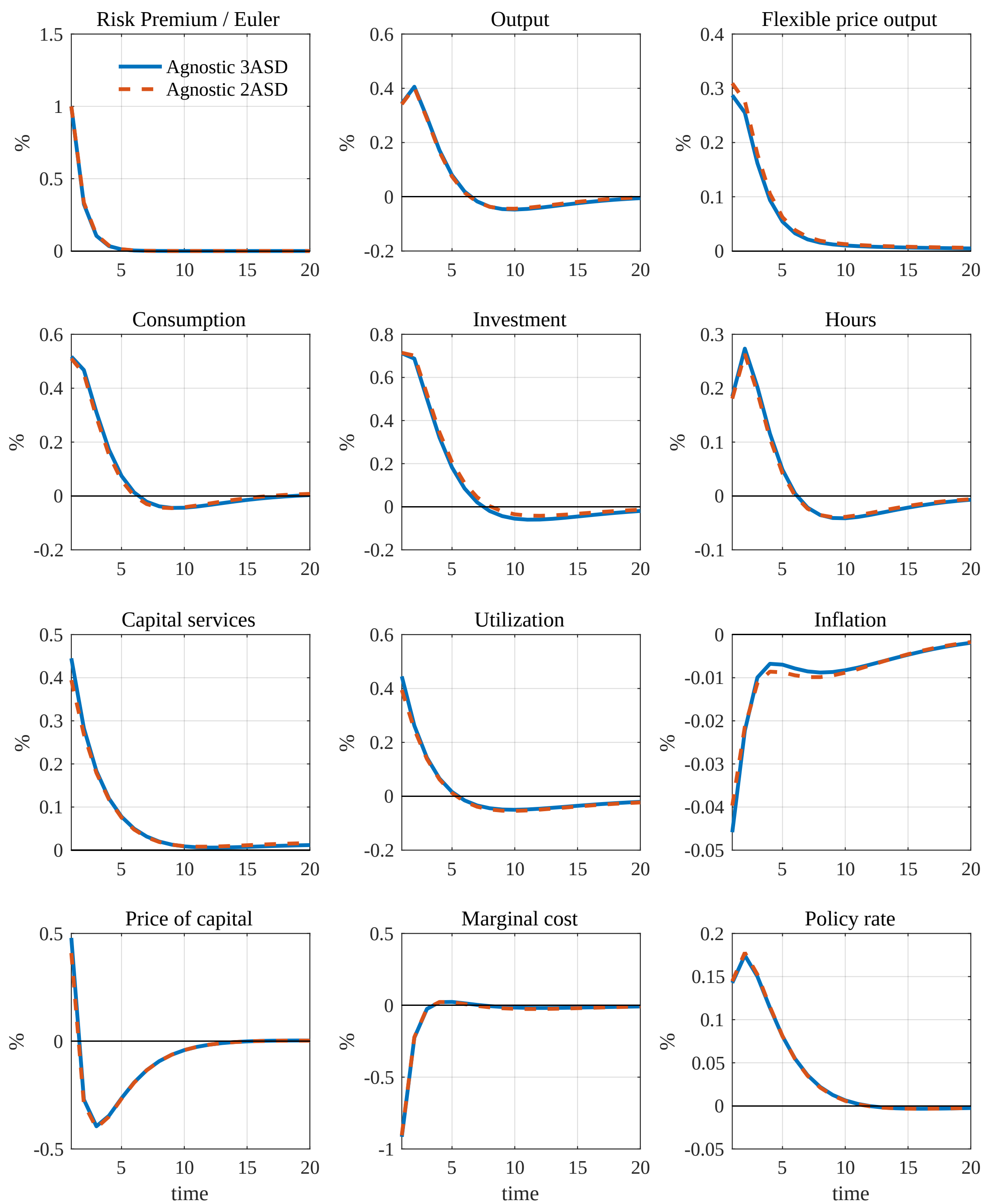
Notes. The table provides the contributions (in percent) of the different structural disturbances to the variance of the observable variables, across different model specifications. The ASD specifications are the ones chosen by our model selection procedure. y stands for log output; c for log consumption; i for log investment; l for hours; w for log wage rate; π for inflation; and r for nominal interest rate. Structural disturbances are defined as follows. ε^a: TFP; ε^g: government expenditures; ε^r: monetary policy; ε^p:price mark-up; ε^w: wage mark-up; ε^b: risk premium; ε^i: investment; $\widetilde{\varepsilon}^A$: agnostic Euler; $\widetilde{\varepsilon}^B$: agnostic investment-modernization; and $\widetilde{\varepsilon}^C$: capital-efficiency wage mark-up.

Table 13: Variance decomposition for additional variables across model specifications

		ε^a	ε^g	ε^r	ε^p	ε^w	$\varepsilon^b/\widetilde{\varepsilon}^A$	$\varepsilon^i/\widetilde{\varepsilon}^B$	$\widetilde{\varepsilon}^C$
y_t	Original SW	29.93	4.09	2.16	6.37	48.58	1.53	7.34	-
	Agnostic: 2 ASDs	26.50	3.02	1.91	7.02	55.93	1.31	4.32	-
	Agnostic: 3 ASDs	21.19	2.13	1.67	5.47	65.95	1.14	2.17	0.28
c_t	Original SW	11.06	8.42	2.08	4.19	69.25	2.18	2.83	-
	Agnostic: 2 ASDs	6.60	6.60	1.78	4.23	78.76	1.81	0.22	-
	Agnostic: 3 ASDs	4.29	4.30	1.52	3.16	84.48	1.51	0.49	0.25
i_t	Original SW	20.37	5.41	1.27	6.93	21.56	0.22	44.23	-
	Agnostic: 2 ASDs	17.22	6.35	1.14	8.25	29.78	1.21	36.04	-
	Agnostic: 3 ASDs	15.31	5.79	1.13	7.75	38.72	1.06	29.25	1.00
r_t^k	Original SW	14.86	17.47	1.63	10.58	19.21	0.86	35.39	-
	Agnostic: 2 ASDs	12.28	20.44	2.65	19.09	29.73	0.92	14.88	-
	Agnostic: 3 ASDs	8.41	14.66	1.73	13.16	30.17	0.67	18.12	13.08
q_t	Original SW	4.65	0.55	9.03	3.11	1.20	45.42	36.04	-
	Agnostic: 2 ASDs	9.78	1.35	21.83	9.30	3.67	19.58	34.49	-
	Agnostic: 3 ASDs	9.72	1.35	19.88	6.50	5.14	18.64	31.56	7.21
z_t	Original SW	14.86	17.47	1.63	10.58	19.21	0.86	35.39	-
	Agnostic: 2 ASDs	12.23	20.36	2.64	19.02	29.61	4.43	11.71	-
	Agnostic: 3 ASDs	8.86	15.43	1.82	13.85	31.76	4.14	9.46	14.68
μ_t^p	Original SW	11.56	0.29	3.27	57.02	23.87	0.87	3.11	-
	Agnostic: 2 ASDs	8.06	0.37	3.38	53.22	18.90	14.59	1.48	-
	Agnostic: 3 ASDs	7.99	0.24	2.13	54.80	11.88	15.22	2.61	5.13
k_t^s	Original SW	23.43	3.92	1.23	11.37	34.19	0.36	25.50	-
	Agnostic: 2 ASDs	21.82	4.90	1.55	16.51	52.66	1.32	1.24	-
	Agnostic: 3 ASDs	15.59	3.60	1.11	14.11	58.20	1.21	0.61	5.57
k_t	Original SW	22.38	8.11	0.50	4.93	31.56	0.04	32.48	-
	Agnostic: 2 ASDs	22.16	11.30	0.55	7.27	55.74	0.21	2.77	-
	Agnostic: 3 ASDs	14.84	8.05	0.42	6.18	58.26	0.12	2.37	9.75
w_t	Original SW	33.03	1.03	1.95	38.38	18.61	0.40	6.60	-
	Agnostic: 2 ASDs	26.99	1.00	2.63	47.34	20.71	0.39	0.92	-
	Agnostic: 3 ASDs	25.35	0.74	1.62	49.34	14.29	0.30	0.44	7.92

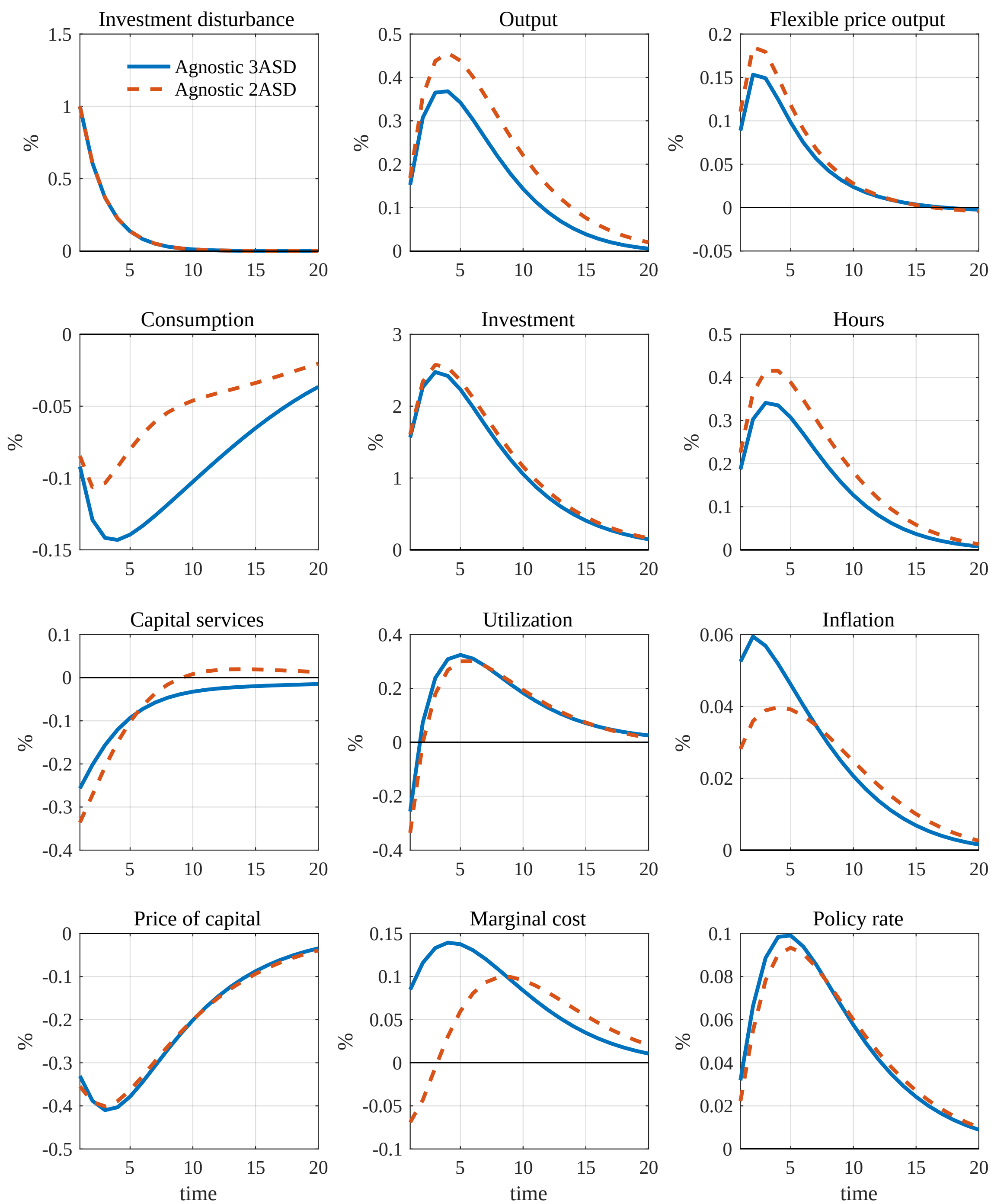
Notes. The table provides the contributions (in percent) of the different structural disturbances to the variance of the observable variables, across different model specifications. The ASD specifications are the ones chosen by our model selection procedure. y stands for log output; c for log consumption; i for log investment; l for hours; w for log wage rate; r^k for rental rate on capital; q for the log price of capital; z for the utilization rate; μ^p for the price mark-up; k^s for log capital used in production; and k for log installed capital. Structural disturbances are defined as follows. ε^a: TFP; ε^g: government expenditures; ε^r: monetary policy; ε^p:price mark-up; ε^w: wage mark-up; ε^b: risk premium; ε^i: investment; $\widetilde{\varepsilon}^A$: agnostic Euler; $\widetilde{\varepsilon}^B$: agnostic investment-modernization; and $\widetilde{\varepsilon}^C$: capital-efficiency wage mark-up.

Figure 9: IRFs of the agnostic Euler disturbance: 2 versus 3 ASDs



Notes. These figures plot the IRFs of the agnostic disturbance $\tilde{\varepsilon}_t^A$ that we interpret as a general Euler disturbance for the empirical specifications with two and three ASDs. Both are chosen with the specific-to-general model selection procedure.

Figure 10: IRFs of the agnostic investment-modernization disturbance: 2 versus 3 ASDs



Notes. These figures plot the IRFs of the agnostic disturbance $\tilde{\varepsilon}_t^B$ that we interpret as an investment-modernization disturbance for the empirical specifications with two and three ASDs. Both are chosen with the specific-to-general model selection procedure.

D.4 Additional results for $\widetilde{\varepsilon}_t^A$

Figure 11 plots the IRFs associated with an innovation in the agnostic Euler disturbance for our 3-ASD benchmark specification and also when the coefficient of this agnostic disturbance in the capital valuation equation is equal to zero. A zero coefficient in this equation means the disturbance is like a preference and not like a bond risk-premium disturbance.[82] The IRFs are very similar, which confirms our claim that the coefficient in the capital valuation equation is quantitatively not very important.

Figure 12 plots the same IRFs when the coefficient of the agnostic Euler disturbance in the Taylor rule is set equal to zero. The figure shows that the direct response of the policy rate to a positive shock to this disturbance dampens the expansion and prevents an upsurge of inflation.

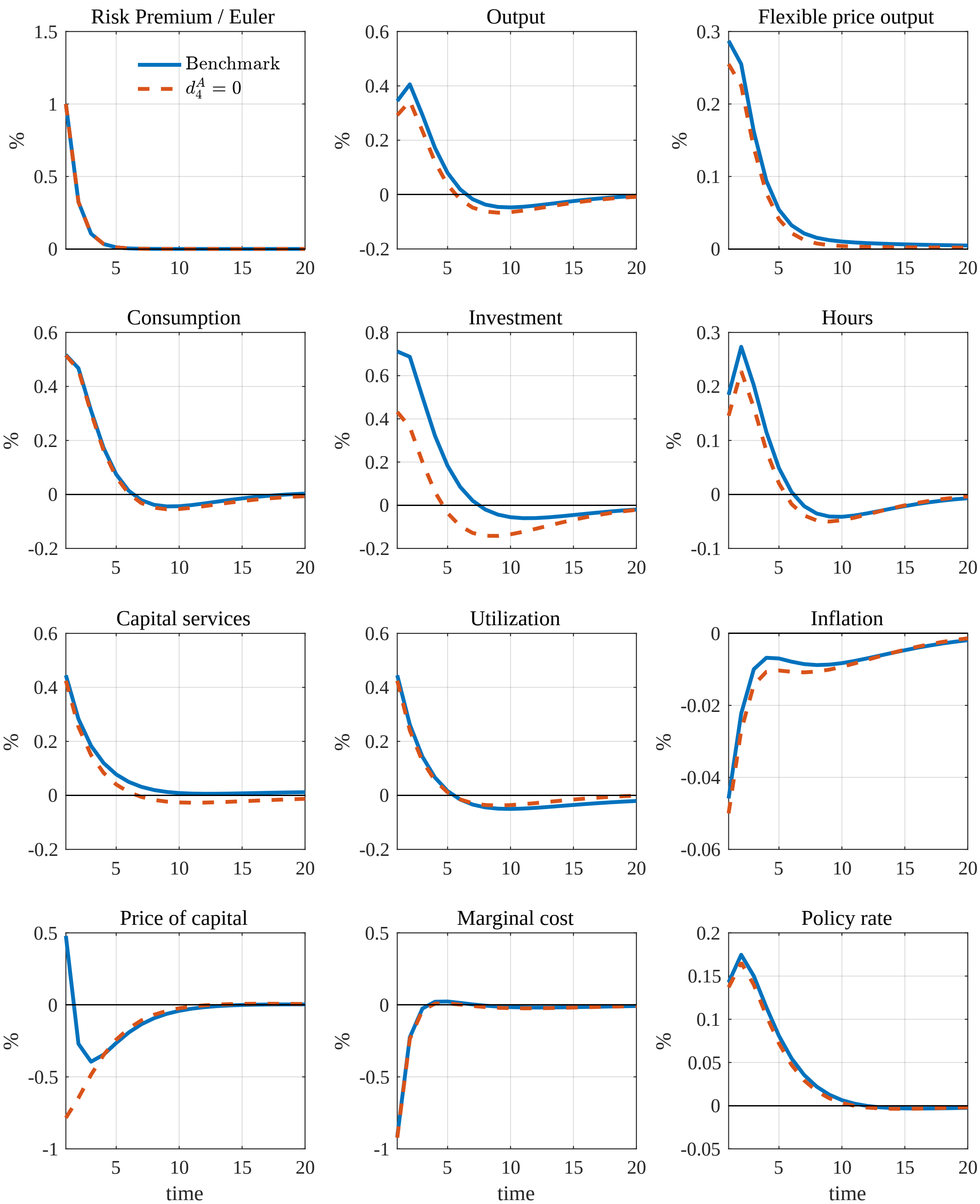
Figure 13 plots the same IRFs when we set equal to zero the coefficients of the disturbance in the four equations that we ignored in the discussion of the agnostic Euler disturbance, namely, the overall budget constraint, the utilization, the price mark-up equation, and the rental rate of capital equation. The figure documents that the role of the agnostic disturbance through these equations is minor since the IRFs are overall quite similar to those of our benchmark specification.

D.5 Additional results for $\widetilde{\varepsilon}_t^C$

Figure 14 plots the IRFs for our agnostic capital-efficiency wage mark-up disturbance when the coefficient of this disturbance in the overall budget constraint is set equal to zero. The figure documents that this has a minor impact on IRFs.

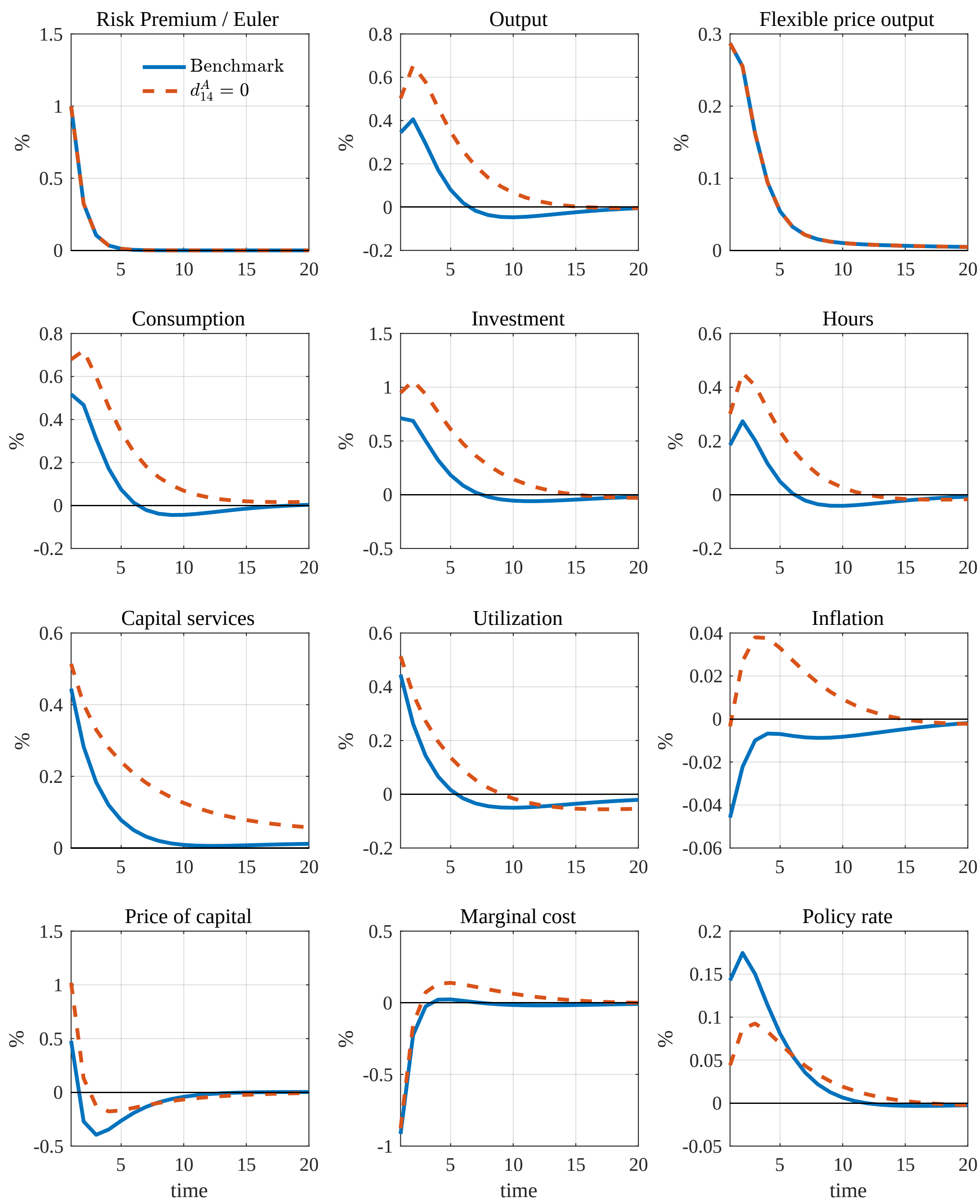
[82] Recall that the MRS has been substituted out of the capital valuation equation using the MRS of the bond Euler equation.

Figure 11: IRFs of the agnostic Euler disturbance with restrictions I



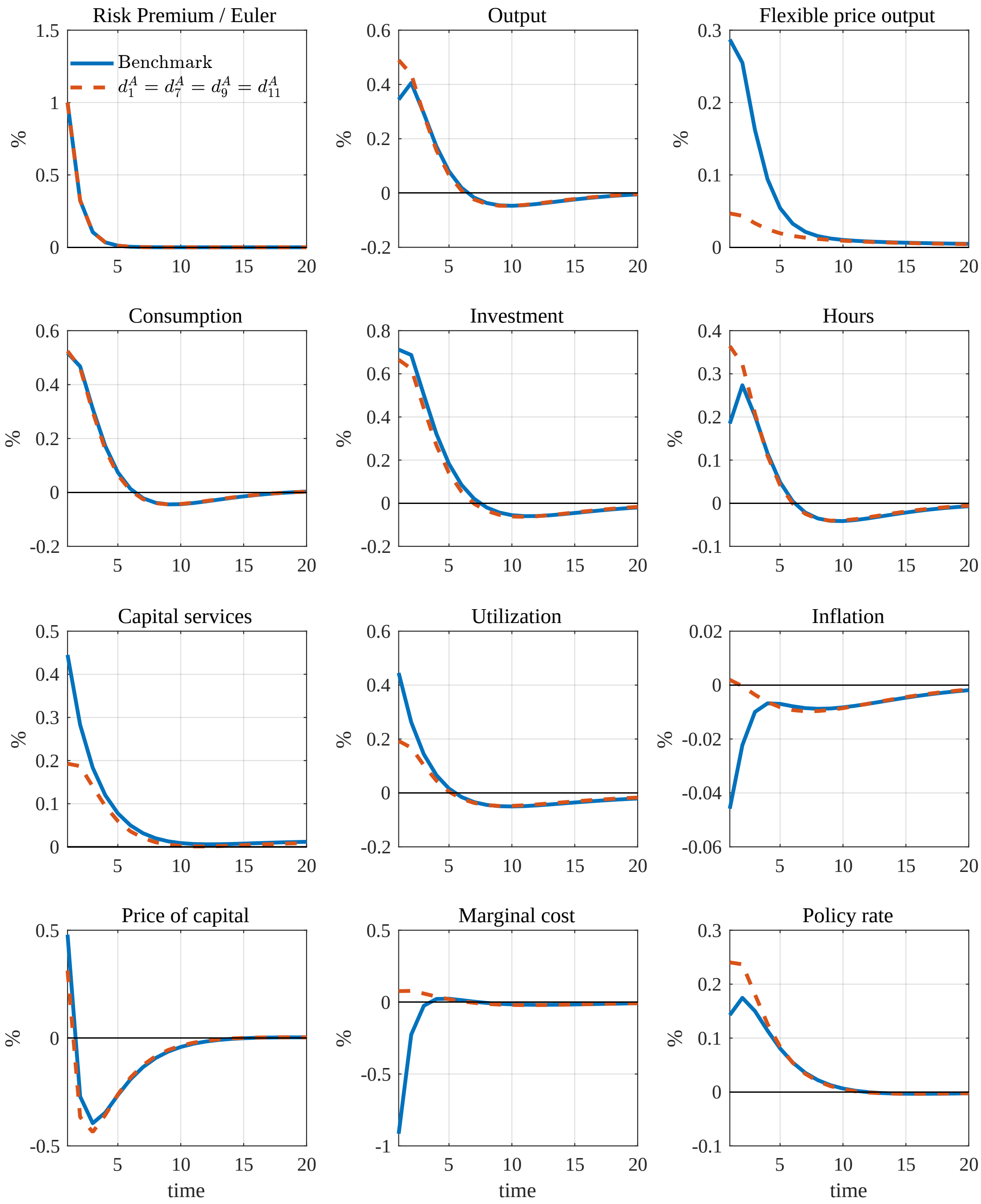
Notes. These figures plot the IRFs of the agnostic Euler disturbance for our benchmark specification and when the impact of this IRF through the capital valuation equation is set equal to zero.

Figure 12: IRFs of the agnostic Euler disturbance with restrictions II



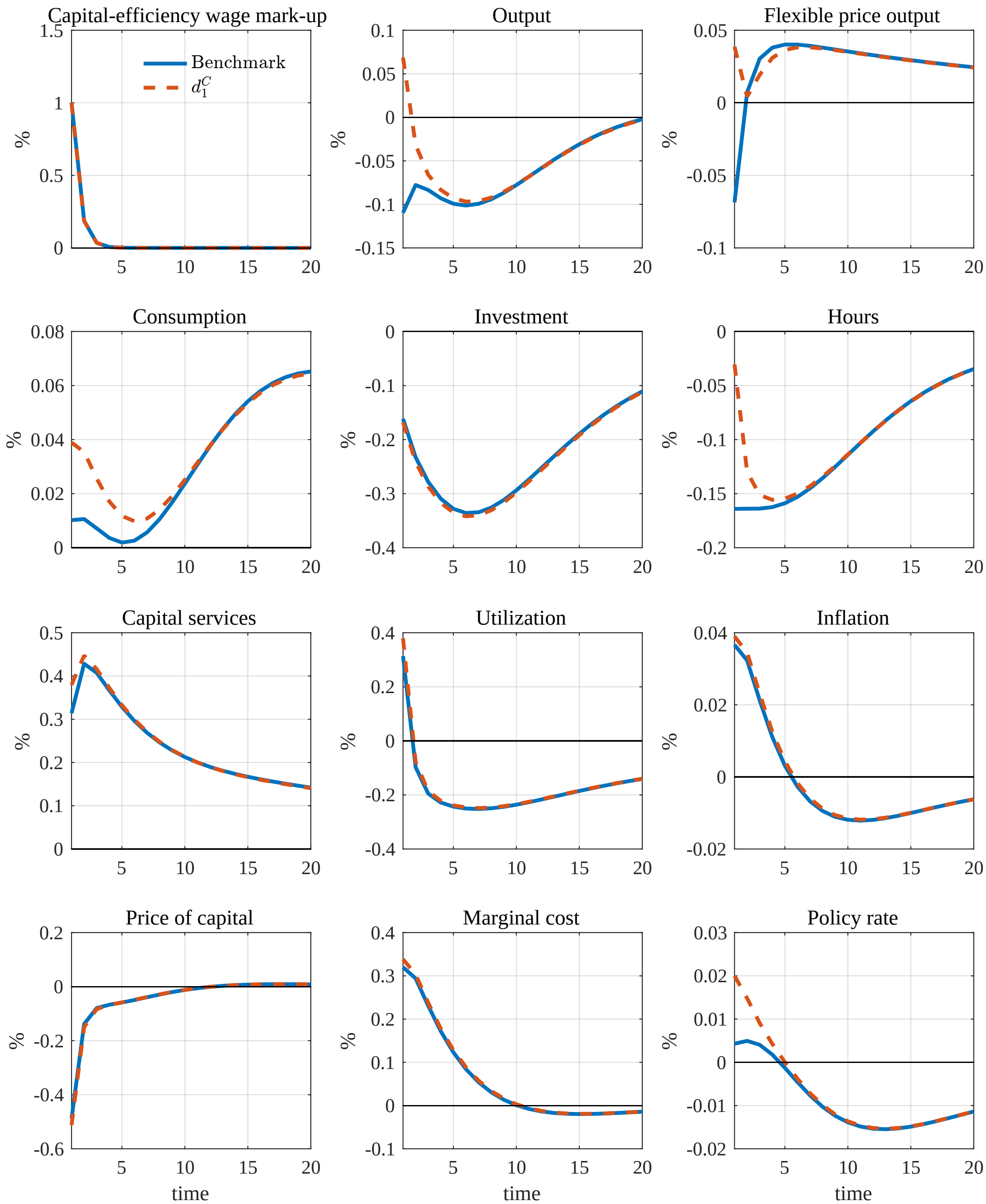
Notes. These figures plot the IRFs of the agnostic Euler disturbance for our benchmark specification and when the impact of this IRF through the Taylor rule is set equal to zero.

Figure 13: IRFs of the agnostic Euler disturbance with restrictions III



Notes. These figures plot the IRFs of the agnostic Euler disturbance for our benchmark specification and when the impact of this IRF through the overall budget constraint, the utilization, the price mark-up equation, and the rental rate of capital equation is set equal to zero.

Figure 14: IRFs of the agnostic capital-efficiency wage mark-up disturbance with restrictions



Notes. These figures plot the IRFs of the agnostic capital-efficiency wage mark-up disturbance for our benchmark specification and when the impact of this IRF through the overall budget constraint is set equal to zero.

E Misspecification: Literature review

Most empirical papers that estimate a dynamic macroeconomic model do not raise the issue of model uncertainty or misspecification, except possibly with some robustness exercises.[83] This does – of course – not mean that the profession is not aware that misspecification is a serious concern. In fact, some of the most prominent researchers in this research area have drawn attention to the risk of misspecification. The first subsection discusses evidence that indicates that misspecification of DSGE models is a serious concern. The second subsection discusses approaches proposed in the literature to deal with misspecfication. See Paccagnini (2017) for a more detailed survey.

E.1 Indications of DSGE misspecification

Del Negro, Schorfheide, Smets, and Wouters (2007) develop a procedure that allows the data to determine the usefulness of a DSGE model relative to a much less restricted VAR. Using a model very similar to the DSGE model of Smets and Wouters (2003), they find that their procedure does put some weight on the DSGE model, which implies that the restrictions of the DSGE model are of some value. However, they also argue that misspecification is a concern that *"... is not small enough to be ignored."* Using the same methodology, Del Negro and Schorfheide (2009) also find *"... strong evidence of DSGE model misspecification."*

There is also more indirect evidence that misspecification of estimated DSGE models is substantive. Using the Smets and Wouters (2003) model for the Euro Area, Beltran and Draper (2015) find that the data prefer implausible estimates for several parameters. For example, *most* of the mass of the marginal likelihood for the parameter of relative risk aversion is above 200, way above the range of values considered reasonable. This information provided by the likelihood is typically not revealed in empirical studies, since only properties of the posterior are reported and the choice of prior ensures that these aspects of the empirical likelihood have little or no weight in the posterior. A similar conclusion can be drawn from Onatski and Williams (2010). They estimate the same model using uniform priors over bounded ranges. These ranges are such that the priors are less informative than the ones typically used in the literature. Consistent with the results in Beltran and Draper (2015), several of the point estimates in Onatski and Williams (2010) are at the prior bounds. Using a new algorithm to deal with the complexity of estimating likelihood functions, Mickelsson (2015) re-estimates the model of Smets and Wouters (2007) and he also finds that several parameter estimates are significantly different from the ones reported in Smets and Wouters (2007).

Another possible reason for misspecification is the assumption that parameters are constant. To get efficient estimates we would like to use long time-series data, but the longer the time series the less likely that all parameters are constant. Canova, Ferroni,

[83] Interestingly, there are quite a few macroeconomic models in which agents – especially agents setting fiscal and monetary policy – face model uncertainty. If policy makers face model uncertainty, then researchers are likely to do so as well.

and Matthes (2015) address this issue and document that this is important for the model of Gertler and Karadi (2010).[84]

E.2 Dealing with misspecification: Other approaches

Richer models. Exogenous random disturbances are typically assumed not to be correlated with each other. This is a convenient assumption, because allowing for interaction between the different exogenous disturbances would substantially increase the number of parameters to be estimated given that DSGE typically have a several exogenous disturbances. However, it seems quite plausible that such disturbances are correlated. Del Negro and Schorfheide (2009) and Cúrdia and Reis (2012) deal with this possible misspecification and allow for more general processes to describe the behavior of the exogenous random disturbances.

Cúrdia and Reis (2012) find that this generalization has nontrivial consequences for the properties of the model. For example, the impact of a monetary policy shock on output is only half as big when the exogenous random variables are allowed to be correlated and the medium-term impact of a government spending shock switches from being positive to negative.[85]

Enriching a model by allowing for additional features and more general specifications is likely to reduce misspecification. However, richer models typically have more parameters, which will reduce the efficiency of the estimation by reducing the number of degrees of freedom.

Multiple models. Another way to deal with potential misspecification is to consider a set of different DSGE models. These could be compared informally or formally using, for example, relative marginal likelihoods or model averaging.[86] However, given the difficulty of modeling macroeconomic phenomena, it seems likely that *all* models in a set of DSGE models are subject to at least some type of misspecification.

Combining structural and reduced-form models. Ireland (2004) is an early paper that proposes a more general procedure to deal with possible misspecification when estimating a DSGE model even though the word misspecification is not used in the paper. Specifically, Ireland (2004) *"... augments the DSGE model so that its residuals – meaning the movements in the data that the theory cannot explain – are described by a VAR."* To understand this procedure, consider the following representation of the

[84]The literature cited in Canova, Ferroni, and Matthes (2015) documents that this is an issue in a variety of DSGE models.

[85]Cúrdia and Reis (2012) still impose that the *innovations* of the shocks are uncorrelated. Thus, the innovations still have a structural interpretation.

[86]See chapter 5 in An and Schorfheide (2007) for a detailed discussion.

linearized solution of a DSGE model:

$$s_t = As_{t-1} + B\eta_t, \tag{57}$$
$$y_t = Cs_{t-1} + D\eta_t, \tag{58}$$

where s_t is a vector containing (endogenous and exogenous) state variables, y_t is a vector containing the observables, and η_t is a vector containing the innovations of the exogenous random variables. Ireland (2004) proposes to augment the observation equation (58) as follows:

$$y_t = Cs_{t-1} + D\eta_t + u_t \tag{59a}$$
$$u_t = Fu_{t-1} + \xi_t \tag{59b}$$

where u_t captures the misspecification or incompleteness of the DSGE model. In his application, the structural equations are the policy rules from a standard Real Business Cycle (RBC) model with total factor productivity (TFP) as the only driving process. If the standard deviation of η_t is equal to 0, then this procedure boils down to estimating a standard VAR.

Note that the presence of the "missing elements" that are captured by u_t is assumed to have no effect on that part of agents' behavior that is described by the DSGE model, that is, the matrices A, B, C, and D. For this to be correct it must be true that the response of the economy to a TFP shock does not depend on the presence of other disturbances. One might think that such independence of a DSGE's policy rule to the presence of other disturbances is only correct if the additional disturbances represent measurement error.[87] However, section 3.2.2 shows that this "independence" property is correct in linear(ized) models in the sense that the specification of the structural part given in equations (57) and (58) does not depend on the presence of not included structural disturbances. It must be noted that the assumption that u_t follows a first-order (or even a finite-order) VAR could very well be restrictive. Thus the reduced-form specification for u_t could be misspecified as well.

The most comprehensive methodology to deal with misspecified DSGE models is put forward in Del Negro, Schorfheide, Smets, and Wouters (2007). Their starting point is a VAR specification of the observables. That is,

$$y_t = \sum_{k=1}^{K} F_k y_{t-1} + G\xi_t \tag{60a}$$
$$\mathbb{E}\left[\xi_t \xi_t'\right] = I. \tag{60b}$$

The key idea of the DSGE-VAR estimation proposed in Del Negro, Schorfheide, Smets,

[87] Although Ireland (2004) does not refer to the residual between model and data as measurement error, other papers in the literature describing his procedure do. Examples are Del Negro and Schorfheide (2009) and Cúrdia and Reis (2012).

and Wouters (2007) is to estimate this time series process with the prior distribution for F and Ω that is centered at the values implied by a DSGE model, $F(\Psi)$ and $G(\Psi)$, where Ψ is the vector containing the parameters of the DSGE model. The estimation procedure consists of jointly estimating Ψ, the structural parameters of the DSGE model, which pin down the prior for the VAR coefficients, and the VAR coefficients themselves.

The precision of the prior of the VAR coefficients is controlled with a scalar parameter, λ. If λ is equal to ∞, then one estimates an unrestricted VAR and if λ is equal to 0, then the procedure boils down to estimating a DSGE without allowing for misspecification. The estimation is executed for different values of λ. To determine the optimal value for λ, the authors propose using the marginal data density, which compares in-sample fit with model complexity.[88] If the restrictions imposed by the DSGE model are incorrect, then the procedure will put more weight on the VAR.

As pointed out in Chari, Kehoe, and McGrattan (2008), DSGE models often do not imply a VAR representation with a finite number of lags, unless all state variables are included. Thus, not only the DSGE, but also the VAR component of the DSGE-VAR procedure could be misspecified.

Wedges. Yet another approach to deal with misspecification is to add "wedges" to specific model equations. This procedure was introduced in Chari, Kehoe, and McGrattan (2007). Inoue, Kuo, and Rossi (2015) use this setup to formally test for model misspecfication. A wedge may have different interpretations or possibly no simple interpretation. From an econometric point a view, wedges are not different from regular structural disturbances in how they affect time series properties of the model. That is, they impose restrictions on the policy functions just as structural disturbances do and it matters crucially how one enters wedges. For example, the assumption that a wedge only enters one and not all model equations is a restriction. Although some wedges can enter more than one equation, wedges used in the literature only enter a few specific model equations and these are chosen by the researcher a priori and – as pointed out in Inoue, Kuo, and Rossi (2015) – wedges can be introduced in different ways. By contrast, ASDs appear in all equations and if one prefers a more concise specification, then our agnostic approach indicates one should use a statistical model selection criterion.

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[88]The DSGE is less complex because it has fewer parameters, but could provide a worse in-sample fit, because of the restrictions it imposes.

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