Exchange Rate Misalignment and External Imbalances: What is the Optimal Monetary Policy Response?
Appendix

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1 The transmission of monetary policy with imperfect capital markets: proof of proposition 1

In this section of the appendix, we analyze how monetary policy impacts the welfare-relevant gaps defined in Section 3.2, and offer the proof of Proposition 1 in the text. As is well known, there are notable differences in the transmission of monetary decisions across LCP and PCP economies. Specifically, a monetary expansion causing nominal depreciation weakens the terms of trade under PCP but tends to strengthen the terms of trade under LCP. Here, our specific interest is to understand how monetary transmission is affected by financial distortions.

1.1 LCP model

Starting with the LCP model, consider for simplicity a Home monetary shock such that CPI inflation follows an autoregressive process, $a_{\mathrm{H}}\pi_{Ht+s}+(1-a_{\mathrm{H}})\,\pi_{Ft+s}=\rho^s\pi>0$, $s\geq 0$—assuming that the Foreign monetary authority responds by keeping CPI price stability, i.e., $a_{\mathrm{H}}\pi^*_{Ft+s}+(1-a_{\mathrm{H}})\,\pi^*_{Ht+s}=0, s\geq 0$. For the reasons explained in the text, we focus on the case $\eta=0$, when the LCP model is relatively straightforward to solve. With $\eta=0$, the responses of key variables to the above monetary policy shock are given in Table A1. In the table, since an expansionary Home monetary policy shock is obviously inefficient (all first-best deviations are equal to zero), the responses of welfare-relevant gaps coincide with the response of actual variables.

Table A1: The effect of a monetary policy shock under LCP

$$\widetilde{\mathcal{W}}_{t+s}=\widetilde{\mathcal{W}}_t=\frac{(\sigma-1)}{2(1-a_{\mathrm{H}})+\sigma\left[2a_{\mathrm{H}}\left((\phi-1)\frac{1-\nu_1}{\nu_2-1}+1\right)-1\right]}\frac{1-\beta}{\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}}\pi$$

$$\widetilde{\mathcal{B}}_t=(1-a_{\mathrm{H}})\left\{2a_{\mathrm{H}}\,(\phi-1)\,\frac{(\beta\nu_2-1)}{\beta\nu_2}\frac{1}{1-\beta\nu_1}\widetilde{\mathcal{W}}_t+\frac{(\sigma-1)}{\sigma}\frac{(1-\rho)}{\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}}\beta\pi\right\}$$

$$\widetilde{\mathcal{T}}_{t+s}+\widetilde{\Delta}_{t+s}=-\frac{1-\nu_1^{s+1}}{1-\nu_1}\frac{(\beta\nu_2-1)}{\beta\nu_2}\widetilde{\mathcal{W}}_{t+s}$$

$$\widetilde{\Delta}_{t+s}=\frac{(1-\rho\beta)}{\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}}\rho^s\pi-(2a_{\mathrm{H}}-1)\left[1-\frac{1-\nu_1^{s+1}}{1-\nu_1}\frac{(\beta\nu_2-1)}{\beta\nu_2}\right]\widetilde{\mathcal{W}}_{t+s}$$

$$\widetilde{\mathcal{Q}}_{t+s}=\frac{(1-\rho\beta)}{\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}}\rho^s\pi-(2a_{\mathrm{H}}-1)\,\widetilde{\mathcal{W}}_{t+s}$$

$$\sigma\widetilde{Y}_{H,t+s}=a_{\mathrm{H}}\frac{(1-\rho\beta)}{\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}}\rho^s\pi-(1-a_{\mathrm{H}})\left[1+2a_{\mathrm{H}}\left(\sigma\phi\frac{1-\nu_1^{s+1}}{1-\nu_1}\frac{(\beta\nu_2-1)}{\beta\nu_2}-1\right)\right]\widetilde{\mathcal{W}}_{t+s}$$

$$\sigma\widetilde{Y}_{F,t+s}=(1-a_{\mathrm{H}})\,\frac{(1-\rho\beta)}{\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}}\rho^s\pi+(1-a_{\mathrm{H}})\left[1+2a_{\mathrm{H}}\left(\sigma\phi\frac{1-\nu_1^{s+1}}{1-\nu_1}\frac{(\beta\nu_2-1)}{\beta\nu_2}-1\right)\right]\widetilde{\mathcal{W}}_{t+s}$$

$$\sigma\widetilde{\mathcal{D}}_{t+s}=\frac{(1-\rho\beta)}{\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}}\rho^s\pi+2\,(1-a_{\mathrm{H}})\,\widetilde{\mathcal{W}}_{t+s}.$$

When markets are incomplete, a monetary shock generally causes the wealth gap $\widetilde{\mathcal{W}}_t$ to deviate from zero (recall that in the bond economy $E_t\widetilde{\mathcal{W}}_{t+1}=\widetilde{\mathcal{W}}_t$)—implying that the effects of a monetary policy shock under incomplete markets are generally different than those under complete markets. A monetary expansion can open a wealth gap in different directions, depending on elasticities, as stated in Proposition 1. By the same token, a monetary expansion can lead to either an external surplus or an external deficit. In turn, a positive $\widetilde{\mathcal{W}}_t$ would attenuate (or amplify) the effects of monetary policy on domestic output and the real exchange rate (domestic consumption and foreign output).

In a few notable special cases, however, the effects of monetary policy are the same as in economies with complete markets. One such case is $\sigma = 1$ (log consumption utility), where $\widetilde{\mathcal{W}}_t = 0$, and neither capital flows $\widetilde{\mathcal{B}}_t$, nor the relative price misalignment, $\widetilde{\mathcal{T}}_t + \widetilde{\Delta}_t$, are affected by monetary policy. In this special case, a monetary easing unambiguously results in positive domestic and foreign output gaps, a positive real exchange rate gap, and a higher relative demand gap. Relative to this benchmark, if the gap $\widetilde{\mathcal{W}}_t$ is positive the effects of monetary policy on the domestic output and the real exchange rate gaps are smaller, while the foreign output and the relative demand gaps react more. These differences reflect the fact that the misalignment $\widetilde{\mathcal{T}}_t + \widetilde{\Delta}_t$ is negative when $\widetilde{\mathcal{W}}_t > 0$, implying "expenditure switching" in favor of Foreign exports. The opposite is true if the wedge is negative: the domestic output and real exchange rate gaps react by more, while the transmission abroad is muted.

Proof of Proposition 1 under LCP. From the first equation in Table A1 it is clear that monetary easing brings about a positive wealth gap $\widetilde{\mathcal{W}}_t > 0$ when $\sigma > 1$ and $\phi \geq 1$, since both the numerator and denominator are positive under home bias ($a_{\mathrm{H}} \geq 1/2$). Under the same conditions it leads also to an (inefficient) capital outflow $\widetilde{\mathcal{B}}_t = \widehat{\mathcal{B}}_t > 0$, since both terms in the second equation in the Table A1 are positive.

1.2 PCP model

The transmission of monetary policy under PCP is shown in Table A2, where we also set $\eta = 0$. Relative to the previous table, monetary easing is now modelled as an increase in domestic PPI inflation $\pi_{Ht+s} = \rho^s \pi > 0$, $s \geq 0$, again under the assumption that the Foreign monetary authority responds by keeping PPI price stability, i.e., $\pi^*_{Ft+s} = 0, s \geq 0$.

Table A2: The effect of a monetary policy shock under PCP

$$\widetilde{\mathcal{W}}_{t+s} = \widetilde{\mathcal{W}}_t = \frac{(2a_{\mathrm{H}}\phi-1)\sigma-(2a_{\mathrm{H}}-1)}{1+(2a_{\mathrm{H}}\phi-1)\sigma-(2a_{\mathrm{H}}-1)} \frac{(1-\beta)}{\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}} \pi$$

$$\widetilde{\mathcal{B}}_t = (1-a_{\mathrm{H}}) \frac{(2a_{\mathrm{H}}\phi-1)\sigma-(2a_{\mathrm{H}}-1)}{\sigma} \frac{1}{\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}} \beta\pi$$

$$\widetilde{\mathcal{Q}}_{t+s} = (2a_{\mathrm{H}}-1)\, \widetilde{\mathcal{T}}_{t+s} = (2a_{\mathrm{H}}-1) \left[\frac{1}{\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}} \rho^s \pi - \widetilde{\mathcal{W}}_{t+s} \right]$$

$$\sigma \widetilde{Y}_{H,t+s} = [1 + 2a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)] \frac{1}{\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}} \rho^s \pi - (1-a_{\mathrm{H}}) \frac{2a_{\mathrm{H}}(\sigma\phi-1)+1}{\sigma} \widetilde{\mathcal{W}}_{t+s}$$

$$\sigma \widetilde{Y}_{F,t+s} = -2a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1) \frac{1}{\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}} \rho^s \pi + (1-a_{\mathrm{H}}) \frac{2a_{\mathrm{H}}(\sigma\phi-1)+1}{\sigma} \widetilde{\mathcal{W}}_{t+s}$$

$$\sigma \widetilde{\mathcal{D}}_{t+s} = \frac{(2a_{\mathrm{H}}-1)}{\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}} \rho^s \pi + 2(1-a_{\mathrm{H}})\, \widetilde{\mathcal{W}}_{t+s}.$$

An expansionary Home monetary policy shock also causes the gap $\widetilde{\mathcal{W}}_t$ to deviate from zero under PCP: under incomplete markets, the effects of a monetary policy shock do not coincide with those under complete markets. Again there are a few notable exceptions: under PCP, the special case in which monetary policy affects neither $\widetilde{\mathcal{W}}_t\,(=0)$ nor capital flows arises when $\phi = \frac{1+\frac{2a_{\mathrm{H}}-1}{\sigma}}{2a_{\mathrm{H}}}$; if $\sigma = 1$, then, this requires $\phi = 1$—a Cobb-Douglas consumption aggregator. In this special case, just like under complete markets, a monetary easing unambiguously results in a higher domestic output, relative demand and real exchange rate gaps. However, foreign output is affected only when $\sigma\phi \neq 1$, and increases if $\sigma\phi < 1$, namely, when goods are Edgeworth-complement. Relative to the benchmark with $\phi = \frac{1+\frac{2a_{\mathrm{H}}-1}{\sigma}}{2a_{\mathrm{H}}}$, similar to LCP, a positive (negative) wealth gap means that the effects of monetary policy on domestic output and the real exchange rate are smaller (larger) than under complete markets, while domestic consumption and foreign output react more (less). These effects reflect the fact that the response of the terms of trade, $\widetilde{\mathcal{T}}_t$, is also smaller (larger), implying a weaker (stronger) expenditure switching in favor of Home goods. Therefore, also under PCP a positive $\widetilde{\mathcal{W}}_t > 0$ may be associated with either outflows or inflows of capital, in turn attenuating or amplifying the effects of monetary policy on domestic output and the real exchange rate (domestic consumption and foreign output).

Proof of Proposition 1 under PCP. From the first equation in Table A2 the wealth gap is positive when the following condition hold:

$$\phi > \frac{1+\frac{2a_{\mathrm{H}}-1}{\sigma}}{2a_{\mathrm{H}}}.$$

From the second equation in the table, it is apparent that, for a monetary easing to lead to an inefficient capital outflow on impact, $\widetilde{\mathcal{B}}_t > 0$, it must also be the case that $\phi > \frac{1+\frac{2a_{\mathrm{H}}-1}{\sigma}}{2a_{\mathrm{H}}}$.

2 Quadratic loss function under LCP and generically incomplete markets: proof of proposition 2

In this section of the appendix we derive the quadratic loss function under LCP and generically incomplete markets. The PCP case can be understood as a special case where law of one price (LOOP) deviations are set to zero.

Write the one-period utility flow:

$$U(C) - V(L) = \zeta_C \frac{C^{1-\sigma} - 1}{1-\sigma} - \varpi \frac{L^{1+\eta}}{1+\eta},$$

Under the assumption of an efficient steady state with subsidy $\frac{(\theta-1)(1-\overline{\tau})}{\theta} = 1$, so that $U'(C) = -V'(L)$, the second order approximation of utility is as follows:

$$\widehat{C}_t - \widehat{Y}_{H,t} + \left(\frac{1-\sigma}{2} \widehat{C}_t + \widehat{\zeta}_{C,t} \right) \widehat{C}_t - (1+\eta) \left(\frac{1}{2} \widehat{Y}_{H,t} - \widehat{\zeta}_{Y,t} \right) \widehat{Y}_{H,t} +$$
$$-\frac{1}{2} \frac{\theta\alpha}{(1-\alpha\beta)(1-\alpha)} \left[a_{\mathrm{H}} \pi_{H,t}^2 + (1-a_{\mathrm{H}}) \pi_{H,t}^{*2} \right] + t.i.p. + o\left(\varepsilon^3\right),$$

where we have used the log-linear approximation to the aggregate production function: $\widehat{Y}_{H,t} = \widehat{\zeta}_{Y,t} + \widehat{L}_t$. Inflation rates appear in this expression because the second order approximation of labor effort is proportional to price dispersion, which in turn is a function of sectoral inflation rates under LCP and Calvo price-setting with symmetric probabilities α (see Engel (2009)).

Similarly, for the Foreign country we have,

$$\widehat{C}_t^* - \widehat{Y}_{F,t} + \left(\frac{1-\sigma}{2} \widehat{C}_t^* + \widehat{\zeta}_{C^*,t} \right) \widehat{C}_t^* - (1+\eta) \left(\frac{1}{2} \widehat{Y}_{F,t} - \widehat{\zeta}_{Y,t}^* \right) \widehat{Y}_{F,t} +$$
$$-\frac{1}{2} \frac{\theta\alpha}{(1-\alpha\beta)(1-\alpha)} \left[a_{\mathrm{H}} \pi_{F,t}^{*2} + (1-a_{\mathrm{H}}) \pi_{F,t}^{2} \right] + t.i.p. + o\left(\varepsilon^3\right),$$

Under cooperation, the global policy objective function $\mathcal{L}_t^W$ will be the sum of the two country-specific terms.

$$\begin{aligned}
\mathcal{L}_t^W = {} & (\widehat{C}_t + \widehat{C}_t^*) - (\widehat{Y}_{H,t} + \widehat{Y}_{F,t}) + \left(\frac{1-\sigma}{2} (\widehat{C}_t + \widehat{\zeta}_{C,t} \right) \widehat{C}_t + \left(\frac{1-\sigma}{2} \widehat{C}_t^* + \widehat{\zeta}_{C^*,t} \right) \widehat{C}_t^* \\
& - (1+\eta) \left(\frac{1}{2} \widehat{Y}_{H,t} - \widehat{\zeta}_{Y,t} \right) \widehat{Y}_{H,t} - (1+\eta) \left(\frac{1}{2} \widehat{Y}_{F,t} - \widehat{\zeta}_{Y,t}^* \right) \widehat{Y}_{F,t} + \\
& -\frac{1}{2} \frac{\theta\alpha}{(1-\alpha\beta)(1-\alpha)} \left(\left[a_{\mathrm{H}} \pi_{H,t}^2 + (1-a_{\mathrm{H}}) \pi_{H,t}^{*2} \right] + \left[a_{\mathrm{H}} \pi_{F,t}^{*2} + (1-a_{\mathrm{H}}) \pi_{F,t}^{2} \right] \right) \\
& + t.i.p. + o\left(\varepsilon^3\right),
\end{aligned}$$

The objective of this appendix is to rewrite the above as a quadratic loss function in terms of gaps and misalignments.

2.1 Useful first order relationships

We begin by writing some useful relations. The real exchange rate is related to the terms of trade and deviations from the law of one price as follows:

$$\widehat{\mathcal{Q}}_t = (2a_{\mathrm{H}} - 1)\,\widehat{\mathcal{T}}_t + 2a_{\mathrm{H}}\widehat{\Delta}_t. \tag{1}$$

The first order approximations of $\widehat{C}_t$ and $\widehat{C}_t^*$, are given by,

$$\begin{aligned} \widehat{C}_t^* &= \widehat{C}_t - \sigma^{-1}\left[\widehat{\mathcal{Q}}_t + \widetilde{\mathcal{W}}_t + \left(\widehat{\zeta}_{C,t} - \widehat{\zeta}^*_{C,t}\right)\right] \qquad (2)\\ \widehat{C}_t &= \frac{1}{2}\left\{\widehat{Y}_{H,t} + \widehat{Y}_{F,t} + \sigma^{-1}\left[\widehat{\mathcal{Q}}_t + \widetilde{\mathcal{W}}_t + \left(\widehat{\zeta}_{C,t} - \widehat{\zeta}^*_{C,t}\right)\right]\right\}, \end{aligned}$$

The first order approximations of $\widehat{C}_t$ and $\widehat{C}_t^*$ imply,

$$\begin{aligned} -(\widehat{C}_t - \widehat{Y}_{H,t}) &= \widehat{C}_t^* - \widehat{Y}_{F,t} = \qquad (3)\\ &\frac{1}{2}\left\{\widehat{Y}_{H,t} - \widehat{Y}_{F,t} - \sigma^{-1}\left[\widehat{\mathcal{Q}}_t + \widetilde{\mathcal{W}}_t + \left(\widehat{\zeta}_{C,t} - \widehat{\zeta}^*_{C,t}\right)\right]\right\} \end{aligned}$$

The first order approximation of aggregate demand yields,

$$\begin{aligned} \widehat{C}_t &= \widehat{Y}_{H,t} - (1 - a_{\mathrm{H}})\,\sigma^{-1}\left[\sigma\phi\widehat{\mathcal{T}}_t + (\sigma\phi - 1)\,\widehat{\mathcal{Q}}_t - \widetilde{\mathcal{W}}_t - \left(\widehat{\zeta}_{C,t} - \widehat{\zeta}^*_{C,t}\right)\right]\\ \widehat{C}_t^* &= \widehat{Y}_{F,t} + (1 - a_{\mathrm{H}})\,\sigma^{-1}\left[\sigma\phi\widehat{\mathcal{T}}_t + (\sigma\phi - 1)\,\widehat{\mathcal{Q}}_t - \widetilde{\mathcal{W}}_t - \left(\widehat{\zeta}_{C,t} - \widehat{\zeta}^*_{C,t}\right)\right] \end{aligned}$$

Combining the first order approximations of aggregate demand, we obtain,

$$\widehat{C}_t = \widehat{Y}_{H,t} - \frac{1 - a_{\mathrm{H}}}{\sigma}\left[2a_{\mathrm{H}}\phi\sigma\left(\widehat{\mathcal{T}}_t + \widehat{\Delta}_t\right) - \widehat{\mathcal{Q}}_t - \widetilde{\mathcal{W}}_t - \left(\widehat{\zeta}_{C,t} - \widehat{\zeta}^*_{C,t}\right)\right].$$

Combining the two expressions for consumption, we obtain the following expression for the terms of trade:

$$\begin{aligned} &[4a_{\mathrm{H}}\,(1 - a_{\mathrm{H}})\,(\sigma\phi - 1) + 1]\left(\widehat{\mathcal{T}}_t + \widehat{\Delta}_t\right) = \qquad (4)\\ &\sigma\left(\widehat{Y}_{H,t} - \widehat{Y}_{F,t}\right) - (2a_{\mathrm{H}} - 1)\left[\widetilde{\mathcal{W}}_t + \widehat{\Delta}_t + \left(\widehat{\zeta}_{C,t} - \widehat{\zeta}^*_{C,t}\right)\right] \end{aligned}$$

In addition, shocks can be expressed in terms of efficient output and the terms of trade,

$$\begin{aligned} &\widehat{\zeta}_{C,t} + (1 + \eta)\,\widehat{\zeta}_{Y,t} = \qquad (5)\\ &(\eta + \sigma)\,\widehat{Y}^{fb}_{H,t} - [2a_{\mathrm{H}}\,(1 - a_{\mathrm{H}})\,(\sigma\phi - 1)]\left(\widehat{\mathcal{T}}^{fb}_t\right) + (1 - a_{\mathrm{H}})\left(\widehat{\zeta}_{C,t} - \widehat{\zeta}^*_{C,t}\right) \end{aligned}$$

Next, using the first order approximation for domestic consumption, we can rewrite domestic marginal costs as follows,

$$\begin{aligned} &\sigma\widehat{C}_t - \widehat{\zeta}_{C,t} + \eta\widehat{Y}_{H,t} - (1 + \eta)\,\widehat{\zeta}_{Y,t} + (1 - a_{\mathrm{H}})\left(\widehat{\mathcal{T}}_t + \Delta_t\right) = \qquad (6)\\ &(\eta + \sigma)\left(\widehat{Y}_{H,t} - \widehat{Y}^{fb}_{H,t}\right) +\\ &-(1 - a_{\mathrm{H}})\cdot\left[(\sigma\phi - 1)\left(\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}^{fb}_t + \widehat{\mathcal{Q}}_t - \widehat{\mathcal{Q}}^{fb}_t\right) - \widetilde{\mathcal{W}}_t - \Delta_t\right] \end{aligned}$$

Rearranging,

$$
\begin{aligned}
&\frac{\sigma}{2}\widehat{C}_t - \widehat{\zeta}_{C,t} + \frac{\eta}{2}\widehat{Y}_{H,t} - (1+\eta)\,\widehat{\zeta}_{Y,t} + \frac{1}{2}\left(1-a_{\mathrm{H}}\right)\left(\widehat{\mathcal{T}}_t + \Delta_t\right) = \qquad (7)\\
&(\eta+\sigma)\left(\frac{1}{2}\widehat{Y}_{H,t} - \widehat{Y}^{fb}_{H,t}\right) - 2a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)(\sigma\phi-1)\left(\frac{1}{2}\left(\widehat{\mathcal{T}}_t+\Delta_t\right) - \widehat{\mathcal{T}}^{fb}_t\right) + \\
&\frac{1}{2}\left(1-a_{\mathrm{H}}\right)\left(\widetilde{\mathcal{W}}_t + \Delta_t - \left(\widehat{\zeta}_{C,t} - \widehat{\zeta}^{*}_{C,t}\right)\right)
\end{aligned}
$$

2.2 Derivation of the global loss function in terms of gaps and misalignments (proof of proposition 2)

To eliminate the linear terms from $\mathcal{L}^W_t$, we proceed as follows. First, we derive a second-order accurate expression for the sum of consumption across countries (the world aggregate demand) by summing up the budget constraints under LCP:

$$
\begin{aligned}
&\frac{P_{\mathrm{H}}}{P}\left(C_{\mathrm{H}} + C^*_{\mathrm{H}}\right) + \frac{SP^*_{\mathrm{F}}}{P}\left(C_{\mathrm{F}} + C^*_{\mathrm{F}}\right) = \frac{P_{\mathrm{H}}}{P}Y_{\mathrm{H}} + \frac{SP^*_{\mathrm{F}}}{P}Y_{\mathrm{F}}\\
&C + \mathcal{Q}C^* + \left(\frac{SP^*_{\mathrm{F}}}{P_{\mathrm{F}}} - 1\right)\frac{P_{\mathrm{F}}}{P}C_{\mathrm{F}} - \left(\frac{SP^*_{\mathrm{H}}}{P_{\mathrm{H}}} - 1\right)\frac{P_{\mathrm{H}}}{P}C^*_{\mathrm{H}} = \frac{P_{\mathrm{H}}}{P}Y_{\mathrm{H}} + \frac{P^*_{\mathrm{F}}}{P^*}\frac{SP^*}{P}Y_{\mathrm{F}}\\
&C + \mathcal{Q}C^* + \left(1-a_{\mathrm{H}}\right)\left[\left(\Delta_{\mathrm{F}} - 1\right)\left(\frac{P_{\mathrm{F}}}{P}\right)^{1-\phi} C + \left(\Delta^{-1}_{\mathrm{H}} - 1\right)\left(\frac{P^*_{\mathrm{H}}}{P^*}\right)^{1-\phi}\mathcal{Q}C^*\right] = \\
&\qquad \frac{P_{\mathrm{H}}}{P}Y_{\mathrm{H}} + \frac{P^*_{\mathrm{F}}}{P^*}\mathcal{Q}Y_{\mathrm{F}}.
\end{aligned}
$$

$$
\begin{aligned}
&C + \mathcal{Q}C^* + \left(1-a_{\mathrm{H}}\right)\left\{\begin{array}{c}\left(\Delta_{\mathrm{F}} - 1\right)\left[a_{\mathrm{H}}\mathcal{T}^{\phi-1}\Delta^{\phi-1}_{\mathrm{H}} + \left(1-a_{\mathrm{H}}\right)\right]^{-1} C + \\ \left(\Delta^{-1}_{\mathrm{H}} - 1\right)\left[a_{\mathrm{H}}\mathcal{T}^{1-\phi}\Delta^{1-\phi}_{\mathrm{F}} + \left(1-a_{\mathrm{H}}\right)\right]^{-1}\mathcal{Q}C^*\end{array}\right\} = \\
&\qquad \left[a_{\mathrm{H}} + \left(1-a_{\mathrm{H}}\right)\mathcal{T}^{1-\phi}\Delta^{1-\phi}_{\mathrm{H}}\right]^{-\frac{1}{1-\phi}} Y_{\mathrm{H}} + \\
&\qquad \left[a_{\mathrm{H}} + \left(1-a_{\mathrm{H}}\right)\mathcal{T}^{\phi-1}\Delta^{\phi-1}_{\mathrm{F}}\right]^{-\frac{1}{1-\phi}}\mathcal{Q}Y_{\mathrm{F}}.
\end{aligned}
$$

The accurate second-order expression for the world demand is:

$$
\begin{gathered}
\widehat{C}_t + \widehat{C}_t^* + \tfrac{1}{2}\left(\widehat{C}_t^2 + \widehat{C}_t^{*2}\right) + \widehat{\mathcal{Q}}_t + \tfrac{1}{2}\widehat{\mathcal{Q}}_t^2 + \widehat{\mathcal{Q}}_t\widehat{C}_t^* + \\
(1-a_{\mathrm{H}})\left[\begin{array}{c}
\widehat{\Delta}_{\mathrm{F},t} + \tfrac{1}{2}\widehat{\Delta}_{\mathrm{F},t}^2 + \Delta_{\mathrm{F},t}\left(\widehat{C}_t + a_{\mathrm{H}}\left(1-\phi\right)\left(\widehat{\mathcal{T}}_t + \widehat{\Delta}_{\mathrm{H},t}\right)\right) - \\
\left(\widehat{\Delta}_{\mathrm{H},t} + \tfrac{1}{2}\widehat{\Delta}_{\mathrm{H},t}^2\right) + \widehat{\Delta}_{\mathrm{H},t}^2 - \widehat{\Delta}_{\mathrm{H},t}\left(\widehat{C}_t^* + \widehat{\mathcal{Q}}_t - a_{\mathrm{H}}\left(1-\phi\right)\left(\widehat{\mathcal{T}}_t + \widehat{\Delta}_{\mathrm{F},t}\right)\right)
\end{array}\right]
\end{gathered}
$$

$$
\begin{aligned}
&= \widehat{Y}_{H,t} + \widehat{Y}_{F,t} + \frac{1}{2}\left(\widehat{Y}_{H,t}^2 + \widehat{Y}_{F,t}^2\right) - (1-a_{\mathrm{H}})\left[\widehat{\mathcal{T}}_t + \widehat{\Delta}_{\mathrm{H},t} + \frac{1}{2}\left(\widehat{\mathcal{T}}_t^2 + \widehat{\Delta}_{\mathrm{H},t}^2\right)\right] - \\
&(1-a_{\mathrm{H}})\,\widehat{Y}_{H,t}\left(\widehat{\mathcal{T}}_t + \widehat{\Delta}_{\mathrm{H},t}\right) + (1-a_{\mathrm{H}})\left[\phi - 1 + (1-a_{\mathrm{H}})(1-\phi)\left(\frac{1}{1-\phi} + 1\right)\right]\widehat{\mathcal{T}}_t\widehat{\Delta}_{\mathrm{H},t} + \\
&\frac{1}{2}(1-a_{\mathrm{H}})\left[\phi + (1-a_{\mathrm{H}})(1-\phi)\left(\frac{1}{1-\phi} + 1\right)\right]\left(\widehat{\mathcal{T}}_t^2 + \widehat{\Delta}_{\mathrm{H},t}^2\right) + \\
&(1-a_{\mathrm{H}})\left[\widehat{\mathcal{T}}_t + \widehat{\Delta}_{\mathrm{F},t} + \frac{1}{2}\left(\widehat{\mathcal{T}}_t^2 + \widehat{\Delta}_{\mathrm{F},t}^2\right)\right] + \widehat{\mathcal{Q}}_t + \frac{1}{2}\widehat{\mathcal{Q}}_t^2 + \widehat{Y}_{F,t}\widehat{\mathcal{Q}}_t + (1-a_{\mathrm{H}})\,\widehat{Y}_{F,t}\left(\widehat{\mathcal{T}}_t + \widehat{\Delta}_{\mathrm{F},t}\right) + \\
&(1-a_{\mathrm{H}})\left(\widehat{\mathcal{T}}_t + \widehat{\Delta}_{\mathrm{F},t}\right)\widehat{\mathcal{Q}}_t + (1-a_{\mathrm{H}})\left[\phi - 1 + (1-a_{\mathrm{H}})(1-\phi)\left(\frac{1}{1-\phi} + 1\right)\right]\widehat{\mathcal{T}}_t\widehat{\Delta}_{\mathrm{F},t} + \\
&\frac{1}{2}(1-a_{\mathrm{H}})\left[(1-a_{\mathrm{H}})\left(\frac{1}{1-\phi} + 1\right)(1-\phi) + \phi - 2\right]\left(\widehat{\mathcal{T}}_t^2 + \widehat{\Delta}_{\mathrm{F},t}^2\right).
\end{aligned}
$$

As the linear terms in relative prices cancel out and under the maintained assumption of symmetry $\widehat{\Delta}_{\mathrm{H},t} = \widehat{\Delta}_{\mathrm{F},t} = \widehat{\Delta}_t$, we get:

$$
\widehat{C}_t + \widehat{C}_t^* + \frac{1}{2}\left(\widehat{C}_t^2 + \widehat{C}_t^{*2}\right) + (1-a_{\mathrm{H}})\left(\widehat{C}_t - \widehat{C}_t^* - \widehat{\mathcal{Q}}_t\right)\widehat{\Delta}_t =
$$

$$
\begin{gathered}
\widehat{Y}_{H,t} + \widehat{Y}_{F,t} + \tfrac{1}{2}\left(\widehat{Y}_{H,t}^2 + \widehat{Y}_{F,t}^2\right) + \left(\widehat{Y}_{F,t} - \widehat{C}_t^*\right)\widehat{\mathcal{Q}}_t + \\
(1-a_{\mathrm{H}})\left(\widehat{Y}_{F,t} - \widehat{Y}_{H,t}\right)\left(\widehat{\mathcal{T}}_t + \widehat{\Delta}_t\right) + a_{\mathrm{H}}(1-a_{\mathrm{H}})\,\phi\left(\widehat{\mathcal{T}}_t + \widehat{\Delta}_t\right)^2 + \\
(1-a_{\mathrm{H}})\left(\left[1 - 2a_{\mathrm{H}}(1-\phi)\right]\widehat{\mathcal{T}}_t - 2a_{\mathrm{H}}(1-\phi)\,\widehat{\Delta}_t\right)\widehat{\Delta}_t,
\end{gathered}
$$

Second, we substitute in the approximation to the sum of consumption—in addition, we subtract $\frac{1}{2}(1-a_{\mathrm{H}})\,\widehat{\mathcal{T}}_t\left(\widehat{Y}_{H,t} - \widehat{Y}_{F,t}\right)$, $\left(\frac{\sigma}{2}\widehat{C}_t - \widehat{\zeta}_{C,t}\right)\widehat{Y}_{H,t}$ and $\left(\dfrac{\sigma}{2}\widehat{C}_t^* - \widehat{\zeta}_{C,t}^*\right)\widehat{Y}_{F,t}$ in order to have a second-order term in the product of output and marginal costs for each country.

$$
\begin{aligned}
&\mathcal{L}_t^W \ltimes \widehat{C}_t + \widehat{C}_t^* - \widehat{Y}_{H,t} - \widehat{Y}_{F,t} + \left(\frac{1-\sigma}{2}\widehat{C}_t + \widehat{\zeta}_{C,t}\right)\widehat{C}_t + \left(\frac{1-\sigma}{2}\widehat{C}_t^* + \widehat{\zeta}_{C,t}^*\right)\widehat{C}_t^* - \\
&(1+\eta)\left(\frac{1}{2}\widehat{Y}_{H,t} - \widehat{\zeta}_{Y,t}\right)\widehat{Y}_{H,t} - (1+\eta)\left(\frac{1}{2}\widehat{Y}_{F,t} - \widehat{\zeta}_{Y,t}^*\right)\widehat{Y}_{F,t} - \\
&\frac{1}{2}\frac{\theta\alpha}{(1-\alpha\beta)(1-\alpha)}\left[a_{\mathrm{H}}\pi_{H,t}^2 + (1-a_{\mathrm{H}})\pi_{H,t}^{*2} + a_{\mathrm{H}}\pi_{F,t}^{*2} + (1-a_{\mathrm{H}})\pi_{F,t}^2\right] \\
&+t.i.p. + o\left(\varepsilon^3\right) \\
&= -\left(\frac{\sigma}{2}\widehat{C}_t - \widehat{\zeta}_{C,t}\right)\widehat{C}_t - \left(\frac{\sigma}{2}\widehat{C}_t^* - \widehat{\zeta}_{C,t}^*\right)\widehat{C}_t^* + \left(\widehat{Y}_{F,t} - \widehat{C}_t^*\right)\widehat{\mathcal{Q}}_t - \\
&(1-a_{\mathrm{H}})\left(\widehat{C}_t - \widehat{C}_t^* - \widehat{\mathcal{Q}}_t\right)\widehat{\Delta}_t - \\
&\frac{1}{2}(1-a_{\mathrm{H}})\left(\widehat{Y}_{H,t} - \widehat{Y}_{F,t}\right)\left(\widehat{\mathcal{T}}_t + \widehat{\Delta}_t\right) + (1-a_{\mathrm{H}})a_{\mathrm{H}}\phi\left(\widehat{\mathcal{T}}_t + \widehat{\Delta}_t\right)^2 + \\
&(1-a_{\mathrm{H}})\left([1-2a_{\mathrm{H}}(1-\phi)]\widehat{\mathcal{T}}_t - 2a_{\mathrm{H}}(1-\phi)\widehat{\Delta}_t\right)\widehat{\Delta}_t - \\
&\left(\frac{\eta}{2}\widehat{Y}_{H,t} - (1+\eta)\widehat{\zeta}_{Y,t} + \frac{1}{2}(1-a_{\mathrm{H}})\left(\widehat{\mathcal{T}}_t + \widehat{\Delta}_t\right)\right)\widehat{Y}_{H,t} - \\
&\left(\frac{\eta}{2}\widehat{Y}_{F,t} - (1+\eta)\widehat{\zeta}_{Y,t}^* - \frac{1}{2}(1-a_{\mathrm{H}})\left(\widehat{\mathcal{T}}_t + \widehat{\Delta}_t\right)\right)\widehat{Y}_{F,t} - \\
&\frac{1}{2}\frac{\theta\alpha}{(1-\alpha\beta)(1-\alpha)}\left[a_{\mathrm{H}}\pi_{H,t}^2 + (1-a_{\mathrm{H}})\pi_{H,t}^{*2} + a_{\mathrm{H}}\pi_{F,t}^{*2} + (1-a_{\mathrm{H}})\pi_{F,t}^2\right] \\
&+t.i.p. + o\left(\varepsilon^3\right)
\end{aligned}
$$

$$
\begin{aligned}
&= -\left(\frac{\sigma}{2}\widehat{C}_t - \widehat{\zeta}_{C,t}\right)\left(\widehat{C}_t - \widehat{Y}_{H,t}\right) - \left(\frac{\sigma}{2}\widehat{C}_t^* - \widehat{\zeta}_{C,t}^* + \widehat{\mathcal{Q}}_t\right)\left(\widehat{C}_t^* - \widehat{Y}_{F,t}\right) - \\
&(1-a_{\mathrm{H}})\left(\widehat{C}_t - \widehat{C}_t^* - \widehat{\mathcal{Q}}_t\right)\widehat{\Delta}_t - \\
&\left(\frac{\sigma}{2}\widehat{C}_t - \widehat{\zeta}_{C,t} + \frac{\eta}{2}\widehat{Y}_{H,t} - (1+\eta)\widehat{\zeta}_{Y,t} + \frac{1}{2}(1-a_{\mathrm{H}})\left(\widehat{\mathcal{T}}_t + \widehat{\Delta}_t\right)\right)\widehat{Y}_{H,t} - \\
&\left(\frac{\sigma}{2}\widehat{C}_t^* - \widehat{\zeta}_{C,t}^* + \frac{\eta}{2}\widehat{Y}_{F,t} - (1+\eta)\widehat{\zeta}_{Y,t}^* - \frac{1}{2}(1-a_{\mathrm{H}})\left(\widehat{\mathcal{T}}_t + \widehat{\Delta}_t\right)\right)\widehat{Y}_{F,t} - \\
&\frac{1}{2}(1-a_{\mathrm{H}})\left(\widehat{\mathcal{T}}_t + \widehat{\Delta}_t\right)\left(\widehat{Y}_{H,t} - \widehat{Y}_{F,t}\right) + \\
&(1-a_{\mathrm{H}})a_{\mathrm{H}}\phi\left(\widehat{\mathcal{T}}_t + \widehat{\Delta}_t\right)^2 + (1-a_{\mathrm{H}})\left([1-2a_{\mathrm{H}}(1-\phi)]\widehat{\mathcal{T}}_t - 2a_{\mathrm{H}}(1-\phi)\widehat{\Delta}_t\right)\widehat{\Delta}_t - \\
&\frac{1}{2}\frac{\theta\alpha}{(1-\alpha\beta)(1-\alpha)}\left[a_{\mathrm{H}}\pi_{H,t}^2 + (1-a_{\mathrm{H}})\pi_{H,t}^{*2} + a_{\mathrm{H}}\pi_{F,t}^{*2} + (1-a_{\mathrm{H}})\pi_{F,t}^2\right] \\
&+t.i.p. + o\left(\varepsilon^3\right).
\end{aligned}
$$

Some more substitutions and algebra follows. Using the expressions for shocks (5) and domestic marginal costs (6) in terms of efficient output and terms of trade, we can express the loss in terms of output gaps, relative price misalignment, including Δ_t, and demand imbalances:

$$
\begin{aligned}
&\mathcal{L}_t^W \ltimes -\left(\frac{\sigma}{2}\widehat{C}_t-\widehat{\zeta}_{C,t}\right)\left(\widehat{C}_t-\widehat{Y}_{H,t}\right)-\left(\frac{\sigma}{2}\widehat{C}_t^*-\widehat{\zeta}_{C,t}^*+\widehat{\mathcal{Q}}_t\right)\left(\widehat{C}_t^*-\widehat{Y}_{F,t}\right)-\\
&\left(1-a_{\mathrm{H}}\right)\left(\widehat{C}_t-\widehat{C}_t^*-\widehat{\mathcal{Q}}_t\right)\widehat{\Delta}_t-\\
&\left[\left(\eta+\sigma\right)\left(\frac{1}{2}\widehat{Y}_{H,t}-\widehat{Y}_{H,t}^{fb}\right)-2a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)\left(\frac{1}{2}\left(\widehat{\mathcal{T}}_t+\widehat{\Delta}_t\right)-\widehat{\mathcal{T}}_t^{fb}\right)\right]\widehat{Y}_{H,t}+-\\
&\left[\left(\eta+\sigma\right)\left(\frac{1}{2}\widehat{Y}_{F,t}-\widehat{Y}_{F,t}^{fb}\right)+2a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)\left(\frac{1}{2}\left(\widehat{\mathcal{T}}_t+\widehat{\Delta}_t\right)-\widehat{\mathcal{T}}_t^{fb}\right)\right]\widehat{Y}_{F,t}+-\\
&\frac{1}{2}\left(1-a_{\mathrm{H}}\right)\left[\left(\widehat{\mathcal{T}}_t+\widehat{\Delta}_t\right)+\widetilde{\mathcal{W}}_t+\widehat{\Delta}_t-\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}_{C,t}^*\right)\right]\left(\widehat{Y}_{H,t}-\widehat{Y}_{F,t}\right)+\\
&\left(1-a_{\mathrm{H}}\right)a_{\mathrm{H}}\phi\left(\widehat{\mathcal{T}}_t+\widehat{\Delta}_t\right)^2+\left(1-a_{\mathrm{H}}\right)\left(\left[1-2a_{\mathrm{H}}\left(1-\phi\right)\right]\widehat{\mathcal{T}}_t-2a_{\mathrm{H}}\left(1-\phi\right)\widehat{\Delta}_t\right)\widehat{\Delta}_t-\\
&\frac{1}{2}\frac{\theta\alpha}{\left(1-\alpha\beta\right)\left(1-\alpha\right)}\left[a_{\mathrm{H}}\pi_{H,t}^2+\left(1-a_{\mathrm{H}}\right)\pi_{H,t}^{*2}+a_{\mathrm{H}}\pi_{F,t}^{*2}+\left(1-a_{\mathrm{H}}\right)\pi_{F,t}^2\right]+t.i.p.+o\left(\varepsilon^3\right),
\end{aligned}
$$

Note that we have also collected all the terms multiplied by $\left(\widehat{C}_t^*-\widehat{Y}_{F,t}\right)$. Collecting the terms in output gaps and the terms multiplied by output differentials yields:

$$
\begin{aligned}
&\mathcal{L}_t^W \ltimes -\left(\frac{\sigma}{2}\widehat{C}_t-\widehat{\zeta}_{C,t}\right)\left(\widehat{C}_t-\widehat{Y}_{H,t}\right)-\left(\frac{\sigma}{2}\widehat{C}_t^*-\widehat{\zeta}_{C,t}^*+\widehat{\mathcal{Q}}_t\right)\left(\widehat{C}_t^*-\widehat{Y}_{F,t}\right)-\\
&\left(1-a_{\mathrm{H}}\right)\left(\widehat{C}_t-\widehat{C}_t^*-\widehat{\mathcal{Q}}_t\right)\widehat{\Delta}_t-\\
&\left(1-a_{\mathrm{H}}\right)\frac{1}{2}\left[\left(\widehat{\mathcal{T}}_t+\widehat{\Delta}_t\right)+\left(\widetilde{\mathcal{W}}_t+\widehat{\Delta}_t\right)-\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}_{C,t}^*\right)\right]\left(\widehat{Y}_{H,t}-\widehat{Y}_{F,t}\right)+\\
&\left(1-a_{\mathrm{H}}\right)a_{\mathrm{H}}\phi\left(\widehat{\mathcal{T}}_t+\widehat{\Delta}_t\right)^2+\left(1-a_{\mathrm{H}}\right)\left(\left[1-2a_{\mathrm{H}}\left(1-\phi\right)\right]\widehat{\mathcal{T}}_t-2a_{\mathrm{H}}\left(1-\phi\right)\widehat{\Delta}_t\right)\widehat{\Delta}_t+\\
&2a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)\left(\frac{1}{2}\left(\widehat{\mathcal{T}}_t+\widehat{\Delta}_t\right)-\widehat{\mathcal{T}}_t^{fb}\right)\left(\widehat{Y}_{H,t}-\widehat{Y}_{F,t}\right)-\\
&\left(\eta+\sigma\right)\left(\frac{1}{2}\widehat{Y}_{H,t}-\widehat{Y}_{H,t}^{fb}\right)\widehat{Y}_{H,t}-\left(\eta+\sigma\right)\left(\frac{1}{2}\widehat{Y}_{F,t}-\widehat{Y}_{F,t}^{fb}\right)\widehat{Y}_{F,t}-\\
&\frac{1}{2}\frac{\theta\alpha}{\left(1-\alpha\beta\right)\left(1-\alpha\right)}\left[a_{\mathrm{H}}\pi_{H,t}^2+\left(1-a_{\mathrm{H}}\right)\pi_{H,t}^{*2}+a_{\mathrm{H}}\pi_{F,t}^{*2}+\left(1-a_{\mathrm{H}}\right)\pi_{F,t}^2\right]+t.i.p.+o\left(\varepsilon^3\right).
\end{aligned}
$$

Using (2) and (3), the first order approximations for $\widehat{C}_t$ and $\widehat{C}_t^*$, we can rearrange further,

$$
\begin{aligned}
&\mathcal{L}_t^W \ltimes \\
&\left[\frac{\sigma}{2}\left(\widehat{C}_t-\widehat{C}_t^*\right)-\widehat{\mathcal{Q}}_t-\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}_{C,t}^*\right)\right]\frac{1}{2}\left\{\widehat{Y}_{H,t}-\widehat{Y}_{F,t}-\sigma^{-1}\left[\widehat{\mathcal{Q}}_t+\widetilde{\mathcal{W}}_t+\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}_{C,t}^*\right)\right]\right\}+ \\
&\left(1-a_{\mathrm{H}}\right)\left(\widehat{C}_t-\widehat{C}_t^*-\widehat{\mathcal{Q}}_t\right)\widehat{\Delta}_t- \\
&\left(1-a_{\mathrm{H}}\right)\frac{1}{2}\left[\left(\widehat{\mathcal{T}}_t+\widehat{\Delta}_t\right)+\left(\widetilde{\mathcal{W}}_t+\widehat{\Delta}_t\right)-\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}_{C,t}^*\right)\right]\left(\widehat{Y}_{H,t}-\widehat{Y}_{F,t}\right)+ \\
&\left(1-a_{\mathrm{H}}\right)a_{\mathrm{H}}\phi\left(\widehat{\mathcal{T}}_t+\widehat{\Delta}_t\right)^2+\left(1-a_{\mathrm{H}}\right)\left(\left[1-2a_{\mathrm{H}}\left(1-\phi\right)\right]\widehat{\mathcal{T}}_t-2a_{\mathrm{H}}\left(1-\phi\right)\widehat{\Delta}_t\right)\widehat{\Delta}_t+ \\
&2a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)\left(\frac{1}{2}\left(\widehat{\mathcal{T}}_t+\widehat{\Delta}_t\right)-\widehat{\mathcal{T}}_t^{fb}\right)\left(\widehat{Y}_{H,t}-\widehat{Y}_{F,t}\right)- \\
&\left(\eta+\sigma\right)\left(\frac{1}{2}\widehat{Y}_{H,t}-\widehat{Y}_{H,t}^{fb}\right)\widehat{Y}_{H,t}-\left(\eta+\sigma\right)\left(\frac{1}{2}\widehat{Y}_{F,t}-\widehat{Y}_{F,t}^{fb}\right)\widehat{Y}_{F,t}- \\
&\frac{1}{2}\frac{\theta\alpha}{\left(1-\alpha\beta\right)\left(1-\alpha\right)}\left[a_{\mathrm{H}}\pi_{H,t}^2+\left(1-a_{\mathrm{H}}\right)\pi_{H,t}^{*2}+a_{\mathrm{H}}\pi_{F,t}^{*2}+\left(1-a_{\mathrm{H}}\right)\pi_{F,t}^2\right]+t.i.p.+o\left(\varepsilon^3\right).
\end{aligned}
$$

Here is a key passage: using the definition of the demand gap $\widetilde{\mathcal{W}}_t=\sigma\left(\widehat{C}_t-\widehat{C}_t^*\right)-\widehat{\mathcal{Q}}_t-\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}_{C,t}^*\right)$, we can eliminate all the terms in consumption:

$$
\begin{aligned}
&\mathcal{L}_t^W \ltimes -\frac{1}{4}\sigma^{-1}\left[\widetilde{\mathcal{W}}_t^2-\left(\widehat{\mathcal{Q}}_t+\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}_{C,t}^*\right)\right)^2\right]+ \\
&\frac{1}{4}\left[\widetilde{\mathcal{W}}_t-\left(\widehat{\mathcal{Q}}_t+\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}_{C,t}^*\right)\right)\right]\left(\widehat{Y}_{H,t}-\widehat{Y}_{F,t}\right)- \\
&\left(1-a_{\mathrm{H}}\right)\sigma^{-1}\left(\widehat{\mathcal{Q}}_t+\widetilde{\mathcal{W}}_t+\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}_{C,t}^*\right)-\sigma\widehat{\mathcal{Q}}_t\right)\widehat{\Delta}_t- \\
&\left(1-a_{\mathrm{H}}\right)\frac{1}{2}\left[\left(\widehat{\mathcal{T}}_t+\widehat{\Delta}_t\right)+\left(\widehat{\mathcal{D}}_t+\widehat{\Delta}_t\right)-\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}_{C,t}^*\right)\right]\left(\widehat{Y}_{H,t}-\widehat{Y}_{F,t}\right)+ \\
&\left(1-a_{\mathrm{H}}\right)a_{\mathrm{H}}\phi\left(\widehat{\mathcal{T}}_t+\widehat{\Delta}_t\right)^2+\left(1-a_{\mathrm{H}}\right)\left[1-2a_{\mathrm{H}}\left(1-\phi\right)\right]\left(\widehat{\mathcal{T}}_t+\widehat{\Delta}_t\right)\widehat{\Delta}_t+ \\
&2a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)\left(\frac{1}{2}\left(\widehat{\mathcal{T}}_t+\widehat{\Delta}_t\right)-\widehat{\mathcal{T}}_t^{fb}\right)\left(\widehat{Y}_{H,t}-\widehat{Y}_{F,t}\right)- \\
&\left(\eta+\sigma\right)\left(\frac{1}{2}\widehat{Y}_{H,t}-\widehat{Y}_{H,t}^{fb}\right)\widehat{Y}_{H,t}-\left(\eta+\sigma\right)\left(\frac{1}{2}\widehat{Y}_{F,t}-\widehat{Y}_{F,t}^{fb}\right)\widehat{Y}_{F,t}- \\
&\frac{1}{2}\frac{\theta\alpha}{\left(1-\alpha\beta\right)\left(1-\alpha\right)}\left[a_{\mathrm{H}}\pi_{H,t}^2+\left(1-a_{\mathrm{H}}\right)\pi_{H,t}^{*2}+a_{\mathrm{H}}\pi_{F,t}^{*2}+\left(1-a_{\mathrm{H}}\right)\pi_{F,t}^2\right] \\
&+t.i.p.+o\left(\varepsilon^3\right).
\end{aligned}
$$

We then collect the terms in output differentials:

$$
\begin{aligned}
&\mathcal{L}_t^W \ltimes -\frac{1}{4}\sigma^{-1}\left[\widetilde{\mathcal{W}}_t^2-\left(\widehat{\mathcal{Q}}_t+\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}_{C,t}^*\right)\right)^2\right]-\\
&\left(1-a_{\mathrm{H}}\right)\sigma^{-1}\left(\widehat{\mathcal{Q}}_t+\widetilde{\mathcal{W}}_t+\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}_{C,t}^*\right)-\sigma\widehat{\mathcal{Q}}_t\right)\Delta_t+\\
&\frac{1}{4}\left[\left(2a_{\mathrm{H}}-1\right)\left(\left(\widetilde{\mathcal{W}}_t+\Delta_t\right)-\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}_{C,t}^*\right)\right)-\left(\widehat{\mathcal{T}}_t+\Delta_t\right)-2\Delta_t\right]\left(\widehat{Y}_{H,t}-\widehat{Y}_{F,t}\right)+\\
&\left(1-a_{\mathrm{H}}\right)a_{\mathrm{H}}\phi\left(\widehat{\mathcal{T}}_t+\Delta_t\right)^2+\left(1-a_{\mathrm{H}}\right)\left(\left[1-2a_{\mathrm{H}}\left(1-\phi\right)\right]\widehat{\mathcal{T}}_t-2a_{\mathrm{H}}\left(1-\phi\right)\widehat{\Delta}_t\right)\widehat{\Delta}_t+\\
&2a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)\left(\frac{1}{2}\left(\widehat{\mathcal{T}}_t+\Delta_t\right)-\widehat{\mathcal{T}}_t^{fb}\right)\left(\widehat{Y}_{H,t}-\widehat{Y}_{F,t}\right)-\\
&\left(\eta+\sigma\right)\left(\frac{1}{2}\widehat{Y}_{H,t}-\widehat{Y}_{H,t}^{fb}\right)\widehat{Y}_{H,t}-\left(\eta+\sigma\right)\left(\frac{1}{2}\widehat{Y}_{F,t}-\widehat{Y}_{F,t}^{fb}\right)\widehat{Y}_{F,t}-\\
&\frac{1}{2}\frac{\theta\alpha}{\left(1-\alpha\beta\right)\left(1-\alpha\right)}\left[a_{\mathrm{H}}\pi_{H,t}^2+\left(1-a_{\mathrm{H}}\right)\pi_{H,t}^{*2}+a_{\mathrm{H}}\pi_{F,t}^{*2}+\left(1-a_{\mathrm{H}}\right)\pi_{F,t}^2\right]+t.i.p.+o\left(\varepsilon^3\right).
\end{aligned}
$$

and use the expression for the terms of trade (4) to obtain,

$$
\begin{aligned}
&\mathcal{L}_t^W \ltimes -\frac{1}{4}\sigma^{-1}\left[\widetilde{\mathcal{W}}_t^2-\left(\widehat{\mathcal{Q}}_t+\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}_{C,t}^*\right)\right)^2\right]-\\
&\left(1-a_{\mathrm{H}}\right)\sigma^{-1}\left(\widehat{\mathcal{Q}}_t+\widetilde{\mathcal{W}}_t+\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}_{C,t}^*\right)-\sigma\widehat{\mathcal{Q}}_t\right)\widehat{\Delta}_t+\\
&\frac{1}{4}\left[\left(2a_{\mathrm{H}}-1\right)\left(\left(\widetilde{\mathcal{W}}_t+\widehat{\Delta}_t\right)-\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}_{C,t}^*\right)\right)-\left(\widehat{\mathcal{T}}_t+\widehat{\Delta}_t\right)-2\widehat{\Delta}_t\right]\left(\widehat{Y}_{H,t}-\widehat{Y}_{F,t}\right)-\\
&-\frac{a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1}\left(2a_{\mathrm{H}}-1\right)\left[\left(\widetilde{\mathcal{W}}_t+\widehat{\Delta}_t\right)-\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}_{C,t}^*\right)\right]\left(\widehat{Y}_{H,t}-\widehat{Y}_{F,t}\right)\\
&\left(1-a_{\mathrm{H}}\right)a_{\mathrm{H}}\phi\left(\widehat{\mathcal{T}}_t+\widehat{\Delta}_t\right)^2+\left(1-a_{\mathrm{H}}\right)\left(\left[1-2a_{\mathrm{H}}\left(1-\phi\right)\right]\widehat{\mathcal{T}}_t-2a_{\mathrm{H}}\left(1-\phi\right)\widehat{\Delta}_t\right)\widehat{\Delta}_t-\\
&\left(\eta+\sigma\right)\left(\frac{1}{2}\widehat{Y}_{H,t}-\widehat{Y}_{H,t}^{fb}\right)\widehat{Y}_{H,t}-\left(\eta+\sigma\right)\left(\frac{1}{2}\widehat{Y}_{F,t}-\widehat{Y}_{F,t}^{fb}\right)\widehat{Y}_{F,t}-\\
&\frac{1}{2}\frac{\theta\alpha)}{\left(1-\alpha\beta\right)\left(1-\alpha\right)}\left[a_{\mathrm{H}}\pi_{H,t}^2+\left(1-a_{\mathrm{H}}\right)\pi_{H,t}^{*2}+a_{\mathrm{H}}\pi_{F,t}^{*2}+\left(1-a_{\mathrm{H}}\right)\pi_{F,t}^2\right]+\\
&\frac{2a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)\sigma}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1}\left(\frac{1}{2}\left(\widehat{Y}_{H,t}-\widehat{Y}_{F,t}\right)-\left(\widehat{Y}_{H,t}^{fb}-\widehat{Y}_{F,t}^{fb}\right)\right)\left(\widehat{Y}_{H,t}-\widehat{Y}_{F,t}\right)\\
&+t.i.p.+o\left(\varepsilon^3\right)
\end{aligned}
$$

The last three lines of the previous expression coincides with the loss function under complete markets, expressed in deviations from the first best ($\widetilde{x}_t = \widehat{x}_t - \widehat{x}_t^{fb}$) when also $\widehat{\Delta}_t = 0$—rewritten below for convenience:

$$\mathcal{L}_t^W - \left(\mathcal{L}_t^W\right)^{fb} \ltimes -\frac{1}{2}\left(\eta+\sigma\right)\left(\widetilde{Y}_{H,t}\right)^2 - \frac{1}{2}\left(\eta+\sigma\right)\left(\widetilde{Y}_{F,t}\right)^2$$

$$-\frac{1}{2}\frac{\theta\alpha}{\left(1-\alpha\beta\right)\left(1-\alpha\right)}\left[a_{\mathrm{H}}\pi_{H,t}^2 + \left(1-a_{\mathrm{H}}\right)\pi_{H,t}^{*2} + a_{\mathrm{H}}\pi_{F,t}^{*2} + \left(1-a_{\mathrm{H}}\right)\pi_{F,t}^2\right] +$$

$$\frac{a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)\sigma}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1}\left(\widetilde{Y}_{H,t}-\widetilde{Y}_{F,t}\right)^2 + t.i.p. + o\left(\varepsilon^3\right).$$

It follows that all the other terms in $\mathcal{L}_t^W$ above must cancel out when $\widetilde{\mathcal{W}}_t = \widehat{\Delta}_t = 0$. The final step in deriving the generic loss function consists of verifying this conjecture, and derive how our expression must change under incomplete markets and LOOP deviations.

Substitute out for $\widehat{\mathcal{Q}}_t$ in terms of $\widehat{\mathcal{T}}_t$ and $\widehat{\Delta}_t$ using (1):

$$-\frac{1}{4}\sigma^{-1}\left[\widetilde{\mathcal{W}}_t^2 - \left(\left(2a_{\mathrm{H}}-1\right)\left(\widehat{\mathcal{T}}_t+\Delta_t\right)+\Delta_t+\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}_{C,t}^*\right)\right)^2\right] -$$

$$\left(1-a_{\mathrm{H}}\right)\sigma^{-1}\left[\left(2a_{\mathrm{H}}-1\right)\left(\widehat{\mathcal{T}}_t+\Delta_t\right)+\left(\Delta_t+\widetilde{\mathcal{W}}_t\right)+\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}_{C,t}^*\right)\right]\Delta_t +$$

$$\left(1-a_{\mathrm{H}}\right)\left(\left(2a_{\mathrm{H}}-1\right)\left(\widehat{\mathcal{T}}_t+\Delta_t\right)+\Delta_t\right)\Delta_t +$$

$$\frac{1}{4}\left[\left(2a_{\mathrm{H}}-1\right)\left(\left(\widetilde{\mathcal{W}}_t+\Delta_t\right)-\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}_{C,t}^*\right)\right)-\left(\widehat{\mathcal{T}}_t+\Delta_t\right)-2\Delta_t\right]\left(\widehat{Y}_{H,t}-\widehat{Y}_{F,t}\right) -$$

$$\frac{a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1}\left(2a_{\mathrm{H}}-1\right)\left[\left(\widetilde{\mathcal{W}}_t+\widehat{\Delta}_t\right)-\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}_{C,t}^*\right)\right]\left(\widehat{Y}_{H,t}-\widehat{Y}_{F,t}\right) +$$

$$\left(1-a_{\mathrm{H}}\right)a_{\mathrm{H}}\phi\left(\widehat{\mathcal{T}}_t+\widehat{\Delta}_t\right)^2 + \left(1-a_{\mathrm{H}}\right)\left(\left[1-2a_{\mathrm{H}}\left(1-\phi\right)\right]\widehat{\mathcal{T}}_t - 2a_{\mathrm{H}}\left(1-\phi\right)\widehat{\Delta}_t\right)\widehat{\Delta}_t$$

and substitute out the output differential using (4), yielding,

$$
\begin{aligned}
= \quad & -\frac{1}{4}\sigma^{-1}\left[\widetilde{\mathcal{W}}_t^2-\left((2a_{\mathrm{H}}-1)\left(\widehat{\mathcal{T}}_t+\Delta_t\right)+\Delta_t+\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}^*_{C,t}\right)\right)^2\right]- \\
& (1-a_{\mathrm{H}})\,\sigma^{-1}\left[(2a_{\mathrm{H}}-1)\left(\widehat{\mathcal{T}}_t+\Delta_t\right)+\left(\Delta_t+\widetilde{\mathcal{W}}_t\right)+\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}^*_{C,t}\right)\right]\Delta_t- \\
& (1-a_{\mathrm{H}})\,\sigma^{-1}\sigma\left((2a_{\mathrm{H}}-1)\left(\widehat{\mathcal{T}}_t+\Delta_t\right)+\Delta_t\right)\Delta_t+ \\
& \frac{1}{4}\sigma^{-1}\left[\begin{array}{c}(2a_{\mathrm{H}}-1)\left(\left(\widetilde{\mathcal{W}}_t+\Delta_t\right)-\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}^*_{C,t}\right)\right)+\\ -\left(\widehat{\mathcal{T}}_t+\Delta_t\right)-2\Delta_t\end{array}\right]\cdot \\
& \left(\begin{array}{c}[4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1]\left(\widehat{\mathcal{T}}_t+\Delta_t\right)+\\ (2a_{\mathrm{H}}-1)\left[\widetilde{\mathcal{W}}_t+\Delta_t+\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}^*_{C,t}\right)\right]\end{array}\right)- \\
& \sigma^{-1}\frac{a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1}(2a_{\mathrm{H}}-1)\left[\left(\widetilde{\mathcal{W}}_t+\widehat{\Delta}_t\right)-\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}^*_{C,t}\right)\right]\cdot \\
& \left(\begin{array}{c}[4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1]\left(\widehat{\mathcal{T}}_t+\Delta_t\right)+\\ (2a_{\mathrm{H}}-1)\left[\widetilde{\mathcal{W}}_t+\Delta_t+\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}^*_{C,t}\right)\right]\end{array}\right)+ \\
& (1-a_{\mathrm{H}})\,a_{\mathrm{H}}\phi\left(\widehat{\mathcal{T}}_t+\widehat{\Delta}_t\right)^2+(1-a_{\mathrm{H}})\left([1-2a_{\mathrm{H}}(1-\phi)]\,\widehat{\mathcal{T}}_t-2a_{\mathrm{H}}(1-\phi)\,\widehat{\Delta}_t\right)\widehat{\Delta}_t
\end{aligned}
$$

After some algebra, the above expression is reduced to:

$$
\begin{aligned}
= \quad & -\frac{a_{\mathrm{H}}(1-a_{\mathrm{H}})\phi}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1}\left(\widetilde{\mathcal{W}}_t+\Delta_t\right)^2+ \\
& (1-a_{\mathrm{H}})\left([1-2a_{\mathrm{H}}(1-\phi)]\,\widehat{\mathcal{T}}_t-2a_{\mathrm{H}}(1-\phi)\,\widehat{\Delta}_t\right)\widehat{\Delta}_t- \\
& (1-a_{\mathrm{H}})\,\sigma^{-1}\left((1-\sigma)\left((2a_{\mathrm{H}}-1)\left(\widehat{\mathcal{T}}_t+\Delta_t\right)+\Delta_t\right)+\widetilde{\mathcal{W}}_t+\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}^*_{C,t}\right)\right)\Delta_t- \\
& \frac{1}{2}\sigma^{-1}\left([4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1]\left(\widehat{\mathcal{T}}_t+\Delta_t\right)+(2a_{\mathrm{H}}-1)\left[\widetilde{\mathcal{W}}_t+\Delta_t+\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}^*_{C,t}\right)\right]\right)\Delta_t+ \\
& \frac{1}{2}\sigma^{-1}\left[(2a_{\mathrm{H}}-1)\left(\widehat{\mathcal{T}}_t+\Delta_t\right)+\widetilde{\mathcal{W}}_t+\Delta_t+\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}^*_{C,t}\right)\right]\Delta_t,
\end{aligned}
$$

which vanishes under complete markets and PCP. Collecting terms we get,

$$
\begin{aligned}
= \quad & -\frac{a_{\mathrm{H}}(1-a_{\mathrm{H}})\phi}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1}\left(\widetilde{\mathcal{W}}_t+\Delta_t\right)^2+ \\
& (1-a_{\mathrm{H}})\left([1-2a_{\mathrm{H}}(1-\phi)]\,\widehat{\mathcal{T}}_t-2a_{\mathrm{H}}(1-\phi)\,\widehat{\Delta}_t\right)\widehat{\Delta}_t+ \\
& \frac{1}{2}\sigma^{-1}\left[2a_{\mathrm{H}}-1-4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)-1-2(1-a_{\mathrm{H}})(1-\sigma)(2a_{\mathrm{H}}-1)\right]\left(\widehat{\mathcal{T}}_t+\Delta_t\right)\Delta_t+ \\
& (1-a_{\mathrm{H}})\,\Delta_t^2+ \\
& \frac{1}{2}\sigma^{-1}\left[1-(2a_{\mathrm{H}}-1)-2(1-a_{\mathrm{H}})\right]\left(\widetilde{\mathcal{W}}_t+\Delta_t+\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}^*_{C,t}\right)\right)\Delta_t,
\end{aligned}
$$

which further simplifies as follows

$$
\begin{aligned}
= \quad & -\frac{a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\phi}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1}\left(\widetilde{\mathcal{W}}_t+\Delta_t\right)^2+ \\
& \left(1-a_{\mathrm{H}}\right)\left(\left[1-2a_{\mathrm{H}}\left(1-\phi\right)\right]\widehat{\mathcal{T}}_t-2a_{\mathrm{H}}\left(1-\phi\right)\widehat{\Delta}_t\right)\widehat{\Delta}_t+ \\
& \left(1-a_{\mathrm{H}}\right)\left[2a_{\mathrm{H}}\left(1-\phi\right)-1\right]\widehat{\mathcal{T}}_t\Delta_t+ \\
& \left(1-a_{\mathrm{H}}\right)2a_{\mathrm{H}}\left[1-\phi\right]\Delta_t^2.
\end{aligned}
$$

Given that the last three lines cancel out, we conclude that with generically incomplete market under LCP the loss function in deviations from the first best can be expressed as:

$$
\begin{gathered}
\mathcal{L}_t^W-\left(\mathcal{L}_t^W\right)^{fb}\ltimes-\frac{1}{2}\left(\eta+\sigma\right)\left(\widetilde{Y}_{H,t}\right)^2-\frac{1}{2}\left(\eta+\sigma\right)\left(\widetilde{Y}_{F,t}\right)^2+ \\
-\frac{1}{2}\frac{\theta\alpha}{\left(1-\alpha\beta\right)\left(1-\alpha\right)}\left[a_{\mathrm{H}}\pi_{H,t}^2+\left(1-a_{\mathrm{H}}\right)\pi_{H,t}^{*2}+a_{\mathrm{H}}\pi_{F,t}^{*2}+\left(1-a_{\mathrm{H}}\right)\pi_{F,t}^2\right]+ \\
\frac{a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1}\left[\left(\sigma\phi-1\right)\sigma\left(\widetilde{Y}_{H,t}-\widetilde{Y}_{F,t}\right)^2-\phi\left(\widehat{\Delta}_t+\widetilde{\mathcal{W}}_t\right)^2\right] \\
+t.i.p.+o\left(\varepsilon^3\right).
\end{gathered}
$$

This completes the derivation of the optimal monetary policy loss function in the LCP economy.

2.3 Generalizations

2.3.1 PCP economy

The loss function under PCP is a special case of the above in which all LOOP deviations $\widehat{\Delta}_t$ are set to zero, which also implies that the inflation term, $\left[a_{\mathrm{H}}\pi_{H,t}^2+\left(1-a_{\mathrm{H}}\right)\pi_{H,t}^{*2}+a_{\mathrm{H}}\pi_{F,t}^{*2}+\left(1-a_{\mathrm{H}}\right)\pi_{F,t}^2\right]$, is equal to $\pi_{H,t}^2+\pi_{F,t}^{*2}$.

2.3.2 Encompassing different specifications of market incompleteness

Observe that maximization of the world welfare subject to the implementability constraints characterizing the competitive equilibrium requires spelling out the exact form of market incompleteness. Taking the difference of the budget constraints for an economy with n traded assets we can generically arrive at the following expression:

$$
\begin{aligned}
& C_t-\mathcal{Q}_tC_t^*= \\
& \frac{P_{\mathrm{H},t}}{P_t}Y_{\mathrm{H},t}+\left(\frac{S_tP_{\mathrm{H},t}^*}{P_{\mathrm{H},t}}-1\right)\frac{P_{\mathrm{H},t}}{P_t}C_{\mathrm{H},t}^*-\left(\frac{P_{\mathrm{F},t}^*}{P_t^*}\mathcal{Q}_tY_{\mathrm{F},t}+\left(1-\frac{SP_{\mathrm{F},t}^*}{P_{\mathrm{F},t}}\right)\frac{P_{\mathrm{F},t}}{P_t}C_{\mathrm{F},t}\right)+ \\
& 2\left[\left(1+r_{t-1}\right)\mathcal{B}_{t-1}+\sum_i\alpha_{i,t-1}\left(R_{i,t}-\left(1+r_{t-1}\right)\right)-\mathcal{B}_t\right],
\end{aligned}
$$

$$C_t - \mathcal{Q}_t C_t^* = \left[a_{\mathrm{H}} + (1-a_{\mathrm{H}})\,\mathcal{T}_t^{1-\phi}\Delta_{\mathrm{H},t}^{1-\phi}\right]^{-\frac{1}{1-\phi}} Y_{\mathrm{H},t} -$$
$$\left[a_{\mathrm{H}} + (1-a_{\mathrm{H}})\,\mathcal{T}_t^{\phi-1}\Delta_{\mathrm{F},t}^{\phi-1}\right]^{-\frac{1}{1-\phi}} \mathcal{Q}_t Y_{\mathrm{F},t} +$$
$$\left(1 - \frac{P_{\mathrm{H},t}}{S_t P_{\mathrm{H},t}^*}\right)\frac{P_{\mathrm{H},t}^*}{P_t^*}\frac{S_t P_t^*}{P_t} C_{\mathrm{H},t}^* + \left(\frac{S P_{\mathrm{F},t}^*}{P_{\mathrm{F},t}} - 1\right)\frac{P_{\mathrm{F},t}}{P_t} C_{\mathrm{F},t} +$$
$$2\left[(1+r_{t-1})\,\mathcal{B}_{t-1} + \sum_i \alpha_{i,t-1}\left(R_{i,t} - (1+r_{t-1})\right) - \mathcal{B}_t\right]$$

$$C_t - \mathcal{Q}_t C_t^* = \left[a_{\mathrm{H}} + (1-a_{\mathrm{H}})\,\mathcal{T}_t^{1-\phi}\Delta_{\mathrm{H},t}^{1-\phi}\right]^{-\frac{1}{1-\phi}} Y_{\mathrm{H},t} -$$
$$\left[a_{\mathrm{H}} + (1-a_{\mathrm{H}})\,\mathcal{T}_t^{\phi-1}\Delta_{\mathrm{F},t}^{\phi-1}\right]^{-\frac{1}{1-\phi}} \mathcal{Q}_t Y_{\mathrm{F},t} +$$
$$(1-a_{\mathrm{H}})\left[\begin{array}{c} \frac{\Delta_{\mathrm{H},t}-1}{\Delta_{\mathrm{H},t}}\left(a_{\mathrm{H}}\mathcal{T}_t^{1-\phi}\Delta_{\mathrm{F},t}^{1-\phi} + (1-a_{\mathrm{H}})\right)^{1-\phi}\mathcal{Q}_t C_t^* + \\ (\Delta_{\mathrm{F}} - 1)\left(a_{\mathrm{H}}\mathcal{T}_t^{\phi-1}\Delta_{\mathrm{H},t}^{\phi-1} + (1-a_{\mathrm{H}})\right)^{1-\phi} C_t \end{array}\right] +$$
$$2\left[(1+r_{t-1})\,\mathcal{B}_{t-1} + \sum_i \alpha_{i,t-1}\left(R_{i,t} - (1+r_{t-1})\right) - \mathcal{B}_t\right]$$

where all ex-post returns are expressed in terms of Home consumption prices — e.g. $1 + r_{t-1} = \dfrac{1+i_t}{P_t/P_{t-1}}$ and $\sum_i \alpha_{i,t} = \mathcal{B}_t$. Around a symmetric steady state with zero real NFA ($\mathcal{B} = 0$) , the consumption differential, up to first order, is given by:

$$\widehat{C}_t - \widehat{C}_t^* - \widehat{\mathcal{Q}}_t =$$
$$\widehat{Y}_{H,t} - \widehat{Y}_{F,t} - \widehat{\mathcal{Q}}_t - 2(1-a_{\mathrm{H}})\,\widehat{\mathcal{T}}_t - (1-a_{\mathrm{H}})\left(\widehat{\Delta}_{\mathrm{F},t} + \widehat{\Delta}_{\mathrm{H},t}\right) +$$
$$(1-a_{\mathrm{H}})\left(\widehat{\Delta}_{\mathrm{F},t} + \widehat{\Delta}_{\mathrm{H},t}\right) + 2\beta^{-1}\left(\widehat{\mathcal{B}}_{t-1} - \beta\widehat{\mathcal{B}}_t + \sum_i \frac{\omega_i}{Y}\left(\widehat{R}_{i,t} - \left(\widehat{1+r_{t-1}}\right)\right)\right).$$

where NFA deviations are defined wrt to steady state output $\widehat{\mathcal{B}}_{t-1} = \dfrac{\mathcal{B}_{t-1} - 0}{Y}$, and ω_i represents the share of gross wealth invested in the i-th asset in the *stochastic* steady state.

For $\widehat{\Delta}_{\mathrm{H},t} = \widehat{\Delta}_{\mathrm{F},t} = \widehat{\Delta}_t$ under symmetry, we get:

$$\begin{aligned}\widehat{C}_t - \widehat{C}_t^* &= \widehat{Y}_{H,t} - \widehat{Y}_{F,t} - 2(1-a_{\mathrm{H}})\,\widehat{\mathcal{T}}_t + \\ &\quad 2\beta^{-1}\left(\widehat{\mathcal{B}}_{t-1} - \beta\widehat{\mathcal{B}}_t + \sum_i \frac{\omega_i}{Y}\left(\widehat{R}_{i,t} - \left(\widehat{1+r_{t-1}}\right)\right)\right).\end{aligned}$$

Under financial autarky, since $\widehat{\mathcal{B}}_{t-1} = 0$, we have the following:

$$\widetilde{\mathcal{W}}_t = \sigma\left[\widehat{Y}_{H,t} - \widehat{Y}_{F,t} - 2(1-a_{\mathrm{H}})\,\widehat{\mathcal{T}}_t\right] - \widehat{\mathcal{Q}}_t - \left(\widehat{\zeta}_{C,t} - \widehat{\zeta}_{C,t}^*\right),$$

whereas, in the case of a bond economy, the wealth gap $\widetilde{\mathcal{W}}_t$ will also reflect net capital flows:

$$\begin{aligned}\widetilde{\mathcal{W}}_t &= \sigma\left[\begin{array}{c}-\left(\left(\widehat{\mathcal{B}}_t-\beta^{-1}\widehat{\mathcal{B}}_{t-1}\right)-\widehat{Y}_{H,t}+(1-a_{\mathrm{H}})\,\widehat{\mathcal{T}}_t\right)+\\ \left(\left(-\widehat{\mathcal{B}}_t-\beta^{-1}\left(-\widehat{\mathcal{B}}_{t-1}\right)\right)-\widehat{Y}_{F,t}-(1-a_{\mathrm{H}})\,\widehat{\mathcal{T}}_t\right)\end{array}\right]+\\ & -\widehat{\mathcal{Q}}_t-\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}^{*}_{C,t}\right).\end{aligned}$$

Finally, rewriting in terms of gaps (useful when characterizing optimal policy)the wealth gap in a bond economy is given by,

$$\begin{aligned}\widetilde{\mathcal{W}}_t &= \sigma\left(\widetilde{C}_t-\widetilde{C}^{*}_t\right)-\widetilde{\mathcal{Q}}_t\\ &= \sigma\left[\widetilde{Y}_{H,t}-\widetilde{Y}_{F,t}+2\beta^{-1}\left(\widehat{\mathcal{B}}_{t-1}-\beta\widehat{\mathcal{B}}_t\right)\right]-2a_{\mathrm{H}}\widehat{\Delta}_t-\left[2\,(1-a_{\mathrm{H}})\,\sigma+(2a_{\mathrm{H}}-1)\right]\widetilde{\mathcal{T}}_t+\\ & 2\,(1-a_{\mathrm{H}})\left[(2a_{\mathrm{H}}\,(\sigma\phi-1)+1-\sigma)\,\widehat{\mathcal{T}}^{fb}_t-\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}^{*}_{C,t}\right)\right],\end{aligned}$$

and under autarky,

$$\begin{aligned}\widetilde{\mathcal{W}}_t &= \sigma\left(\widetilde{C}_t-\widetilde{C}^{*}_t\right)-\widetilde{\mathcal{Q}}_t\\ &= \sigma\left[\widetilde{Y}_{H,t}-\widetilde{Y}_{F,t}\right]-2a_{\mathrm{H}}\widehat{\Delta}_t-\left[2\,(1-a_{\mathrm{H}})\,\sigma+(2a_{\mathrm{H}}-1)\right]\widetilde{\mathcal{T}}_t+\\ & 2\,(1-a_{\mathrm{H}})\left[(2a_{\mathrm{H}}\,(\sigma\phi-1)+1-\sigma)\,\widehat{\mathcal{T}}^{fb}_t-\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}^{*}_{C,t}\right)\right].\end{aligned}$$

3 Characterizing optimal monetary targeting rules and optimal allocations under incomplete markets: proofs of propositions 3, 4, 5, 6 9 and 10

In this section we work out the constrained efficient allocation in our model economy—this is found by maximizing the expected discounted value of the following loss function in deviation from first best,

$$\begin{aligned}\mathcal{L}_t^W - \left(\mathcal{L}_t^W\right)^{fb} \ltimes & -\frac{1}{2}(\eta+\sigma)\left(\widehat{Y}_{H,t} - \widehat{Y}_{H,t}^{fb}\right)^2 - \frac{1}{2}(\eta+\sigma)\left(\widehat{Y}_{F,t} - \widehat{Y}_{F,t}^{fb}\right)^2 - \\ & \frac{1}{2}\frac{\theta\alpha}{(1-\alpha\beta)(1-\alpha)}\left[a_{\mathrm{H}}\pi_{H,t}^2 + (1-a_{\mathrm{H}})\pi_{H,t}^{*2} + a_{\mathrm{H}}\pi_{F,t}^{*2} + (1-a_{\mathrm{H}})\pi_{F,t}^2\right] + \\ & \frac{a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)\sigma}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1}\left[\left(\widehat{Y}_{H,t} - \widehat{Y}_{H,t}^{fb}\right) - \left(\widehat{Y}_{F,t} - \widehat{Y}_{F,t}^{fb}\right)\right]^2 - \\ & \frac{a_{\mathrm{H}}(1-a_{\mathrm{H}})\phi}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1}\left(\widehat{\Delta}_t + \widehat{\mathcal{W}}_t\right)^2 + t.i.p. + o\left(\varepsilon^3\right),\end{aligned} \tag{8}$$

with respect to its arguments $\widehat{Y}_{H,t}$, $\widehat{Y}_{F,t}$, $\widehat{\mathcal{W}}_t$, $\widehat{\Delta}_t$ and $\pi_{H,t}, \pi_{F,t}, \pi_{H,t}^*, \pi_{F,t}^*$ subject to the NK Phillips curves, the equilibrium condition linking relative prices to output gap differentials and demand gaps, the definition of the wealth gap, and the Euler equation characterizing the evolution of the wealth gap. In the case of non-trivial portfolio decisions (not covered here), higher order Euler equations characterizing these choices would have also to be considered.

We treat the cases of PCP and LCP separately as some of the constraints differ significantly.

3.1 LCP economy

3.1.1 Proofs of propositions 3 and 4

In the LCP case, the monetary authority minimizes (1), with respect to its arguments $\widehat{Y}_{H,t}$, $\widehat{Y}_{F,t}$, $\widehat{\Delta}_t$, $\widehat{\mathcal{W}}_t$, and $\pi_{H,t}$, $\pi_{H,t}^*$, $\pi_{F,t}^*$, $\pi_{F,t}$, subject to the following constraints arising from the competitive equilibrium:

1. NK Phillips curves determining inflation rates

$$\begin{aligned} & \pi_{H,t} - \beta E_t \pi_{H,t+1} = \\ \frac{(1-\alpha\beta)(1-\alpha)}{\alpha} & \left[\begin{array}{c} (\sigma+\eta)\left(\widehat{Y}_{H,t} - \widehat{Y}_{H,t}^{fb}\right) + \widehat{\mu}_t + \\ -(1-a_{\mathrm{H}})\left[2a_{\mathrm{H}}(\sigma\phi-1)\left(\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}_t^{fb} + \widehat{\Delta}_t\right) - \left(\widehat{\Delta}_t + \widehat{\mathcal{W}}_t\right)\right] \end{array}\right] \\ & = \pi_{H,t}^* - \beta E_t \pi_{H,t+1}^* + \frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\widehat{\Delta}_t, \end{aligned}$$

$$\pi^*_{F,t} - \beta E_t \pi^*_{F,t+1} =$$

$$\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\left[\begin{array}{c}(\sigma+\eta)\left(\widehat{Y}_{F,t} - \widehat{Y}^{fb}_{F,t}\right) + \widehat{\mu}^*_t + \\ (1-a_{\mathrm{H}})\left[2a_{\mathrm{H}}(\sigma\phi-1)\left(\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}^{fb}_t + \widehat{\Delta}_t\right) - \left(\widehat{\Delta}_t + \widehat{\mathcal{W}}_t\right)\right]\end{array}\right]$$

$$= \pi_{F,t} - \beta E_t \pi_{F,t+1} - \frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\widehat{\Delta}_t,$$

and the constraint on inflation differentials in the same currency:

$$\pi_{F,t} - \pi_{H,t} - \left(\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}_{t-1} + \widehat{\Delta}_t - \widehat{\Delta}_{t-1}\right) = 0,$$

where the equilibrium relations for first best outcomes $\widehat{Y}^{fb}_{H,t}$, $\widehat{Y}^{fb}_{F,t}$, $\widehat{\mathcal{T}}^{fb}_t$ in terms of fundamental shocks are as follows:

$$(\eta+\sigma)\widehat{Y}^{fb}_{H,t} =$$
$$[2a_{\mathrm{H}}\ (1-a_{\mathrm{H}})(\sigma\phi-1)]\left(\widehat{\mathcal{T}}^{fb}_t\right) - (1-a_{\mathrm{H}})\left(\widehat{\zeta}_{C,t} - \widehat{\zeta}^*_{C,t}\right) + \widehat{\zeta}_{C,t} + (1+\eta)\widehat{\zeta}_{Y,t}$$
$$(\eta+\sigma)\widehat{Y}^{fb}_{F,t} =$$
$$[2a_{\mathrm{H}}\ (1-a_{\mathrm{H}})(\sigma\phi-1)]\left(-\widehat{\mathcal{T}}^{fb}_t\right) + (1-a_{\mathrm{H}})\left(\widehat{\zeta}_{C,t} - \widehat{\zeta}^*_{C,t}\right) + \widehat{\zeta}^*_{C,t} + (1+\eta)\widehat{\zeta}^*_{Y,t},$$
$$[4(1-a_{\mathrm{H}})a_{\mathrm{H}}(\phi\sigma-1)+1]\widehat{\mathcal{T}}^{fb}_t = \sigma\left(\widehat{Y}^{fb}_{H,t} - \widehat{Y}^{fb}_{F,t}\right) - (2a_{\mathrm{H}}-1)\left(\widehat{\zeta}_{C,t} - \widehat{\zeta}^*_{C,t}\right);$$

2. the equilibrium condition linking relative prices to output gap differentials, $\widehat{\Delta}_t$ and demand gaps:

$$\widehat{\mathcal{T}}_t + \widehat{\Delta}_t - \widehat{\mathcal{T}}^{fb}_t = \frac{\sigma\left[\left(\widehat{Y}_{H,t} - \widehat{Y}_{F,t}\right) - \left(\widehat{Y}^{fb}_{H,t} - \widehat{Y}^{fb}_{F,t}\right)\right] - (2a_{\mathrm{H}}-1)\left(\widehat{\mathcal{W}}_t + \widehat{\Delta}_t\right)}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1};$$

3. the definition of wealth gap $\widehat{\mathcal{W}}_t$ from the difference in budget constraints, depending also on net wealth $\widehat{\mathcal{B}}_t$:

$$\begin{aligned}\widehat{\mathcal{W}}_t &= \sigma\left[\left(\widehat{C}_t - \widehat{C}^*_t\right) - \left(\widehat{C}^{fb}_t - \widehat{C}^{*fb}_t\right)\right] - \left(\widehat{\mathcal{Q}}_t - \widehat{\mathcal{Q}}^{fb}_t\right) \\ &= \sigma\left[\begin{array}{c}\left(\widehat{Y}_{H,t} - \widehat{Y}_{F,t}\right) - \left(\widehat{Y}^{fb}_{H,t} - \widehat{Y}^{fb}_{F,t}\right) + \\ 2\beta^{-1}\left(\widehat{\mathcal{B}}_{t-1} - \beta\widehat{\mathcal{B}}_t + \sum_i \frac{\omega_i}{Y}\left(\widehat{R}_{i,t} - \left(\widehat{1+r_t}\right)\right)\right)\end{array}\right] + \\ &\quad -[2(1-a_{\mathrm{H}})\sigma + (2a_{\mathrm{H}}-1)]\left(\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}^{fb}_t\right) - 2a_{\mathrm{H}}\widehat{\Delta}_t + \\ &\quad 2(1-a_{\mathrm{H}})\left[(2a_{\mathrm{H}}(\sigma\phi-1)+1-\sigma)\widehat{\mathcal{T}}^{fb}_t - \left(\widehat{\zeta}_{C,t} - \widehat{\zeta}^*_{C,t}\right)\right],\end{aligned}$$

4. the Euler equation characterizing the evolution of $\widehat{\mathcal{W}}_t$ (and thus net wealth $\widehat{\mathcal{B}}_t$):

$$E_t\widehat{\mathcal{W}}_{t+1} - \widehat{\mathcal{W}}_t = 0.$$

Bond economy

Observe that in the case of a bond economy, the program amounts to choosing $\widehat{Y}_{H,t}$, $\widehat{Y}_{F,t}$, $\widehat{\Delta}_t$, $\widehat{\mathcal{W}}_t$, $\pi_{H,t}$, $\pi^*_{H,t}$, $\pi^*_{F,t}$, $\pi_{F,t}$, and $\widehat{\mathcal{B}}_t$, subject to the following expression for $\widehat{\mathcal{W}}_t$ in terms of differences of budget constraints, namely:

$$
\begin{gathered}
(1-a_{\mathrm{H}})\left[2a_{\mathrm{H}}(\phi-1)+1\right]\widehat{\mathcal{W}}_t = \left[4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1\right]\left(\beta^{-1}\widehat{\mathcal{B}}_{t-1}-\widehat{\mathcal{B}}_t\right)+ \\
(1-a_{\mathrm{H}})\left[2a_{\mathrm{H}}(\sigma\phi-1)+1-\sigma\right]\left[\left(\widehat{Y}_{H,t}-\widehat{Y}^{fb}_{H,t}\right)-\left(\widehat{Y}_{F,t}-\widehat{Y}^{fb}_{F,t}\right)\right]+ \\
2a_{\mathrm{H}}(1-a_{\mathrm{H}})\left[2(1-a_{\mathrm{H}})(\sigma\phi-1)+1-\phi\right]\widehat{\Delta}_t+ \\
(1-a_{\mathrm{H}})\left[4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1\right]\cdot \\
\sigma^{-1}\left[(2a_{\mathrm{H}}(\sigma\phi-1)+1-\sigma)\widehat{\mathcal{T}}^{fb}_t-\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}^*_{C,t}\right)\right]
\end{gathered}
$$

The necessary FOC's with respect to inflation are given by:

$$
\begin{aligned}
\pi_{H,t} &: \quad 0=-\theta\frac{\alpha}{(1-\alpha\beta)(1-\alpha)}a_{\mathrm{H}}\pi_{H,t}-\gamma_{H,t}+\gamma_{H,t-1}-\gamma_t \\
\pi^*_{H,t} &: \quad 0=-\theta\frac{\alpha}{(1-\alpha\beta)(1-\alpha)}(1-a_{\mathrm{H}})\pi^*_{H,t}-\gamma^*_{H,t}+\gamma^*_{H,t-1} \\
\pi_{F,t} &: \quad 0=-\theta\frac{\alpha}{(1-\alpha\beta)(1-\alpha)}(1-a_{\mathrm{H}})\pi_{F,t}-\gamma_{F,t}+\gamma_{F,t-1}+\gamma_t \\
\pi^*_{F,t} &: \quad 0=-\theta\frac{\alpha}{(1-\alpha\beta)(1-\alpha)}a_{\mathrm{H}}\pi^*_{F,t}-\gamma^*_{F,t}+\gamma^*_{F,t-1},
\end{aligned}
$$

where $\gamma_{H,t}$, $\gamma_{F,t}$, $\gamma^*_{H,t}$and $\gamma^*_{F,t}$ are the multipliers associated with the Phillips curves — whose lags appear reflecting the assumption of commitment, implying the following solutions for the multipliers:

$$
\begin{aligned}
-\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\left(\gamma_{H,t}+\gamma_{F,t}\right) &= \theta\left(a_{\mathrm{H}}\widehat{p}_{H,t}+(1-a_{\mathrm{H}})\widehat{p}_{F,t}\right) \\
-\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\left(\gamma^*_{H,t}+\gamma^*_{F,t}\right) &= \theta\left(a_{\mathrm{H}}\widehat{p}^*_{F,t}+(1-a_{\mathrm{H}})\widehat{p}^*_{H,t}\right) \\
-2\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\gamma_t &= \theta\left[a_{\mathrm{H}}\left(\pi_{H,t}-\pi^*_{F,t}\right)+(1-a_{\mathrm{H}})\left(\pi^*_{H,t}-\pi_{F,t}\right)\right]+ \\
&\quad -\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\left[\begin{array}{c}\left(-\gamma_{H,t}-\gamma^*_{H,t}+\gamma^*_{H,t-1}+\gamma_{H,t-1}\right)+ \\ -\left(-\gamma^*_{F,t}-\gamma_{F,t}+\gamma_{F,t-1}+\gamma^*_{F,t-1}\right)\end{array}\right].
\end{aligned}
$$

The FOC with respect to output is given by:

$$
\begin{aligned}
\widehat{Y}_{H,t} \quad : \quad & 0=(\sigma+\eta)\left(\widehat{Y}_{H,t}-\widehat{Y}^{fb}_{H,t}\right)+ \\
& -\frac{2a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)\sigma}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1}\left[\left(\widehat{Y}_{H,t}-\widehat{Y}^{fb}_{H,t}\right)-\left(\widehat{Y}_{F,t}-\widehat{Y}^{fb}_{F,t}\right)\right]+ \\
& \frac{2a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\phi}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1}\frac{2a_{\mathrm{H}}\left(\sigma\phi-1\right)+1-\sigma}{2a_{\mathrm{H}}\left(\phi-1\right)+1}\left(\widehat{\Delta}_t+\widehat{\mathcal{W}}_t\right)+ \\
& -\left[\sigma+\eta-\frac{\left(1-a_{\mathrm{H}}\right)\left(\sigma-1\right)}{2a_{\mathrm{H}}\left(\phi-1\right)+1}\right]\frac{\left(1-\alpha\beta\right)\left(1-\alpha\right)}{\alpha}\left(\gamma_{H,t}+\gamma^*_{H,t}\right)+ \\
& -\frac{\left(1-a_{\mathrm{H}}\right)\left(\sigma-1\right)}{2a_{\mathrm{H}}\left(\phi-1\right)+1}\frac{\left(1-\alpha\beta\right)\left(1-\alpha\right)}{\alpha}\left(\gamma_{F,t}+\gamma^*_{F,t}\right)+ \\
& -\frac{1}{2a_{\mathrm{H}}\left(\phi-1\right)+1}\left(\beta E_t\gamma_{t+1}-\gamma_t\right)+ \\
& \frac{2a_{\mathrm{H}}\left(\sigma\phi-1\right)+1-\sigma}{2a_{\mathrm{H}}\left(\phi-1\right)+1}\left(\lambda_t-\beta^{-1}\lambda_{t-1}\right);
\end{aligned}
$$

$$
\begin{aligned}
\widehat{Y}_{F,t} \quad : \quad & 0=(\sigma+\eta)\left(\widehat{Y}_{F,t}-\widehat{Y}^{fb}_{F,t}\right)+ \\
& \frac{2a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)\sigma}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1}\left[\left(\widehat{Y}_{H,t}-\widehat{Y}^{fb}_{H,t}\right)-\left(\widehat{Y}_{F,t}-\widehat{Y}^{fb}_{F,t}\right)\right]+ \\
& -\frac{2a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\phi}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1}\frac{2a_{\mathrm{H}}\left(\sigma\phi-1\right)+1-\sigma}{2a_{\mathrm{H}}\left(\phi-1\right)+1}\left(\widehat{\Delta}_t+\widehat{\mathcal{W}}_t\right)+ \\
& -\left[\sigma+\eta-\frac{\left(1-a_{\mathrm{H}}\right)\left(\sigma-1\right)}{2a_{\mathrm{H}}\left(\phi-1\right)+1}\right]\frac{\left(1-\alpha\beta\right)\left(1-\alpha\right)}{\alpha}\left(\gamma_{F,t}+\gamma^*_{F,t}\right)+ \\
& -\frac{\left(1-a_{\mathrm{H}}\right)\left(\sigma-1\right)}{2a_{\mathrm{H}}\left(\phi-1\right)+1}\frac{\left(1-\alpha\beta\right)\left(1-\alpha\right)}{\alpha}\left(\gamma_{H,t}+\gamma^*_{H,t}\right)+ \\
& \frac{1}{2a_{\mathrm{H}}\left(\phi-1\right)+1}\left(\beta E_t\gamma_{t+1}-\gamma_t\right)+ \\
& -\frac{2a_{\mathrm{H}}\left(\sigma\phi-1\right)+1-\sigma}{2a_{\mathrm{H}}\left(\phi-1\right)+1}\left(\lambda_t-\beta^{-1}\lambda_{t-1}\right);
\end{aligned}
$$

where we have used the fact that

$$
\begin{aligned}
\frac{\partial\widehat{\mathcal{W}}_t}{\partial\widehat{Y}_{H,t}} &= -\frac{\partial\widehat{\mathcal{W}}_t}{\partial\widehat{Y}_{F,t}}=\frac{2a_{\mathrm{H}}\left(\sigma\phi-1\right)+1-\sigma}{2a_{\mathrm{H}}\left(\phi-1\right)+1} \\
\frac{\partial\widehat{\mathcal{T}}_t}{\partial\widehat{Y}_{H,t}} &= \frac{\sigma-\left(2a_{\mathrm{H}}-1\right)\dfrac{\partial\widehat{\mathcal{W}}_t}{\partial\widehat{Y}_{H,t}}}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1} \\
&= -\frac{\partial\widehat{\mathcal{T}}_t}{\partial\widehat{Y}_{F,t}}=\frac{1}{2a_{\mathrm{H}}\left(\phi-1\right)+1};
\end{aligned}
$$

The FOC with respect to LOOP deviations is given by:

$$
\begin{aligned}
\widehat{\Delta}_t \quad : \quad & 0 = -\frac{2a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\phi}{2a_{\mathrm{H}}\left(\phi-1\right)+1}\left(\widehat{\Delta}_t+\widehat{\mathcal{W}}_t\right)+ \\
& \frac{\left(1-\alpha\beta\right)\left(1-\alpha\right)}{\alpha}\frac{1}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\phi\sigma-1\right)+1}\cdot \\
& \frac{1}{2}\left[\begin{array}{c}\left(4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\phi\sigma-1\right)+1\right)\left(\gamma_{H,t}+\gamma_{F,t}-\left(\gamma^*_{F,t}+\gamma^*_{H,t}\right)\right)- \\ \left(\left(2a_{\mathrm{H}}-1\right)-2\left(1-a_{\mathrm{H}}\right)\left[2a_{\mathrm{H}}\left(\sigma\phi-1\right)+1\right]\frac{2a_{\mathrm{H}}\left[2\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1-\phi\right]}{2a_{\mathrm{H}}\left(\phi-1\right)+1}\right)\cdot \\ \left(\gamma_{H,t}+\gamma^*_{H,t}-\gamma_{F,t}-\gamma^*_{F,t}\right)\end{array}\right]- \\
& \frac{2a_{\mathrm{H}}-1}{2a_{\mathrm{H}}\left(\phi-1\right)+1}\left(\beta E_t\gamma_{t+1}-\gamma_t\right)- \\
& \frac{2a_{\mathrm{H}}\left[2\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1-\phi\right]}{2a_{\mathrm{H}}\left(\phi-1\right)+1}\left(\lambda_t-\beta^{-1}\lambda_{t-1}\right),
\end{aligned}
$$

where we have used the fact that:

$$
\begin{aligned}
\frac{\partial\widehat{\mathcal{W}}_t}{\partial\widehat{\Delta}_t} &= \frac{2a_{\mathrm{H}}\left[2\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1-\phi\right]}{2a_{\mathrm{H}}\left(\phi-1\right)+1} \\
&= -1+\frac{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1}{2a_{\mathrm{H}}\left(\phi-1\right)+1} \\
\frac{\partial\widehat{\mathcal{T}}_t}{\partial\widehat{\Delta}_t} &= -1-\left(2a_{\mathrm{H}}-1\right)\frac{1+\frac{\partial\widehat{\mathcal{W}}_t}{\partial\widehat{\Delta}_t}}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1} \\
&= -1-\frac{\left(2a_{\mathrm{H}}-1\right)}{2a_{\mathrm{H}}\left(\phi-1\right)+1}=-\frac{2a_{\mathrm{H}}\phi}{2a_{\mathrm{H}}\left(\phi-1\right)+1}
\end{aligned}
$$

Finally, the FOC with respect to net wealth is given by:

$$
\begin{aligned}
\widehat{\mathcal{B}}_t \quad : \quad & 0 = 2a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\phi\left[E_t\widehat{\mathcal{W}}_{t+1}-\widehat{\mathcal{W}}_t\right]+ \\
& -\left(1-a_{\mathrm{H}}\right)\left[2a_{\mathrm{H}}\left(\sigma\phi-1\right)+1\right]\frac{\left(1-\alpha\beta\right)\left(1-\alpha\right)}{\alpha}\cdot \\
& \left[\left(E_t\left(\gamma_{H,t+1}+\gamma^*_{H,t+1}\right)-\left(\gamma_{H,t}+\gamma^*_{H,t}\right)\right)-\left(E_t\left(\gamma_{F,t+1}+\gamma^*_{F,t+1}\right)-\left(\gamma_{F,t}+\gamma^*_{F,t}\right)\right)\right]+ \\
& \left(2a_{\mathrm{H}}-1\right)\left[\beta E_t\gamma_{t+2}-E_t\gamma_{t+1}+\beta E_t\gamma_{t+1}-\gamma_t\right]+ \\
& \left[4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1\right]\left[\left(E_t\lambda_{t+1}-\beta^{-1}\lambda_t\right)-\left(\lambda_t-\beta^{-1}\lambda_{t-1}\right)\right],
\end{aligned}
$$

which simplifies as follows:

$$
\begin{aligned}
0 \quad = \quad & -\left(1-a_{\mathrm{H}}\right)\left[2a_{\mathrm{H}}\left(\sigma\phi-1\right)+1\right]\frac{\left(1-\alpha\beta\right)\left(1-\alpha\right)}{\alpha}\cdot \\
& \left[\left(E_t\left(\gamma_{H,t+1}+\gamma^*_{H,t+1}\right)-\left(\gamma_{H,t}+\gamma^*_{H,t}\right)\right)-\left(E_t\left(\gamma_{F,t+1}+\gamma^*_{F,t+1}\right)-\left(\gamma_{F,t}+\gamma^*_{F,t}\right)\right)\right]+ \\
& \left(2a_{\mathrm{H}}-1\right)\left[\beta E_t\gamma_{t+2}-E_t\gamma_{t+1}+\beta E_t\gamma_{t+1}-\gamma_t\right]+ \\
& \left[4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1\right]\left[\left(E_t\lambda_{t+1}-\beta^{-1}\lambda_t\right)-\left(\lambda_t-\beta^{-1}\lambda_{t-1}\right)\right].
\end{aligned}
$$

Proof of Proposition 3: Sum rule. By summing the FOCs for inflation rates and output the solution can be expressed in terms of a familiar sum rule for (the change in) world output gaps and CPI inflation rates (where observe that we have switched to the gap notation, e.g. $\widetilde{Y}_{H,t} = \widehat{Y}_{H,t} - \widehat{Y}^{fb}_{H,t}$):

$$\begin{aligned} 0 &= \widetilde{Y}_{H,t} + \widetilde{Y}_{F,t} + \theta \left(a_{\mathrm{H}} \widehat{p}_{H,t} + (1 - a_{\mathrm{H}}) \widehat{p}_{F,t} + a_{\mathrm{H}} \widehat{p}^{*}_{F,t} + (1 - a_{\mathrm{H}}) \widehat{p}^{*}_{H,t} \right) \\ &= \left[\widetilde{Y}_{H,t} - \widetilde{Y}_{H,t-1} \right] + \left[\widetilde{Y}_{F,t} - \widetilde{Y}_{F,t-1} \right] + \\ &\quad \theta \left[a_{\mathrm{H}} \pi_{H,t} + (1 - a_{\mathrm{H}}) \pi_{F,t} + a_{\mathrm{H}} \pi^{*}_{F,t} + (1 - a_{\mathrm{H}}) \pi^{*}_{H,t} \right], \end{aligned}$$

the same as under complete markets.

Proof of Proposition 4: Difference rule. The difference rule is difficult to characterize analytically, but for the special case of $\eta = 0$. From the FOC for output solve for the term $\left(\beta E_t \gamma_{t+1} - \gamma_t \right)$:

$$\begin{aligned} \frac{1}{2a_{\mathrm{H}}(\phi - 1) + 1} \left(\beta E_t \gamma_{t+1} - \gamma_t \right) &= (\sigma + \eta) \left(\widehat{Y}_{H,t} - \widehat{Y}^{fb}_{H,t} \right) \\ &\quad - \frac{2a_{\mathrm{H}}(1 - a_{\mathrm{H}})(\sigma\phi - 1)\sigma}{4a_{\mathrm{H}}(1 - a_{\mathrm{H}})(\sigma\phi - 1) + 1} \left[\left(\widehat{Y}_{H,t} - \widehat{Y}^{fb}_{H,t} \right) - \left(\widehat{Y}_{F,t} - \widehat{Y}^{fb}_{F,t} \right) \right] + \\ &\quad \frac{2a_{\mathrm{H}}(1 - a_{\mathrm{H}})\phi}{4a_{\mathrm{H}}(1 - a_{\mathrm{H}})(\sigma\phi - 1) + 1} \frac{2a_{\mathrm{H}}(\sigma\phi - 1) + 1 - \sigma}{2a_{\mathrm{H}}(\phi - 1) + 1} \left(\widehat{\Delta}_t + \widehat{\mathcal{W}}_t \right) + \\ &\quad - \left\{ \begin{array}{c} (\sigma + \eta) \frac{(1-\alpha\beta)(1-\alpha)}{\alpha} \left(\gamma_{H,t} + \gamma^{*}_{H,t} \right) + \\ \frac{(1-a_{\mathrm{H}})(\sigma-1)}{2a_{\mathrm{H}}(\phi-1)+1} \frac{(1-\alpha\beta)(1-\alpha)}{\alpha} \left[\left(\gamma_{F,t} + \gamma^{*}_{F,t} \right) - \left(\gamma_{H,t} + \gamma^{*}_{H,t} \right) \right] \end{array} \right\} + \\ &\quad \frac{2a_{\mathrm{H}}(\sigma\phi - 1) + 1 - \sigma}{2a_{\mathrm{H}}(\phi - 1) + 1} \left(\lambda_t - \beta^{-1} \lambda_{t-1} \right) \end{aligned}$$

$$\begin{aligned} \frac{1}{2a_{\mathrm{H}}(\phi - 1) + 1} \left(\beta E_t \gamma_{t+1} - \gamma_t \right) &= -(\sigma + \eta) \left(\widehat{Y}_{F,t} - \widehat{Y}^{fb}_{F,t} \right) - \\ &\quad \frac{2a_{\mathrm{H}}(1 - a_{\mathrm{H}})(\sigma\phi - 1)\sigma}{4a_{\mathrm{H}}(1 - a_{\mathrm{H}})(\sigma\phi - 1) + 1} \left[\left(\widehat{Y}_{H,t} - \widehat{Y}^{fb}_{H,t} \right) - \left(\widehat{Y}_{F,t} - \widehat{Y}^{fb}_{F,t} \right) \right] + \\ &\quad \frac{2a_{\mathrm{H}}(1 - a_{\mathrm{H}})\phi}{4a_{\mathrm{H}}(1 - a_{\mathrm{H}})(\sigma\phi - 1) + 1} \frac{2a_{\mathrm{H}}(\sigma\phi - 1) + 1 - \sigma}{2a_{\mathrm{H}}(\phi - 1) + 1} \left(\widehat{\Delta}_t + \widehat{\mathcal{W}}_t \right) + \\ &\quad (\sigma + \eta) \frac{(1 - \alpha\beta)(1 - \alpha)}{\alpha} \left(\gamma_{F,t} + \gamma^{*}_{F,t} \right) + \\ &\quad \frac{(1 - a_{\mathrm{H}})(\sigma - 1)}{2a_{\mathrm{H}}(\phi - 1) + 1} \frac{(1 - \alpha\beta)(1 - \alpha)}{\alpha} \left[\left(\gamma_{H,t} + \gamma^{*}_{H,t} \right) - \left(\gamma_{F,t} + \gamma^{*}_{F,t} \right) \right] + \\ &\quad \frac{2a_{\mathrm{H}}(\sigma\phi - 1) + 1 - \sigma}{2a_{\mathrm{H}}(\phi - 1) + 1} \left(\lambda_t - \beta^{-1} \lambda_{t-1} \right); \end{aligned}$$

Summing up we obtain:

$$\begin{aligned}
\frac{2}{2a_{\mathrm{H}}(\phi-1)+1}\left(\beta E_t\gamma_{t+1}-\gamma_t\right) &= (\sigma+\eta)\left[\left(\widehat{Y}_{H,t}-\widehat{Y}^{fb}_{H,t}\right)-\left(\widehat{Y}_{F,t}-\widehat{Y}^{fb}_{F,t}\right)\right]- \\
&\quad \frac{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)\sigma}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1}\left[\left(\widehat{Y}_{H,t}-\widehat{Y}^{fb}_{H,t}\right)-\left(\widehat{Y}_{F,t}-\widehat{Y}^{fb}_{F,t}\right)\right]+ \\
&\quad \frac{4a_{\mathrm{H}}(1-a_{\mathrm{H}})\phi}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1}\frac{2a_{\mathrm{H}}(\sigma\phi-1)+1-\sigma}{2a_{\mathrm{H}}(\phi-1)+1}\left(\widehat{\Delta}_t+\widehat{\mathcal{W}}_t\right)- \\
&\quad \left\{(\sigma+\eta)-\tfrac{2(1-a_{\mathrm{H}})(\sigma-1)}{2a_{\mathrm{H}}(\phi-1)+1}\right\}\tfrac{(1-\alpha\beta)(1-\alpha)}{\alpha}\left[\left(\gamma_{H,t}+\gamma^*_{H,t}\right)-\left(\gamma_{F,t}+\gamma^*_{F,t}\right)\right]+ \\
&\quad 2\frac{2a_{\mathrm{H}}(\sigma\phi-1)+1-\sigma}{2a_{\mathrm{H}}(\phi-1)+1}\left(\lambda_t-\beta^{-1}\lambda_{t-1}\right)
\end{aligned}$$

Consider now the FOC wrt LOOP:

$$\begin{aligned}
\widehat{\Delta}_t \quad : \quad & 0=-\frac{2a_{\mathrm{H}}(1-a_{\mathrm{H}})\phi}{2a_{\mathrm{H}}(\phi-1)+1}\left(\widehat{\Delta}_t+\widehat{\mathcal{W}}_t\right)+ \\
& \frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\frac{1}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\phi\sigma-1)+1}\cdot \\
& \frac{1}{2}\left[\begin{array}{c}
\left(4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\phi\sigma-1)+1\right)\left(\gamma_{H,t}+\gamma_{F,t}-\left(\gamma^*_{F,t}+\gamma^*_{H,t}\right)\right)+ \\
-\left[(2a_{\mathrm{H}}-1)-2(1-a_{\mathrm{H}})\left[2a_{\mathrm{H}}(\sigma\phi-1)+1\right]\frac{2a_{\mathrm{H}}[2(1-a_{\mathrm{H}})(\sigma\phi-1)+1-\phi]}{2a_{\mathrm{H}}(\phi-1)+1}\right]\cdot \\
\left(\gamma_{H,t}+\gamma^*_{H,t}-\gamma_{F,t}-\gamma^*_{F,t}\right)
\end{array}\right]+ \\
& -\frac{2a_{\mathrm{H}}-1}{2a_{\mathrm{H}}(\phi-1)+1}\left(\beta E_t\gamma_{t+1}-\gamma_t\right)+ \\
& -\frac{2a_{\mathrm{H}}\left[2(1-a_{\mathrm{H}})(\sigma\phi-1)+1-\phi\right]}{2a_{\mathrm{H}}(\phi-1)+1}\left(\lambda_t-\beta^{-1}\lambda_{t-1}\right);
\end{aligned}$$

From the FOC for $\widehat{\mathcal{B}}_t$

$$\begin{aligned}
0 \quad = \quad & -(1-a_{\mathrm{H}})\left[2a_{\mathrm{H}}(\sigma\phi-1)+1\right]\tfrac{(1-\alpha\beta)(1-\alpha)}{\alpha}\cdot \\
& \left[E_t\left(\gamma_{H,t+1}+\gamma^*_{H,t+1}\right)-\left(\gamma_{H,t}+\gamma^*_{H,t}\right)-\left(E_t\left(\gamma_{F,t+1}+\gamma^*_{F,t+1}\right)-\left(\gamma_{F,t}+\gamma^*_{F,t}\right)\right)\right]+ \\
& (2a_{\mathrm{H}}-1)\left[\left(\beta E_t\gamma_{t+2}-E_t\gamma_{t+1}\right)-\left(\beta E_t\gamma_{t+1}-\gamma_t\right)\right]+ \\
& \left[4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1\right]\left[\left(E_t\lambda_{t+1}-\beta^{-1}\lambda_t\right)-\left(\lambda_t-\beta^{-1}\lambda_{t-1}\right)\right],
\end{aligned}$$

we get the following solution for $\left(\lambda_t-\beta^{-1}\lambda_{t-1}\right)$:

$$\begin{aligned}
\left[4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1\right]\left(\lambda_t-\beta^{-1}\lambda_{t-1}\right) \quad = \quad & -(2a_{\mathrm{H}}-1)\left(\beta E_t\gamma_{t+1}-\gamma_t\right) \\
& +(1-a_{\mathrm{H}})\left[2a_{\mathrm{H}}(\sigma\phi-1)+1\right]\tfrac{(1-\alpha\beta)(1-\alpha)}{\alpha}\cdot \\
& \left[\left(\gamma_{H,t}+\gamma^*_{H,t}\right)-\left(\gamma_{F,t}+\gamma^*_{F,t}\right)\right]
\end{aligned}$$

where we have also used the fact that:

$$\frac{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1}{2a_{\mathrm{H}}(\phi-1)+1}+(2a_{\mathrm{H}}-1)\frac{2a_{\mathrm{H}}(\sigma\phi-1)+1-\sigma}{2a_{\mathrm{H}}(\phi-1)+1}=\sigma.$$

Thus we can write

$$
\begin{aligned}
2\sigma\left(\beta E_t\gamma_{t+1}-\gamma_t\right) &= \left[\sigma+\eta\left[4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1\right]\right]\left[\left(\widehat{Y}_{H,t}-\widehat{Y}_{H,t}^{fb}\right)-\left(\widehat{Y}_{F,t}-\widehat{Y}_{F,t}^{fb}\right)\right]+\\
&\quad 4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\phi\frac{2a_{\mathrm{H}}\left(\sigma\phi-1\right)+1-\sigma}{2a_{\mathrm{H}}\left(\phi-1\right)+1}\left(\widehat{\Delta}_t+\widehat{\mathcal{W}}_t\right)+\\
&\quad -\left(\sigma+\eta\left[4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1\right]\right)\frac{\left(1-\alpha\beta\right)\left(1-\alpha\right)}{\alpha}\\
&\quad \left[\left(\gamma_{H,t}+\gamma_{H,t}^{*}\right)-\left(\gamma_{F,t}+\gamma_{F,t}^{*}\right)\right]
\end{aligned}
$$

Set $\eta=0$ and solve for $\left(\gamma_{H,t}+\gamma_{H,t}^{*}-\gamma_{F,t}-\gamma_{F,t}^{*}\right)$

$$
\begin{aligned}
2\sigma\left(\beta E_t\gamma_{t+1}-\gamma_t\right) &= \sigma\left[\left(\widehat{Y}_{H,t}-\widehat{Y}_{H,t}^{fb}\right)-\left(\widehat{Y}_{F,t}-\widehat{Y}_{F,t}^{fb}\right)\right]+\\
&\quad 4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\phi\frac{2a_{\mathrm{H}}\left(\sigma\phi-1\right)+1-\sigma}{2a_{\mathrm{H}}\left(\phi-1\right)+1}\left(\widehat{\Delta}_t+\widehat{\mathcal{W}}_t\right)+\\
&\quad -\sigma\frac{\left(1-\alpha\beta\right)\left(1-\alpha\right)}{\alpha}\left[\left(\gamma_{H,t}+\gamma_{H,t}^{*}\right)-\left(\gamma_{F,t}+\gamma_{F,t}^{*}\right)\right]
\end{aligned}
$$

also using the FOC for $\widehat{\Delta}_t$ after substituting out for $\left(\lambda_t-\beta^{-1}\lambda_{t-1}\right)$:

$$
\begin{aligned}
&2\frac{2a_{\mathrm{H}}-1}{\left[4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1\right]}\left(\beta E_t\gamma_{t+1}-\gamma_t\right)=-\frac{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\phi}{2a_{\mathrm{H}}\left(\phi-1\right)+1}\left(\widehat{\Delta}_t+\widehat{\mathcal{W}}_t\right)+\\
&\frac{\left(1-\alpha\beta\right)\left(1-\alpha\right)}{\alpha}\left(\gamma_{H,t}+\gamma_{F,t}-\left(\gamma_{F,t}^{*}+\gamma_{H,t}^{*}\right)\right)-\left(2a_{\mathrm{H}}-1\right)\frac{\left(1-\alpha\beta\right)\left(1-\alpha\right)}{\alpha}\\
&\frac{\left(\gamma_{H,t}+\gamma_{H,t}^{*}-\gamma_{F,t}-\gamma_{F,t}^{*}\right)}{\left[4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1\right]}.
\end{aligned}
$$

The following equality holds:

$$
\begin{aligned}
&\frac{2a_{\mathrm{H}}-1}{\left[4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1\right]}\sigma\left[\left(\widehat{Y}_{H,t}-\widehat{Y}_{H,t}^{fb}\right)-\left(\widehat{Y}_{F,t}-\widehat{Y}_{F,t}^{fb}\right)\right]+\\
&\frac{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\phi}{2a_{\mathrm{H}}\left(\phi-1\right)+1}\left[\left(2a_{\mathrm{H}}-1\right)\frac{2a_{\mathrm{H}}\left(\sigma\phi-1\right)+1-\sigma}{\left[4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1\right]}+\sigma\right]\left(\widehat{\Delta}_t+\widehat{\mathcal{W}}_t\right)=\\
&\sigma\frac{\left(1-\alpha\beta\right)\left(1-\alpha\right)}{\alpha}\left(\gamma_{H,t}+\gamma_{F,t}-\left(\gamma_{F,t}^{*}+\gamma_{H,t}^{*}\right)\right),
\end{aligned}
$$

which further simplifies as follows:

$$
\begin{aligned}
&\frac{2a_{\mathrm{H}}-1}{\left[4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1\right]}\sigma\left[\left(\widehat{Y}_{H,t}-\widehat{Y}_{H,t}^{fb}\right)-\left(\widehat{Y}_{F,t}-\widehat{Y}_{F,t}^{fb}\right)\right]+\\
&\left[\left(2a_{\mathrm{H}}-1\right)\frac{2a_{\mathrm{H}}\left(\sigma\phi-1\right)+1-\sigma}{\left[4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1\right]}+\sigma\right]\frac{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\phi}{2a_{\mathrm{H}}\left(\phi-1\right)+1}\left(\widehat{\Delta}_t+\widehat{\mathcal{W}}_t\right)\\
=\;&\frac{\left(1-\alpha\beta\right)\left(1-\alpha\right)}{\alpha}\sigma\left(\gamma_{H,t}+\gamma_{F,t}-\left(\gamma_{F,t}^{*}+\gamma_{H,t}^{*}\right)\right).
\end{aligned}
$$

In turn we can rewrite the left hand side of the above expression as follows:

$$
\begin{aligned}
& (2a_{\mathrm{H}}-1)\left[\widehat{\mathcal{T}}_t-\widehat{\mathcal{T}}_t^{fb}+\widehat{\Delta}_t+\frac{(2a_{\mathrm{H}}-1)\left(\widehat{\mathcal{W}}_t+\widehat{\Delta}_t\right)}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)(\sigma\phi-1)+1}\right]+ \\
& \left[(2a_{\mathrm{H}}-1)\frac{2a_{\mathrm{H}}(\sigma\phi-1)+1-\sigma}{\left[4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)(\sigma\phi-1)+1\right]}+\sigma\right]\frac{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\phi}{2a_{\mathrm{H}}(\phi-1)+1}\left(\widehat{\Delta}_t+\widehat{\mathcal{W}}_t\right) \\
= \; & \widehat{\mathcal{Q}}_t-\widehat{\mathcal{Q}}_t^{fb}-\widehat{\Delta}_t+\left(\widehat{\mathcal{W}}_t+\widehat{\Delta}_t\right)+\frac{(2a_{\mathrm{H}}-1)^2-1-4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)(\sigma\phi-1)}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)(\sigma\phi-1)+1}\left(\widehat{\mathcal{W}}_t+\widehat{\Delta}_t\right)+ \\
& \left[(2a_{\mathrm{H}}-1)\frac{2a_{\mathrm{H}}(\sigma\phi-1)+1-\sigma}{\left[4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)(\sigma\phi-1)+1\right]}+\sigma\right]\frac{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\phi}{2a_{\mathrm{H}}(\phi-1)+1}\left(\widehat{\Delta}_t+\widehat{\mathcal{W}}_t\right) \\
= \; & \widehat{\mathcal{Q}}_t-\widehat{\mathcal{Q}}_t^{fb}-\widehat{\Delta}_t+\left(\widehat{\mathcal{W}}_t+\widehat{\Delta}_t\right)+\frac{-4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\sigma\phi}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)(\sigma\phi-1)+1}\left(\widehat{\mathcal{W}}_t+\widehat{\Delta}_t\right)+ \\
& \left[(2a_{\mathrm{H}}-1)\frac{2a_{\mathrm{H}}(\sigma\phi-1)+1-\sigma}{\left[4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)(\sigma\phi-1)+1\right]}+\sigma\right]\frac{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\phi}{2a_{\mathrm{H}}(\phi-1)+1}\left(\widehat{\Delta}_t+\widehat{\mathcal{W}}_t\right) \\
= \; & \widehat{\mathcal{Q}}_t-\widehat{\mathcal{Q}}_t^{fb}-\widehat{\Delta}_t+\left(\widehat{\mathcal{W}}_t+\widehat{\Delta}_t\right)+\frac{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\phi(\sigma-1)}{2a_{\mathrm{H}}(\phi-1)+1}\left(\widehat{\mathcal{W}}_t+\widehat{\Delta}_t\right).
\end{aligned}
$$

Finally, using the FOC for inflation to substitute out $\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\left(\gamma_{H,t}+\gamma_{F,t}-\left(\gamma_{F,t}^*+\gamma_{H,t}^*\right)\right)$, we arrive at the following expression for the optimal difference rule in levels:

$$
\begin{aligned}
0 &= \sigma\theta\left[\left(a_{\mathrm{H}}\widehat{p}_{H,t}+\left(1-a_{\mathrm{H}}\right)\widehat{p}_{F,t}\right)-\left(a_{\mathrm{H}}\widehat{p}_{F,t}^*+\left(1-a_{\mathrm{H}}\right)\widehat{p}_{H,t}^*\right)\right]+ \\
& \widehat{\mathcal{Q}}_t-\widehat{\mathcal{Q}}_t^{fb}+\widehat{\mathcal{W}}_t+\frac{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\phi}{2a_{\mathrm{H}}(\phi-1)+1}(\sigma-1)\left(\widehat{\Delta}_t+\widehat{\mathcal{W}}_t\right),
\end{aligned}
$$

which is straightforward to write in terms of inflation and growth rates of the other variables as in Proposition 4:

$$
\begin{gathered}
0=\theta\left[\left(a_{\mathrm{H}}\pi_{H,t}+\left(1-a_{\mathrm{H}}\right)\pi_{F,t}\right)-\left(a_{\mathrm{H}}\pi_{H,t}^*+\left(1-a_{\mathrm{H}}\right)\pi_{H,t}^*\right)\right]+ \\
\left[\left(\widehat{C}_t-\widehat{C}_t^*\right)-\left(\widehat{C}_t^{fb}-\widehat{C}_t^{*fb}\right)\right]-\left[\left(\widehat{C}_{t-1}-\widehat{C}_{t-1}^*\right)-\left(\widehat{C}_{t-1}^{fb}-\widehat{C}_{t-1}^{*fb}\right)\right]+ \\
\frac{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\phi}{2a_{\mathrm{H}}(\phi-1)+1}\frac{(\sigma-1)}{\sigma}\left(\widehat{\Delta}_t-\widehat{\Delta}_{t-1}+\widehat{\mathcal{W}}_t-\widehat{\mathcal{W}}_{t-1}\right).
\end{gathered}
$$

An alternative way of expressing the targeting criterion. The targeting criterion could also be expressed as a combination of the CPI-inflation and consumption differentials:

$$
\begin{gathered}
0=\theta\left[\left(a_{\mathrm{H}}\pi_{H,t}+\left(1-a_{\mathrm{H}}\right)\pi_{F,t}\right)-\left(a_{\mathrm{H}}\pi_{F,t}^*+\left(1-a_{\mathrm{H}}\right)\pi_{H,t}^*\right)\right]+ \\
\left[\widehat{\mathcal{W}}_t-\widehat{\mathcal{W}}_{t-1}\right]+\left[\left(\widehat{\mathcal{Q}}_t-\widehat{\mathcal{Q}}_t^{fb}\right)-\left(\widehat{\mathcal{Q}}_{t-1}-\widehat{\mathcal{Q}}_{t-1}^{fb}\right)\right] \\
0=\theta\left[\left(a_{\mathrm{H}}\pi_{H,t}+\left(1-a_{\mathrm{H}}\right)\pi_{F,t}\right)-\left(a_{\mathrm{H}}\pi_{H,t}^*+\left(1-a_{\mathrm{H}}\right)\pi_{H,t}^*\right)\right]+ \\
\left[\left(\widehat{C}_t-\widehat{C}_t^*\right)-\left(\widehat{C}_t^{fb}-\widehat{C}_t^{*fb}\right)\right]-\left[\left(\widehat{C}_{t-1}-\widehat{C}_{t-1}^*\right)-\left(\widehat{C}_{t-1}^{fb}-\widehat{C}_{t-1}^{*fb}\right)\right].
\end{gathered}
$$

Taking again the difference in CPI inflation using the NKPC:

$$
\begin{aligned}
& a_{\mathrm{H}} \pi_{H,t} + (1-a_{\mathrm{H}}) \pi_{F,t} - \left(a_{\mathrm{H}} \pi^{*}_{F,t} + (1-a_{\mathrm{H}}) \pi^{*}_{H,t} \right) - \\
& \beta E_t \left(a_{\mathrm{H}} \pi_{H,t+1} + (1-a_{\mathrm{H}}) \pi_{F,t+1} \right) - \beta E_t \left(a_{\mathrm{H}} \pi^{*}_{F,t+1} + (1-a_{\mathrm{H}}) \pi^{*}_{H,t+1} \right) = \\
= & \frac{(1-\alpha\beta)(1-\alpha)}{\alpha} \left\{ (2a_{\mathrm{H}}-1) \left[\begin{array}{c} \left(\widehat{Y}_{H,t} - \widehat{Y}^{fb}_{H,t} \right) - \left(\widehat{Y}_{F,t} - \widehat{Y}^{fb}_{F,t} \right) - \\ (2a_{\mathrm{H}}-1) \left(\widehat{\mathcal{D}}_t + \widehat{\Delta}_t \right) + \widehat{\mu}_t - \widehat{\mu}^{*}_t + \\ -4a_{\mathrm{H}} (1-a_{\mathrm{H}}) (\phi-1) \left(\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}^{fb}_t + \widehat{\Delta}_t \right) \\ + \left(\widehat{\Delta}_t + \widehat{\mathcal{D}}_t \right) \end{array} \right] + 2(1-a_{\mathrm{H}}) \widehat{\Delta}_t \right\} \\
= & \frac{(1-\alpha\beta)(1-\alpha)}{\alpha} \left\{ \widehat{\mathcal{Q}_t} - \widehat{\mathcal{Q}}^{fb}_t + (2a_{\mathrm{H}}-1) \left[\widehat{\mu}_t - \widehat{\mu}^{*}_t + \widehat{\mathcal{D}}_t \right] \right\},
\end{aligned}
$$

where we have used the following relation:

$$
\begin{aligned}
\widehat{\mathcal{Q}_t} - \widehat{\mathcal{Q}}^{fb}_t &= (2a_{\mathrm{H}}-1) \left(\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}^{fb}_t + \widehat{\Delta}_t \right) + \widehat{\Delta}_t \\
&= (2a_{\mathrm{H}}-1) \frac{\left[\left(\widehat{Y}_{H,t} - \widehat{Y}_{F,t} \right) - \left(\widehat{Y}^{fb}_{H,t} - \widehat{Y}^{fb}_{F,t} \right) \right] - (2a_{\mathrm{H}}-1) \left(\widehat{\mathcal{D}}_t + \widehat{\Delta}_t \right)}{4a_{\mathrm{H}} (1-a_{\mathrm{H}}) (\phi-1) + 1} + \widehat{\Delta}_t.
\end{aligned}
$$

In contrast to a complete markets (CM) economy, a policy that sets CPI inflation rates to zero in response to efficient shocks is not optimal.

Finally, notice that we can also write the CPI inflation differential as a function of consumption differentials:

$$
\begin{aligned}
& a_{\mathrm{H}} \pi_{H,t} + (1-a_{\mathrm{H}}) \pi_{F,t} - \left(a_{\mathrm{H}} \pi^{*}_{F,t} + (1-a_{\mathrm{H}}) \pi^{*}_{H,t} \right) - \\
& \beta E_t \left(a_{\mathrm{H}} \pi_{H,t+1} + (1-a_{\mathrm{H}}) \pi_{F,t+1} \right) - \beta E_t \left(a_{\mathrm{H}} \pi^{*}_{F,t+1} + (1-a_{\mathrm{H}}) \pi^{*}_{H,t+1} \right) \\
= \; & \frac{(1-\alpha\beta)(1-\alpha)}{\alpha} \left\{ \left[\left(\widehat{C}_t - \widehat{C}^{*}_t \right) - \left(\widehat{C}^{fb}_t - \widehat{C}^{*fb}_t \right) \right] - 2(1-a_{\mathrm{H}}) \widehat{\mathcal{W}}_t + (2a_{\mathrm{H}}-1) \left[\widehat{\mu}_t - \widehat{\mu}^{*}_t \right] \right\}
\end{aligned}
$$

3.1.2 Proof of Proposition 5

We start by first proving Proposition 5 in the text, namely that $\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}^{fb}_t + \widehat{\Delta}_t$, $\widehat{\mathcal{W}}_t$ and $\widehat{\mathcal{B}}_t$ are independent of monetary policy under the maintained parametric assumptions $\sigma = 1$ and $\eta = 0$. Next, we proceed to solve for the optimal allocations.

We can solve for net foreign assets $\widehat{\mathcal{B}}_t$ and (the permanent shift in) $\widehat{\mathcal{W}}_t$ by using the budget constraint:

$$
\begin{aligned}
\widehat{\mathcal{W}}_t &= \widehat{\mathcal{W}}_t - \widehat{\mathcal{W}}^{fb}_t = \\
&= \sigma \left[\left(\widehat{C}_t - \widehat{C}^{*}_t \right) - \left(\widehat{C}^{fb}_t - \widehat{C}^{*fb}_t \right) \right] - \left(\widehat{\mathcal{Q}}_t - \widehat{\mathcal{Q}}^{fb}_t \right) \\
&= \sigma \left[\left(\widehat{Y}_{H,t} - \widehat{Y}^{fb}_{H,t} \right) - \left(\widehat{Y}_{F,t} - \widehat{Y}^{fb}_{F,t} \right) + 2\beta^{-1} \left(\widehat{\mathcal{B}}_{t-1} - \beta \widehat{\mathcal{B}}_t \right) \right] + \\
&\quad - \left[2(1-a_{\mathrm{H}}) \sigma + (2a_{\mathrm{H}}-1) \right] \left(\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}^{fb}_t \right) - 2a_{\mathrm{H}} \widehat{\Delta}_t + \\
&\quad 2(1-a_{\mathrm{H}}) \left[\left(2a_{\mathrm{H}} (\sigma\phi - 1) + 1 - \sigma \right) \widehat{\mathcal{T}}^{fb}_t - \left(\widehat{\zeta}_{C,t} - \widehat{\zeta}^{*}_{C,t} \right) \right],
\end{aligned}
$$

where we used the fact that

$$\left(\widehat{\mathcal{Q}}_t - \widehat{\mathcal{Q}}_t^{fb}\right) = 2a_{\mathrm{H}}\left(\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}_t^{fb} + \widehat{\Delta}_t\right) - \left(\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}_t^{fb}\right),$$

and the link between the output gap and relative prices:

$$\sigma\left[\left(\widehat{Y}_{H,t} - \widehat{Y}_{H,t}^{fb}\right) - \left(\widehat{Y}_{F,t} - \widehat{Y}_{F,t}^{fb}\right)\right] = \left[4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi - 1\right) + 1\right]\left(\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}_t^{fb} + \widehat{\Delta}_t\right) + \left(2a_{\mathrm{H}} - 1\right)\left(\widehat{\mathcal{W}}_t + \widehat{\Delta}_t\right)$$

we obtain the following simplification:

$$\begin{aligned}
\left(1-a_{\mathrm{H}}\right)\widehat{\mathcal{W}}_t &= \sigma\beta^{-1}\left(\widehat{\mathcal{B}}_{t-1} - \beta\widehat{\mathcal{B}}_t\right) + \left(1-a_{\mathrm{H}}\right)\left(\sigma-1\right)\widehat{\Delta}_t + \\
&\quad \left(1-a_{\mathrm{H}}\right)\left[2a_{\mathrm{H}}\left(\sigma\phi-1\right) - \left(\sigma-1\right)\right]\left(\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}_t^{fb} + \widehat{\Delta}_t\right) + \\
&\quad \left(1-a_{\mathrm{H}}\right)\left[\left(2a_{\mathrm{H}}\left(\sigma\phi-1\right) + 1 - \sigma\right)\widehat{\mathcal{T}}_t^{fb} - \left(\widehat{\zeta}_{C,t} - \widehat{\zeta}_{C,t}^{*}\right)\right];
\end{aligned}$$

when $\sigma = 1$ the expression becomes:

$$\begin{aligned}
\left(1-a_{\mathrm{H}}\right)\widehat{\mathcal{W}}_t &= \beta^{-1}\left(\widehat{\mathcal{B}}_{t-1} - \beta\widehat{\mathcal{B}}_t\right) + \\
&\quad 2a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\phi-1\right)\left(\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}_t^{fb} + \widehat{\Delta}_t\right) + \\
&\quad \left(1-a_{\mathrm{H}}\right)\left[2a_{\mathrm{H}}\left(\phi-1\right)\widehat{\mathcal{T}}_t^{fb} - \left(\widehat{\zeta}_{C,t} - \widehat{\zeta}_{C,t}^{*}\right)\right].
\end{aligned}$$

Using the consumption Euler equation we get the following difference equation for NFAs:

$$\begin{gathered}
\beta^{-1}\left[E_t\left(\beta\widehat{\mathcal{B}}_{t+1} - \widehat{\mathcal{B}}_t\right) - \left(\beta\widehat{\mathcal{B}}_t - \widehat{\mathcal{B}}_{t-1}\right)\right] = \\
2a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\phi-1\right)E_t\left(\left(\widehat{\mathcal{T}}_{t+1} - \widehat{\mathcal{T}}_{t+1}^{fb} + \widehat{\Delta}_{t+1}\right) - \left(\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}_t^{fb} + \widehat{\Delta}_t\right)\right) + \\
\left(1-a_{\mathrm{H}}\right)\left[2a_{\mathrm{H}}\left(\phi-1\right)E_t\left(\widehat{\mathcal{T}}_{t+1}^{fb} - \widehat{\mathcal{T}}_t^{fb}\right) - E_t\left(\left(\widehat{\zeta}_{C,t+1} - \widehat{\zeta}_{C,t+1}^{*}\right) - \left(\widehat{\zeta}_{C,t} - \widehat{\zeta}_{C,t}^{*}\right)\right)\right].
\end{gathered}$$

In order to solve it, observe first that we can solve for the expression for $\left(\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}_t^{fb} + \widehat{\Delta}_t\right)$ by using the relation

$$\pi_{F,t} - \pi_{H,t} = \left(\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}_{t-1} + \widehat{\Delta}_t - \widehat{\Delta}_{t-1}\right),$$

and taking the difference between the NKPC for $\pi_{F,t} - \pi_{H,t}$ with $\sigma = 1$ and $\eta = 0$ to get the following difference equation:

$$\begin{gathered}
\pi_{F,t} - \pi_{H,t} - \beta E_t\left(\pi_{F,t+1} - \pi_{H,t+1}\right) = \\
\left(\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}_{t-1} + \widehat{\Delta}_t - \widehat{\Delta}_{t-1}\right) - \beta E_t\left(\widehat{\mathcal{T}}_{t+1} - \widehat{\mathcal{T}}_t + \widehat{\Delta}_{t+1} - \widehat{\Delta}_t\right) = \\
-\frac{\left(1-\alpha\beta\right)\left(1-\alpha\right)}{\alpha}\left[\begin{array}{c}
\left(\widehat{Y}_{H,t} - \widehat{Y}_{H,t}^{fb}\right) - \left(\widehat{Y}_{F,t} - \widehat{Y}_{F,t}^{fb}\right) - \widehat{\Delta}_t + \\
-2\left(1-a_{\mathrm{H}}\right)\left[2a_{\mathrm{H}}\left(\phi-1\right)\left(\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}_t^{fb} + \widehat{\Delta}_t\right) - \left(\widehat{\Delta}_t + \widehat{\mathcal{W}}_t\right)\right]
\end{array}\right].
\end{gathered}$$

Using again the equilibrium relation between the output gap and relative prices also when $\sigma = 1$ and $\eta = 0$:

$$\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}_t^{fb} + \widehat{\Delta}_t = \frac{\left[\left(\widehat{Y}_{H,t} - \widehat{Y}_{H,t}^{fb}\right) - \left(\widehat{Y}_{F,t} - \widehat{Y}_{F,t}^{fb}\right)\right] - (2a_{\mathrm{H}} - 1)\left(\widehat{\mathcal{W}}_t + \widehat{\Delta}_t\right)}{4a_{\mathrm{H}}\left(1 - a_{\mathrm{H}}\right)(\phi - 1) + 1},$$

we can simplify the above difference equation as follows:

$$\begin{gathered}\beta E_t\left[\left(\widehat{\mathcal{T}}_{t+1} - \widehat{\mathcal{T}}_{t+1}^{fb} + \widehat{\Delta}_{t+1}\right) - \left(\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}_t^{fb} + \widehat{\Delta}_t\right)\right] - \\ \left[\left(\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}_t^{fb} + \widehat{\Delta}_t\right) - \left(\widehat{\mathcal{T}}_{t-1} - \widehat{\mathcal{T}}_{t-1}^{fb} + \widehat{\Delta}_{t-1}\right)\right] - \\ \frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\left(\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}_t^{fb} + \widehat{\Delta}_t\right) = \\ \frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\widehat{\mathcal{W}}_t - E_t\left[\beta\left(\widehat{\mathcal{T}}_{t+1}^{fb} - \widehat{\mathcal{T}}_t^{fb}\right) - \left(\widehat{\mathcal{T}}_t^{fb} - \widehat{\mathcal{T}}_{t-1}^{fb}\right)\right].\end{gathered}$$

We solve this difference equation for $\left(\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}_t^{fb} + \widehat{\Delta}_t\right)$:

$$\begin{aligned}\left(\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}_t^{fb} + \widehat{\Delta}_t\right) &= \nu_1\left(\widehat{\mathcal{T}}_{t-1} - \widehat{\mathcal{T}}_{t-1}^{fb} + \widehat{\Delta}_{t-1}\right) - \frac{(1-\alpha\beta)(1-\alpha)}{\alpha\beta}\sum_{j=0}^{\infty}\nu_2^{-j-1}\widehat{\mathcal{W}}_t + \\ &\quad \sum_{j=0}^{\infty}\nu_2^{-j-1}E_t\left[\left(\widehat{\mathcal{T}}_{t+j+1}^{fb} - \widehat{\mathcal{T}}_{t+j}^{fb}\right) - \beta^{-1}\left(\widehat{\mathcal{T}}_{t+j}^{fb} - \widehat{\mathcal{T}}_{t+j-1}^{fb}\right)\right],\end{aligned}$$

where $0 < \nu_1 < 1 < \beta^{-1} < \nu_2$ are the eigenvalues of the difference equation, solving the standard characteristic equation:

$$\beta\nu^2 - \left[1 + \beta + \frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\right]\nu + 1 = 0,$$

namely

$$\nu = \frac{1 + \beta + \frac{(1-\alpha\beta)(1-\alpha)}{\alpha} \pm \sqrt{\left[1 + \beta + \frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\right]^2 - 4\beta}}{2\beta}.$$

We simplify further using the fact that $\widehat{\mathcal{W}}_t$ is a martingale:

$$\begin{gathered}\left(\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}_t^{fb} + \widehat{\Delta}_t\right) = \nu_1\left(\widehat{\mathcal{T}}_{t-1} - \widehat{\mathcal{T}}_{t-1}^{fb} + \widehat{\Delta}_{t-1}\right) - \frac{(\beta\nu_2 - 1)}{\beta\nu_2}\widehat{\mathcal{W}}_t + \\ \sum_{s=0}^{\infty}\nu_2^{-s-1}E_t\left[\left(\widehat{\mathcal{T}}_{t+j+1}^{fb} - \widehat{\mathcal{T}}_{t+j}^{fb}\right) - \beta^{-1}\left(\widehat{\mathcal{T}}_{t+j}^{fb} - \widehat{\mathcal{T}}_{t+j-1}^{fb}\right)\right],\end{gathered}$$

where we have also used the fact that: $\frac{(\beta\nu_2-1)}{\beta\nu_2} = \frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\frac{1}{\beta(\nu_2-1)}$.

Observe that we have only used equilibrium relations that are independent of monetary policy. Therefore, the three variables $\left(\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}_t^{fb} + \widehat{\Delta}_t\right)$, $\widehat{\mathcal{B}}_t$ and $\widehat{\mathcal{W}}_t$

are all related and can be solved independently of monetary policy as a function of exogenous shocks only.

To complete the proof of Proposition 5 we thus need to show that net foreign assets $\widehat{\mathcal{B}}_t$ do not depend on monetary policy. This is straightforward, as by using the consumption Euler equation and substituting out the solution for the terms involving $\left(\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}_t^{fb} + \widehat{\Delta}_t\right)$, namely

$$E_t\left(\left(\widehat{\mathcal{T}}_{t+1} - \widehat{\mathcal{T}}_{t+1}^{fb} + \widehat{\Delta}_{t+1}\right) - \left(\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}_t^{fb} + \widehat{\Delta}_t\right)\right) = \\ -\left(1-\nu_1\right)\left(\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}_t^{fb} + \widehat{\Delta}_t\right) + \\ -\frac{\left(\beta\nu_2 - 1\right)}{\beta\nu_2}\widehat{\mathcal{W}}_t + \sum_{s=0}^{\infty}\nu_2^{-s-1}E_t\left[\left(\widehat{\mathcal{T}}_{t+s+2}^{fb} - \widehat{\mathcal{T}}_{t+s+1}^{fb}\right) - \beta^{-1}\left(\widehat{\mathcal{T}}_{t+s+1}^{fb} - \widehat{\mathcal{T}}_{t+s}^{fb}\right)\right],$$

we get the following difference equation for $\widehat{\mathcal{B}}_t$ that we can solve explicitly for NFAs independently of monetary policy:

$$\widehat{\mathcal{B}}_t - \widehat{\mathcal{B}}_{t-1} = -2a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\phi-1\right)\cdot \\ \left[\beta\sum_{j=0}^{\infty}\beta^j E_t\big[\left(\widehat{\mathcal{T}}_{t+j+1} - \widehat{\mathcal{T}}_{t+j+1}^{fb} + \widehat{\Delta}_{t+j+1}\right) - \left(\widehat{\mathcal{T}}_{t+j} - \widehat{\mathcal{T}}_{t+j}^{fb} + \widehat{\Delta}_{t+j}\right)\big]\right] - \\ \left(1-a_{\mathrm{H}}\right)\left[2a_{\mathrm{H}}\left(\phi-1\right)\right]\beta\sum_{j=0}^{\infty}\beta^j E_t\left[\left(\widehat{\mathcal{T}}_{t+j+1}^{fb} - \widehat{\mathcal{T}}_{t+j}^{fb}\right)\right] + \\ \left(1-a_{\mathrm{H}}\right)\beta\sum_{j=0}^{\infty}\beta^j E_t\left[\left(\widehat{\zeta}_{C,t+j+1} - \widehat{\zeta}_{C,t+j+1}^{*}\right) - \left(\widehat{\zeta}_{C,t+j} - \widehat{\zeta}_{C,t+j}^{*}\right)\right].$$

We can further simplify the latter expression using the above solutions for relative price misalignments; namely we have that for $j \geq 0$:

$$E_t\left(\widehat{\mathcal{T}}_{t+j} - \widehat{\mathcal{T}}_{t+j}^{fb} + \widehat{\Delta}_{t+j}\right) = \nu_1^{j+1}\left(\widehat{\mathcal{T}}_{t-1} - \widehat{\mathcal{T}}_{t-1}^{fb} + \widehat{\Delta}_{t-1}\right) - \frac{1-\nu_1^{j+1}}{1-\nu_1}\frac{\left(\beta\nu_2-1\right)}{\beta\nu_2}\widehat{\mathcal{W}}_t \\ + \sum_{s=0}^{j}\nu_1^{j-s}\left(\sum_{h=0}^{\infty}\nu_2^{-h-1}E_t\left[\left(\widehat{\mathcal{T}}_{t+h+s+1}^{fb} - \widehat{\mathcal{T}}_{t+h+s}^{fb}\right) - \beta^{-1}\left(\widehat{\mathcal{T}}_{t+h+s}^{fb} - \widehat{\mathcal{T}}_{t+h+s-1}^{fb}\right)\right]\right),$$

Putting the above together we can find the following solution for NFAs only as a function of exogenous shocks and $\widehat{\mathcal{W}}_t$, which is also independent of monetary policy:

$$\begin{aligned}
&\widehat{\mathcal{B}}_t - \widehat{\mathcal{B}}_{t-1} = 2a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\phi-1\right)\left(1-\nu_1\right)\cdot \\
&\qquad\qquad \beta\textstyle\sum_{j=0}^{\infty}\beta^j\nu_1^{j+1}\left(\widehat{\mathcal{T}}_{t-1}-\widehat{\mathcal{T}}_{t-1}^{fb}+\widehat{\Delta}_{t-1}\right)+ \\
&\beta\textstyle\sum_{j=0}^{\infty}\beta^j\left\{\sum_{s=0}^{j}\nu_1^{j-s}\left(\sum_{h=0}^{\infty}\nu_2^{-h-1}E_t\left[\begin{array}{c}\left(\widehat{\mathcal{T}}_{t+h+s+1}^{fb}-\widehat{\mathcal{T}}_{t+h+s}^{fb}\right)\\ -\beta^{-1}\left(\widehat{\mathcal{T}}_{t+h+s}^{fb}-\widehat{\mathcal{T}}_{t+h+s-1}^{fb}\right)\end{array}\right]\right)\right\} \\
&+2a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\phi-1\right)\frac{\left(\beta\nu_2-1\right)\nu_1}{\nu_2\left(1-\beta\nu_1\right)}\widehat{\mathcal{W}}_t \\
&-2a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\phi-1\right)\cdot \\
&\beta\sum_{j=0}^{\infty}\beta^j\left[\sum_{s=0}^{\infty}\nu_2^{-s-1}E_t\left[\left(\widehat{\mathcal{T}}_{t+j+s+2}^{fb}-\widehat{\mathcal{T}}_{t+j+s+1}^{fb}\right)-\beta^{-1}\left(\widehat{\mathcal{T}}_{t+j+s+1}^{fb}-\widehat{\mathcal{T}}_{t+j+s}^{fb}\right)\right]\right] \\
&-\left(1-a_{\mathrm{H}}\right)\left[2a_{\mathrm{H}}\left(\phi-1\right)\right]\beta\sum_{j=0}^{\infty}\beta^jE_t\left[\left(\widehat{\mathcal{T}}_{t+j+1}^{fb}-\widehat{\mathcal{T}}_{t+j}^{fb}\right)\right] \\
&+\left(1-a_{\mathrm{H}}\right)\beta\sum_{j=0}^{\infty}\beta^jE_t\left[\left(\widehat{\zeta}_{C,t+j+1}-\widehat{\zeta}_{C,t+j+1}^{*}\right)-\left(\widehat{\zeta}_{C,t+j}-\widehat{\zeta}_{C,t+j}^{*}\right)\right].
\end{aligned}$$

This completes the proof of Proposition 5.

3.1.3 Elasticity thresholds in Section 5.2

Here we derive the threshold shown in equation (39) of the text, that we reproduce below for convenience. Under LCP, for $\sigma = 1$ and $\eta = 0$, conditional on anticipated taste shocks, the wealth gap $\widetilde{\mathcal{W}}_t$ and $\widehat{\mathcal{B}}_t$ have the opposite sign if

$$\phi > \frac{2a_{\mathrm{H}} - \frac{\beta\nu_2}{(\beta\nu_2-1)}}{2a_{\mathrm{H}}} < 1,$$

As shown in Table 1 in the text, with $\sigma = 1$ and $\eta = 0$, the terms-of-trade response to *(current or anticipated) taste shocks* in the first-best allocation is $\widetilde{\mathcal{T}}_t^{fb} = 0$. So, the expressions in Table 5 simplify as follows:

$$\left(1-a_{\mathrm{H}}\right)\left[1+2a_{\mathrm{H}}\left(\phi-1\right)\tfrac{(\beta\nu_2-1)}{\beta\nu_2}\right]\widetilde{\mathcal{W}}_t = -\left(\widehat{\mathcal{B}}_t-\beta^{-1}\widehat{\mathcal{B}}_{t-1}\right)$$

$$\begin{aligned}
&\widehat{\mathcal{B}}_t-\widehat{\mathcal{B}}_{t-1} = \tfrac{2a_{\mathrm{H}}(\phi-1)\frac{(\beta\nu_2-1)\nu_1}{\nu_2(1-\beta\nu_1)}}{1+2a_{\mathrm{H}}(\phi-1)\frac{(\beta\nu_2-1)}{\beta\nu_2(1-\beta\nu_1)}}\left(\beta^{-1}\widehat{\mathcal{B}}_{t-1}-\widehat{\mathcal{B}}_{t-1}\right) \\
&+\left[\tfrac{1+2a_{\mathrm{H}}(\phi-1)\frac{(\beta\nu_2-1)}{\beta\nu_2}}{1+2a_{\mathrm{H}}(\phi-1)\frac{(\beta\nu_2-1)}{\beta\nu_2(1-\beta\nu_1)}}\right]\beta\textstyle\sum_{j=0}^{\infty}\beta^jE_t\left[\left(\widehat{\zeta}_{C,t+j+1}-\widehat{\zeta}_{C,t+j+1}^{*}\right)-\left(\widehat{\zeta}_{C,t+j}-\widehat{\zeta}_{C,t+j}^{*}\right)\right].
\end{aligned}$$

From this, it is easy to derive the threshold above. Note that the threshold is decreasing in openness ($a_{\mathrm{H}} \to 1/2, \phi \geq 0$) and the degree of price stickiness ($\nu_2 \to 1/\beta$, and $\frac{\beta\nu_2}{(\beta\nu_2-1)} \to 1, \phi \geq 0$), and is smaller than the threshold in the natural allocation.

Since first-best terms of trade $\widetilde{\mathcal{T}}_{t+s}^{fb}$ are different from zero for productivity shocks, deriving a threshold requires taking a stand on the term $\mathcal{Z}_t$ in Table 5, rewritten below:

$$\begin{array}{l}\mathcal{Z}_t = 2a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)(\phi-1)\sum_{j=0}^{\infty}\nu_2^{-j-1}E_t\left[\left(\widehat{\mathcal{T}}_{t+j+1}^{fb}-\widehat{\mathcal{T}}_{t+j}^{fb}\right)-\beta^{-1}\left(\widehat{\mathcal{T}}_{t+j}^{fb}-\widehat{\mathcal{T}}_{t+j-1}^{fb}\right)\right] \\ -2a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)(\phi-1)\left[\frac{1+2a_{\mathrm{H}}(\phi-1)\frac{(\beta\nu_2-1)}{\beta\nu_2}}{1+2a_{\mathrm{H}}(\phi-1)\frac{(\beta\nu_2-1)}{\beta\nu_2(1-\beta\nu_1)}}\right]\cdot\left\{\beta\sum_{j=0}^{\infty}\beta^jE_t\left[\left(\widehat{\mathcal{T}}_{t+j+1}^{fb}-\widehat{\mathcal{T}}_{t+j}^{fb}\right)\right]+\right. \\ \left.\sum_{j=0}^{\infty}\beta^j\left[\begin{array}{c}\sum_{s=0}^{\infty}\nu_2^{-s-1}E_t\left[\left(\widehat{\mathcal{T}}_{t+j+s+1}^{fb}-\widehat{\mathcal{T}}_{t+j+s}^{fb}\right)-\beta^{-1}\left(\widehat{\mathcal{T}}_{t+j+s}^{fb}-\widehat{\mathcal{T}}_{t+j+s-1}^{fb}\right)\right] \\ -\left(1-\nu_1\right)\beta\left[\sum_{s=0}^{j}\nu_1^{j-s}\left(\sum_{h=0}^{\infty}\nu_2^{-h-1}E_t\left[\begin{array}{c}\left(\widehat{\mathcal{T}}_{t+h+s+1}^{fb}-\widehat{\mathcal{T}}_{t+h+s}^{fb}\right) \\ -\beta^{-1}\left(\widehat{\mathcal{T}}_{t+h+s}^{fb}-\widehat{\mathcal{T}}_{t+h+s-1}^{fb}\right)\end{array}\right]\right)\right]\end{array}\right]\right\}.\end{array}$$

Specifically, $\widehat{\mathcal{B}}_t < 0$ only if $\mathcal{Z}_t < 0$, which in turn implies the following restrictions on parameters and productivity shocks:

$$\begin{array}{l}\sum_{j=0}^{\infty}\nu_2^{-j-1}E_t\left[\left(\widehat{\mathcal{T}}_{t+j+1}^{fb}-\widehat{\mathcal{T}}_{t+j}^{fb}\right)-\beta^{-1}\left(\widehat{\mathcal{T}}_{t+j}^{fb}-\widehat{\mathcal{T}}_{t+j-1}^{fb}\right)\right] < \\ \left[\frac{1+2a_{\mathrm{H}}(\phi-1)\frac{(\beta\nu_2-1)}{\beta\nu_2}}{1+2a_{\mathrm{H}}(\phi-1)\frac{(\beta\nu_2-1)}{\beta\nu_2(1-\beta\nu_1)}}\right]\cdot\left\{\beta\sum_{j=0}^{\infty}\beta^jE_t\left[\left(\widehat{\mathcal{T}}_{t+j+1}^{fb}-\widehat{\mathcal{T}}_{t+j}^{fb}\right)\right]\right. \\ \left.+\sum_{j=0}^{\infty}\beta^j\left[\begin{array}{c}\sum_{s=0}^{\infty}\nu_2^{-s-1}E_t\left[\left(\widehat{\mathcal{T}}_{t+j+s+1}^{fb}-\widehat{\mathcal{T}}_{t+j+s}^{fb}\right)-\beta^{-1}\left(\widehat{\mathcal{T}}_{t+j+s}^{fb}-\widehat{\mathcal{T}}_{t+j+s-1}^{fb}\right)\right] \\ -\left(1-\nu_1\right)\beta\left[\sum_{s=0}^{j}\nu_1^{j-s}\left(\sum_{h=0}^{\infty}\nu_2^{-h-1}E_t\left[\begin{array}{c}\left(\widehat{\mathcal{T}}_{t+h+s+1}^{fb}-\widehat{\mathcal{T}}_{t+h+s}^{fb}\right) \\ -\beta^{-1}\left(\widehat{\mathcal{T}}_{t+h+s}^{fb}-\widehat{\mathcal{T}}_{t+h+s-1}^{fb}\right)\end{array}\right]\right)\right]\end{array}\right]\right\}.\end{array}$$

Using the expression for $\widetilde{\mathcal{W}}_t$ in Table 5, we can deriving an expression highlighting the conditions under which a capital inflows due to anticipated productivity shocks lead to a positive or a negative wealth gap:

$$\begin{array}{l}\left(1-a_{\mathrm{H}}\right)\widetilde{\mathcal{W}}_t = \left[\dfrac{2a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)(\phi-1)}{1+2a_{\mathrm{H}}(\phi-1)\frac{(\beta\nu_2-1)}{\beta\nu_2(1-\beta\nu_1)}}\right]\cdot\left\{\beta\displaystyle\sum_{j=0}^{\infty}\beta^jE_t\left[\left(\widehat{\mathcal{T}}_{t+j+1}^{fb}-\widehat{\mathcal{T}}_{t+j}^{fb}\right)\right]+\right. \\ \left.+\displaystyle\sum_{j=0}^{\infty}\beta^j\left[\begin{array}{c}\sum_{s=0}^{\infty}\nu_2^{-s-1}E_t\left[\left(\widehat{\mathcal{T}}_{t+j+s+1}^{fb}-\widehat{\mathcal{T}}_{t+j+s}^{fb}\right)-\beta^{-1}\left(\widehat{\mathcal{T}}_{t+j+s}^{fb}-\widehat{\mathcal{T}}_{t+j+s-1}^{fb}\right)\right] \\ -\left(1-\nu_1\right)\beta\left[\sum_{s=0}^{j}\nu_1^{j-s}\left(\sum_{h=0}^{\infty}\nu_2^{-h-1}E_t\left[\begin{array}{c}\left(\widehat{\mathcal{T}}_{t+h+s+1}^{fb}-\widehat{\mathcal{T}}_{t+h+s}^{fb}\right) \\ -\beta^{-1}\left(\widehat{\mathcal{T}}_{t+h+s}^{fb}-\widehat{\mathcal{T}}_{t+h+s-1}^{fb}\right)\end{array}\right]\right)\right]\end{array}\right]\right\}.\end{array}$$

Provided that $\frac{2a_{\mathrm{H}}(1-a_{\mathrm{H}})(\phi-1)}{1+2a_{\mathrm{H}}(\phi-1)\frac{(\beta\nu_2-1)}{\beta\nu_2(1-\beta\nu_1)}} > 0$ (which is the case for $\phi > 1$ and $\phi < 1-\frac{\beta\nu_2(1-\beta\nu_1)}{2a_{\mathrm{H}}(\beta\nu_2-1)} < 1$), the sign of $\widetilde{\mathcal{W}}_t$ depends on the sign of the expression in curly brackets on the right hand side. For the parameterization on the second column of Figure 2, with $\phi = 0.3$, we are assuming that the expression in curly bracket is negative.

3.1.4 Constrained optimal allocation under LCP and proof of Proposition 9

In order to derive the optimal allocation, consider again the difference of the sum of the within-country NKPC with $\sigma = 1$ and $\eta = 0$:

$$\begin{gathered} a_{\mathrm{H}}\pi_{H,t} + (1 - a_{\mathrm{H}})\,\pi_{F,t} - \left(a_{\mathrm{H}}\pi^*_{F,t} + (1 - a_{\mathrm{H}})\,\pi^*_{H,t}\right) - \\ \beta E_t\left(a_{\mathrm{H}}\pi_{H,t+1} + (1 - a_{\mathrm{H}})\,\pi_{F,t+1}\right) - \beta E_t\left(a_{\mathrm{H}}\pi^*_{F,t+1} + (1 - a_{\mathrm{H}})\,\pi^*_{H,t+1}\right) = \\ = \frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\left\{\widehat{\mathcal{Q}}_t - \widehat{\mathcal{Q}}^{fb}_t + (2a_{\mathrm{H}} - 1)\left[\widehat{\mu}_t - \widehat{\mu}^*_t + \widehat{\mathcal{W}}_t\right]\right\}. \end{gathered}$$

We next substitute the relative target rule and derive a difference equation in the misalignment and demand gaps:

$$\begin{gathered} a_{\mathrm{H}}\pi_{H,t} + (1 - a_{\mathrm{H}})\,\pi_{F,t} - \left(a_{\mathrm{H}}\pi^*_{F,t} + (1 - a_{\mathrm{H}})\,\pi^*_{H,t}\right) - \beta E_t\left(a_{\mathrm{H}}\pi_{H,t+1} + (1 - a_{\mathrm{H}})\,\pi_{F,t+1}\right) - \\ \beta E_t\left(a_{\mathrm{H}}\pi^*_{F,t+1} + (1 - a_{\mathrm{H}})\,\pi^*_{H,t+1}\right) = \\ \theta^{-1}\Bigg\{\beta E_t\left[\left(\widehat{\mathcal{W}}_{t+1} - \widehat{\mathcal{W}}_t\right) + \left(\widehat{\mathcal{Q}}_{t+1} - \widehat{\mathcal{Q}}^{fb}_{t+1}\right) - \left(\widehat{\mathcal{Q}}_t - \widehat{\mathcal{Q}}^{fb}_t\right)\right] - \\ \left(\widehat{\mathcal{W}}_t - \widehat{\mathcal{W}}_{t-1}\right) - \left[\left(\widehat{\mathcal{Q}}_t - \widehat{\mathcal{Q}}^{fb}_t\right) - \left(\widehat{\mathcal{Q}}_{t-1} - \widehat{\mathcal{Q}}^{fb}_{t-1}\right)\right]\Bigg\} = \\ = \frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\left\{\widehat{\mathcal{Q}}_t - \widehat{\mathcal{Q}}^{fb}_t + (2a_{\mathrm{H}} - 1)\,\widehat{\mathcal{W}}_t\right\}. \end{gathered}$$

The equation admits the following solution as a function of both current and future values of $\widehat{\mathcal{W}}_t$:

$$\begin{aligned} \left(\widehat{\mathcal{Q}}_t - \widehat{\mathcal{Q}}^{fb}_t\right) &= \varkappa_1\left(\widehat{\mathcal{Q}}_{t-1} - \widehat{\mathcal{Q}}^{fb}_{t-1}\right) - \frac{1}{\beta\varkappa_2}\sum_{j=0}^{\infty}\varkappa_2{}^{-j}E_t\left(\widehat{\mathcal{W}}_{t+j} - \widehat{\mathcal{W}}_{t+j-1}\right) + \\ &\quad - (2a_{\mathrm{H}} - 1)\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\frac{\theta}{\beta\varkappa_2}\sum_{j=0}^{\infty}\varkappa_2^{-j}E_t\widehat{\mathcal{W}}_{t+j}. \end{aligned}$$

where $0 < \varkappa_1 < \beta < 1 < \beta^{-1} < \varkappa_2$ are the eigenvalues of the difference equation, solving the standard characteristic equation:

$$\beta\varkappa^2 - \left[1 + \beta + \frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\theta\right]\varkappa + 1 = 0,$$

namely

$$\varkappa_{1,2} = \frac{1 + \beta + \frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\theta \pm \sqrt{\left[1 + \beta + \frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\theta\right]^2 - 4\beta}}{2\beta}.$$

As in the PCP case, we can simplify further by using the law of motion for the wealth gap $\widehat{\mathcal{W}}_t$, $E_t\widehat{\mathcal{W}}_{t+j} = \widehat{\mathcal{W}}_t$:

$$\begin{aligned}\left(\widehat{\mathcal{Q}}_{t+j} - \widehat{\mathcal{Q}}^{fb}_{t+j}\right) &= \varkappa_1 \left(\widehat{\mathcal{Q}}_{t+j-1} - \widehat{\mathcal{Q}}^{fb}_{t+j-1}\right) - \frac{1}{\beta\varkappa_2}\left(\widehat{\mathcal{W}}_{t+j} - \widehat{\mathcal{W}}_{t+j-1}\right) + \\ &\quad - (2a_{\mathrm{H}} - 1)\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\frac{\theta}{\beta(\varkappa_2 - 1)}\widehat{\mathcal{W}}_t.\end{aligned}$$

The first term $\widehat{\mathcal{W}}_{t+j} - \widehat{\mathcal{W}}_{t+j-1} = 0$ for $j \geq 1$, while it is equal to $\widehat{\mathcal{W}}_t$ for $j = 0$; instead the last term represents a constant shifter proportional to $\widehat{\mathcal{W}}_t$ for any $j \geq 0$. Furthermore, recalling that,

$$\begin{gathered}\left[\widehat{\mathcal{W}}_t - \widehat{\mathcal{W}}_{t-1}\right] + \left[\left(\widehat{\mathcal{Q}}_t - \widehat{\mathcal{Q}}^{fb}_t\right) - \left(\widehat{\mathcal{Q}}_{t-1} - \widehat{\mathcal{Q}}^{fb}_{t-1}\right)\right] = \\ \left[\left(\widehat{C}_t - \widehat{C}^*_t\right) - \left(\widehat{C}^{fb}_t - \widehat{C}^{*fb}_t\right)\right] - \left[\left(\widehat{C}_{t-1} - \widehat{C}^*_{t-1}\right) - \left(\widehat{C}^{fb}_{t-1} - \widehat{C}^{*fb}_{t-1}\right)\right],\end{gathered}$$

we have that inefficient deviations in cross-country consumption differentials (and thus in CPI inflation) are given by:

$$\begin{gathered}\left[\left(\widehat{C}_t - \widehat{C}^*_t\right) - \left(\widehat{C}^{fb}_t - \widehat{C}^{*fb}_t\right)\right] - \left[\left(\widehat{C}_{t-1} - \widehat{C}^*_{t-1}\right) - \left(\widehat{C}^{fb}_{t-1} - \widehat{C}^{*fb}_{t-1}\right)\right] = \\ \left(\frac{\beta\delta_2 - 1}{\beta\delta_2}\right)\left(\widehat{\mathcal{W}}_t - \widehat{\mathcal{W}}_{t-1}\right) - (2a_{\mathrm{H}} - 1)\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\frac{\theta}{\beta(\delta_2 - 1)}\widehat{\mathcal{W}}_t \\ - (1-\delta_1)\left(\widehat{\mathcal{Q}}_{t-1} - \widehat{\mathcal{Q}}^{fb}_{t-1}\right) = \\ -2\theta\left(a_{\mathrm{H}}\pi_{H,t} + (1-a_{\mathrm{H}})\pi_{F,t}\right),\end{gathered}$$

which, interestingly, does not depend on the trade elasticity ϕ.

Thus, we also reach a solution for the deviations from the law of one price:

$$\widehat{\Delta}_t = \left(\widehat{\mathcal{Q}}_t - \widehat{\mathcal{Q}}^{fb}_t\right) - (2a_{\mathrm{H}} - 1)\left(\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}^{fb}_t + \widehat{\Delta}_t\right).$$

Finally, we can solve for the permanent response of $\widehat{\mathcal{W}}_t$ as a function only of exogenous shocks:

$$\begin{aligned}(1-a_{\mathrm{H}})\widehat{\mathcal{W}}_t &= \beta^{-1}\left(\widehat{\mathcal{B}}_{t-1} - \beta\widehat{\mathcal{B}}_t\right) + \\ &\quad 2a_{\mathrm{H}}(1-a_{\mathrm{H}})(\phi - 1)\left(\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}^{fb}_t + \widehat{\Delta}_t\right) + \\ &\quad (1-a_{\mathrm{H}})\left[2a_{\mathrm{H}}(\phi-1)\widehat{\mathcal{T}}^{fb}_t - \left(\widehat{\zeta}_{C,t} - \widehat{\zeta}^*_{C,t}\right)\right].\end{aligned}$$

Using again

$$\begin{aligned}\left(\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}^{fb}_t + \widehat{\Delta}_t\right) &= \nu_1\left(\widehat{\mathcal{T}}_{t-1} - \widehat{\mathcal{T}}^{fb}_{t-1} + \widehat{\Delta}_{t-1}\right) - \frac{(\beta\nu_2 - 1)}{\beta\nu_2}\widehat{\mathcal{W}}_t + \\ &\quad \sum_{s=0}^{\infty}\nu_2^{-s-1}E_t\left[\left(\widehat{\mathcal{T}}^{fb}_{t+s+1} - \widehat{\mathcal{T}}^{fb}_{t+s}\right) - \beta^{-1}\left(\widehat{\mathcal{T}}^{fb}_{t+s} - \widehat{\mathcal{T}}^{fb}_{t+s-1}\right)\right],\end{aligned}$$

$$
\begin{aligned}
&(1-a_{\mathrm{H}})\left[1+2a_{\mathrm{H}}(\phi-1)\frac{(\beta\nu_2-1)}{\beta\nu_2}\right]\widehat{\mathcal{W}}_t = \\
&\left(\beta^{-1}\widehat{\mathcal{B}}_{t-1}-\widehat{\mathcal{B}}_t\right)+ \\
&2a_{\mathrm{H}}(1-a_{\mathrm{H}})(\phi-1)\sum_{s=0}^{\infty}\nu_2^{-s-1}E_t\left[\left(\widehat{\mathcal{T}}^{fb}_{t+s+1}-\widehat{\mathcal{T}}^{fb}_{t+s}\right)-\beta^{-1}\left(\widehat{\mathcal{T}}^{fb}_{t+s}-\widehat{\mathcal{T}}^{fb}_{t+s-1}\right)\right]- \\
&(1-a_{\mathrm{H}})\left[2a_{\mathrm{H}}(\phi-1)\widehat{\mathcal{T}}^{fb}_t-\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}^{*}_{C,t}\right)\right].
\end{aligned}
$$

Recalling the solution for capital flows

$$
\begin{aligned}
&\widehat{\mathcal{B}}_t-\widehat{\mathcal{B}}_{t-1}= \\
&2a_{\mathrm{H}}(1-a_{\mathrm{H}})(\phi-1)(1-\nu_1)\left\{\beta\sum_{j=0}^{\infty}\beta^j\nu_1^{j+1}\left(\widehat{\mathcal{T}}_{t-1}-\widehat{\mathcal{T}}^{fb}_{t-1}+\widehat{\Delta}_{t-1}\right)+\right. \\
&\left.\beta\sum_{j=0}^{\infty}\beta^j\sum_{s=0}^{j}\nu_1^{j-s}\sum_{h=0}^{\infty}\nu_2^{-h-1}E_t\left[\begin{array}{c}\left(\widehat{\mathcal{T}}^{fb}_{t+h+s+1}-\widehat{\mathcal{T}}^{fb}_{t+h+s}\right)\\ -\beta^{-1}\left(\widehat{\mathcal{T}}^{fb}_{t+h+s}-\widehat{\mathcal{T}}^{fb}_{t+h+s-1}\right)\end{array}\right]\right\}+ \\
&2a_{\mathrm{H}}(1-a_{\mathrm{H}})(\phi-1)\frac{(\beta\nu_2-1)\nu_1}{\nu_2(1-\beta\nu_1)}\widehat{\mathcal{W}}_t- \\
&2a_{\mathrm{H}}(1-a_{\mathrm{H}})(\phi-1)\beta\sum_{j=0}^{\infty}\beta^j\sum_{s=0}^{\infty}\nu_2^{-s-1}E_t\left[\begin{array}{c}\left(\widehat{\mathcal{T}}^{fb}_{t+j+s+2}-\widehat{\mathcal{T}}^{fb}_{t+j+s+1}\right)-\\ \beta^{-1}\left(\widehat{\mathcal{T}}^{fb}_{t+j+s+1}-\widehat{\mathcal{T}}^{fb}_{t+j+s}\right)\end{array}\right]- \\
&(1-a_{\mathrm{H}})\left[2a_{\mathrm{H}}(\phi-1)\right]\beta\sum_{j=0}^{\infty}\beta^jE_t\left[\left(\widehat{\mathcal{T}}^{fb}_{t+j+1}-\widehat{\mathcal{T}}^{fb}_{t+j}\right)\right]+ \\
&(1-a_{\mathrm{H}})\beta\sum_{j=0}^{\infty}\beta^jE_t\left[\left(\widehat{\zeta}_{C,t+j+1}-\widehat{\zeta}^{*}_{C,t+j+1}\right)-\left(\widehat{\zeta}_{C,t+j}-\widehat{\zeta}^{*}_{C,t+j}\right)\right].
\end{aligned}
$$

$$
\begin{gathered}
\left(1-a_{\mathrm{H}}\right)\left[1+2a_{\mathrm{H}}\left(\phi-1\right)\frac{\left(\beta\nu_{2}-1\right)}{\beta\nu_{2}\left(1-\beta\nu_{1}\right)}\right]\widehat{\mathcal{W}}_{t}= \\
\beta^{-1}\widehat{\mathcal{B}}_{t-1}-\widehat{\mathcal{B}}_{t-1}+2a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\phi-1\right)\beta\sum_{j=0}^{\infty}\beta^{j}\cdot \\
\left\{\begin{array}{c}
\sum_{s=0}^{\infty}\nu_{2}^{-s-1}E_{t}\Big[\left(\widehat{\mathcal{T}}_{t+j+s+2}^{fb}-\widehat{\mathcal{T}}_{t+j+s+1}^{fb}\right)- \\
\beta^{-1}\left(\widehat{\mathcal{T}}_{t+j+s+1}^{fb}-\widehat{\mathcal{T}}_{t+j+s}^{fb}\right)\Big]- \\
\left(1-\nu_{1}\right)\sum_{s=0}^{j}\nu_{1}^{j-s}\sum_{h=0}^{\infty}\nu_{2}^{-h-1}E_{t}\Big[\left(\widehat{\mathcal{T}}_{t+h+s+1}^{fb}-\widehat{\mathcal{T}}_{t+h+s}^{fb}\right)- \\
\beta^{-1}\left(\widehat{\mathcal{T}}_{t+h+s}^{fb}-\widehat{\mathcal{T}}_{t+h+s-1}^{fb}\right)\Big]
\end{array}\right\} \\
+\left(1-a_{\mathrm{H}}\right)\beta\sum_{j=0}^{\infty}\beta^{j}\left\{\begin{array}{c}
2a_{\mathrm{H}}\left(\phi-1\right)E_{t}\left[\left(\widehat{\mathcal{T}}_{t+j+1}^{fb}-\widehat{\mathcal{T}}_{t+j}^{fb}\right)\right]+ \\
-E_{t}\left[\left(\widehat{\zeta}_{C,t+j+1}-\widehat{\zeta}_{C,t+j+1}^{*}\right)-\left(\widehat{\zeta}_{C,t+j}-\widehat{\zeta}_{C,t+j}^{*}\right)\right]
\end{array}\right\} \\
+2a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\phi-1\right)\sum_{s=0}^{\infty}\nu_{2}^{-s-1}E_{t}\left[\left(\widehat{\mathcal{T}}_{t+s+1}^{fb}-\widehat{\mathcal{T}}_{t+s}^{fb}\right)-\beta^{-1}\left(\widehat{\mathcal{T}}_{t+s}^{fb}-\widehat{\mathcal{T}}_{t+s-1}^{fb}\right)\right] \\
+\left(1-a_{\mathrm{H}}\right)\left[2a_{\mathrm{H}}\left(\phi-1\right)\widehat{\mathcal{T}}_{t}^{fb}-\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}_{C,t}^{*}\right)\right].
\end{gathered}
$$

$$
\begin{gathered}
2a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\phi-1\right)\frac{\left(\beta\nu_{2}-1\right)\nu_{1}}{\nu_{2}\left(1-\beta\nu_{1}\right)}\widehat{\mathcal{W}}_{t}= \\
\frac{2a_{\mathrm{H}}\left(\phi-1\right)\frac{\left(\beta\nu_{2}-1\right)\nu_{1}}{\nu_{2}\left(1-\beta\nu_{1}\right)}}{1+2a_{\mathrm{H}}\left(\phi-1\right)\frac{\left(\beta\nu_{2}-1\right)}{\beta\nu_{2}\left(1-\beta\nu_{1}\right)}}\Bigg\{\beta^{-1}\widehat{\mathcal{B}}_{t-1}-\widehat{\mathcal{B}}_{t-1}+2a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\phi-1\right)\beta\cdot \\
\sum_{j=0}^{\infty}\beta^{j}\left[\begin{array}{c}
\sum_{s=0}^{\infty}\nu_{2}^{-s-1}E_{t}\left[\begin{array}{c}\left(\widehat{\mathcal{T}}_{t+j+s+2}^{fb}-\widehat{\mathcal{T}}_{t+j+s+1}^{fb}\right)- \\ \beta^{-1}\left(\widehat{\mathcal{T}}_{t+j+s+1}^{fb}-\widehat{\mathcal{T}}_{t+j+s}^{fb}\right)\end{array}\right]- \\
\left(1-\nu_{1}\right)\sum_{s=0}^{j}\nu_{1}^{j-s}\left(\sum_{h=0}^{\infty}\nu_{2}^{-h-1}E_{t}\left[\begin{array}{c}\left(\widehat{\mathcal{T}}_{t+h+s+1}^{fb}-\widehat{\mathcal{T}}_{t+h+s}^{fb}\right)- \\ \beta^{-1}\left(\widehat{\mathcal{T}}_{t+h+s}^{fb}-\widehat{\mathcal{T}}_{t+h+s-1}^{fb}\right)\end{array}\right]\right)
\end{array}\right]+ \\
2a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\phi-1\right)\sum_{s=0}^{\infty}\nu_{2}^{-s-1}E_{t}\left[\begin{array}{c}\left(\widehat{\mathcal{T}}_{t+s+1}^{fb}-\widehat{\mathcal{T}}_{t+s}^{fb}\right)- \\ \beta^{-1}\left(\widehat{\mathcal{T}}_{t+s}^{fb}-\widehat{\mathcal{T}}_{t+s-1}^{fb}\right)\end{array}\right]+ \\
\left(1-a_{\mathrm{H}}\right)\beta\sum_{j=0}^{\infty}\beta^{j}\left[\begin{array}{c}2a_{\mathrm{H}}\left(\phi-1\right)E_{t}\left[\left(\widehat{\mathcal{T}}_{t+j+1}^{fb}-\widehat{\mathcal{T}}_{t+j}^{fb}\right)\right]- \\ E_{t}\left[\left(\widehat{\zeta}_{C,t+j+1}-\widehat{\zeta}_{C,t+j+1}^{*}\right)-\left(\widehat{\zeta}_{C,t+j}-\widehat{\zeta}_{C,t+j}^{*}\right)\right]\end{array}\right]+ \\
\left(1-a_{\mathrm{H}}\right)\left[2a_{\mathrm{H}}\left(\phi-1\right)\widehat{\mathcal{T}}_{t}^{fb}-\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}_{C,t}^{*}\right)\right]\Bigg\}.
\end{gathered}
$$

$$
\begin{aligned}
&\widehat{\mathcal{B}}_t - \widehat{\mathcal{B}}_{t-1} = \\
&2a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\phi-1\right)\left(1-\nu_1\right)\beta\sum_{j=0}^{\infty}\beta^j\nu_1^{j+1}\left(\widehat{\mathcal{T}}_{t-1}-\widehat{\mathcal{T}}_{t-1}^{fb}+\widehat{\Delta}_{t-1}\right)+ \\
&\frac{2a_{\mathrm{H}}\left(\phi-1\right)\frac{(\beta\nu_2-1)\nu_1}{\nu_2(1-\beta\nu_1)}}{1+2a_{\mathrm{H}}\left(\phi-1\right)\frac{(\beta\nu_2-1)}{\beta\nu_2(1-\beta\nu_1)}}\left(\beta^{-1}\widehat{\mathcal{B}}_{t-1}-\widehat{\mathcal{B}}_{t-1}\right)- \\
&\left[\frac{1+2a_{\mathrm{H}}\left(\phi-1\right)\frac{(\beta\nu_2-1)}{\beta\nu_2}}{1+2a_{\mathrm{H}}\left(\phi-1\right)\frac{(\beta\nu_2-1)}{\beta\nu_2(1-\beta\nu_1)}}\right]2a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\phi-1\right)\cdot \\
&\sum_{j=0}^{\infty}\beta^j\left\{\begin{array}{c}\sum_{s=0}^{\infty}\nu_2^{-s-1}E_t\left[\begin{array}{c}\left(\widehat{\mathcal{T}}_{t+j+s+1}^{fb}-\widehat{\mathcal{T}}_{t+j+s}^{fb}\right)- \\ \beta^{-1}\left(\widehat{\mathcal{T}}_{t+j+s}^{fb}-\widehat{\mathcal{T}}_{t+j+s-1}^{fb}\right)\end{array}\right]- \\ \left(1-\nu_1\right)\beta\sum_{s=0}^{j}\nu_1^{j-s}\sum_{h=0}^{\infty}\nu_2^{-h-1}E_t\left[\begin{array}{c}\left(\widehat{\mathcal{T}}_{t+h+s+1}^{fb}-\widehat{\mathcal{T}}_{t+h+s}^{fb}\right)- \\ \beta^{-1}\left(\widehat{\mathcal{T}}_{t+h+s}^{fb}-\widehat{\mathcal{T}}_{t+h+s-1}^{fb}\right)\end{array}\right]\end{array}\right\}- \\
&\left[\frac{1+2a_{\mathrm{H}}\left(\phi-1\right)\frac{(\beta\nu_2-1)}{\beta\nu_2}}{1+2a_{\mathrm{H}}\left(\phi-1\right)\frac{(\beta\nu_2-1)}{\beta\nu_2(1-\beta\nu_1)}}\right]\left(1-a_{\mathrm{H}}\right)\beta\cdot \\
&\sum_{j=0}^{\infty}\beta^j\left[\begin{array}{c}2a_{\mathrm{H}}\left(\phi-1\right)E_t\left[\left(\widehat{\mathcal{T}}_{t+j+1}^{fb}-\widehat{\mathcal{T}}_{t+j}^{fb}\right)\right]- \\ E_t\left[\left(\widehat{\zeta}_{C,t+j+1}-\widehat{\zeta}_{C,t+j+1}^{*}\right)-\left(\widehat{\zeta}_{C,t+j}-\widehat{\zeta}_{C,t+j}^{*}\right)\right]\end{array}\right]+ \\
&2a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\phi-1\right)\sum_{s=0}^{\infty}\nu_2^{-s-1}E_t\left[\begin{array}{c}\left(\widehat{\mathcal{T}}_{t+s+1}^{fb}-\widehat{\mathcal{T}}_{t+s}^{fb}\right)- \\ \beta^{-1}\left(\widehat{\mathcal{T}}_{t+s}^{fb}-\widehat{\mathcal{T}}_{t+s-1}^{fb}\right)\end{array}\right]+ \\
&\frac{2a_{\mathrm{H}}\left(\phi-1\right)\frac{(\beta\nu_2-1)\nu_1}{\nu_2(1-\beta\nu_1)}}{1+2a_{\mathrm{H}}\left(\phi-1\right)\frac{(\beta\nu_2-1)}{\beta\nu_2(1-\beta\nu_1)}}\left(1-a_{\mathrm{H}}\right)\left[2a_{\mathrm{H}}\left(\phi-1\right)\widehat{\mathcal{T}}_t^{fb}-\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}_{C,t}^{*}\right)\right].
\end{aligned}
$$

Furthermore,

$$(1-a_{\rm H})\left[1+2a_{\rm H}(\phi-1)\frac{(\beta\nu_2-1)}{\beta\nu_2(1-\beta\nu_1)}\right]\widehat{\mathcal{W}}_t =$$

$$\beta^{-1}\widehat{\mathcal{B}}_{t-1}-\widehat{\mathcal{B}}_{t-1}+2a_{\rm H}(1-a_{\rm H})(\phi-1)\sum_{j=0}^{\infty}\beta^j\cdot$$

$$\left\{\begin{array}{c} \sum_{s=0}^{\infty}\nu_2^{-s-1}E_t\left[\begin{array}{c}\left(\widehat{\mathcal{T}}^{fb}_{t+j+s+1}-\widehat{\mathcal{T}}^{fb}_{t+j+s}\right)-\\ \beta^{-1}\left(\widehat{\mathcal{T}}^{fb}_{t+j+s}-\widehat{\mathcal{T}}^{fb}_{t+j+s-1}\right)\end{array}\right]- \\ (1-\nu_1)\beta\sum_{s=0}^{j}\nu_1^{j-s}\sum_{h=0}^{\infty}\nu_2^{-h-1}E_t\left[\begin{array}{c}\left(\widehat{\mathcal{T}}^{fb}_{t+h+s+1}-\widehat{\mathcal{T}}^{fb}_{t+h+s}\right)-\\ \beta^{-1}\left(\widehat{\mathcal{T}}^{fb}_{t+h+s}-\widehat{\mathcal{T}}^{fb}_{t+h+s-1}\right)\end{array}\right]\end{array}\right\}$$

$$(1-a_{\rm H})\beta\sum_{j=0}^{\infty}\beta^j\left\{\begin{array}{c}2a_{\rm H}(\phi-1)E_t\left[\left(\widehat{\mathcal{T}}^{fb}_{t+j+1}-\widehat{\mathcal{T}}^{fb}_{t+j}\right)\right]-\\ E_t\left[\left(\widehat{\zeta}_{C,t+j+1}-\widehat{\zeta}^*_{C,t+j+1}\right)-\left(\widehat{\zeta}_{C,t+j}-\widehat{\zeta}^*_{C,t+j}\right)\right]\end{array}\right\}+$$

$$(1-a_{\rm H})\left[2a_{\rm H}(\phi-1)\widehat{\mathcal{T}}^{fb}_t-\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}^*_{C,t}\right)\right].$$

Lastly, we derive the link between the demand gap and capital flows shown in Section 5 in the main text:

$$(1-a_{\rm H})\left[1+2a_{\rm H}(\phi-1)\frac{(\beta\nu_2-1)}{\beta\nu_2}\right]\widehat{\mathcal{W}}_t=-\widehat{\mathcal{B}}_t+$$

$$(1-a_{\rm H})2a_{\rm H}(\phi-1)\sum_{s=0}^{\infty}\nu_2^{-s-1}E_t\left[\left(\widehat{\mathcal{T}}^{fb}_{t+s+1}-\widehat{\mathcal{T}}^{fb}_{t+s}\right)-\beta^{-1}\left(\widehat{\mathcal{T}}^{fb}_{t+s}-\widehat{\mathcal{T}}^{fb}_{t+s-1}\right)\right]+$$

$$\left[\frac{1+2a_{\rm H}(\phi-1)\frac{(\beta\nu_2-1)\nu_1}{\nu_2(1-\beta\nu_1)}}{1+2a_{\rm H}(\phi-1)\frac{(\beta\nu_2-1)}{\beta\nu_2(1-\beta\nu_1)}}\right](1-a_{\rm H})\left[2a_{\rm H}(\phi-1)\widehat{\mathcal{T}}^{fb}_t-\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}^*_{C,t}\right)\right].$$

Proof of Proposition 9: Derivation of the output gap.

We can finally derive the output gap under the constrained optimal allocation as follows:

$$\left(\widehat{Y}_{H,t}-\widehat{Y}^{fb}_{H,t}\right)-\left(\widehat{Y}_{F,t}-\widehat{Y}^{fb}_{F,t}\right)=2\left(\widehat{Y}_{H,t}-\widehat{Y}^{fb}_{H,t}\right)=$$

$$=\left[4a_{\rm H}(1-a_{\rm H})(\phi-1)+1\right]\left(\widehat{\mathcal{T}}_t-\widehat{\mathcal{T}}^{fb}_t+\widehat{\Delta}_t\right)+(2a_{\rm H}-1)\left(\widehat{\mathcal{W}}_t+\widehat{\Delta}_t\right)$$

$$=4a_{\rm H}(1-a_{\rm H})\phi\left(\widehat{\mathcal{T}}_t-\widehat{\mathcal{T}}^{fb}_t+\widehat{\Delta}_t\right)+(2a_{\rm H}-1)\left(\widehat{\mathcal{W}}_t+\left(\widehat{\mathcal{Q}}_t-\widehat{\mathcal{Q}}^{fb}_t\right)\right),$$

namely:

$$2\left(\widehat{Y}_{H,t}-\widehat{Y}^{fb}_{H,t}\right)=$$

$$(2a_{\rm H}-1)\left(\widehat{\mathcal{W}}_t+\left(\widehat{\mathcal{Q}}_t-\widehat{\mathcal{Q}}^{fb}_t\right)\right)-4a_{\rm H}(1-a_{\rm H})\phi\frac{(\beta\nu_2-1)}{\beta\nu_2}\widehat{\mathcal{W}}_t\cdot$$

$$4a_{\rm H}(1-a_{\rm H})\phi\left[\begin{array}{c}\sum_{j=0}^{\infty}\nu_2^{-j-1}E_t\left[\left(\widehat{\mathcal{T}}^{fb}_{t+j+1}-\widehat{\mathcal{T}}^{fb}_{t+j}\right)-\beta^{-1}\left(\widehat{\mathcal{T}}^{fb}_{t+j}-\widehat{\mathcal{T}}^{fb}_{t+j-1}\right)\right]+\\ \nu_1\left(\widehat{\mathcal{T}}_{t-1}-\widehat{\mathcal{T}}^{fb}_{t-1}+\widehat{\Delta}_{t-1}\right)\end{array}\right]$$

This completes the derivation of the output gap in Proposition 9.

3.2 PCP economy

3.2.1 Proof Proposition 6

The PCP loss function is given by (1) subject to $(\widehat{\Delta}_t) = 0$ and $\left[a_{\mathrm{H}}\pi^2_{H,t} + (1-a_{\mathrm{H}})\,\pi^{*2}_{H,t} + a_{\mathrm{H}}\pi^{*2}_{F,t} + (1-a_{\mathrm{H}})\,\pi^2_{F,t}\right] = \pi^2_{H,t} + \pi^{*2}_{F,t}$. Under PCP optimal monetary policy minimizes the loss function subject to:

1. NK Phillips curves determining inflation rates

$$\begin{aligned}\pi_{H,t} &= \beta E_t \pi_{H,t+1} + \frac{(1-\alpha\beta)(1-\alpha)}{\alpha} \\ &\cdot \left\{ \begin{array}{c} (\eta+\sigma)\left(\widehat{Y}_{H,t} - \widehat{Y}^{fb}_{H,t}\right) + \widehat{\mu}_t + \\ -(1-a_{\mathrm{H}})\cdot\left[2a_{\mathrm{H}}(\sigma\phi-1)\left(\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}^{fb}_t\right) - \widetilde{\mathcal{W}}_t\right] \end{array} \right\}\end{aligned}$$

$$\begin{aligned}\pi^*_{F,t} &= \beta E_t \pi^*_{F,t+1} + \frac{(1-\alpha\beta)(1-\alpha)}{\alpha} \\ &\cdot \left\{ \begin{array}{c} (\eta+\sigma)\left(\widehat{Y}_{F,t} - \widehat{Y}^{fb}_{F,t}\right) + \widehat{\mu}^*_t + \\ +(1-a_{\mathrm{H}})\cdot\left[2a_{\mathrm{H}}(\sigma\phi-1)\left(\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}^{fb}_t\right) - \widetilde{\mathcal{W}}_t\right] \end{array} \right\},\end{aligned}$$

where the equilibrium relations for first best outcomes $\widehat{Y}^{fb}_{H,t}$, $\widehat{Y}^{fb}_{F,t}$, $\widehat{\mathcal{T}}^{fb}_t$ in terms of fundamental shocks are as follows:

$$\begin{aligned}&(\eta+\sigma)\,\widehat{Y}^{fb}_{H,t} = \\ &[2a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)]\left(\widehat{\mathcal{T}}^{fb}_t\right) - (1-a_{\mathrm{H}})\left(\widehat{\zeta}_{C,t} - \widehat{\zeta}^*_{C,t}\right) + \widehat{\zeta}_{C,t} + (1+\eta)\,\widehat{\zeta}_{Y,t}, \\ &(\eta+\sigma)\,\widehat{Y}^{fb}_{F,t} = \\ &[2a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)]\left(-\widehat{\mathcal{T}}^{fb}_t\right) + (1-a_{\mathrm{H}})\left(\widehat{\zeta}_{C,t} - \widehat{\zeta}^*_{C,t}\right) + \widehat{\zeta}^*_{C,t} + (1+\eta)\,\widehat{\zeta}^*_{Y,t},\end{aligned}$$

whereas the terms of trade can in turn be written as a function of relative output and preference shocks

$$\left[4(1-a_{\mathrm{H}})\,a_{\mathrm{H}}\phi\sigma + (2a_{\mathrm{H}}-1)^2\right]\widehat{\mathcal{T}}^{fb}_t = \sigma\left(\widehat{Y}^{fb}_{H,t} - \widehat{Y}^{fb}_{F,t}\right) - (2a_{\mathrm{H}}-1)\left(\widehat{\zeta}_{C,t} - \widehat{\zeta}^*_{C,t}\right);$$

2. The equilibrium condition linking relative prices to output differentials and the wealth gap:

$$\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}^{fb}_t = \frac{\sigma\left[\left(\widehat{Y}_{H,t} - \widehat{Y}_{F,t}\right) - \left(\widehat{Y}^{fb}_{H,t} - \widehat{Y}^{fb}_{F,t}\right)\right] - (2a_{\mathrm{H}}-1)\,\widetilde{\mathcal{W}}_t}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1};$$

3. The definition of demand gap $\widetilde{\mathcal{W}}_t$ in terms of differences in budget constraints and real net wealth $\widehat{\mathcal{B}}_t$:

$$
\begin{aligned}
\widetilde{\mathcal{W}}_t &= \widehat{\mathcal{W}}_t = \sigma\left(\widehat{C}_t - \widehat{C}_t^*\right) - \widehat{\mathcal{Q}}_t - \left(\widehat{\zeta}_{C,t} - \widehat{\zeta}_{C,t}^*\right) \\
&= \sigma\left[\begin{array}{c} \widehat{Y}_{H,t} - \widehat{Y}_{F,t} - 2\left(1 - a_{\mathrm{H}}\right)\widehat{\mathcal{T}}_t + \\ 2\beta^{-1}\left(\widehat{\mathcal{B}}_{t-1} - \beta\widehat{\mathcal{B}}_t\right) \end{array}\right] + \\
&\quad - \left(2a_{\mathrm{H}} - 1\right)\widehat{\mathcal{T}}_t - \left(\widehat{\zeta}_{C,t} - \widehat{\zeta}_{C,t}^*\right);
\end{aligned}
$$

4. the Euler equations characterizing the evolution of $\widetilde{\mathcal{W}}_t$ (and net wealth $\widehat{\mathcal{B}}_t$):

$$E_t\widetilde{\mathcal{W}}_{t+1} = \widetilde{\mathcal{W}}_t.$$

Bond economy

Observe that in the case of a bond economy, the program amounts to choosing $\widehat{Y}_{H,t}$, $\widehat{Y}_{F,t}$, $\widehat{\mathcal{D}}_t$, $\pi_{H,t}$, $\pi^*_{F,t}$ and $\widehat{\mathcal{B}}_t$ subject to the following expression for $\widetilde{\mathcal{W}}_t$ in terms of differences of budget constraints:

$$
\begin{aligned}
&\left(1 - a_{\mathrm{H}}\right)\left[1 + 2a_{\mathrm{H}}\left(\phi - 1\right)\right]\widetilde{\mathcal{W}}_t = \left[4a_{\mathrm{H}}\left(1 - a_{\mathrm{H}}\right)\left(\sigma\phi - 1\right) + 1\right]\left(\beta^{-1}\widehat{\mathcal{B}}_{t-1} - \widehat{\mathcal{B}}_t\right) + \\
&\quad \left(1 - a_{\mathrm{H}}\right)\left[2a_{\mathrm{H}}\left(\sigma\phi - 1\right) + 1 - \sigma\right]\left[\left(\widehat{Y}_{H,t} - \widehat{Y}^{fb}_{H,t}\right) - \left(\widehat{Y}_{F,t} - \widehat{Y}^{fb}_{F,t}\right)\right] + \\
&\left(1 - a_{\mathrm{H}}\right)\left[4a_{\mathrm{H}}\left(1 - a_{\mathrm{H}}\right)\left(\sigma\phi - 1\right) + 1\right]\sigma^{-1}\left[\left(2a_{\mathrm{H}}\left(\sigma\phi - 1\right) + 1 - \sigma\right)\widehat{\mathcal{T}}^{fb}_t - \left(\widehat{\zeta}_{C,t} - \widehat{\zeta}^*_{C,t}\right)\right];
\end{aligned}
$$

The necessary FOC's with respect to inflation are given by:

$$
\begin{aligned}
\pi_{H,t} &: \quad 0 = -\theta\frac{\alpha}{\left(1 - \alpha\beta\right)\left(1 - \alpha\right)}\pi_{H,t} - \gamma_{H,t} + \gamma_{H,t-1} \\
\pi^*_{F,t} &: \quad 0 = -\theta\frac{\alpha}{\left(1 - \alpha\beta\right)\left(1 - \alpha\right)}\pi^*_{F,t} - \gamma^*_{F,t} + \gamma^*_{F,t-1},
\end{aligned}
$$

implying

$$
\begin{aligned}
-\frac{\left(1 - \alpha\beta\right)\left(1 - \alpha\right)}{\alpha}\left(\gamma_{H,t} - \gamma_{H,t-1}\right) &= \theta\pi_{H,t} = \theta\left(\widehat{p}_{H,t} - \widehat{p}_{H,t-1}\right) \\
-\frac{\left(1 - \alpha\beta\right)\left(1 - \alpha\right)}{\alpha}\left(\gamma^*_{F,t} - \gamma^*_{F,t-1}\right) &= \theta\left(\widehat{p}^*_{F,t} - \widehat{p}^*_{F,t-1}\right),
\end{aligned}
$$

where $\gamma_{H,t}$ and $\gamma^*_{F,t}$ are the multipliers associated with the Phillips curves — whose lags appear reflecting the assumption of commitment; and with respect to output (where observe that we have switched to the gap notation, e.g. $\widetilde{Y}_{H,t} =$

$\widehat{Y}_{H,t} - \widehat{Y}^{fb}_{H,t}$):

$$\begin{aligned}
\widehat{Y}_{H,t} \quad : \quad & 0 = (\sigma+\eta)\,\widetilde{Y}_{H,t} - \\
& \frac{2a_{\mathrm{H}}\,(1-a_{\mathrm{H}})\,(\sigma\phi-1)\,\sigma}{4a_{\mathrm{H}}\,(1-a_{\mathrm{H}})\,(\sigma\phi-1)+1}\left[\widetilde{Y}_{H,t}-\widetilde{Y}_{F,t}\right] + \\
& \frac{2a_{\mathrm{H}}\,(1-a_{\mathrm{H}})\,\phi}{4a_{\mathrm{H}}\,(1-a_{\mathrm{H}})\,(\sigma\phi-1)+1}\frac{2a_{\mathrm{H}}\,(\sigma\phi-1)+1-\sigma}{2a_{\mathrm{H}}\,(\phi-1)+1}\widetilde{\mathcal{W}}_t + \\
& \frac{2a_{\mathrm{H}}\,(\sigma\phi-1)+1-\sigma}{2a_{\mathrm{H}}\,(\phi-1)+1}\left(\lambda_t-\beta^{-1}\lambda_{t-1}\right) - \\
& \left[\sigma+\eta-\frac{(1-a_{\mathrm{H}})\,(\sigma-1)}{2a_{\mathrm{H}}\,(\phi-1)+1}\right]\frac{(1-\alpha\beta)\,(1-\alpha)}{\alpha}\gamma_{H,t} + \\
& \frac{(1-a_{\mathrm{H}})\,(\sigma-1)}{2a_{\mathrm{H}}\,(\phi-1)+1}\frac{(1-\alpha\beta)\,(1-\alpha)}{\alpha}\gamma^*_{F,t};
\end{aligned}$$

$$\begin{aligned}
\widehat{Y}_{F,t} \quad : \quad & 0 = (\sigma+\eta)\left(\widetilde{Y}_{F,t}\right) + \\
& \frac{2a_{\mathrm{H}}\,(1-a_{\mathrm{H}})\,(\sigma\phi-1)\,\sigma}{4a_{\mathrm{H}}\,(1-a_{\mathrm{H}})\,(\sigma\phi-1)+1}\left[\widetilde{Y}_{H,t}-\widetilde{Y}_{F,t}\right] - \\
& \frac{2a_{\mathrm{H}}\,(1-a_{\mathrm{H}})\,\phi}{4a_{\mathrm{H}}\,(1-a_{\mathrm{H}})\,(\sigma\phi-1)+1}\frac{2a_{\mathrm{H}}\,(\sigma\phi-1)+1-\sigma}{2a_{\mathrm{H}}\,(\phi-1)+1}\widetilde{\mathcal{W}}_t - \\
& \frac{2a_{\mathrm{H}}\,(\sigma\phi-1)+1-\sigma}{2a_{\mathrm{H}}\,(\phi-1)+1}\left(\lambda_t-\beta^{-1}\lambda_{t-1}\right) - \\
& \left[\sigma+\eta+\frac{(1-a_{\mathrm{H}})\,(\sigma-1)}{2a_{\mathrm{H}}\,(\phi-1)+1}\right]\frac{(1-\alpha\beta)\,(1-\alpha)}{\alpha}\gamma^*_{F,t} - \\
& \frac{(1-a_{\mathrm{H}})\,(\sigma-1)}{2a_{\mathrm{H}}\,(\phi-1)+1}\frac{(1-\alpha\beta)\,(1-\alpha)}{\alpha}\gamma_{H,t};
\end{aligned}$$

Furthermore,

$$\begin{aligned}
\widehat{\mathcal{B}}_t \quad : \quad & 0 = 2a_{\mathrm{H}}\,(1-a_{\mathrm{H}})\,\phi\left[E_t\widetilde{\mathcal{W}}_{t+1}-\widetilde{\mathcal{W}}_t\right] + \\
& \left[4a_{\mathrm{H}}\,(1-a_{\mathrm{H}})\,(\sigma\phi-1)+1\right]\left[(E_t\lambda_{t+1}-\lambda_t)-\beta^{-1}\,(\lambda_t-\lambda_{t-1})\right] - \\
& (1-a_{\mathrm{H}})\left[2a_{\mathrm{H}}\,(\sigma\phi-1)+1\right]\frac{(1-\alpha\beta)\,(1-\alpha)}{\alpha}\cdot \\
& \left[\left((E_t\gamma_{H,t+1}-\gamma_{H,t}\right)\left((E_t\gamma^*_{F,t+1}-\gamma^*_{F,t}\right)\right]
\end{aligned}$$

implying

$$\begin{aligned}
0 \quad = \quad & \left[(\beta E_t\lambda_{t+1}-\lambda_t)-(\beta\lambda_t-\lambda_{t-1})\right] + \\
& (1-a_{\mathrm{H}})\left(\frac{2a_{\mathrm{H}}\,(\sigma\phi-1)+1}{4a_{\mathrm{H}}\,(1-a_{\mathrm{H}})\,(\sigma\phi-1)+1}\right)\theta\left(\beta E_t\pi_{H,t+1}-\beta E_t\pi^*_{F,t+1}\right).
\end{aligned}$$

As stated in Proposition 3 and already shown above for the LCP case, the solution can be expressed in terms of a familiar sum rule for (the change in) world output gaps and inflation rates:

$$\begin{aligned} 0 &= \widetilde{Y}_{H,t} + \widetilde{Y}_{F,t} + \theta\left(\widehat{p}_{H,t} + \widehat{p}^{*}_{F,t}\right) \\ &= \left[\widetilde{Y}_{H,t} - \widetilde{Y}_{H,t-1}\right] + \left[\widetilde{Y}_{F,t} - \widetilde{Y}_{F,t-1}\right] + \\ &\quad \theta\left[\pi_{H,t} + \pi^{*}_{F,t}\right], \end{aligned}$$

and a difference rule.

Proof of Proposition 6: Difference rule. The difference rule under PCP can be obtained by subtracting the output FOC's to solve for λ_t :

$$-2\frac{2a_{\mathrm{H}}\left(\sigma\phi-1\right)+1-\sigma}{2a_{\mathrm{H}}\left(\phi-1\right)+1}\beta^{-1}\left(\beta\lambda_t-\lambda_{t-1}\right) =$$
$$\left[(\sigma+\eta)-\frac{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)\sigma}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1}\right]\left(\widetilde{Y}_{H,t}-\widetilde{Y}_{F,t}\right)+$$
$$\frac{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\phi}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1}\frac{2a_{\mathrm{H}}\left(\sigma\phi-1\right)+1-\sigma}{2a_{\mathrm{H}}\left(\phi-1\right)+1}\widehat{\mathcal{W}}_t+$$
$$\left[\sigma+\eta-2\frac{\left(1-a_{\mathrm{H}}\right)\left(\sigma-1\right)}{2a_{\mathrm{H}}\left(\phi-1\right)+1}\right]\theta\left(\widehat{p}_{H,t}-\widehat{p}^{*}_{F,t}\right).$$

We can solve for $(\beta\lambda_t - \lambda_{t-1})$ from the first order condition for $\widehat{\mathcal{B}}_t$

$$0 = \left[\left(\beta E_t\lambda_{t+1}-\lambda_t\right)-\left(\beta\lambda_t-\lambda_{t-1}\right)\right]+$$
$$\left(1-a_{\mathrm{H}}\right)\left(\frac{2a_{\mathrm{H}}\left(\sigma\phi-1\right)+1}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1}\right)\theta\beta\left(E_t\pi_{H,t+1}-E_t\pi^{*}_{F,t+1}\right),$$

$$-\left[E_t\left(\beta\lambda_{t+1}-\lambda_t\right)-\left(\beta\lambda_t-\lambda_{t-1}\right)\right] =$$
$$\left(1-a_{\mathrm{H}}\right)\left(\frac{2a_{\mathrm{H}}\left(\sigma\phi-1\right)+1}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1}\right)\theta\beta E_t\left[\begin{array}{c}\left(\widehat{p}_{H,t+1}-\widehat{p}_{H,t}\right)\\ -\left(\widehat{p}^{*}_{F,t+1}-\widehat{p}^{*}_{F,t}\right)\end{array}\right].$$

A solution to the above equation is given by the following:

$$-\left(\beta\lambda_t-\lambda_{t-1}\right)=\left(1-a_{\mathrm{H}}\right)\left(\frac{2a_{\mathrm{H}}\left(\sigma\phi-1\right)+1}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1}\right)\theta\beta\left(\widehat{p}_{H,t}-\widehat{p}^{*}_{F,t}\right).$$

Effectively this assumes that the growth rate in the (quasi-change $(\beta\lambda_t - \lambda_{t-1})$ of the) Lagrange multiplier of relative wealth depends on contemporaneous shocks only via their effects on inflation differentials.

In turn, this implies the following difference rule:

$$\begin{aligned} 0 = &\left[(\sigma+\eta)-\frac{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)\sigma}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1}\right]\left(\widetilde{Y}_{H,t}-\widetilde{Y}_{F,t}\right)+ \\ &\frac{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\phi}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1}\frac{2a_{\mathrm{H}}\left(\sigma\phi-1\right)+1-\sigma}{2a_{\mathrm{H}}\left(\phi-1\right)+1}\widehat{\mathcal{W}}_t+ \\ &\left[\sigma+\eta-\frac{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)\sigma}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1}\right]\theta\left(\widehat{p}_{H,t}-\widehat{p}^{*}_{F,t}\right). \end{aligned}$$

Therefore, in terms of inflation rates and growth rates the "difference" rule is the following:

$$0=\left[(\sigma+\eta)-\frac{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)(\sigma\phi-1)\sigma}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)(\sigma\phi-1)+1}\right]\left\{\begin{array}{c}\left(\widetilde{Y}_{H,t}-\widetilde{Y}_{H,t-1}\right)-\left(\widetilde{Y}_{F,t}-\widetilde{Y}_{F,t-1}\right)+\\ \theta\left(\pi_{H,t}-\pi^{*}_{F,t}\right)\end{array}\right\}+$$
$$\frac{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\phi}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)(\sigma\phi-1)+1}\frac{2a_{\mathrm{H}}(\sigma\phi-1)+1-\sigma}{2a_{\mathrm{H}}(\phi-1)+1}\left(\widehat{\mathcal{W}}_t-\widehat{\mathcal{W}}_{t-1}\right).$$

This complete the proof of Proposition 6.

Alternative formulation of the difference rule. Alternatively, we can substitute the above expression for the Lagrange multiplier λ_t

$$\begin{array}{c}-2\frac{2a_{\mathrm{H}}(\sigma\phi-1)+1-\sigma}{2a_{\mathrm{H}}(\phi-1)+1}\beta^{-1}\left(\beta\lambda_t-\lambda_{t-1}\right)=\\ \left[(\sigma+\eta)-\frac{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)(\sigma\phi-1)\sigma}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)(\sigma\phi-1)+1}\right]\left(\widetilde{Y}_{H,t}-\widetilde{Y}_{F,t}\right)+\\ \frac{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\phi}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)(\sigma\phi-1)+1}\frac{2a_{\mathrm{H}}(\sigma\phi-1)+1-\sigma}{2a_{\mathrm{H}}(\phi-1)+1}\widehat{\mathcal{W}}_t+\\ \left[\sigma+\eta-2\frac{\left(1-a_{\mathrm{H}}\right)(\sigma-1)}{2a_{\mathrm{H}}(\phi-1)+1}\right]\theta\left(\widehat{p}_{H,t}-\widehat{p}^{*}_{F,t}\right),\end{array}$$

into the first order condition for $\widehat{\mathcal{B}}_t$,

$$\begin{array}{rcl}0&=&\left[\left(\beta E_t\lambda_{t+1}-\lambda_t\right)-\left(\beta\lambda_t-\lambda_{t-1}\right)\right]+\\ &&\left(1-a_{\mathrm{H}}\right)\left(\frac{2a_{\mathrm{H}}(\sigma\phi-1)+1}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)(\sigma\phi-1)+1}\right)\theta\left(\beta E_t\pi_{H,t+1}-\beta E_t\pi^{*}_{F,t+1}\right).\end{array}$$

yielding:

$$\begin{array}{rcl}0&=&-2\left(1-a_{\mathrm{H}}\right)\frac{2a_{\mathrm{H}}(\sigma\phi-1)+1-\sigma}{2a_{\mathrm{H}}(\phi-1)+1}\cdot\\ &&\left(\frac{2a_{\mathrm{H}}(\sigma\phi-1)+1}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)(\sigma\phi-1)+1}\right)\theta\left(E_t\pi_{H,t+1}-E_t\pi^{*}_{F,t+1}\right)+\\ &&\left[(\sigma+\eta)-\frac{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)(\sigma\phi-1)\sigma}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)(\sigma\phi-1)+1}\right]\left[\begin{array}{c}E_t\left(\widetilde{Y}_{H,t+1}-\widetilde{Y}_{H,t}\right)+\\ -E_t\left(\widetilde{Y}_{F,t+1}-\widetilde{Y}_{F,t}\right)\end{array}\right]+\\ &&\frac{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\phi}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)(\sigma\phi-1)+1}\frac{2a_{\mathrm{H}}(\sigma\phi-1)+1-\sigma}{2a_{\mathrm{H}}(\phi-1)+1}\left(E_t\widehat{\mathcal{W}}_{t+1}-\widehat{\mathcal{W}}_t\right)+\\ &&\left[\sigma+\eta-2\frac{\left(1-a_{\mathrm{H}}\right)(\sigma-1)}{2a_{\mathrm{H}}(\phi-1)+1}\right]\theta\left(E_t\pi_{H,t+1}-E_t\pi^{*}_{F,t+1}\right).\end{array}$$

Recalling the law of motion for the wealth gap, $E_t\widehat{\mathcal{W}}_{t+1}=\widehat{\mathcal{W}}_t$ and the expression,

$$\frac{2(1-a_{\mathrm{H}})(\sigma-1)}{2a_{\mathrm{H}}(\phi-1)+1} = \frac{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)\sigma}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1} - 2(1-a_{\mathrm{H}})\left(\frac{2a_{\mathrm{H}}(\sigma\phi-1)+1}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1}\right)\frac{2a_{\mathrm{H}}(\sigma\phi-1)+1-\sigma}{2a_{\mathrm{H}}(\phi-1)+1},$$

we obtain the following targeting rule:

$$\begin{aligned} 0 \;=\; & \left[(\sigma+\eta) - \frac{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)\sigma}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1}\right]\cdot \\ & \left\{ \begin{array}{c} E_t\left[\left(\widetilde{Y}_{H,t+1}-\widetilde{Y}_{H,t}\right)\right] - E_t\left[\left(\widetilde{Y}_{F,t+1}-\widetilde{Y}_{F,t}\right)\right] + \\ \theta\left(E_t\pi_{H,t+1} - E_t\pi^*_{F,t+1}\right) \end{array} \right\}. \end{aligned}$$

Interestingly, this rule is a forward-looking version of the one which prevails under complete markets:

$$\begin{aligned} 0 \;=\; & \left[\left(\widetilde{Y}_{H,t}-\widetilde{Y}_{H,t-1}\right)\right] - \left[\left(\widetilde{Y}_{F,t}-\widetilde{Y}_{F,t-1}\right)\right] + \\ & \theta\left(\pi_{H,t}-\pi^*_{F,t}\right). \end{aligned}$$

3.2.2 Solving explicitly for the constrained optimal allocation under PCP

The targeting rule can thus be written:

$$0 = \left[\eta + \frac{\sigma}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1}\right]\left[\left(\widetilde{Y}_{H,t}-\widetilde{Y}_{H,t-1}\right)+\theta\pi_{H,t}\right] + \frac{2a_{\mathrm{H}}(1-a_{\mathrm{H}})\phi}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1}\,\frac{2a_{\mathrm{H}}(\sigma\phi-1)+1-\sigma}{2a_{\mathrm{H}}(\phi-1)+1}\left(\widehat{\mathcal{W}}_t-\widehat{\mathcal{W}}_{t-1}\right).$$

Using it to solve for inflation and substituting into the Phillips curve:

$$\theta\pi_{H,t} = -\left(\widetilde{Y}_{H,t}-\widetilde{Y}_{H,t-1}\right) + -\frac{2a_{\mathrm{H}}(1-a_{\mathrm{H}})\phi}{\eta\left[4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1\right]+\sigma}\,\frac{2a_{\mathrm{H}}(\sigma\phi-1)+1-\sigma}{2a_{\mathrm{H}}(\phi-1)+1}\left(\widehat{\mathcal{W}}_t-\widehat{\mathcal{W}}_{t-1}\right),$$

and recalling the following relation for $\widehat{\mathcal{W}}_t$:

$$\begin{aligned} (1-a_{\mathrm{H}})\left[\frac{2a_{\mathrm{H}}(\phi-1)+1}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1}\right]\widehat{\mathcal{W}}_t = -\beta^{-1}\left(\beta\widehat{\mathcal{B}}_t-\widehat{\mathcal{B}}_{t-1}\right) + \\ (1-a_{\mathrm{H}})\left[\frac{2a_{\mathrm{H}}(\sigma\phi-1)-(\sigma-1)}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1}\right]\left[2\widetilde{Y}_{H,t}+\left(\widehat{Y}^{fb}_{H,t}-\widehat{Y}^{fb}_{F,t}\right)\right] - \\ (1-a_{\mathrm{H}})\left[\frac{2a_{\mathrm{H}}(\phi-1)+1}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1}\right]\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}^*_{C,t}\right) \end{aligned}$$

we obtain the following system of difference equations in $\widetilde{Y}_{H,t}$ and $\widehat{\mathcal{B}}_t$:

$$\beta^{-1}\left[E_t\left(\beta\widehat{\mathcal{B}}_{t+1}-\widehat{\mathcal{B}}_t\right)-\left(\beta\widehat{\mathcal{B}}_t-\widehat{\mathcal{B}}_{t-1}\right)\right]-$$
$$2\left(1-a_{\mathrm{H}}\right)\left[\frac{2a_{\mathrm{H}}\left(\sigma\phi-1\right)-\left(\sigma-1\right)}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1}\right]\left[E_t\left(\widetilde{Y}_{H,t+1}-\widetilde{Y}_{H,t}\right)\right]=$$
$$\left(1-a_{\mathrm{H}}\right)\left[\frac{2a_{\mathrm{H}}\left(\sigma\phi-1\right)-\left(\sigma-1\right)}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1}\right]\left[E_t\left(\widehat{Y}^{fb}_{H,t+1}-\widehat{Y}^{fb}_{F,t+1}\right)-\left(\widehat{Y}^{fb}_{H,t}-\widehat{Y}^{fb}_{F,t}\right)\right]-$$
$$\left(1-a_{\mathrm{H}}\right)\left[\frac{2a_{\mathrm{H}}\left(\phi-1\right)+1}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1}\right]\left[E_t\left(\widehat{\zeta}_{C,t+1}-\widehat{\zeta}^*_{C,t+1}\right)-\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}^*_{C,t}\right)\right],$$

and,

$$\left\{\begin{array}{c}-\left[\eta+\frac{\sigma}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1}\right]\left[\widetilde{Y}_{H,t}-\widetilde{Y}_{H,t-1}\right]-\\ \frac{2a_{\mathrm{H}}(1-a_{\mathrm{H}})\phi}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1}\frac{2a_{\mathrm{H}}(\sigma\phi-1)+1-\sigma}{2a_{\mathrm{H}}(\phi-1)+1}\left(\widehat{\mathcal{W}}_t-\widehat{\mathcal{W}}_{t-1}\right)\end{array}\right\}$$
$$+\beta\left[\eta+\frac{\sigma}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1}\right]E_t\left[\widetilde{Y}_{H,t+1}-\widetilde{Y}_{H,t}\right]=$$
$$\frac{\left(1-\alpha\beta\right)\left(1-\alpha\right)}{\alpha}\theta\left[\eta+\frac{\sigma}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1}\right]^2\widetilde{Y}_{H,t}$$
$$+\frac{\left(1-\alpha\beta\right)\left(1-\alpha\right)}{\alpha}\left(1-a_{\mathrm{H}}\right)\theta\left[\frac{2a_{\mathrm{H}}\left(\sigma\phi-1\right)+1}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1}\right]\cdot$$
$$\left[\eta+\frac{\sigma}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1}\right]\widehat{\mathcal{W}}_t.$$

We use the method of undetermined coefficients to solve this system, exploiting the martingale nature of the variable $\widehat{\mathcal{W}}_t$, namely $E_t\widehat{\mathcal{W}}_{t+j}=\widehat{\mathcal{W}}_t$.

Rearranging the last difference equation for the output gap as follows:

$$\beta E_t\left[\widetilde{Y}_{H,t+1}-\widetilde{Y}_{H,t}\right]-\left[\widetilde{Y}_{H,t}-\widetilde{Y}_{H,t-1}\right]-$$
$$\left[\eta+\frac{\sigma}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1}\right]\frac{\left(1-\alpha\beta\right)\left(1-\alpha\right)}{\alpha}\theta\widetilde{Y}_{H,t}$$
$$=\frac{2a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\phi}{\eta\left[4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1\right]+\sigma}\frac{2a_{\mathrm{H}}\left(\sigma\phi-1\right)+1-\sigma}{2a_{\mathrm{H}}\left(\phi-1\right)+1}\left(\widehat{\mathcal{W}}_t-\widehat{\mathcal{W}}_{t-1}\right)+$$
$$\left(1-a_{\mathrm{H}}\right)\frac{2a_{\mathrm{H}}\left(\sigma\phi-1\right)+1}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1}\frac{\left(1-\alpha\beta\right)\left(1-\alpha\right)}{\alpha}\theta\widehat{\mathcal{W}}_t,$$

we can solve for $\widetilde{Y}_{H,t}$ as function of current and future values of $\widehat{\mathcal{W}}_t$:

$$\begin{aligned}\widetilde{Y}_{H,t}-\delta_1\widetilde{Y}_{H,t-1} &= \\ &-\left[\begin{array}{c}\left(1-a_{\mathrm{H}}\right)\frac{2a_{\mathrm{H}}\phi}{\eta[4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1]+\sigma}\frac{2a_{\mathrm{H}}(\sigma\phi-1)+1-\sigma}{2a_{\mathrm{H}}(\phi-1)+1}\frac{1}{\beta\delta_2}\cdot\\ \sum_{j=0}^{\infty}\delta_2^{-j}E_t\left(\widehat{\mathcal{W}}_{t+j}-\widehat{\mathcal{W}}_{t+j-1}\right)\end{array}\right]\\ &-\left(1-a_{\mathrm{H}}\right)\frac{2a_{\mathrm{H}}\left(\sigma\phi-1\right)+1}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1}\frac{\left(1-\alpha\beta\right)\left(1-\alpha\right)}{\alpha}\frac{\theta}{\beta\delta_2}\sum_{j=0}^{\infty}\delta_2^{-j}E_t\widehat{\mathcal{W}}_{t+j}.\end{aligned}$$

where $0 < \delta_1 < 1 < \beta^{-1} < \delta_2$ are the eigenvalues of the difference equation, solving the standard characteristic equation:

$$\beta\delta^2 - \left\{1 + \beta + \left[\eta + \frac{\sigma}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi - 1\right) + 1}\right]\frac{\left(1-\alpha\beta\right)\left(1-\alpha\right)}{\alpha}\theta\right\}\delta + 1 = 0,$$

namely,

$$\begin{aligned}\delta \quad &= \quad \frac{1}{2\beta}\left(1 + \beta + \left[\eta + \frac{\sigma}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi - 1\right) + 1}\right]\frac{\left(1-\alpha\beta\right)\left(1-\alpha\right)}{\alpha}\theta\right) \pm \\ &\quad\ \frac{1}{2\beta}\sqrt{\left\{1 + \beta + \left[\eta + \frac{\sigma}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi - 1\right) + 1}\right]\frac{\left(1-\alpha\beta\right)\left(1-\alpha\right)}{\alpha}\theta\right\}^2 - 4\beta},\end{aligned}$$

We can simplify the above solution which is solely a function of $\widehat{\mathcal{W}}_t$, as $E_t\widehat{\mathcal{W}}_{t+j} = \widehat{\mathcal{W}}_t$:

$$\begin{aligned}&\left(\widehat{Y}_{H,t+j} - \widehat{Y}^{fb}_{H,t+j}\right) = \\ &\delta_1\left(\widehat{Y}_{H,t+j-1} - \widehat{Y}^{fb}_{H,t+j-1}\right) - \\ &\left(1-a_{\mathrm{H}}\right)\frac{2a_{\mathrm{H}}\phi}{\eta\left[4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1\right]+\sigma}\frac{2a_{\mathrm{H}}\left(\sigma\phi-1\right)+1-\sigma}{2a_{\mathrm{H}}\left(\phi-1\right)+1}\frac{1}{\beta\delta_2}E_t\left(\widehat{\mathcal{W}}_{t+j} - \widehat{\mathcal{W}}_{t+j-1}\right) - \\ &\left(1-a_{\mathrm{H}}\right)\frac{2a_{\mathrm{H}}\left(\sigma\phi-1\right)+1}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1}\frac{\left(1-\alpha\beta\right)\left(1-\alpha\right)}{\alpha}\frac{\theta}{\beta\left(\delta_2-1\right)}\widehat{\mathcal{W}}_t;\end{aligned}$$

and we have that

$$\frac{\left(1-\alpha\beta\right)\left(1-\alpha\right)}{\alpha}\theta\left[\eta + \frac{\sigma}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1}\right] = \frac{\left(\delta_2-1\right)\left(\beta\delta_2-1\right)}{\delta_2}$$

Furthermore,

$$\begin{aligned}&E_t\widetilde{Y}_{H,t+s} = \\ &\delta_1\left[\left(\widehat{Y}_{H,t+s-1} - \widehat{Y}^{fb}_{H,t+s-1}\right)\right] - \left(1-a_{\mathrm{H}}\right)\frac{2a_{\mathrm{H}}\left(\sigma\phi-1\right)+1}{\eta\left[4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1\right]+\sigma}\frac{\left(\beta\delta_2-1\right)}{\beta\delta_2}\widehat{\mathcal{W}}_t + \\ &-\left(1-a_{\mathrm{H}}\right)\frac{2a_{\mathrm{H}}\phi}{\eta\left[4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1\right]+\sigma}\frac{2a_{\mathrm{H}}\left(\sigma\phi-1\right)+1-\sigma}{2a_{\mathrm{H}}\left(\phi-1\right)+1}\frac{1}{\beta\delta_2}E_t\left(\widehat{\mathcal{W}}_{t+s} - \widehat{\mathcal{W}}_{t+s-1}\right)\end{aligned}$$

Notice that the second term $E_t\left(\widehat{\mathcal{W}}_{t+j} - \widehat{\mathcal{W}}_{t+j-1}\right) = 0$ for $j \geq 1$, while it is equal to $\widehat{\mathcal{W}}_t$ for $j = 0$. The last term represents a constant shifter proportional to $\widehat{\mathcal{W}}_t$ for any $j \geq 0$.

We can compare the above with the allocation under $\pi_{H,t} = \pi_{F,t} = 0$, characterized as follows:

$$\begin{aligned}\left(\widehat{\mathcal{T}}_t - \widehat{\mathcal{T}}^{fb}_t\right) \quad &= \quad -\frac{\sigma + \left(2a_{\mathrm{H}}-1\right)\eta}{\eta\left[4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1\right]+\sigma}\widehat{\mathcal{W}}_t \\ \left(\widehat{Y}_{H,t} - \widehat{Y}^{fb}_{H,t}\right) \quad &= \quad -\left(1-a_{\mathrm{H}}\right)\frac{1+2a_{\mathrm{H}}\left(\sigma\phi-1\right)}{\eta\left[4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1\right]+\sigma}\widehat{\mathcal{W}}_t.\end{aligned}$$

We can also solve for inflation using the targeting rule:

$$\theta\pi_{H,t} = -\left[\left(\widehat{Y}_{H,t} - \widehat{Y}^{fb}_{H,t}\right) - \left(\widehat{Y}_{H,t-1} - \widehat{Y}^{fb}_{H,t-1}\right)\right] +$$
$$-\frac{2a_{\mathrm{H}}(1-a_{\mathrm{H}})\phi}{\eta\left[4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1\right]+\sigma}\frac{2a_{\mathrm{H}}(\sigma\phi-1)+1-\sigma}{2a_{\mathrm{H}}(\phi-1)+1}\left(\widehat{\mathcal{W}}_t - \widehat{\mathcal{W}}_{t-1}\right),$$

which implies:

$$\theta E_t\pi_{H,t+j} = (1-\delta_1)\left(\widehat{Y}_{H,t+j-1} - \widehat{Y}^{fb}_{H,t+j-1}\right) +$$
$$(1-a_{\mathrm{H}})\frac{2a_{\mathrm{H}}(\sigma\phi-1)+1}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1}\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\frac{\theta}{\beta(\delta_2-1)}\widehat{\mathcal{W}}_t +$$
$$-\frac{2a_{\mathrm{H}}(1-a_{\mathrm{H}})\phi}{\eta\left[4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1\right]+\sigma}\frac{2a_{\mathrm{H}}(\sigma\phi-1)+1-\sigma}{2a_{\mathrm{H}}(\phi-1)+1}\left(\frac{\beta\delta_2-1}{\beta\delta_2}\right)E_t\left(\widehat{\mathcal{W}}_{t+j} - \widehat{\mathcal{W}}_{t+j-1}\right).$$

Likewise, armed with the above solution for $\left(\widehat{Y}_{H,t+j} - \widehat{Y}^{fb}_{H,t+j}\right)$, we can solve the following difference equation for $\widehat{\mathcal{B}}_t$:

$$\beta E_t\left(\widehat{\mathcal{B}}_{t+1} - \widehat{\mathcal{B}}_t\right) - \left(\widehat{\mathcal{B}}_t - \widehat{\mathcal{B}}_{t-1}\right) =$$
$$2(1-a_{\mathrm{H}})\left[\frac{2a_{\mathrm{H}}(\sigma\phi-1)-(\sigma-1)}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1}\right]\beta\left[E_t\widetilde{Y}_{H,t+1} - \widetilde{Y}_{H,t}\right] +$$
$$(1-a_{\mathrm{H}})\left[\frac{2a_{\mathrm{H}}(\sigma\phi-1)-(\sigma-1)}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1}\right]\beta\left[E_t\left(\widehat{Y}^{fb}_{H,t+1} - \widehat{Y}^{fb}_{F,t+1}\right) - \left(\widehat{Y}^{fb}_{H,t} - \widehat{Y}^{fb}_{F,t}\right)\right] +$$
$$-(1-a_{\mathrm{H}})\left[\frac{2a_{\mathrm{H}}(\phi-1)+1}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1}\right]\beta\left[E_t\left(\widehat{\zeta}_{C,t+1} - \widehat{\zeta}^*_{C,t+1}\right) - \left(\widehat{\zeta}_{C,t} - \widehat{\zeta}^*_{C,t}\right)\right].$$

The eigenvalues of this difference equation are 1 and $1/\beta$, yielding the following standard solution:

$$\widehat{\mathcal{B}}_t = \left[\begin{array}{c}\widehat{\mathcal{B}}_{t-1} - 2(1-a_{\mathrm{H}})\left[\frac{2a_{\mathrm{H}}(\sigma\phi-1)-(\sigma-1)}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1}\right]\beta \\ \sum_{j=0}^{\infty}\beta^j E_t\left[\left(\widehat{Y}_{H,t+j+1} - \widehat{Y}^{fb}_{H,t+j+1}\right) - \left(\widehat{Y}_{H,t+j} - \widehat{Y}^{fb}_{H,t+j}\right)\right]\end{array}\right] -$$
$$(1-a_{\mathrm{H}})\left[\frac{2a_{\mathrm{H}}(\sigma\phi-1)-(\sigma-1)}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1}\right]\beta\sum_{j=0}^{\infty}\beta^j E_t\left[\left(\widehat{Y}^{fb}_{H,t+j+1} - \widehat{Y}^{fb}_{F,t+j+1}\right) - \left(\widehat{Y}^{fb}_{H,t+j} - \widehat{Y}^{fb}_{F,t+j}\right)\right] -$$
$$(1-a_{\mathrm{H}})\left[\frac{2a_{\mathrm{H}}(\phi-1)+1}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1}\right]\beta\sum_{j=0}^{\infty}\beta^j E_t\left[\left(\widehat{\zeta}_{C,t+j+1} - \widehat{\zeta}^*_{C,t+j+1}\right) - \left(\widehat{\zeta}_{C,t+j} - \widehat{\zeta}^*_{C,t+j}\right)\right].$$

Using the above solution for the output gap, we have that for $j \geq 0$:

$$E_t\left[\left(\widehat{Y}_{H,t+j+1} - \widehat{Y}^{fb}_{H,t+j+1}\right) - \left(\widehat{Y}_{H,t+j} - \widehat{Y}^{fb}_{H,t+j}\right)\right] =$$
$$-(1-\delta_1)E_t\left(\widehat{Y}_{H,t+j} - \widehat{Y}^{fb}_{H,t+j}\right) +$$
$$-(1-a_{\mathrm{H}})\frac{2a_{\mathrm{H}}(\sigma\phi-1)+1}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1}\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\frac{\theta}{\beta(\delta_2-1)}\widehat{\mathcal{W}}_t,$$

where

$$E_t\left(\widehat{Y}_{H,t+j}-\widehat{Y}^{fb}_{H,t+j}\right)=$$

$$\delta_1^j\left[\delta_1\left(\widehat{Y}_{H,t-1}-\widehat{Y}^{fb}_{H,t-1}\right)-\frac{2a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\phi}{\eta\left[4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1\right]+\sigma}\frac{2a_{\mathrm{H}}\left(\sigma\phi-1\right)+1-\sigma}{2a_{\mathrm{H}}\left(\phi-1\right)+1}\frac{1}{\beta\delta_2}\widehat{\mathcal{W}}_t\right]-$$

$$\left(1-a_{\mathrm{H}}\right)\frac{2a_{\mathrm{H}}\left(\sigma\phi-1\right)+1}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1}\frac{\left(1-\alpha\beta\right)\left(1-\alpha\right)}{\alpha}\frac{\theta}{\beta\left(\delta_2-1\right)}\sum_{s=0}^{j}\delta_1^s\widehat{\mathcal{W}}_t,$$

which also implies that:

$$E_t\left[\left(\widehat{Y}_{H,t+j+1}-\widehat{Y}^{fb}_{H,t+j+1}\right)-\left(\widehat{Y}_{H,t+j}-\widehat{Y}^{fb}_{H,t+j}\right)\right]=$$

$$-\left(1-\delta_1\right)\delta_1^j\left\{\delta_1\left(\widehat{Y}_{H,t-1}-\widehat{Y}^{fb}_{H,t-1}\right)\right\}$$

$$-\left(1-\delta_1\right)\delta_1^j\left\{\frac{2a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\phi}{\eta\left[4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1\right]+\sigma}\frac{2a_{\mathrm{H}}\left(\sigma\phi-1\right)+1-\sigma}{2a_{\mathrm{H}}\left(\phi-1\right)+1}\frac{1}{\beta\delta_2}\widehat{\mathcal{W}}_t\right\}+$$

$$\left(1-a_{\mathrm{H}}\right)\frac{2a_{\mathrm{H}}\left(\sigma\phi-1\right)+1}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1}\frac{\left(1-\alpha\beta\right)\left(1-\alpha\right)}{\alpha}\frac{\theta}{\beta\left(\delta_2-1\right)}\left[\left(1-\delta_1\right)\frac{1-\delta_1^{j+1}}{1-\delta_1}-1\right]\widehat{\mathcal{W}}_t.$$

As a result we have that:

$$\beta\sum_{j=0}^{\infty}\beta^jE_t\left[\left(\widehat{Y}_{H,t+j+1}-\widehat{Y}^{fb}_{H,t+j+1}\right)-\left(\widehat{Y}_{H,t+j}-\widehat{Y}^{fb}_{H,t+j}\right)\right]=$$

$$-\left(1-\delta_1\right)\beta\sum_{j=0}^{\infty}\beta^j\delta_1^j\left[\begin{array}{c}\delta_1\left(\widehat{Y}_{H,t-1}-\widehat{Y}^{fb}_{H,t-1}\right)-\\ \frac{2a_{\mathrm{H}}(1-a_{\mathrm{H}})\phi}{\eta[4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1]+\sigma}\frac{2a_{\mathrm{H}}(\sigma\phi-1)+1-\sigma}{2a_{\mathrm{H}}(\phi-1)+1}\frac{1}{\beta\delta_2}\widehat{\mathcal{W}}_t\end{array}\right]-$$

$$\left(1-a_{\mathrm{H}}\right)\frac{2a_{\mathrm{H}}\left(\sigma\phi-1\right)+1}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1}\frac{\left(1-\alpha\beta\right)\left(1-\alpha\right)}{\alpha}\frac{\theta}{\beta\left(\delta_2-1\right)}\sum_{j=0}^{\infty}\beta^{j+1}\delta_1^{j+1}\widehat{\mathcal{W}}_t$$

$$=-\frac{\left(1-\delta_1\right)\beta}{1-\beta\delta_1}\left[\begin{array}{c}\delta_1\left(\widehat{Y}_{H,t-1}-\widehat{Y}^{fb}_{H,t-1}\right)-\\ \frac{2a_{\mathrm{H}}(1-a_{\mathrm{H}})\phi}{\eta[4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1]+\sigma}\frac{2a_{\mathrm{H}}(\sigma\phi-1)+1-\sigma}{2a_{\mathrm{H}}(\phi-1)+1}\frac{1}{\beta\delta_2}\widehat{\mathcal{W}}_t\end{array}\right]-$$

$$\left(1-a_{\mathrm{H}}\right)\frac{2a_{\mathrm{H}}\left(\sigma\phi-1\right)+1}{\eta\left[4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1\right]+\sigma}\frac{\beta\delta_2-1}{\beta\delta_2}\frac{\beta\delta_1}{1-\beta\delta_1}\widehat{\mathcal{W}}_t$$

Therefore the solution for NFA is the following:

$$\begin{aligned}
\widehat{\mathcal{B}}_t &= \widehat{\mathcal{B}}_{t-1} + 2\left(1-a_{\mathrm{H}}\right)\left[\frac{2a_{\mathrm{H}}\left(\sigma\phi-1\right)-\left(\sigma-1\right)}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1}\right]\frac{\beta}{1-\beta\delta_1}\cdot \\
&\left\{\begin{array}{c} \left(1-\delta_1\right)\delta_1\left(\widehat{Y}_{H,t-1}-\widehat{Y}^{fb}_{H,t-1}\right)-\left(1-a_{\mathrm{H}}\right)\frac{2a_{\mathrm{H}}\phi}{\eta\left[4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1\right]+\sigma}\frac{2a_{\mathrm{H}}(\sigma\phi-1)+1-\sigma}{2a_{\mathrm{H}}(\phi-1)+1}\frac{(1-\delta_1)}{\beta\delta_2}\widehat{\mathcal{W}}_t+ \\ \left(1-a_{\mathrm{H}}\right)\delta_1\frac{2a_{\mathrm{H}}(\sigma\phi-1)+1}{\eta\left[4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1\right]+\sigma}\frac{\beta\delta_2-1}{\beta\delta_2}\widehat{\mathcal{W}}_t \end{array}\right\}- \\
&\left(1-a_{\mathrm{H}}\right)\left[\frac{2a_{\mathrm{H}}\left(\sigma\phi-1\right)-\left(\sigma-1\right)}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1}\right]\beta\sum_{j=0}^{\infty}\beta^j E_t\left[\left(\widehat{Y}^{fb}_{H,t+j+1}-\widehat{Y}^{fb}_{F,t+j+1}\right)-\left(\widehat{Y}^{fb}_{H,t+j}-\widehat{Y}^{fb}_{F,t+j}\right)\right]+ \\
&\left(1-a_{\mathrm{H}}\right)\left[\frac{2a_{\mathrm{H}}\left(\phi-1\right)+1}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1}\right]\beta\sum_{j=0}^{\infty}\beta^j E_t\left[\left(\widehat{\zeta}_{C,t+j+1}-\widehat{\zeta}^*_{C,t+j+1}\right)-\left(\widehat{\zeta}_{C,t+j}-\widehat{\zeta}^*_{C,t+j}\right)\right].
\end{aligned}$$

Finally, recalling the following relation for $\widehat{\mathcal{W}}_t$:

$$\begin{aligned}
&\left(1-a_{\mathrm{H}}\right)\left[\frac{2a_{\mathrm{H}}\left(\phi-1\right)+1}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1}\right]\widehat{\mathcal{W}}_t = -\beta^{-1}\left(\beta\widehat{\mathcal{B}}_t-\widehat{\mathcal{B}}_{t-1}\right)+ \\
&\left(1-a_{\mathrm{H}}\right)\left[\frac{2a_{\mathrm{H}}\left(\sigma\phi-1\right)-\left(\sigma-1\right)}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1}\right]\left[2\left(\widehat{Y}_{H,t}-\widehat{Y}^{fb}_{H,t}\right)+\left(\widehat{Y}^{fb}_{H,t}-\widehat{Y}^{fb}_{F,t}\right)\right]- \\
&\left(1-a_{\mathrm{H}}\right)\left[\frac{2a_{\mathrm{H}}\left(\phi-1\right)+1}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1}\right]\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}^*_{C,t}\right),
\end{aligned}$$

we can solve for the impact response on $\widehat{\mathcal{W}}_t$ for $j=0$ as a function only of exogenous shocks. The permanent response of the wealth gap under the optimal policy is given by:

$$\begin{aligned}
&\widehat{\mathcal{W}}_t\left[2a_{\mathrm{H}}\left(\phi-1\right)+1+\frac{2\left(1-a_{\mathrm{H}}\right)\left[2a_{\mathrm{H}}\left(\sigma\phi-1\right)-\left(\sigma-1\right)\right]}{\eta\left[4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1\right]+\sigma}\frac{1}{\beta\delta_2\left(1-\beta\delta_1\right)}\cdot\right. \\
&\left.\left(\begin{array}{c} 2a_{\mathrm{H}}\phi\frac{2a_{\mathrm{H}}(\sigma\phi-1)+1-\sigma}{2a_{\mathrm{H}}(\phi-1)+1}\left(1-\beta\right)+ \\ \left[2a_{\mathrm{H}}\left(\sigma\phi-1\right)+1\right]\left(\beta\delta_2-1\right)\end{array}\right)\right]= \\
&\left[2a_{\mathrm{H}}\left(\sigma\phi-1\right)+1-\sigma\right]\beta\sum_{j=0}^{\infty}\beta^j E_t\left[\left(\widehat{Y}^{fb}_{H,t+j+1}-\widehat{Y}^{fb}_{F,t+j+1}\right)-\left(\widehat{Y}^{fb}_{H,t+j}-\widehat{Y}^{fb}_{F,t+j}\right)\right]+ \\
&-\left[2a_{\mathrm{H}}\left(\sigma\phi-1\right)+1\right]\beta\sum_{j=0}^{\infty}\beta^j E_t\left[\left(\widehat{\zeta}_{C,t+j+1}-\widehat{\zeta}^*_{C,t+j+1}\right)-\left(\widehat{\zeta}_{C,t+j}-\widehat{\zeta}^*_{C,t+j}\right)\right]+ \\
&\left[2a_{\mathrm{H}}\left(\sigma\phi-1\right)+1-\sigma\right]\left(\widehat{Y}^{fb}_{H,t}-\widehat{Y}^{fb}_{F,t}\right)-\left[2a_{\mathrm{H}}\left(\sigma\phi-1\right)+1\right]\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}^*_{C,t}\right).
\end{aligned}$$

Similarly, we can derive the response of NFAs as a function of exogenous shocks.

3.2.3 Comparison with strict PPI price stability and proof of Proposition 10

Under PPI price stability the output gap obeys the following relation,

$$\left(\widehat{Y}_{H,t}-\widehat{Y}^{fb}_{H,t}\right)=-\left(1-a_{\mathrm{H}}\right)\frac{1+2a_{\mathrm{H}}\left(\sigma\phi-1\right)}{\eta\left[4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\sigma\phi-1\right)+1\right]+\sigma}\widehat{\mathcal{W}}_t,$$

and capital flows are given by:

$$\widehat{\mathcal{B}}_t = \widehat{\mathcal{B}}_{t-1} - \frac{(1-a_{\mathrm{H}})}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1}\beta\sum_{j=0}^{\infty}\beta^j$$

$$\left\{\begin{array}{c} (2a_{\mathrm{H}}(\sigma\phi-1)+1-\sigma)E_t\left(\left(\widehat{Y}^{fb}_{H,t+j+1}-\widehat{Y}^{fb}_{F,t+j+1}\right)-\left(\widehat{Y}^{fb}_{H,t+j}-\widehat{Y}^{fb}_{F,t+j}\right)\right)+ \\ -(2a_{\mathrm{H}}(\phi-1)+1)E_t\left(\left(\widehat{\zeta}_{C,t+1+j}-\widehat{\zeta}^*_{C,t+1+j}\right)-\left(\widehat{\zeta}_{C,t+j}-\widehat{\zeta}^*_{C,t+j}\right)\right) \end{array}\right\}.$$

As a result, the wealth gap is given by

$$\left[[2a_{\mathrm{H}}(\phi-1)+1]+2(1-a_{\mathrm{H}})\frac{(2a_{\mathrm{H}}(\sigma\phi-1)+1-\sigma)([2a_{\mathrm{H}}(\sigma\phi-1)+1])}{\eta[4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1]+\sigma}\right]\widehat{\mathcal{W}}_t =$$

$$\beta\sum_{j=0}^{\infty}\beta^j\left[\begin{array}{c} (2a_{\mathrm{H}}(\sigma\phi-1)+1-\sigma)E_t\left(\left(\widehat{Y}^{fb}_{H,t+j+1}-\widehat{Y}^{fb}_{F,t+j+1}\right)-\left(\widehat{Y}^{fb}_{H,t+j}-\widehat{Y}^{fb}_{F,t+j}\right)\right)+ \\ -(2a_{\mathrm{H}}(\phi-1)+1)E_t\left(\left(\widehat{\zeta}_{C,t+1+j}-\widehat{\zeta}^*_{C,t+1+j}\right)-\left(\widehat{\zeta}_{C,t+j}-\widehat{\zeta}^*_{C,t+j}\right)\right) \end{array}\right]$$

$$[2a_{\mathrm{H}}(\sigma\phi-1)+1-\sigma]\left(\widehat{Y}^{fb}_{H,t}-\widehat{Y}^{fb}_{F,t}\right)-[2a_{\mathrm{H}}(\sigma\phi-1)+1]\left(\widehat{\zeta}_{C,t}-\widehat{\zeta}^*_{C,t}\right)$$

Proof of Proposition 10: $\left|\widetilde{\mathcal{W}}^{na}_t\right| > \left|\widetilde{\mathcal{W}}_t\right|$ Compare the coefficient multiplying the wealth gap under PPI price stability and the optimal policy for the case $\eta = 0$ and $\sigma = 1$:

$$\begin{aligned} PPI\ coefficient &= [2a_{\mathrm{H}}(\phi-1)+1][4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\phi-1)+1] \\ Optimal\ coefficient &= [2a_{\mathrm{H}}(\phi-1)+1] \\ &\quad \left[\begin{array}{c} 4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\phi-1)+1+ \\ 4a_{\mathrm{H}}(1-a_{\mathrm{H}})\phi\frac{4a_{\mathrm{H}}^2(\phi-1)^2}{[2a_{\mathrm{H}}(\phi-1)+1]^2}\frac{(1-\beta)}{\beta\delta_2(1-\beta\delta_1)} \end{array}\right], \end{aligned}$$

where we also used the fact that:

$$\begin{aligned} & 1-\beta^2\delta_2\delta_1 \\ = \; & 1-\frac{1}{4}\left(1+\beta+\left[\frac{1}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\phi-1)+1}\right]\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\theta\right)^2 - \\ & \frac{1}{4}\left\{1+\beta+\left[\eta+\frac{\sigma}{4a_{\mathrm{H}}(1-a_{\mathrm{H}})(\sigma\phi-1)+1}\right]\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\theta\right\}^2+\beta \\ = \; & 1-\beta > 0. \end{aligned}$$

The first term in square bracket $[2a_{\mathrm{H}}(\phi-1)+1]$ is positive for $\phi > 1-1/2a_{\mathrm{H}}$, while the term in the second square bracket is always positive for both the PPI and the optimal policy coefficients, but larger under the optimal policy for $\phi \neq 1$. Hence, for given shocks, the wealth gap has always the same sign under both policies. Moreover, as its coefficient is larger when positive and smaller when negative, the wealth gap is always smaller in absolute value under the optimal policy.

We next proceed to derive expressions for the output gap under PPI price stability and under the optimal policy such that we compare outcomes. Under PPI price stability the output gap is given by,

$$E_t\left(\widehat{Y}_{H,t+j}-\widehat{Y}^{fb}_{H,t+j}\right)=-\left(1-a_{\mathrm{H}}\right)\left[1+2a_{\mathrm{H}}\left(\phi-1\right)\right]\widehat{\mathcal{W}}_t,$$

In contrast, under the optimal policy, the output gap is given by,

$$\begin{aligned}E_t\left(\widehat{Y}_{H,t+j}-\widehat{Y}^{fb}_{H,t+j}\right)&=-\left(1-a_{\mathrm{H}}\right)\left[2a_{\mathrm{H}}\left(\phi-1\right)+1\right]\cdot\\&\left[2a_{\mathrm{H}}\phi\frac{2a_{\mathrm{H}}\left(\phi-1\right)}{\left[2a_{\mathrm{H}}\left(\phi-1\right)+1\right]^2}\frac{\delta_1^j}{\beta\delta_2}+\frac{\beta\delta_2-1}{\beta\delta_2}\frac{1-\delta_1^{j+1}}{1-\delta_1}\right]\widehat{\mathcal{W}}_t,\end{aligned}$$

under the optimal policy, respectively. Therefore for given shocks, the responses of the output gap are given by

$$\begin{aligned}\left(\widehat{Y}_{H,t}-\widehat{Y}^{fb}_{H,t}\right)&=\frac{-\left(1-a_{\mathrm{H}}\right)}{\left[4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\phi-1\right)+1\right]}\cdot shocks\\\left(\widehat{Y}_{H,t}-\widehat{Y}^{fb}_{H,t}\right)&=\frac{-\left(1-a_{\mathrm{H}}\right)}{\left[4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\phi-1\right)+1+4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\phi\frac{4a_{\mathrm{H}}^2\left(\phi-1\right)^2}{\left[2a_{\mathrm{H}}\left(\phi-1\right)+1\right]^2}\frac{\left(1-\beta\right)}{\beta\delta_2\left(1-\beta\delta_1\right)}\right]}\cdot\\&\left[2a_{\mathrm{H}}\phi\frac{2a_{\mathrm{H}}\left(\phi-1\right)}{\left[2a_{\mathrm{H}}\left(\phi-1\right)+1\right]^2}\frac{\delta_1^j}{\beta\delta_2}+\frac{\beta\delta_2-1}{\beta\delta_2}\frac{1-\delta_1^{j+1}}{1-\delta_1}\right]\cdot shocks.\end{aligned}$$

On impact, the output gap is smaller in absolute value under the optimal policy. Observe that this is generically true since,

$$\begin{aligned}&\frac{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\phi-1\right)+1+4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\phi\frac{4a_{\mathrm{H}}^2\left(\phi-1\right)^2}{\left[2a_{\mathrm{H}}\left(\phi-1\right)+1\right]^2}\frac{\left(1-\beta\right)}{\beta\delta_2\left(1-\beta\delta_1\right)}}{4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\phi-1\right)+1}\\>\quad&1-\frac{1+4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\phi-1\right)}{\beta\delta_2\left[2a_{\mathrm{H}}\left(\phi-1\right)+1\right]^2}.\end{aligned}$$

is always satisfied. The left-hand side is always larger than 1, while the right hand side is positive but lower than 1, since $\beta\delta_2\geq 1$ and

$$\left[2a_{\mathrm{H}}\left(\phi-1\right)+1\right]^2\geq 1+4a_{\mathrm{H}}\left(1-a_{\mathrm{H}}\right)\left(\phi-1\right).$$

This establishes that for any value of the elasticity, the optimal policy trades-off more inflation volatility for more stability in the output gap and in the demand gap.

Proof of Lemma 2. Finally, also for the case $\eta=0$ and $\sigma=1$ we can derive expressions for capital flows under PPI stability and under the optimal policy,

$$\widehat{\mathcal{B}}_t = \widehat{\mathcal{B}}_{t-1} - \frac{(1-a_{\rm H})}{4a_{\rm H}(1-a_{\rm H})(\phi-1)+1}\beta\sum_{j=0}^{\infty}\beta^j$$

$$\left\{\begin{array}{c} (2a_{\rm H}(\phi-1))\, E_t\left(\left(\widehat{Y}^{fb}_{H,t+j+1}-\widehat{Y}^{fb}_{F,t+j+1}\right)-\left(\widehat{Y}^{fb}_{H,t+j}-\widehat{Y}^{fb}_{F,t+j}\right)\right)+ \\ -(2a_{\rm H}(\phi-1)+1)\, E_t\left(\left(\widehat{\zeta}_{C,t+1+j}-\widehat{\zeta}^*_{C,t+1+j}\right)-\left(\widehat{\zeta}_{C,t+j}-\widehat{\zeta}^*_{C,t+j}\right)\right)\end{array}\right\},$$

and under the optimal policy:

$$\begin{aligned}
&\widehat{\mathcal{B}}_t = \widehat{\mathcal{B}}_{t-1} - \frac{(1-a_{\rm H})}{4a_{\rm H}(1-a_{\rm H})(\phi-1)+1}\beta\sum_{j=0}^{\infty}\beta^j \\
&\left\{\begin{array}{c} (2a_{\rm H}(\phi-1))\, E_t\left(\left(\widehat{Y}^{fb}_{H,t+j+1}-\widehat{Y}^{fb}_{F,t+j+1}\right)-\left(\widetilde{Y}^{fb}_{H,t+j}-\widetilde{Y}^{fb}_{F,t+j}\right)\right)- \\ (2a_{\rm H}(\phi-1)+1)\, E_t\left(\left(\widehat{\zeta}_{C,t+1+j}-\widehat{\zeta}^*_{C,t+1+j}\right)-\left(\widehat{\zeta}_{C,t+j}-\widehat{\zeta}^*_{C,t+j}\right)\right)\end{array}\right\}+ \\
&\frac{(1-a_{\rm H})}{4a_{\rm H}(1-a_{\rm H})(\phi-1)+1}\frac{\beta}{1-\beta\delta_1}4a_{\rm H}(1-a_{\rm H})(\phi-1)\cdot \\
&\left\{\begin{array}{l} -2a_{\rm H}\phi\frac{2a_{\rm H}(\phi-1)}{2a_{\rm H}(\phi-1)+1}\frac{(1-\delta_1)}{\beta\delta_2}+ \\ \delta_1\left[2a_{\rm H}(\phi-1)+1\right]\frac{\beta\delta_2-1}{\beta\delta_2}\end{array}\right\}\widehat{\mathcal{W}}_t+ \\
&2(1-a_{\rm H})\left[\frac{2a_{\rm H}(\phi-1)}{4a_{\rm H}(1-a_{\rm H})(\phi-1)+1}\right]\frac{\beta}{1-\beta\delta_1}(1-\delta_1)\delta_1\left(\widehat{Y}_{H,t-1}-\widehat{Y}^{fb}_{H,t-1}\right).
\end{aligned}$$

Given the optimal solution for $\widehat{\mathcal{W}}_t$, relative to PPI price stability, expected shocks are now multiplied by the term

$$-\frac{(1-a_{\rm H})}{4a_{\rm H}(1-a_{\rm H})(\phi-1)+1}\left[\left[\, 1-\frac{1}{(1-\beta\delta_1)\delta_2}\cdot\right.\right.$$

$$\left.\frac{4a_{\rm H}(1-a_{\rm H})(\phi-1)\left\{4a_{\rm H}^2(\phi-1)\left[(\phi-1)(\beta\delta_2\delta_1-1)-(1-\delta_1)\right]+(1+4a_{\rm H}(\phi-1))(\beta\delta_2-1)\delta_1\right\}}{\left[2a_{\rm H}(\phi-1)+1\right]^2\left[4a_{\rm H}(1-a_{\rm H})(\phi-1)+1+4a_{\rm H}(1-a_{\rm H})\phi\frac{4a_{\rm H}^2(\phi-1)^2}{[2a_{\rm H}(\phi-1)+1]^2}\frac{(1-\beta)}{\beta\delta_2(1-\beta\delta_1)}\right]}\right];$$

since $\beta\delta_2\delta_1 = 1$ the above further simplifies:

$$\begin{aligned}
&-\frac{(1-a_{\rm H})}{4a_{\rm H}(1-a_{\rm H})(\phi-1)+1}\left[1-\frac{1-\delta_1}{\delta_2-1}\frac{4a_{\rm H}(1-a_{\rm H})(\phi-1)}{\left[2a_{\rm H}(\phi-1)+1\right]^2}\cdot\right. \\
&\left.\frac{4a_{\rm H}(1-a_{\rm H})(\phi-1)+1}{4a_{\rm H}(1-a_{\rm H})(\phi-1)+1+4a_{\rm H}(1-a_{\rm H})\phi\frac{4a_{\rm H}^2(\phi-1)^2}{[2a_{\rm H}(\phi-1)+1]^2}\frac{(1-\beta)}{\beta(\delta_2-1)}}\right].
\end{aligned}$$

The second term in brackets is positive for $\phi > 1$ and always less than 1 in absolute value, since $\delta_2 - 1 > 1 - \delta_1$:

$$\delta_2 - 1 = \frac{1}{2\beta}\left(1 - \beta + \left[\eta + \frac{\sigma}{4a_{\mathrm{H}}\left(1 - a_{\mathrm{H}}\right)\left(\sigma\phi - 1\right) + 1}\right]\frac{\left(1 - \alpha\beta\right)\left(1 - \alpha\right)}{\alpha}\theta\right) +$$
$$\frac{1}{2\beta}\sqrt{\left\{1 + \beta + \left[\eta + \frac{\sigma}{4a_{\mathrm{H}}\left(1 - a_{\mathrm{H}}\right)\left(\sigma\phi - 1\right) + 1}\right]\frac{\left(1 - \alpha\beta\right)\left(1 - \alpha\right)}{\alpha}\theta\right\}^2 - 4\beta},$$
$$1 - \delta_1 = \frac{1}{2\beta}\left(\beta - 1 - \left[\eta + \frac{\sigma}{4a_{\mathrm{H}}\left(1 - a_{\mathrm{H}}\right)\left(\sigma\phi - 1\right) + 1}\right]\frac{\left(1 - \alpha\beta\right)\left(1 - \alpha\right)}{\alpha}\theta\right) +$$
$$\frac{1}{2\beta}\sqrt{\left\{1 + \beta + \left[\eta + \frac{\sigma}{4a_{\mathrm{H}}\left(1 - a_{\mathrm{H}}\right)\left(\sigma\phi - 1\right) + 1}\right]\frac{\left(1 - \alpha\beta\right)\left(1 - \alpha\right)}{\alpha}\theta\right\}^2 - 4\beta},$$

which implies that,

$$\delta_2 - 1 \geq 1 - \delta_1 <=>$$
$$1 + \left[\eta + \frac{\sigma}{4a_{\mathrm{H}}\left(1 - a_{\mathrm{H}}\right)\left(\sigma\phi - 1\right) + 1}\right]\frac{\left(1 - \alpha\beta\right)\left(1 - \alpha\right)}{\alpha}\theta > \beta.$$

Therefore, optimal policy dampens capital flows for $\phi > 1$ and makes them larger in absolute value for $\phi < 1$.

4 Costly intermediation and stationarity of net foreign assets

Our results so far have been derived in a specification of the model in which both $\widehat{\mathcal{B}}_t$ and $\widehat{\mathcal{W}}_t$ are not stationary. In this subsection, we show that nonstationarity does not play any substantive role. In the literature, a standard approach to ensure that $\widehat{\mathcal{B}}_t$ is stationary in bond economies is to assume that its changes are subject to some (portfolio) adjustment costs; Gabaix and Maggiori [2015] have recently shown that this sluggish adjustment can result from costly intermediation of cross-border flows when financial intermediaries operate under borrowing constraints. In our framework, a simple way to capture the same idea is to posit deviations from the uncovered interest rate parity condition that are proportional to net foreign assets:

$$E_t\widehat{\mathcal{W}}_{t+1} - \widehat{\mathcal{W}}_t = -\delta\widehat{\mathcal{B}}_t.$$

With this modification, the solutions for $\widehat{\mathcal{B}}_t$ and $\widehat{\mathcal{W}}_t$ in the CO economy become:

$$\widehat{\mathcal{B}}_t = \gamma_1\widehat{\mathcal{B}}_t + (1-a_{\mathrm{H}})\sum_{j=0}^{\infty}\gamma_2^{-j-1}E_t\left[\left(\widehat{\zeta}_{C,t+1+j} - \widehat{\zeta}^*_{C,t+1+j}\right) - \left(\widehat{\zeta}_{C,t+j} - \widehat{\zeta}^*_{C,t+j}\right)\right],$$

$$\begin{aligned}\widehat{\mathcal{W}}_t &= \left(\frac{\widehat{\mathcal{B}}_{t-1} - \beta\widehat{\mathcal{B}}_t}{(1-a_{\mathrm{H}})\,\beta}\right) - \left(\widehat{\zeta}_{C,t} - \widehat{\zeta}^*_{C,t}\right)\\ &= -\left[\left(\widehat{\zeta}_{C,t} - \widehat{\zeta}^*_{C,t}\right) + \sum_{j=0}^{\infty}\gamma_2^{-j-1}E_t\left[\left(\widehat{\zeta}_{C,t+1+j} - \widehat{\zeta}^*_{C,t+1+j}\right) - \left(\widehat{\zeta}_{C,t+j} - \widehat{\zeta}^*_{C,t+j}\right)\right] - \frac{\gamma_1 - \beta}{(1-a_{\mathrm{H}})\,\beta}\widehat{\mathcal{B}}_{t-1}\right].\end{aligned}$$

where $\beta < \gamma_1 < 1 < \gamma_2$ are the roots of the characteristic equation associated with the above second-order difference equation:

$$\beta\gamma^2 - (1+\beta+\beta\delta)\,\gamma + 1 = 0.$$

Both $\widehat{\mathcal{W}}_t$ and $\widehat{\mathcal{B}}_t$ are now stationary, but still functions of exogenous shocks only, so the optimal targeting rules are the same as those derived above under both LCP and PCP. Therefore, optimal monetary policy will react in the same way to a capital inflow, by tightening under LCP and easing under PCP (although of course with a different strength). Clearly, setting $\delta = 0$ in the last expression leads to $\gamma_1 = 1$ and $\gamma_2 = 1/\beta$, which yields expressions (31) and (32) in the subsection 4.1.1 of the main text.