News Uncertainty in Brexit U.K.*

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December 4, 2019

Abstract

After the Brexit referendum the behavior of the U.K. economy defied expectations, as it did not exhibit a V-shaped recession, but a slow decline in production. We address this puzzle through the lens of a setup with heterogeneous firms facing non-convex capital adjustment costs. We model the referendum as a news shock, with the time horizon and the content of the news being uncertain. Brexit uncertainty is informed by expectation data from the Decision Maker Panel, a novel survey of U.K. businesses, where each CFO provides probability distributions over the expected Brexit date and the long-run expected outcome of Brexit on firm-level sales, for different Brexit scenarios. We show that the long expected duration of the negotiations is key for the model to reproduce the post-referendum behavior of the U.K. economy. Intuitively, if the chances that uncertainty resolves in the short run are small, only relatively few firms find it worth to pay the inefficiency cost associated with an investment freeze. Concurrently, if the expected horizon of the news is longer, anticipation effects are smaller. The long-run effects of Brexit implied by U.K. business expectations are large, entailing losses of 4.8% and 7.7% of GDP for Soft and Hard Brexit, respectively. The transitional dynamics under policy uncertainty show that the referendum has produced significant economic damage, with a three-year cumulative loss of about 2% of GDP.

Keywords: news shocks, uncertainty, firms heterogeneity, long-run productivity, survey-data.
JEL codes: E22, E32, E65, O04

We are very grateful to Tatsuro Senga for his advice. We also thank Stefania Albanesi, Francisco Buera, Martin Eichenbaum, Federico Ravenna, Richard Rogerson, and seminar participants at Queen Mary University of London, the University College of London, the University of Copenhagen, the Bank of England, the workshop on "the changing economy and the business cycle" at Oslo and the annual meetings of the European Economic Association and the Royal Economic Society for their comments and suggestions. Renato Faccini is grateful to the Bank of England for the award of the George Fellowship and for discussions with members of the staff and the Monetary Policy Committee. However, the views expressed in this paper do not necessarily represent those of the Bank of England, Danmarks National Bank or the European System of Central Banks.

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1 Introduction

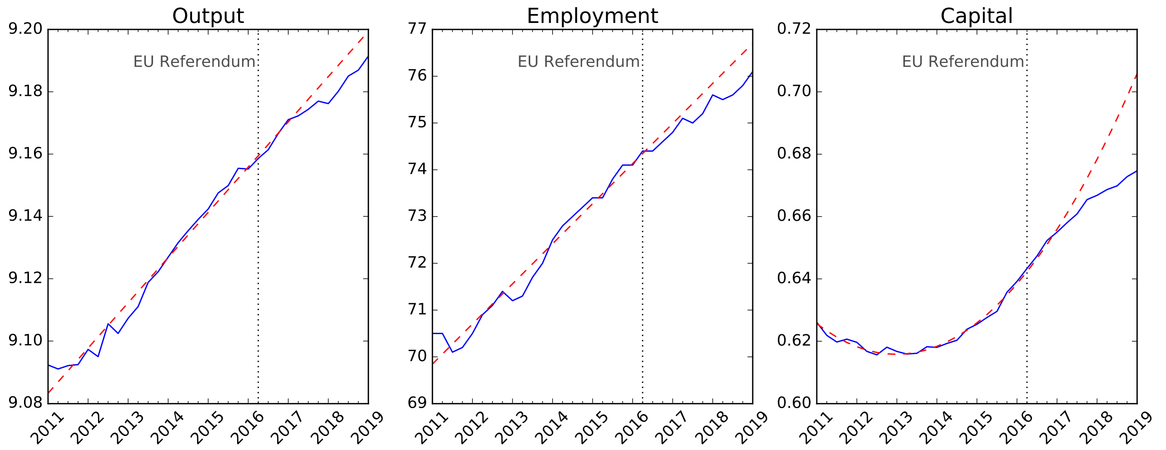
Does uncertainty matter for the business cycle? After the vote to leave the E.U. in the Brexit referendum of June 2016, the behavior of the U.K. economy has instilled in many economists the doubt that maybe uncertainty is not as important as previously thought. This is so because the widespread and very pessimistic expectations about GDP growth that preceded the vote failed to materialize. Indeed, the academic literature on economic uncertainty has shown that uncertainty shocks propagate in a V-shaped fashion, inducing a sharp and short-lived contraction in aggregate production (Bloom, 2009). Consistent with this view, prior to the referendum, a number of policy institutions, including HM treasury, predicted a sharp recession conditional on a victory of the leave campaign (HM Treasury, 2016). But this contraction failed to materialize, leading to dramatic upward revisions in the short-term forecast of economic activity. The Brexit referendum is an ideal case study to gauge the importance of uncertainty, for the sheer magnitude of uncertainty sparked, which involves not only the future legislation of trade agreements with the E.U. and the rest of the world, but also the future regulation of migration flows, domestic industrial policy, banking regulation, taxation policy etc. If uncertainty really matters for business cycles, then we should clearly see it in the U.K. data after the Brexit referendum.

Figure 1 shows that soon after the Brexit referendum, the employment rate, as well as capital and output per capita slowly started to fall below trend growth. So the figure suggests that the Brexit referendum has likely produced a material impact over the last three years. But if this is the case, these damaging effects did not come through a sudden contraction in economic activity, the manifestation of conventional uncertainty shocks, but via a gradual decline in output and in the factors of production.[1]

The starting point of our analysis is the observation that Brexit uncertainty hardly fits the notion of a canonical uncertainty shock, and therefore it should not be expected to propagate accordingly. An uncertainty shock is defined as a second-moment shock to aggregate and idiosyncratic fundamentals over the short run. But in the case of Brexit, uncertainty is primarily about fundamentals in the long run. Moreover, Brexit will affect fundamentals directly only after it takes place; the outcome of the referendum instead does not affect fundamentals directly, but only indirectly, through expectations. We therefore model the referendum as a news shock, with the time horizon and the content of the news being uncertain. We will make use of survey data on firm-level distributions of beliefs about both the expected Brexit date and expected impact of Brexit on sales, to inform uncertainty in the model. The aim is to

[1] Born et al. (2019) provide evidence for the gradual slowdown of GDP using a more sophisticated methodology to compute the trend.

Figure 1: U.K. Macroeconomic Aggregates and Their Pre-Referendum Trend



Notes: *The Figure shows GDP per capita, the employment rate and capital per capita in deviations form their trend. Output and capital per capita are expressed in logs, the rate of unemployment is in level. Trend growth is linear for output and employment and quadratic for capital, and is calculated from 2011Q1 to 2016Q2.*

disentangle the separate roles of the news and the uncertainty associated with the news in the response of the U.K. economy to the referendum. More broadly, we take Brexit as a laboratory to learn about the role played by uncertainty in news shocks.

Specifically, we setup an economic environment *à la* Khan and Thomas (2008), in which heterogeneous firms face fixed costs of investment and partial investment irreversibility. These assumptions are necessary to give uncertainty a chance to bite quantitatively, as they can potentially generate material option values of waiting, which in turn induce firms to freeze investments as they wait for uncertainty to resolve. We further assume that the stationary equilibrium of the economy is hit by the unexpected news that a package of policy reforms may be implemented in the future, with firms being uncertain about which policy will be implemented. A specific Brexit policy, say Hard or Soft Brexit, is defined as a combination of aggregate and firm-level TFP shocks that are permanent. Because the model is real, these shocks should not be interpreted literally as technology shocks, but more broadly as permanent sales shocks induced by the reforms, or as the outcome of idiosyncratic policy wedges in the spirit of Restuccia and Rogerson (2008) and Buera and Shin (2013, 2018). Whether these shocks will lead to a permanent increase or a permanent decrease in aggregate production after Brexit will depend on the data that we use to inform the calibration.

We distinguish between three sources of uncertainty in the model. First, we allow for timing uncertainty as it is not clear *when* Brexit will take place, if at all. Specifically, every period, one of the following three stochastic outcomes can take place: with some probability the reforms are either implemented, scrapped once and for all, or else the negotiations drag on to the next period. This source of uncertainty plays a key role in the analysis, as it allows us to run counterfactual experiments in which we change the expected duration of the negotiations. Second, timing uncertainty induces aggregate uncertainty as GDP will differ depending on whether Brexit happens or not and, conditional on Brexit taking place, on whether the U.K. will leave with or without a deal. We refer to these two outcomes as Soft and Hard Brexit, respectively. Finally, for a given Brexit scenario, firms are also uncertain about the precise effect of Brexit on their sales. So the model also features idiosyncratic uncertainty.

We calibrate the model to assess the effects of the Brexit referendum on the behavior of key macroeconomic outcomes before Brexit uncertainty is resolved. To inform uncertainty about firms' sales and the timing of Brexit in the model, we make use of a novel survey of U.K. businesses, the Decision Maker Panel (DMP), which was specifically designed to gather detailed information on the uncertainty surrounding the effects of Brexit on individual U.K. businesses. Specifically, the Chief Financial Officer of every respondent firm provides a probability distribution on the expected Brexit date, as well as two probability distributions about the expected impact of Brexit on their sales, relative to the case where the Brexit referendum did not happen: one is unconditional, and the other is conditional on the U.K. leaving the E.U. without a deal (Hard Brexit). This information allows us to pin down the aggregate and idiosyncratic shocks that characterize Soft and Hard Brexit as well as the parameters governing the duration of the negotiations.

The long-run effects of Brexit implied by the firms' expectations are large, entailing losses of approximately 4.8% in the case of Soft Brexit, and 7.7% in the case of Hard. These estimates are remarkably close to those obtained by the government using an open economy general equilibrium model (see HM Government, 2018): their central estimates are a fall of 4.9% in the average case of a Free Trade Agreement and 7.6% in the case of no deal. When we work out the transitional dynamics we find that the news shock of the Brexit referendum can explain most of the deviation of output, labor and capital from their pre-referendum trend reported in Figure 1. In line with the data, the model gives rise to a gradual decline in GDP and in the factors of production. We then disentangle the various economic mechanisms that lead to such propagation. By shutting down investment frictions, we tell apart the decline in production that is due to the expected convergence to a steady state with lower productivity and capital, from the effects of the *'wait and see'* channel, which leads to an expansion of the inaction region, as firms freeze investments in the hope that uncertainty resolves. We find both channels to be quantitatively important, with the relative contribution of uncertainty becoming dominant over time. By shutting down idiosyncratic uncertainty we then find that most of the expansion in the investment inaction region is driven by aggregate uncertainty.

Importantly, we find that timing uncertainty plays a key role in driving investment decisions. If the expected duration of the Brexit negotiations was short, say about one year, the referendum would have produced a much more immediate and pronounced fall in production. We therefore conclude that the long expected duration of the negotiations is key to producing the gradual decline that we observe in Figure 1. This result is the product of two forces operating in the same direction. On the one hand, the duration of the negotiations coincides with the duration of uncertainty. In turn, uncertainty propagates by inducing firms to freeze their investments, as they wait for it to resolve. But freezing investment is costly, as it implies operating at a sub-optimal scale of production. If the chances that uncertainty resolves in a given period are tiny, most firms do not find it worthwhile to "wait and see". On the other hand, the expected duration of the negotiations also coincides with the expected time horizon of the news shock. If the horizon is longer, anticipation effects are smaller.

The paper is closely related to the literature on uncertainty, sparked by the seminal work by Bloom (2009).[2] The focus of this literature is on uncertainty shocks, which are defined as temporary shocks to the second-moment of an exogenous process, typically TFP, be it at the aggregate or firm-level or both.[3] The nature of uncertainty, in this context, is about the realization of technology shocks at business cycle frequencies and is typically interpreted as uncertainty on the states of idiosyncratic and aggregate demand over the *short run*. We explore the role of uncertainty in news shocks, which is connected with the state of fundamentals in the *long run*. The role of the expected duration of uncertainty in the propagation of the news shock in our model is different from the role of persistence in the propagation of a typical uncertainty shock. In the latter case, the share of inactive firms increases with the persistence of the shock; in our model instead it falls with the expected duration of uncertainty, for the reasons explained above. So it matters a lot whether uncertainty is about current or future fundamentals, just as it matters how long uncertainty is expected to last. We believe that many episodes of economic policy uncertainty, including the threat of a trade war, the possible dissolution of the Euro area, as well as fiscal or institutional reforms, spark from the news about a future that may be years ahead. In all such episodes, aggregate, idiosyncratic and timing uncertainties are likely to be intertwined. We develop a

[2] See, among others, Baker and Bloom (2013), Baker et al. (2015), Fernandez-Villaverde et al. (2015), Barrero et al. (2017), Cacciatore and Ravenna (2018) and Bloom et al. (2018).

[3] Bloom et al. (2018) provide evidence that economic recessions are systematically associated with a sudden and short-lived increase in the variance of aggregate and firm-level shocks to fundamentals.

simple framework that we believe may speak more generally about these issues, beyond the specific case of Brexit.

From a modeling perspective, our work is perhaps more closely related to the literature on news shocks sparked by Beaudry and Portier (2004, 2006), which has revived the Pigouvian hypothesis that changes in beliefs may be an important source of economic fluctuations, even in the absence of actual changes in fundamentals. We introduce news in the context of a heterogeneous firms model *à la* Khan and Thomas (2008), where firms face non convex capital adjustment costs, and we discipline expectations using survey data on the distribution of beliefs. Our result that the Brexit referendum accounts for most of the deviation from trend in output and the factors of production represents a concrete illustration of the importance of beliefs in business cycles. Our results also link to the literature on disaster risks, in the tradition of Barro (2006, 2008), Rietz (1988) and Gourio (2012). What generates fluctuations in production after the Brexit referendum is an unexpected change in the probability of Brexit happening, which in the light of business expectations can be interpreted as an economic disaster. According to our model, the disaster risk generated by the Brexit referendum has been a key source of macroeconomic fluctuations for the U.K. economy.

Our work is also very closely connected to the macro development literature on resource misallocation, pioneered by Restuccia and Rogerson (2008) and further developed by Buera and Shin (2013, 2018). In these models, large-scale reforms that impinge on disparate fields of regulation, including trade policy, competition and industrial policy, labor market and migration policy and so on, are parsimoniously modeled as affecting idiosyncratic distortions at the firm-level. The productivity shocks induced by Brexit in the calibrated model could be interpreted as reflecting the introduction of idiosyncratic distortions, which produce resource misallocation in the long run. In line with the spirit of this literature, our approach does not disentangle the precise channel through which Brexit introduces distortions, i.e. whether it is via a trade channel, a change in migration flows, tax policy, industrial policy or more generally through the broad set of regulations that would characterize the economic environment after Brexit. Our approach is to take the sales expectations of businesses as given from the data, without taking a stance on the precise mechanism that produces them.

Finally, our model relates to the recent literature that has studied the effects of Brexit. Some research has focused on the long-term impact on trade including Van Reenen (2016), Sampson (2017) and Dinghra et al. (2017), and on foreign direct investment, McGrattan and Waddle (2019). Others have studied the effects of uncertainty. For instance, Steinberg (2019) studies the effects of Brexit uncertainty within a

Melitz (2003)-type model, while Bloom et al. (2019a) investigate the microdata of the Decision Makers Panel. Born et al. (2019) use a synthetic control method to gauge the effects of the Brexit referendum on U.K. growth, while Broadbent et al. (2019), develop an open economy DSGE model to work out the effect of the referendum, which is interpreted as news about lower productivity growth in the tradable sector.

In the next section, we present the model and in Section 3 we discuss how we exploit expectation data from the DMP survey to assign parameter values. Section 4 presents the quantitative analysis, discussing the long-run effects of Brexit under different policy scenarios and the short-term effects of policy uncertainty. Section 5 concludes.

2 Model

We present a simple, discrete-time economy with heterogeneous firms facing non-convex costs of adjusting the capital stock, comprising a fixed cost and partial investment irreversibility. These assumptions are required to allow for potentially meaningful effects of policy uncertainty (Bloom, 2009 and Bloom et al., 2018). We further assume that the stationary equilibrium of this economy is hit by the news that a policy may be implemented at some future stochastic point in time. The details of the policy, described below, imply that firms face permanent state-contingent productivity shocks, both aggregate and idiosyncratic. As a result, the model features timing uncertainty as well as aggregate and idiosyncratic state uncertainty. We deliberately take a partial equilibrium approach and abstract from the explicit modeling of the household sector. Indeed, we do not want to impose equality between domestic savings and investment for the U.K. economy.[4]

2.1 Environment

The economy is populated by a unit mass of heterogeneous firms. Each period, firms produce by renting labour services at the wage rate w and using their capital stock, which evolves according to $k' = (1-\delta)k + i$, where $\delta \in (0,1)$ is a rate of depreciation, i denotes investment and $k \in \mathbb{R}_+$. Firms are characterised by a four-tuple of idiosyncratic state variables: *i)* idiosyncratic productivity; *ii)* an idiosyncratic policy state; *iii)* the capital stock; *iv)* a fixed cost associated with investment. We examine

[4]Introducing in the model a consumption and savings decision has implications for the transition, but not for the long-run expected effects of Brexit, which are pinned down by the survey data. Adding the household sector implies that the permanent income shock induced by the Brexit referendum generates a massive increase in precautionary savings. If savings equals investment, the news of the Brexit referendum induces an investment boom, which is at odds with the data.

each element in turn.

The evolution of individual productivity, denoted by $a \in \mathcal{A} = \{a_1, ..., a_{N_a}\}$, is governed by an exogenous Markov stochastic AR(1) process: $log(a') = \psi log(a) + \epsilon$, where $\epsilon \sim \mathcal{N}(0, \sigma)$ and $\psi \in [0, 1)$. Each firm is also characterized by an idiosyncratic policy state $z \in \mathcal{Z} = \{z_+, z_-, z_0\}$, which captures whether the implementation of the Brexit policy contributes a positive, negative or neutral permanent component to firm-level productivity, respectively. This state becomes relevant only after the policy is implemented, at which time firms take an independent draw of this idiosyncratic policy state. Specifically, firms will draw z_+ and z_- with associated probability q_+ and q_-, respectively, and z_0 with the complement probability $1 - q_+ - q_-$. These assumptions generate idiosyncratic uncertainty over the outcome of the policy. Finally, following Khan and Thomas (2008) we assume that actively changing the capital stock is subject to a fixed cost of adjustment ξ, which is stochastically drawn each period from the uniform distribution $G(\xi) \sim \mathcal{U}\left[0, \overline{\xi}\right]$.

In addition to these idiosyncratic states, we introduce a fifth state variable, which captures the aggregate state of Brexit, and is denoted by $\zeta \in \{\zeta^P, \zeta^N, \zeta^S, \zeta^H\}$. We let ζ^P denote the pre-referendum aggregate state of the economy. For simplicity, we assume that this steady state does not contemplate the possibility that the Brexit referendum may happen. This assumption comes with little sacrifice of realism, since the outcome of the referendum was largely unexpected (see Born et al. 2018 and Bloom, 2019a for a discussion). ζ^N denotes instead the aggregate state of the negotiations, which starts when the news is unexpectedly announced and ends either if Brexit happens, or if it is ruled out once and for all. Specifically, we assume that while the economy is in the state ζ^N, Brexit takes place at Poisson rate θ. In this case, the economy transitions either to the absorbing state ζ^H, with probability γ^H, or to the other absorbing state ζ^S, with probability $\gamma^S = 1 - \gamma^H$, depending on which policy is implemented, Hard or Soft. In a given period, we assume that there is also the possibility that a remain shock strikes with probability γ^R, in which case the economy reverts to the initial absorbing state ζ^P. We interpret this shock as the outcome of a second referendum, which overrules the first one with the decision to remain in the E.U. once and for all. If Brexit is neither implemented nor scrapped, the negotiations drag on to the next period with period probability $1 - \theta - \gamma^R$. We note that the state of the negotiations can be interpreted as an aggregate state of uncertainty, since the presence of aggregate, idiosyncratic and timing uncertainty accompanies the negotiations. The transition matrix for the aggregate policy state of the economy is therefore given by:

$$
\Gamma_\zeta(\zeta_{t+1} = \zeta^i | \zeta_t = \zeta^q) =
$$

$$
\begin{array}{lc}
\downarrow \zeta_t^q, \zeta_{t+1}^i \rightarrow & \begin{array}{cccc} \zeta^N & \zeta^S & \zeta^H & \zeta^P \end{array} \\
\begin{array}{l} \zeta^N \\ \zeta^S \\ \zeta^H \\ \zeta^P \end{array} & \begin{pmatrix} 1-\gamma^R-\theta & \theta(1-\gamma^H) & \theta\gamma^H & \gamma^R \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.
\end{array} \tag{1}
$$

2.2 The Production Function

The production function is given by

$$
f = \tilde{\tau} a (k^\alpha l^{1-\alpha})^{(1-\nu)}, \tag{2}
$$

where $1-\nu$ is a span of control parameter, which represents the share of output that remunerates the variable factors and governs the degree of decreasing returns to scale. The term $\tilde{\tau}$ captures instead the effects of Brexit on firm-level productivity. We assume that unless Brexit happens, that is, in either of the states ζ^P and ζ^N, all firms face $\tilde{\tau} = 1$. Conditional on Brexit happening instead, firms will be subject to a distribution of values for $\tilde{\tau}$. Specifically, it is assumed that:

$$
\tilde{\tau} \equiv \tau(z_i, \zeta^j, a) = \begin{cases} 1 & \text{for} \quad j = \{P, N\}, \\ (1+X^j)(1+x_i^j)a^\omega & \text{for} \quad j = \{S, H\}, \quad i \in \{z_+, z_-, z_0\}, \end{cases} \tag{3}
$$

where $X^j \in \mathbb{R}$ is the parameter capturing the permanent aggregate productivity shock associated with the aggregate policy state $j \in \{\zeta^S, \zeta^H\}$ and $x_i^j \in \mathbb{R}$ is the parameter associated with the permanent idiosyncratic TFP shock, which is contingent on both the aggregate policy state $j \in \{\zeta^S, \zeta^H\}$ and the idiosyncratic policy state $i \in \{z_+, z_-, z_0\}$. We assume that $x_i^j = 0$ for $i = z_0$. In this case, Brexit affects the firms with idiosyncratic state z_0 only through the aggregate shocks X^j. Moreover, it is assumed that the effect of Brexit on sales depends on productivity via the elasticity parameter $\omega \in \mathbb{R}$. A negative value of ω implies that, everything else equal, firms with higher productivity lose more or gain less from Brexit. This specification is the most parsimonious one that allows us to match the empirical moments that we discuss in Section (3). We emphasize that because the model is real, a change in $\tilde{\tau}$ should not be interpreted literally as a technology shock, but more broadly as a permanent sales shock induced by the reforms, or as the outcome of idiosyncratic policy wedges in the spirit of Restuccia and Rogerson (2008) and Buera and Shin (2013, 2018).

2.3 The Problem of the Firms

Every period, a firm can invest in any future level of capital k' only upon payment of the fixed adjustment cost ξ. Because ξ is drawn independently every period from the same uniform distribution G, for a given end-of-period stock of capital, a firm's current adjustment cost has no implication for its future adjustment. As a result, it is sufficient to describe differences across firms by their productivity a, individual policy state z and capital k. We therefore summarize the joint distribution of firms over (a, z, k) by a probability measure $\mu(a, z, k)$ defined on the Borel algebra $\mathbb{S}$ for the product space $\mathbb{S} = \mathcal{A} \times \mathcal{Z} \times \mathbb{R}_+$.

At the beginning of the period, after observing the aggregate policy state of the economy ζ, and inheriting its capital stock k from the previous period, the firm takes a new draw of productivity, a, and, conditional on Brexit taking place, also a new draw of the idiosyncratic policy state z. It then chooses its current level of employment, produces and pays its workers. Next, it draws its fixed adjustment cost ξ. The current period value of a firm before drawing the fixed adjustment cost can therefore be represented as the expected value of the firm over all possible realizations of ξ:

$$
\begin{aligned}
v(a, z, k; \zeta) &= \mathbb{E}_{\xi} \tilde{v}(a, z, k, \xi; \zeta) \\
&= \int_0^{\bar{\xi}} \tilde{v}(a, z, k, \xi; \zeta) \; G(d\xi).
\end{aligned} \tag{4}
$$

Once the firm observes the draw of the fixed adjustment cost ξ, it decides whether to pay the cost and adjust its capital to the desired level k', or avoid the cost and retain its previous capital stock k. The optimal choice solves:

$$
\tilde{v}(a, z, k, \xi; \zeta) = \quad \max \left\{ -\xi + v^A(a, z, k; \zeta) \, , \, v^{NA}(a, z, k; \zeta) \right\}, \tag{5}
$$

where v^A and v^{NA} represent the value functions of adjusting and non-adjusting capital, respectively. In turn, given the current aggregate state ζ, the value upon adjustment of capital solves the following dynamic problem:

$$v^A(a,z,k;\zeta) = \max_{k',l} \Big[f(a,z,k,l;\zeta) - wl - k' + (1-\delta)k - \mathbb{I}(i<0)\,|i|\,\chi \qquad (6)$$

$$+ \mathbb{E}_{\Gamma_a,\Gamma_z,\Gamma_\zeta} \beta v(a',z',k';\zeta'|a,z,\zeta)\Big],$$

subject to the laws of motion for the aggregate policy state Γ_ζ, for idiosyncratic productivity Γ_a, and the individual policy state Γ_z. In the above maximisation problem β is the discount factor, and $\mathbb{I}$ is an indicator function that takes the value of one if the firm disinvests and zero otherwise. The parameter χ governs the partial irreversibility of investment (i.e. the resale of capital occurs at a price that is only a share $(1-\chi)$ of its purchase price). The expression in the first line represents the period flow of profits, after substituting investment using the law of motion for capital.

The value function $v^{NA}(a,z,k;\zeta,\mu)$ is instead the solution to the following equation:

$$v^{NA}(a,z,k;\zeta) = \max_{l} \Big[f(a,z,k,l;\zeta) - wl \qquad (7)$$

$$+ \mathbb{E}_{\Gamma_a,\Gamma_z,\Gamma_\zeta} \beta v(a',z',(1-\delta)k;\zeta'|a,z,\zeta)\Big],$$

subject to the laws of motion for the states a, z, and ζ, which implies that the firm neither invests nor disinvests, bringing into the next period its current stock of capital k, after depreciation.

It is possible to define for every firm a threshold adjustment cost $\xi^T(a,z,k;\zeta)$ that makes a firm indifferent between adjusting and not adjusting its capital stock:

$$v^A(a,z,k;\zeta) - \xi^T(a,z,k;\zeta) = v^{NA}(a,z,k;\zeta). \qquad (8)$$

In this framework, the policy function for the next period choice of capital can be represented by the following piecewise function:

$$K(a,z,k,\xi;\zeta) = \begin{cases} k'(a,z,k;\zeta) & \text{if} \quad \xi < \xi^T(a,z,k;\zeta) \\ (1-\delta)k & \text{otherwise,} \end{cases}$$

which implies that $K(a,z,k,\xi;\zeta)$ is equal to the optimal level of capital if the draw of the adjustment cost is below the threshold, and equal to a fraction $1-\delta$ of the current capital stock otherwise. As a result, the optimal investment decision will exhibit an inaction region for any value of $\xi > \xi^T$. In this setup, aggregate and idiosyncratic uncertainty about the future policy states widens this region, with firms postponing investment, or disinvestment, as they wait for uncertainty to resolve. Finally, we denote by $L(a, z, k; \zeta)$ the policy function for employment. Note that this function is independent of the fixed costs of adjustment ξ since the optimization of labor is a static choice.

3 Calibration

The model comprises three main sets of parameters: those that affect the pre-referendum steady state of the U.K. economy, those that affect the stochastic process for Brexit and those representing the policy shocks that characterize the post-Brexit steady states. The first set comprises relatively conventional parameter values, which are assigned following the literature. The other two sets of parameters are instead less conventional. In order to assign values, we make use of expectation data regarding the timing and the impact of Brexit, which are obtained from the survey of U.K. firms in the Decision Makers Panel (DMP). The parameters governing the stochastic process of Brexit are set to the values directly inferred from the survey. The parameters governing the permanent shocks associated with Brexit are instead calibrated to minimize deviations from key moments of the data. In what follows, we present the DMP data and discuss separately how we tackle the calibration of the three sets of parameter values.

3.1 The Decision Maker Panel

The DMP is a new, large and representative survey of U.K. businesses that was launched in August 2016 by the Bank of England, together with Stanford University and the University of Nottingham. This survey was specifically designed to investigate how the uncertainty sparked by the Brexit referendum would affect U.K. businesses. While the survey features a large array of questions, aimed at gauging the effect of uncertainty on both economic activity and inflation, a particularly appealing feature for our purpose is that individual businesses are asked to provide probability distributions about the expected effects of Brexit on their sales, and not just a point estimate. As a result, the survey incorporates valuable information on Brexit uncertainty about various outcomes of the negotiations. We note that the questions that are relevant for our purpose have only been asked very sporadically, so we do not have a time series that allows us to track how the distribution of beliefs has been changing over time. Our exercise will therefore abstract from possible time variation in beliefs.

The sampling pool of firms for the DMP was selected from the registry of all active companies in

Table 1: Calibration of the Pre-Referendum Steady-State

Parameter	*Value*	*Description/Target*	*Source*
Technology & Prices			
β	0.995	Annual interest rate of 1.8%	ONS
w	1.621	Labor hours 33.7%	ONS
α	0.333	Capital income share of $1/3$	Bloom et al. (2018)
ν	0.250	DRTS parameter	Bloom et al. (2018)
Idiosyncratic Productivity Process & Adjustment Costs			
ψ	0.950	Idio. productivity persistence	Khan and Thomas (2008)
σ	0.022	St. dev. of idio. productivity	Khan and Thomas (2008)
δ	0.020	Annual depreciation of 8.2%	ONS
$\bar{\xi}$	$1.4e^{-4}$	Annual inaction of 8%	Khan and Thomas (2008)
χ	0.339	Resale loss of capital	Bloom et al. (2018)

the U.K. and comprises all those businesses - about 42,000 - who were not a subsidiary of a U.K. parent company, who had a complete set of company accounts and at least ten employees. For firms that were randomly sampled, the Chief Financial Officer (CFO) was contacted and asked to participate. Those who agreed were asked to fill in a survey every month. The survey has a rotating panel structure with each member being asked one-third of the quarterly questionnaire each month, with the random ordering of the three monthly questionnaires in each quarter. The average monthly response rate was 53% and has ranged between 40% and 65%, including on average 2,500 companies. In our calibration we make use of the results from the survey, which are published online and are therefore made publicly available.[5] For further details about the survey we refer to Bloom et al. (2019a). In what follows we discuss the specific questions of the DMP that are key, in the context of our model, to identify the parameters.

3.2 Calibration of the Pre-Referendum Steady State Economy

In this section we calibrate the steady state of the model prior to the Brexit referendum under the assumption that one period equals one quarter. The calibration follows the literature on models with heterogeneous firms (see Kahn and Thomas, 2008, and Bloom et al., 2018). Table 1 displays the model parameters, their values, and the calibration targets. Whenever specific targets for the U.K. economy are not available we use the corresponding U.S. values that are customary in the firm dynamics literature.

Using data from the U.K. Office of National Statistics (ONS) over the period 1992-2015, we calibrate the prices in the model such that the discount factor β matches an average annual real interest rate of

[5] The results can be downloaded from https://decisionmakerpanel.co.uk/

Table 2: Firms' Expectations about Brexit Date

"U.K.'s expected withdrawal date from the E.U., after any transition period, average probability (%)"					
2019	2020	2021	2022	2023 or later	Never
19	18	29	15	9	9

Notes: *Question S.18 from the Decision Maker Panel, Nov.2017-Jan.2018, accessible via* `https://www.bankofengland.co.uk/statistics/research-datasets`. *The probabilities are an average of the firms' responses weighted by their employment shares.*

1.8%, and the wage implies that working hours are 33.7% of the time endowment. The parameters of the production function, namely α and ν, are set to standard values, in accordance with Bloom et al. (2018).

As for the parameters governing the persistence and the variance of the stochastic idiosyncratic productivity process, we rely on the values set by Khan and Thomas (2008) for the U.S. economy. The capital depreciation rate δ is fixed to achieve an annual capital depreciation of 8.2% as calculated by the ONS. The upper support of the uniform distribution from which the fixed cost is drawn, $\overline{\xi}$, is set to match the findings of Cooper and Haltiwanger (2006) and Khan and Thomas (2008), which observe that annually 8% of U.S. firms remain inactive with respect to their investment. Finally, the parameter governing partial investment irreversibility, χ, is calibrated to match a resale loss of capital of 33.9% as in Bloom (2009) and Bloom et al. (2018).

3.3 Calibration of the Brexit Stochastic Process

The stochastic transition set in motion by the Brexit referendum is governed by the following three parameters: *(i)* the probability that in a given quarter Brexit takes place, θ; *(ii)* the probability that, conditional on Brexit occurring, Brexit is Hard, γ^H; *(iii)* the probability that in a given quarter the decision of the Brexit referendum is reversed, γ^R.

We first calibrate θ and γ^R using question S.18 of the DMP, which asks: *"What do you think is the percentage likelihood (probability) of the U.K. leaving the E.U. (after the end of any transitional arrangements) in each of the following years?:* i*) 2019,* ii) *2020,* iii) *2021,* iv) *2022,* v) *2023 or later,* vi) *Never"*. Table 2 shows the average probability attached to each date. The answers to this question indicate that the probability of the U.K. not having left the E.U. by the end of 2022 is 18%. In turn, this probability comprises a 9% chance of Brexit happening any time after 2022 and a 9% chance of Brexit never taking place, thus capturing the possibility that a second referendum eventually reverses the Brexit

Table 3: Firms' Expectations about the Probability of Hard Brexit

"What probability (%), do you attach to a disorderly Brexit, whereby no-deal is reached by the end of March 2019?"				
$< 20\%$	$\geq 20-40\%$	$\geq 40-60\%$	$\geq 60-80\%$	$\geq 80\%$
17	31	25	17	10

Source: *Question S.25 from the Decision Maker Panel, Feb.2018-Apr.2018, accessible via* `https://www.bankofengland.co.uk/statistics/research-datasets`.
Notes: *The probabilities reported in the Table are an average of the firms' responses, weighted by their employment shares. The term "disorderly Brexit" refers to the case in which the U.K. leaves the E.U. without any formal arrangement, and thus transitions to the W.T.O. trading conditions. We refer to this scenario as "Hard Brexit" in the model.*

decision.

In the model, the probability of the U.K. not having left the E.U. by the end of 2022 can be expressed as the probability of remaining in the negotiation state for 26 consecutive quarters (from 2016Q3 up to and including 2022Q4):

$$Pr(\text{U.K. not having left E.U. by end of 2022}) = (1-\theta-\gamma^R)^{26}. \tag{9}$$

Moreover, the probability that the U.K. never leaves the E.U. can be represented in the model by the probability that the decision of the Brexit referendum is eventually reversed at any future point in time:

$$\begin{aligned} &Pr(\text{U.K. never leaving the E.U.}) = \\ &\sum_{i=1}^{\infty}(1-\theta-\gamma^R)^{i-1}\gamma^R = \frac{\gamma^R}{1-(1-\theta-\gamma^R)}. \end{aligned} \tag{10}$$

After assigning the values of 0.18 and 0.09 to the L.H.S. of equations (9) and (10), we can solve for $\theta = 0.0581$ and $\gamma^R = 0.0057$.

To calibrate the conditional probability of Hard Brexit, γ^H, and the implied probability of Soft Brexit, γ^S, we make use of Question S.25, reported in Table 3, which is informative about the unconditional probability of Hard Brexit. The question asks: *"What probability, in percent, do you attach to a disorderly Brexit, whereby no-deal is reached by the end of March 2019?"*. Attributing mid-points of $10, 30, 50, 70$ and 90 to the five bins in Table 3, we get that the average perceived unconditional probability of Hard Brexit is 44%.

In the model, Hard Brexit is not restricted to take place before any particular date but follows a Poisson process. The unconditional probability of Hard Brexit happening at any future point in time is given by:

$$Pr(\text{Hard Brexit}) = \sum_{i=1}^{\infty}(1-\theta-\gamma^R)^{i-1}\theta\gamma^H = \frac{\theta\gamma^H}{1-(1-\theta-\gamma^R)}. \tag{11}$$

Given the values for γ^R and θ inferred from Table 2, and making use of the probability $Pr(\text{Hard Brexit}) = 0.44$, from Table 3, we can solve equation (11) for the conditional probability of Hard Brexit, and get $\gamma^H = 0.4835$, which implies $\gamma^S = 0.5165$.

3.4 Calibration of the Brexit Policy Parameters

In this section we discuss how we assign values to the following nine Brexit policy parameters: the aggregate shocks in Soft and Hard Brexit, X^j for $j = S, H$; the positive and negative idiosyncratic shocks x^j_+ and x^j_- for $j = S, H$, the probabilities associated with drawing the positive and negative idiosyncratic shock, q_+ and q_-, respectively, and the parameter ω, which governs how post-Brexit sales vary with firm-level productivity. To calibrate these parameters we make use of two questions from the DMP survey, whose results, averaged across all respondent firms, are reported in Table 4. The first question asks: *"The Prime Minister has said that the U.K. government does 'not seek membership of the Single Market. Instead, we seek the greatest possible access to it through a new, comprehensive, bold and ambitious Free Trade Agreement.' How likely do you think it is that the eventual agreement will have the following effects, compared to what would have been the case had the U.K. remained a member of the E.U., with five scenarios provided about the effect on sales at home and abroad: i) a large positive effect adding* 10% *or more, ii) modest positive effect adding less than* 10%*, iii) make little difference, iv) modest negative effect subtracting less than* 10%*, v) large negative effect subtracting* 10% *or more."* [6]

The second question asks about the expected effects of Brexit on sales, conditional on the specific case where the U.K. economy exits the E.U. without a deal (Hard Brexit). The question asks: *"The Prime Minister has said that Brexit negotiations will be tough and 'no-deal is better than a bad deal'. If the U.K. leaves the E.U. without a deal then there could be an increase in non-tariff barriers to trade with the E.U. (for example from a higher cost of meeting required standards and regulation in E.U. markets, or an inability to acquire the necessary permissions). How likely do you think it is that this outcome will have the following effect on the sales of your business, compared to what would have been the case had the U.K. remained a member of the E.U.: i) a large positive effect adding* 10% *or more, ii) modest positive*

[6] Note that the question asks about the expected effects of "the eventual agreement" and not about the effects of a possible Free Trade Agreement, and is therefore intended to capture average individual expectations over all possible Brexit outcomes, including the case where no deal is reached. In this case the eventual agreement is the fall-back position dictated by WTO terms. (cf. Bloom et al. 2019b).

Table 4: Calibration of the Brexit Policy Parameters

"Expected impact of Brexit on sales, average probability"			*"Expected impact of disorderly Brexit on sales, average probability"*		
	Data	*Model*		*Data*	*Model*
$< -10\%$	17.5	17.6	$< -10\%$	19.7	19.9
$> -10\%$	27.6	28.9	$> -10\%$	26.9	27.6
$\simeq 0\%$	37.2	36.3	$\simeq 0\%$	43.5	42.4
$< 10\%$	12.3	12.1	$< 10\%$	6.9	7.1
$> 10\%$	5.4	5.1	$> 10\%$	2.9	3.0
	Data	*Model*		*Data*	*Model*
Average st. dev. of sales expectations	5.9	6.6	*Elasticity of expected sales to productivity*	-0.37	-0.37

Source: *The question on the top left panel is S.8 from the Decision Maker Panel, Nov. 2017-Jan. 2018, accessible via* `https://www.bankofengland.co.uk/statistics/research-datasets`*. The question in the top right panel was not published online and we thank the Bank of England for sharing the data.*
Notes: *The table shows the moments of the survey data used for the calibration of the Brexit policy and the corresponding moments in the model. The top left panel reports the average across firms of the unconditional probability distributions about the effects of Brexit on their sales; the top right panel reflects the effects of a no deal outcome on their sales, i.e. Hard Brexit. The probabilities are an average of the firms' responses weighted by their respective employment shares. Notice that the unconditional distribution should be interpreted as the outcome of firms taking their sales expectations over all possible Brexit scenarios, including the case of no deal. The average standard deviation of sales expectations and the elasticity of expected sales to productivity have been calculated by Bloom et al. (2019c).*

effect adding less than 10%*, iii) make little difference, iv) modest negative effect subtracting less than* 10%*, v) large negative effect subtracting* 10% *or more."* In both questions every respondent is therefore asked to attach a probability to each of these five outcomes, thereby providing an individual probability distribution on the expected effect of Brexit on their sales. The first question elicits information on the unconditional distribution, while the second is conditional on Hard Brexit.

Each question provides us with four independent moments, which can be used as calibration targets. The four moments in the aggregate distribution about the sales impact of Hard Brexit elicit direct information on the Hard Brexit policy parameters X^H, x^H_-, x^H_+. Given this information, the probability bins of the unconditional distribution, together with our assumption that there can be only two types of Brexit, Hard or Soft, directly identify the policy parameters of the Soft Brexit policy, X^S, x^S_-, x^S_+. Together, these two distributions also pin down the probability parameters q_- and q_+.

To elicit information that is useful to identify how post-Brexit sales correlate with firm-level productivity, as captured by the parameter ω, we make use of estimates by Bloom et al. (2019c) on the elasticity of expected sales to firm-level productivity in the DMP data. Their finding of a negative correlation of -0.37 implies that more productive firms expect heavier losses from Brexit.[7] Finally, we also target the

[7] We thank the authors for sharing their results.

average dispersion of the histograms reported in the answers to the first question, the one on the unconditional post-Brexit-sales expectations. Because we have nine parameters for ten calibration targets, our model is over-identified. How well the model can capture this measure of average dispersion in firms' expectation is, therefore, a useful validation test for the formulation of the policy in eq.(3).

To compare model outcomes with the data, we compute artificial probability distributions for the firms in our model in the same way as they are computed in the DMP survey, that is, we produce five-bin histograms, both unconditional and conditional on Hard Brexit. Specifically, we generate a large number of artificial panels of $2,500$ firms, the same size as the number of average respondents in the DMP survey, where every firm (i) is characterized by a productivity draw (a_i) from the time-invariant distribution of ability $p(a)$. When computing the unconditional probability distribution of the expected effects of Brexit on sales, we consider that every firm may end up in different aggregate policy states with probabilities implied by the transition matrix in equation (1). Moreover, conditional on the states of Soft and Hard Brexit occurring, each firm may end up in a different idiosyncratic policy state z_i for $i \in \{z_+, z_-, z_0\}$. We compute for each firm the steady-state sales changes contingent on each of these aggregate and idiosyncratic states, and then measure the probability associated with every bin of the histogram. We compute the Hard Brexit distribution similarly, just noting that the only uncertainty over the states is idiosyncratic, i.e. related to the draws of z.

With the individual probability distributions at hand, we then average every bin of the distribution across all firms in every panel, and then across all panels, to produce the aggregate moments that correspond to the data reported in Table 4. We calibrate the parameters of the policy using the method of simulated moments (MSM), that is, we minimise the squared deviation of model moments from the targets (see Appendix A for details on the numerical algorithm).

Table 4 reports the moments targeted in our calibration exercise and their corresponding values in our model. Looking at the data on the expected effect of Brexit on sales, we see that companies placed more weight on Brexit reducing sales than on it increasing them. This is even more so in the case of Hard Brexit. The average unconditional probability attached to Brexit increasing sales was 17.7%, while the average probability of a negative impact was 45.1%; the average probability of it having no effect was 37.2%. The average probability of experiencing sales losses conditional on Hard Brexit is only slightly larger, 46.6%, but within that, we observe a relative increase in the probability of experiencing sales losses above 10%. The probability of experiencing gains from Hard Brexit, 9.8% is nearly half that of the unconditional probability. So a good chunk of the mass in the unconditional probability distribution

Table 5: Calibration of the Brexit Policy Parameters

Aggregate Policy Parameters		*Idiosyncratic Policy Parameters*				*Elasticity to Productivity*	*Idiosyncratic Probabilities*	
X^S	X^H	x_+^S	x_-^S	x_+^H	x_-^H	ω	q_+	q_-
0.009	-0.005	0.037	-0.054	0.044	-0.042	-0.003	0.101	0.457

Notes: *The table reports the values of the policy parameters calibrated using the moments from Table 4. Note that the parameters with the superscripts S and H relate to the aggregate state states of Soft (ζ^S) and Hard Brexit (ζ^H), respectively. The parameters with subscripts $-$ and $+$ refer to the idiosyncratic policy states z_- and z_+, respectively.*

assigned to positive effects on sales shifts to the left in the case of Hard Brexit, raising the probability of receiving no material impact to 43.5%. When mapping the probability distribution of the data to those constructed in the model, we assume that expected changes in sales that are lower than 2% in absolute value correspond to firms reporting in the survey that Brexit would make ”little difference”.

The results of the calibration can be observed in Table 4. The model does remarkably well at matching all targeted moments. Even the average standard deviation of the individual histograms is very close to the data. The calibrated parameter values are reported in Table 5.

4 Quantitative Analysis

Equipped with the calibrated parameter values, we first compute the effects of Brexit in the long run. We do so by comparing how key macroeconomic aggregates change in the Hard and Soft Brexit steady states relative to the pre-referendum steady state. We then work out the transitional dynamics under policy uncertainty, focusing on the period that precedes the eventual resolution of Brexit uncertainty, and compare model outcomes to the actual behavior of the U.K. economy. We then investigate the different forces at play in the response of the economy to the Brexit referendum, disentangling anticipation effects from the effects of idiosyncratic and aggregate uncertainty. Next, we explore the role of the expected duration of the negotiations in the propagation of the news of the referendum. Finally, we compute the transitional dynamics to the Soft and Hard Brexit steady states in order to investigate the effects of a resolution of uncertainty. We refer to Appendix B for technical details about the computational strategy.

Table 6: Brexit Steady-States Comparison

Variable	*Pre-Referendum*	*Soft Brexit*	*Hard Brexit*
Output	1.096	1.044 *(-4.8%)*	1.012 *(-7.7%)*
Employment	0.332	0.316 *(-4.8%)*	0.306 *(-7.7%)*
Capital	11.414	10.869 *(-4.8%)*	10.542 *(-7.6%)*
TFP	1.036	1.024 *(-1.2%)*	1.015 *(-2.0%)*

4.1 The Effects of Brexit in the Long Run

Table 6 shows the long-run effects of Soft and Hard Brexit on aggregate production, labor, capital and TFP, as implied by the calibrated model. The results reveal large economic losses, with GDP falling by 7.7% in the case of Hard Brexit and 4.8% in the case of Soft. The effects on the inputs of production are quantitatively similar to those of GDP, while aggregate TFP falls by 1.2% in the case of Soft Brexit and 2.0% in the case of Hard.[8] We emphasize that the result that Brexit implies large economic losses in the long run is not implicit in the assumptions of the model, but is rather implied by the sales expectations of U.K. businesses. In particular, two features of the data, discussed above, drive this quantitative result: the unconditional distribution of sales expectations being heavily skewed to the left, and expected post-brexit sales being negatively correlated with productivity.

The DMP data only allows the model to identify two Brexit scenarios: Hard and Soft. The Hard Brexit scenario neatly corresponds to the no-deal Brexit case in the DMP dataset. In this scenario the model delivers a fall in GDP that is in line with the government's estimates, which range from -6.3% to -9.0%, with a central case of -7.6% (see HM Government, 2018). However, Soft Brexit is not identified as neatly. Indeed, Soft Brexit represents the average case in which the U.K. and the E.U. strike a deal, which includes different scenarios like the Single Market or a Free Trade Agreement. The long-run estimates of the Soft Brexit scenario in our model also fall within the range of estimates from the HM Government assessment, which span from -0.9% in the case of a European Economic Area arrangement, to -6.4% in the case of the worst Free Trade Agreement (see HM Government, 2018), with an estimate of -4.9% in the case of an average FTA. The fall in GDP conditional on Soft Brexit implied by businesses expectations (-4.8%) is therefore very close to the Government's estimates of the average impact of a FTA agreement, reflecting the declared political intentions.

We note that these steady-state calculations are based on the beliefs of current firms, some of which may well be pushed out of business by Brexit, as the new economic environment makes room for new

[8] Aggregate TFP is computed as $Y/(K^{\alpha}L^{1-\alpha})^{(1-\nu)}$, where Y, K and L denote aggregates.

entrepreneurial activities to emerge in a process of creative destruction. Because these future potential entrepreneurs are not part of the survey, the above estimates may not accurately reflect the true long-run effects of Brexit, but may be biased downwards. Moreover, the individual expectations of U.K. businesses may potentially be affected by individual ideological biases, and hence may not correctly reflect the true effects of Brexit on sales. However, because we are ultimately interested in assessing the effects of Brexit uncertainty in the short-run, it is the expectations of the *current* U.K. businesses that matter for their investment decisions, however biased they may be. We now move on to discuss the short-term dynamics generated under policy uncertainty.

4.2 The Effects of the Brexit Referendum

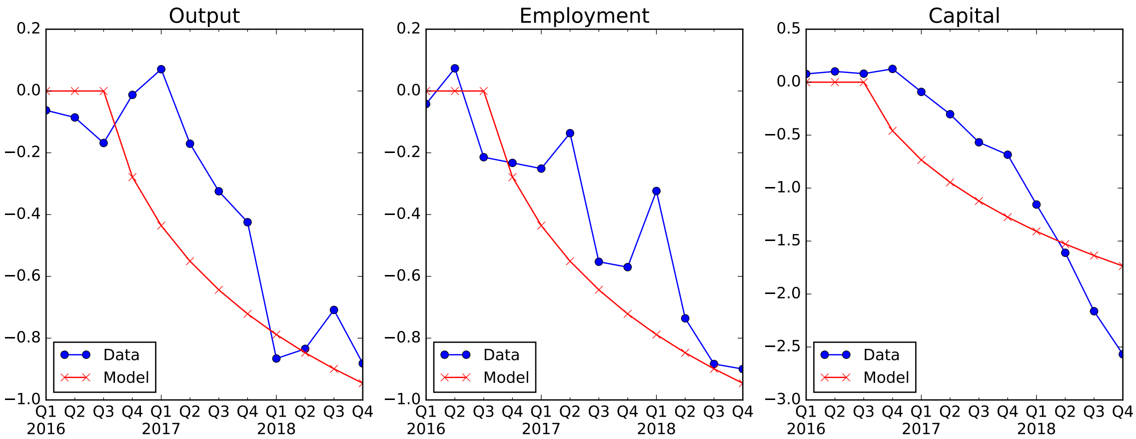
4.2.1 Comparing Model vs. Data

To assess the short-term impact of the Brexit referendum, we trace the responses of the model economy under policy uncertainty and compare them to the data. Figure 2 shows the behavior of output, employment and capital in the model and in the data, over the period 2016Q1-2018Q4, where 2016Q2 is the quarter of the referendum. The response of the model is shown by the red solid line with x-markers, and all variables are expressed in percentage deviations from the steady state. The blue solid line with circle markers shows the behavior of the data reported in Figure 1, in percentage deviations from their trend. So while the response of the model is by construction conditional on the news shock of the referendum, the behavior of the data is unconditional.

The figure reveals that the news of the Brexit referendum alone can account remarkably well for the deviation of output and employment from the counterfactual pre-referendum trend. Three years on, output is 1% below trend and the cumulative loss over this period has been around 2% of annual GDP. These quantitative results align well with the empirical findings in Born et al. (2019) and NIESR (2019).[9] The model also explains much of the behavior of capital, though it front-loads part of the disinvestment. This is due to the assumption of a constant Poisson rate at which Brexit happens, θ. This assumption is motivated purely by computational reasons. A more realistic approach would be to assume a Brexit Poisson rate that is increasing over time, starting close to zero and then rising materially when approaching the first cliff edge of the negotiations in March 2019. We will show below that in our model the effects of the news are smaller, the longer the expected duration of the negotiations. Accounting

[9]Born et al. (2019) estimate an output loss of 2.4% by year-end 2018 (two and a half years after the referendum), while NIESR (2019) estimate a cumulative loss of GDP of 2.5% up to 2019Q3.

Figure 2: Economy's Response to the Brexit Referendum



Notes: *Model response to the news of the Brexit referendum (red line with x-markers) vs. the behavior of output per capita, capital per capita and the employment rate in the data (blue line with circle markers). The responses of the model are in percentage deviations from steady state; the behavior of the data represents percentage deviations from their pre-referendum trend, as reported in Figure 1.*

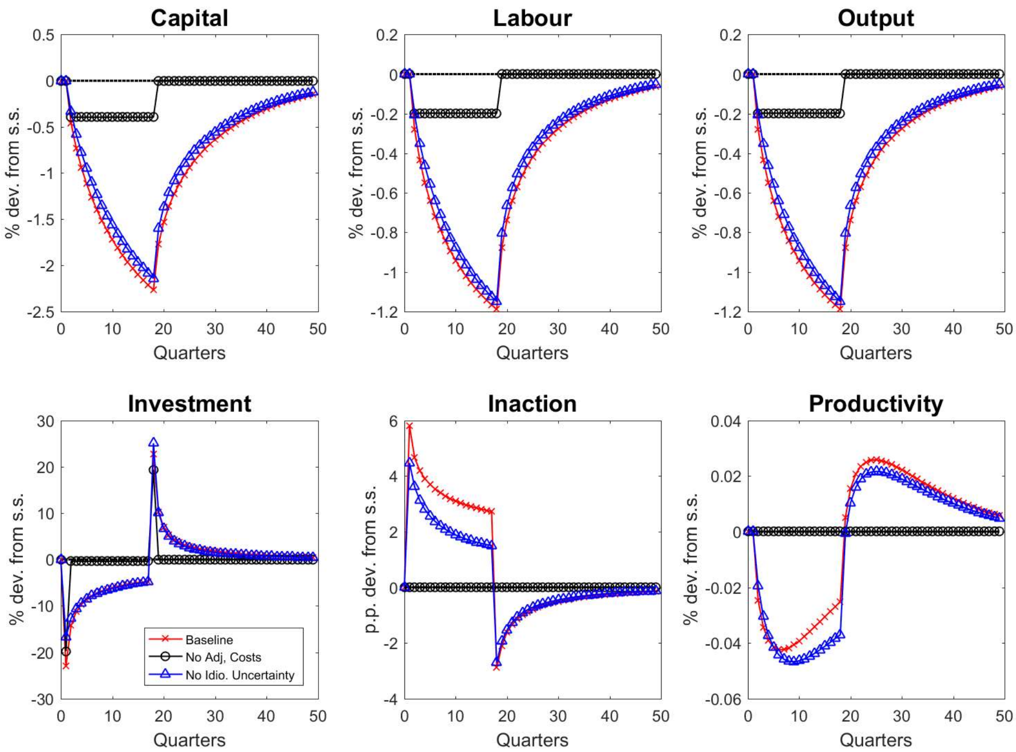
for a Brexit Poisson rate increasing over time would therefore imply that the impact of uncertainty on investment intensifies as the expected date of the resolution gets closer. In this case the response of investment would therefore be backloaded, thereby producing the same slope in the behavior of capital that we see in the data.

Overall, the model seems to do a rather good job at capturing the behavior of GDP and the production inputs over the post-referendum period, lending validity to the assumptions made in the model and its calibration. Indeed, the model can account for the initially subdued response of the U.K. economy to the news of the referendum, and for much of the subsequent gradual decline in economic activity. In the next section we disentangle the different forces underlying the response to the news of the referendum, highlighting the role played by uncertainty. We will then investigate the role that the expected duration of the negotiations plays in the propagation of the news.

4.2.2 Disentangling the Roles of News, Aggregate Uncertainty and Idiosyncratic Uncertainty

The response of the model economy to the news of the referendum is driven both by the expectation that the economy is likely to converge to a new steady-state with lower capital, and by the uncertainty associated with the news. In turn, uncertainty has both an aggregate and an idiosyncratic component. In this section we disentangle the roles of these three forces in the propagation of the news of the referendum before uncertainty is resolved.

Figure 3: The Role of Aggregate Uncertainty, Idiosyncratic Uncertainty and News



Notes: *The figure shows the propagation of the Brexit referendum shock in the baseline case (the red line with x-markers), against the counterfactual scenarios where we exclude adjustment costs (the black line with circle markers) and where we shut down idiosyncratic uncertainty (the blue line with triangle markers). All variables are expressed in percentage deviations from the initial steady state, apart from the share of inactive firms (inaction), which is in percentage points deviations. In all cases it is assumed that after eighteen quarters the second referendum shock reverts the economy to the pre-referendum steady state.*

Uncertainty, whether aggregate or idiosyncratic, propagates by inducing firms to freeze their investment and save on the fixed cost of adjustment, as they wait for uncertainty to resolve. To separate the role of uncertainty from that of anticipation, we shut down the *'wait and see'* channel of propagation by restricting the fixed cost of adjustment to equal zero. The transitional dynamics under policy uncertainty, generated by the baseline model with capital adjustment costs and the one without, are represented in Figure 3 by the red line with x-markers and the black line with circle markers, respectively. In both cases, it is assumed that the remain shock unexpectedly hits the economy after eighteen quarters, which is the time after which, on average, the U.K. firms surveyed in the DMP expect Brexit to take place (see Table 2). The Figure shows that in the absence of investment frictions, aggregate investment still falls, but less than in the benchmark case where both propagation channels are active (the red line). Figure 3 suggests that both the roles of uncertainty and anticipation are quantitatively important, with the relative importance of uncertainty becoming dominant over time, as it leads to a prolonged fall in output and in the factors of production.

Because the firms that are most productive are also the ones that are most exposed to Brexit uncertainty, they tend to drive the increase in the share of inactive firms, as they freeze investments more than the less productive ones. As a result, their relative scale of production falls, thereby leading to an increase in capital misallocation, which is reflected in the fall of aggregate productivity. These dynamics are reversed when the remain shock hits, with the most productive firms boosting the aggregate level of investment, leading to an increase in aggregate productivity and to a convergence back to the pre-referendum steady state.

In Figure 3 we also report the case where we shut down idiosyncratic uncertainty (the blue line with triangle markers). Specifically, we assume that the idiosyncratic policy state z is known to every individual firm since the news of the referendum. The results reveal that the transmission of the referendum shock with and without idiosyncratic uncertainty are not that different (compare the red and the blue line). Hence, we conclude that aggregate uncertainty is the single most important determinant of uncertainty in the transmission of the Brexit referendum shock.

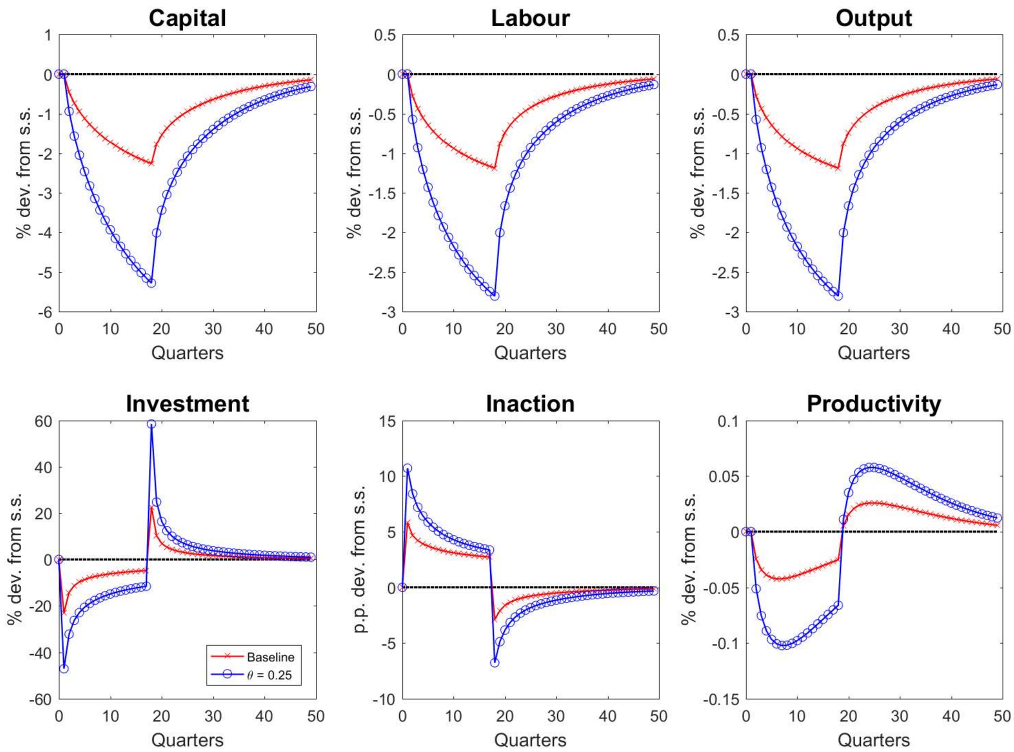
4.2.3 The Role of the Expected Duration of the Brexit Negotiations

We now investigate the role of the expected duration of the Brexit negotiations by simultaneously changing the Poisson rates θ and γ^R, while keeping the unconditional probability of Brexit unchanged. These two parameters govern the probability that the negotiations end in a given period, either because Brexit happens, or because a second referendum takes place. Figure 4 reports transitional dynamics for the baseline case, where the Brexit shock is expected to strike after approximately eighteen quarters - $1/\theta$ -, and compares to the case where this duration is shortened to four quarters (the red line with x-markers and the blue line with circle markers, respectively).[10] In both cases of long and short duration of the negotiations, we assume that after eighteen quarters the second referendum shock γ^R unexpectedly brings the economy back to the pre-referendum steady state.

The results indicate that the impact of the referendum is stronger when the expected duration of the state of the negotiations ζ^N is shorter. Reducing this expected duration increases the investment inaction region, leading to a sharp contraction in investment and a faster decumulation of capital. Note that in the

[10] In the case of $\theta = 0.25$, we set $\gamma^R = 0.024725$ to satisfy eq.(10), which implies that despite the rise in θ relative to the baseline calibration, there still is a 9% unconditional probability that Brexit would never happen.

Figure 4: The Effects of the Expected Duration of Brexit Negotiations



Notes: *The figure shows the model's response to the Brexit referendum under alternative assumptions on the value of* θ*: the baseline case (red line with x-markers), with a Poisson Brexit rate of* $\theta = 0.0581$*, and the case of* $\theta = 0.25$ *(blue line with circle markers). In the case in which we set* $\theta = 0.25$ *we also set* $\gamma^R = 0.024725$ *to keep the unconditional probability that Brexit happens unchanged. In both cases of low and high expected duration of the negotiations it is assumed that after eighteen quarters the remain shock* γ^R *reverts the economy to the pre-referendum steady state.*

case of a quick expected resolution of the negotiations (the blue line with circle markers), the contraction of output in the first quarter, -0.6% is below the level of trend growth observed prior to the referendum, which was around 0.5% per quarter, so it is sufficient to generate an actual contraction in production.

The increase in inaction is driven by two forces operating in the same direction. On the one hand, the expected duration of the negotiations coincides with the expected duration of the state of uncertainty. Faced with uncertainty, firms have an incentive to freeze their investment, waiting for it to resolve. But freezing investment is costly, as it implies operating at a sub-optimal scale of production. If the chances that uncertainty resolves in a given period are higher, more firms find it worthwhile to "wait and see". On the other hand, the expected duration of the negotiations also coincides with the expected time horizon of the news. Shortening the expected horizon implies larger anticipation effects.

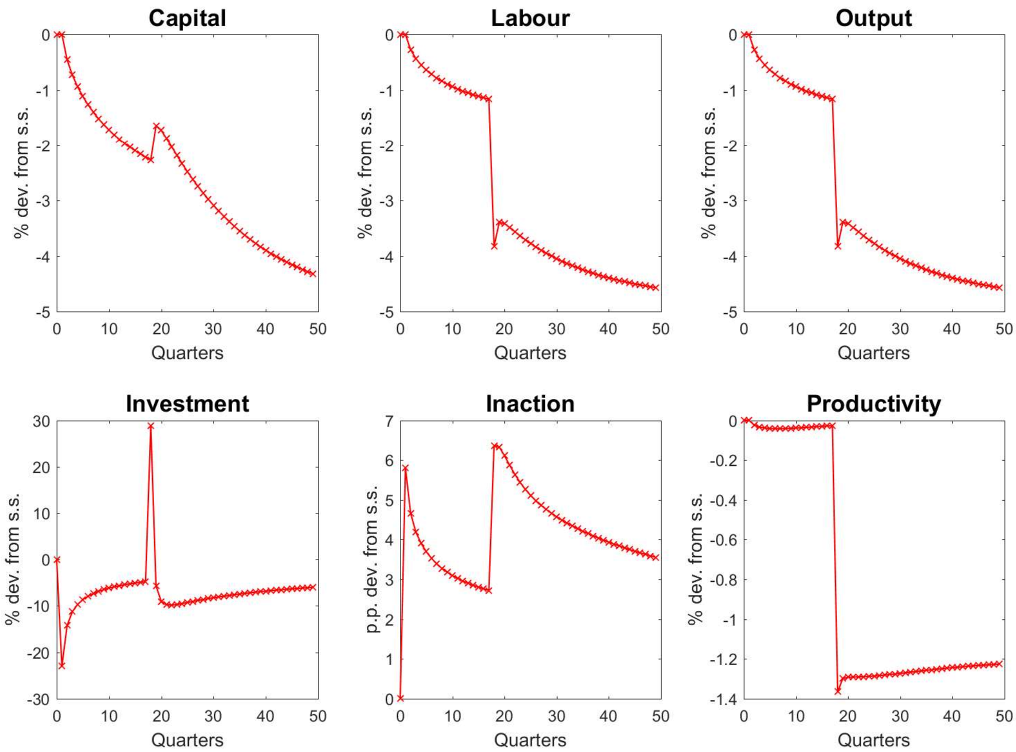
The policy implications substantiate the arguments made in a speech by the deputy governor of monetary policy at the Bank of England, Broadbent (2019), who reflects on the adverse consequences of keeping postponing the Brexit deadline, which has happened several times since March 2019. In his words, *"the effects of a repeated series of cliff-edges, each of which is expected to be decisive but in reality just gives way to the next cliff, is more damaging for investment than if it had been clear at the outset that the process will take time... For a given duration of uncertainty, it's better that people are aware of it from the start, rather than being repeatedly surprised at how long the process takes"*.

So these results suggest that a possible reason why the Brexit referendum has not produced as dramatic effects as anticipated is that the projections failed to account for the role of the expected duration of the negotiations on investment decisions. We also note that the role of the expected duration of uncertainty in the propagation of the news shock in our model is different from the role of persistence in the propagation of a typical uncertainty shock, which is defined as a second-moment shock to current fundamentals. In the latter case, the share of inactive firms increases with the persistence of the shock (Bloom et al., 2007); in our model instead it falls with the average duration of uncertainty. The difference is due to the assumption that in our model uncertainty sparks from a news shock, which by definition will have a direct impact only on future fundamentals. The longer the expected duration of uncertainty, the further away in time is the expected moment in which fundamentals will be affected. As a result, the more subdued the response to the shock, because of the two channels discussed above. An uncertainty shock instead has a direct impact on the variance of current fundamentals. The longer its persistence, the stronger the response. So it matters a lot whether uncertainty is about current or future fundamentals, just as it matters how long uncertainty is expected to last.

We believe that many episodes of economic policy uncertainty, like Brexit, the threat of trade wars, the possible dissolution of the Euro area, or more generally proposals of institutional or fiscal reforms, can be interpreted as news about a future that may be several years away. In these cases, the economic policy uncertainty associated with the news may contribute to a gradual cooling down of the economy, depending on the expected duration of uncertainty, with very different propagation relative to the canonical uncertainty shocks studied in the literature.

We note that the responses in Figure 4 represent a concrete illustration of the importance of beliefs in business cycles. The key hypothesis that is common to the literature on news shocks, sparked by the work of Beaudry and Portier (2004, 2006), is that changes in beliefs unrelated to future fundamentals may be an important source of economic fluctuations. The thought experiment in Figure 4, in which a second referendum offsets the change in beliefs triggered by the first one, delivers just that. Our results also link

Figure 5: Soft Brexit Simulation



Notes: *The figure illustrates the model's simulation when transitioning to the Soft Brexit steady state. All variables are expressed in percentage deviations from the pre-referendum steady state, apart from the share of inactive firms (inaction), which is expressed in percentage point deviations.*

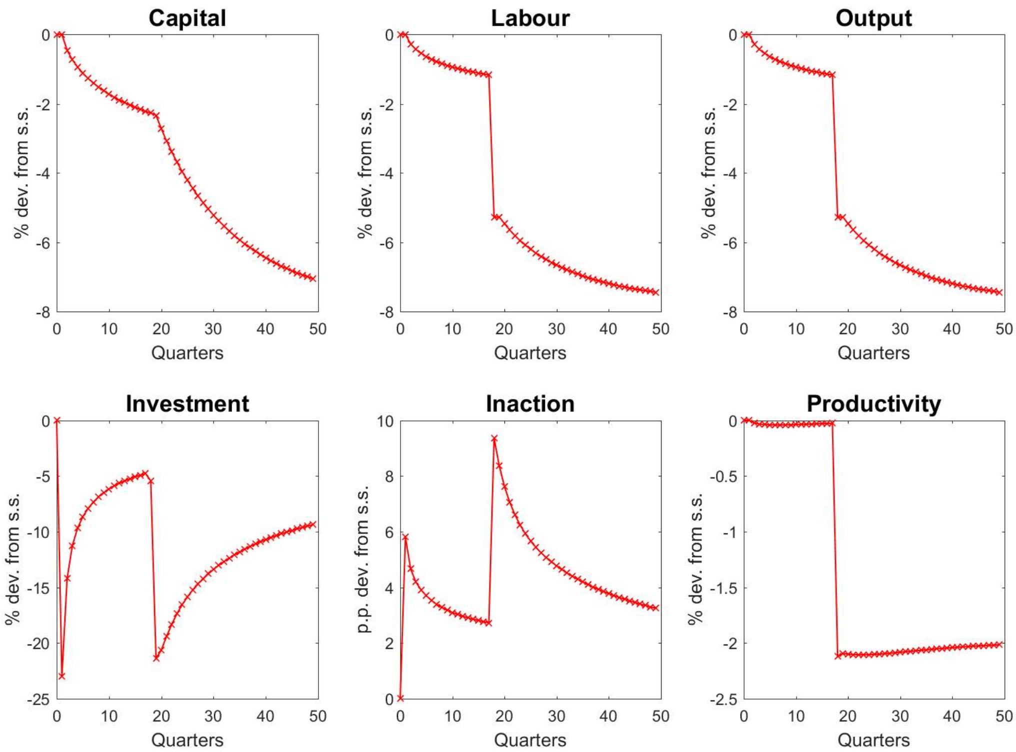
to the literature on disaster risks, in the tradition of Barro (2006, 2008), Rietz (1988) and Gourio (2012). What generates fluctuations in production after the Brexit referendum is a change in the probability of Brexit happening, which in the light of business expectations can be interpreted as an economic disaster. According to our model, the disaster risk generated by the Brexit referendum has been a key source of macroeconomic fluctuations for the U.K. economy.

4.3 The Resolution of Brexit Uncertainty and Transitions to the Long Run

In this section we illustrate the transitional dynamics leading to both the Soft and Hard Brexit steady states, assuming again that uncertainty resolves eighteen quarters after the referendum. These dynamics are reported in Figures 5 and 6 for the cases of Soft and Hard Brexit, respectively.

Up to the eighteenth quarter, the dynamics computed under policy uncertainty are the same as the ones investigated so far in Figures 3 and 4. When Brexit happens, the firms immediately learn whether

Figure 6: Hard Brexit Simulation



Notes: *The figure displays the model's simulation when transitioning to the Hard Brexit steady state. All variables are expressed in percentage deviations from the pre-referendum steady state, apart from the share of inactive firms (inaction), which is expressed in percentage point deviations.*

they will converge to the Soft or Hard steady-state. Aggregate productivity falls sharply, as can be seen in the last panel of Figures 5 and 6, inducing a sudden fall in production and labor and, because of investment frictions, a gradual decumulation of capital.

An interesting finding that emerges from figure 5 is that in the case of Soft Brexit, even if the economy will eventually converge to a steady state with lower productivity and capital, in the quarter when Brexit happens investment and capital may actually increase, leading employment and output to bounce back towards their steady-state values. This behavior is due to the positive idiosyncratic productivity shocks that affect a subset of firms. Under policy uncertainty, these productivity shocks come as a surprise, as a fraction of firms suddently realize that they have actually benefited from Brexit, or that their losses were not as bad as anticipated. This leads to massive investment on the part of these firms since, after four and a half years of prolonged uncertainty, they have substantially decumulated their capital stock. So the model shows that in the case of Soft Brexit, pent-up demand for investment arises as uncertainty is resolved. The same type of effects are not visible in the case of Hard Brexit reported in Figure 6, because they are dwarfed by the massive disinvestment of those firms, particularly the largest and most productive ones, that face large losses from Brexit. These results nicely formalize, within the context of a calibrated structural model, the arguments made in a speech by Haskel (2019), a member of the Bank of England Monetary Policy Committee, who explores the question whether investment will eventually bounce back after uncertainty is resolved. In line with our findings, he concludes that the answer to this question should depend on what deal is struck, with a pick-up in investment being more likely in the case of a softer Brexit being negotiated.

5 Conclusions

We have investigated the effects of the Brexit referendum through the lenses of a heterogeneous firms model where the uncertainty about the beliefs associated with the possible Brexit outcomes is informed by expectation data. The quantitative analysis reveals that the recent slowdown of the U.K. economy can be explained by the referendum. This economic decline can be attributed in part to the news that the economy is likely to converge to a steady state with lower capital, and in part to the uncertainty associated with the news, which induces firms to freeze investment, waiting for it to resolve. We find that the relative contribution of uncertainty has become dominant over time. Our results also indicate that the expected duration of the negotiations is key for the propagation of the news, and that its effects are larger, the sooner uncertainty is expected to resolve. The policy implication is that to keep postponing the Brexit deadline generates a succession of cliff edges in the negotiations that, by setting up expectations of a quick resolution of uncertainty, maximizes its damage.

Moving beyond the specific case of Brexit, our results indicate that uncertainty may propagate differently depending on whether it concerns current or future fundamentals and on how long it is expected to last. While typical episodes of uncertainty, like those observed during recessions, are about the state of fundamentals over the short run and are expected to resolve relatively soon, economic policy uncertainty often stems from news about fundamentals in the long run, and may be expected to last for longer. More work remains to be done to sharpen our understanding of how these propagation mechanisms differ.

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Appendix A Method of Simulated Moments

The Method of Simulated Moments (MSM) retrieves the parameters that minimize the sum of squared residuals between the moments of the data and those of the model. The minimization problem reads as follows:

$$\Theta = \arg\min_{\Theta} \mathbf{d}(\Theta)' W \mathbf{d}(\Theta), \tag{12}$$

where Θ is a $N \times 1$ vector of parameters, $\mathbf{d}(\Theta)$ is a $M \times 1$ vector of residuals, and W is a $M \times M$ weighting matrix. It is required that there should be at least as many parameters (N) as moments (M), that is $N \geq M$. In the case of $N = M$ the model is just-identified, whereas if $N > M$ the model is over-identified. Note that setting W as a matrix with the reciprocal of the squared data moments on the diagonal and zero elsewhere implies that solving Equation 12 is equivalent to minimizing the sum of squared residuals between the moments of the data and those of the model.

In order to solve the MSM we rely on the root-finding method of Nelder and Mead (1965). Since we use a local root-finding method, we conduct robustness checks by altering both the initial starting values and the step factor. We find that the results are not sensitive to such modifications.

Appendix B Computational Strategy

This section describes the numerical methods used to compute the model's steady state, as well as the transitional dynamics under policy uncertainty. Before proceeding, we first describe how the state space is discretized in order to solve the model numerically.

B.1 State Space Discretization

The model contains a total of four states: idiosyncratic productivity (a), idiosyncratic policy state (z), idiosyncratic capital (k), aggregate state (ζ). The discretization of the four states is as follows:

- The idiosyncratic productivity (a) is discretized into a grid $\mathbf{a} \in \{\overline{a}_1, ..., \overline{a}_{N_a}\}$ comprising of $N_a = 15$ log-linearly spaced points.
- The idiosyncratic policy states (z) is discretized into a grid containing $N_z = 3$ idiosyncratic states represented by $\mathbf{z} \in \{\overline{z}_+, \overline{z}_-, \overline{z}_0\}$.
- The idiosyncratic capital (k) is discretized into a grid $\mathbf{k} \in \{\overline{k}_1, ..., \overline{k}_{N_k}\}$ containing of $N_k = 25$ points spaced log-linearly between 1×10^{-5} and 5×20^2.
- The aggregate states (ζ) are four ($N_\zeta = 4$): the pre-referendum state (ζ^P), the negotiations state (ζ^N), the Soft Brexit state (ζ^S) and the Hard Brexit state (ζ^H). These aggregate states can be represented into the following grid $\mathbf{Z} \in \{\overline{\zeta^P}, \overline{\zeta^N}, \overline{\zeta^S}, \overline{\zeta^H}\}$.

Overall, the state space used for the numerical method used for the computational purposes of the model is $N_a \times N_k \times N_z \times N_\zeta$, or more specifically $15 \times 25 \times 3 \times 4$.

We also discretize the following three exogenous stochastic processes:

- The stochastic process of the aggregate states can be represented by the transition matrix Γ_ζ of size $N_\zeta \times N_\zeta$ where $\sum_{l=1}^{N_\zeta} \pi^{\zeta}_{j,l} = 1$ for all $j \in \{1, ..., N_\zeta\}$. The transition matrix probabilities are displayed in Equation 1.
- The stochastic process of idiosyncratic productivity can be represented by the transition matrix Γ_a of size $N_a \times N_a$ discretized using the Tauchen's method.
- The stochastic process of the idiosyncratic policy states can be represented by the transition matrix Γ_z of size $N_a \times N_a \times N_z \times N_z$. Where $\Gamma_z(\zeta' = \zeta^j, z' | \zeta = \zeta^i, z) = \mathbb{I}$ if $i = \{P, S, H\}$ and

$j = \{P, N, S, H\}$, as these states do not entail the draw of z. Moreover, if $i = \{N\}$, $j = \{N, P\}$, and $z_n = \{z_+, z_0, z_-\}$: $\Gamma_z(\zeta' = \zeta^j, z'|\zeta = \zeta^i, z) = \mathbb{I}$. However, if $i = \{N\}$, $j = \{S, H\}$, and $z_n = \{z_+, z_0, z_-\}$:

$$\begin{array}{cc} \downarrow z, z' \rightarrow & \begin{array}{ccc} z_+ & z_0 & z_- \end{array} \\ \Gamma_z(\zeta' = \zeta^j, z'|\zeta = \zeta^i, z) = \begin{array}{c} z_+ \\ z_0 \\ z_- \end{array} & \begin{pmatrix} q_+ & (1 - q_+ - q_-) & q_- \\ q_+ & (1 - q_+ - q_-) & q_- \\ q_+ & (1 - q_+ - q_-) & q_- \end{pmatrix}. \end{array} \tag{13}$$

B.2 Steady State

We compute the stochastic steady states of the pre-referendum economy abstracting from the possibility that Brexit may happen, i.e. we do not calculate the *'risky'* steady state. In what follows we describe the solution algorithm based on value function iteration.

B.3 Steady State Solution Algorithm:

1. Solve the problem of the firms using value function iteration, given the prices β and w:
   (a) guess an initial value function $V_g(a, z, k)$, for instance $V_g(e, z, k) = 0$;
   (b) solve for $V^{NA}(a, z, k)$ and $V^A(a, z, k)$ by taking expectations over the exogenous processes of a and z and using $V'(a, z, k) = V_g(a, z, k)$, and obtain the policy functions $K(a, z, k)$ and $L(a, z, k)$;
   (c) using $V^{NA}(a, z, k)$ and $V^A(a, z, k)$ find $\tilde{V}(a, z, k, \xi)$;
   (d) then find the policy function for the fixed capital adjustment cost threshold $\xi^T(a, z, k)$;
   (e) calculate $V(a, z, k)$ by taking expectations of $\tilde{V}(a, z, k, \xi)$ over ξ using the threshold $\xi^T(a, z, k)$;
   (f) check whether the absolute percentage deviation between the guessed value function $V_g(a, z, k)$ and the obtained value function $V(a, z, k)$ is within a pre-set tolerance. If the absolute deviation is smaller than the tolerance then exit the algorithm and save the optimal policy functions ($K(a, z, k)$,$L(a, z, k)$,$\xi^T(a, z, k)$), otherwise update the guess $V_g(a, z, k) = V(a, z, k)$ and repeat steps (a)-(e) until convergence.
2. Using the policy functions $K(a, z, k)$ and $\xi^T(a, z, k)$ solve for the stationary distribution as a fixed point, defined as $\mu'(a', z', k') = \mu(a, z, k)$, by iterating on the distribution of firms over idiosyn-

cratic productivity, idiosyncratic policy, and idiosyncratic capital holdings. In doing so, the transitional probability matrices, Γ_a and Γ_z, for the exogenous processes for a and z, respectively, are used for the evolution of the distribution:

$$\mu'(a',z',k') = \sum_{a\in\mathbf{a}}^{N_a}\sum_{z\in\mathbf{z}}^{N_z}\mu(a,z,k)\Gamma_a(a'=a_l|a=a_q)\Gamma_z(z'=z_i|z=z_j)\mathbb{I}(k',a,z,k), \tag{14}$$

where $\mathbb{I}(k',a,z,k)=1$ if $k'=K(a,z,k)$ and 0 otherwise.

3. Once the stationary distribution is obtained, it is possible to multiply it by the relevant policy decision to obtain the aggregates K,L,Y,I.

B.4 Transitional Dynamics under Policy Uncertainty

We solve for the transitional dynamics under policy uncertainty. The model features policy uncertainty, coming through the stochastic processes represented by the transition matrices Γ_ζ and Γ_z.

We set $T = 100$ and $N^* = 19$, where T is the total number of periods in the simulation and N^* denotes the first period in which uncertainty is resolved. For the transition from ζ^P to ζ^j where $j\in\{S,H,P\}$:

1. Solve the model for the initial steady state (ζ^P) using value function iteration and obtain the initial distribution $\mu_0(a,z,k)$ by solving the fixed point of the stationary distribution.

2. Solve the model for all the aggregate states with the aggregate policy stochastic process (Γ_ζ) and the idiosyncratic policy stochastic process (Γ_z) and obtain the optimal policy functions $K_t(a,z,k;\zeta^i)$, $L_t(a,z,k;\zeta^i)$, and $\xi_t^T(a,z,k;\zeta^i)$ where $i\in\{P,N,S,H\}$ using value function iteration.

3. Using the optimal policy functions and $\mu_{t-1}(a,z,k)$, obtain aggregates and solve for the next period distribution $\mu_t(a,z,k)$ for $t=1,...,N^*$ under the aggregate state ζ^N.

4. Again, using the optimal policy functions and $\mu_{t-1}(a,z,k)$, obtain aggregates and solve for the next period distribution $\mu_t(a,z,k)$ for $t=N^*+1,...,T$ under the aggregate state ζ^j.

We have used alternative maximum time periods for the algorithm, namely, $T = 200, 300$ and checked that the results do not change.