On the costs of sovereign default in quantitative models*

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Abstract

Quantitative models building on Eaton and Gersovitz (1981) have become the workhorse in the literature of sovereign default. The vast majority of this work assumes that in case of default, output falls according to an exogenous function. This paper argues that these models' predictions strongly depend on the default cost function, and commonly used functions yield entirely different results.

JEL CLASSIFICATIONS: F34, F32

KEYWORDS: sovereign debt, default costs, rollover risk, robustness.

1 Introduction

A large and growing literature employs quantitative models building on the early contributions of Eaton and Gersovitz (1981), Arellano (2008) and Aguiar and Gopinath (2006) to study sovereign debt and default. The framework considers a small open economy that can borrow from abroad, cannot commit to repay lenders, and decides at every period whether to default or not and how much debt to issue. In the vast majority of those models, sovereign default implies an exogenous drop in output from y to $h(Y)$. The function $h(y)$ is typically chosen to match some targets, such as the frequency of default and the volatility of spreads. Several functional forms for $h(y)$ have been adopted. Figure 1 presents some of the most common specifications.

The top left corner of Figure 1 displays the output costs of default in the model of Arellano (2008). When output is high, default brings it down to a constant low level, but when output is low, default does not affect it. Many papers in the literature employ this functional form.[1]

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[1]Examples include Cuadra and Sapriza (2008), Cuadra et al. (2010), Boz (2011), Arellano and Ramanarayanan (2012), Kim and Zhang (2012), Lizarazo (2013), Durdu et al. (2013), Fink and Scholl (2016), Arellano and Bai (2017), Salomao (2017), Scholl (2017), Pancrazi et al. (2020), Andreasen et al. (2019) and Kaas et al. (2020). Sánchez et al. (2018) use a similar specification.

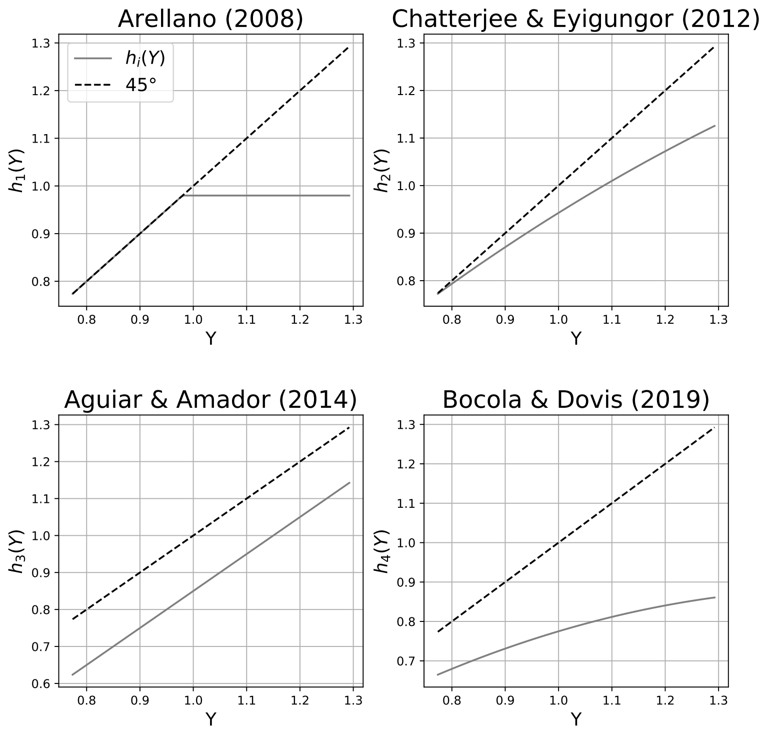

Figure 1: Default cost functions $h(Y)$

The bottom left corner of Figure 1 shows the function used in the handbook chapter by Aguiar and Amador (2014). Default leads to a constant drop in output. A similar specification is used by Roch and Uhlig (2018). Several papers in the literature assume a cost proportional to the level of output, which leads to similar results.[2]

The functional forms employed by Chatterjee and Eyigungor (2012) (top right graph of Figure 1) and Bocola and Dovis (2019) (bottom right graph of Figure 1) are the same, but the parameters are not, leading to different effects of default on output. Many papers in the literature employ this same functional form, with a variety of parameters.[3] Several other functional forms have been used, and the default cost is typically increasing in output, but not as sharply as in Arellano (2008).[4]

[2]Examples include Aguiar and Gopinath (2006), Alfaro and Kanczuk (2009), Hatchondo and Martinez (2009), Bai and Zhang (2010), Yue (2010), Guimaraes (2011), Rebelo et al. (2019) and Phan and Schwartzman (2020). Similar specifications are a natural choice for models that do not perform quantitative analysis (e.g., Gonçalves and Guimaraes (2015)).

[3]Examples include Hatchondo et al. (2014), Chatterjee and Eyigungor (2015), Hatchondo et al. (2016), Pouzo and Presno (2016), Kirsch and Ruhmkorf (2017), Jeon and Kabukcuoglu (2018), Na et al. (2018), Bocola et al. (2019), Mihalache (2020), Passadore and Xu (2018) and Onder and Sunel (2020).

[4]Examples include the functional forms in Gordon (2015) (also used in Gordon and Guerron-Quintana (2018)), Bianchi

This paper shows that the predictions of quantitative models of sovereign default strongly depend on the default cost function. We study a standard model, with standard calibration, and one of the four default cost functions depicted in Figure 1. With an average debt maturity of 2 years, the average debt-to-output ratio varies between 11% and 163%. The default frequency varies between 0.02% and 5% per year. The correlation between debt-to-output and GDP varies between -0.98 and 0.89. Similar differences are found in the model with short-term debt.

We then study how the effect of rollover risk in the model depends on the default cost function. In theory, liquidity issues might induce sovereigns to default on their outstanding debt. Hence, an individual's decision about buying a sovereign bond might depend on her expectations of what others will do. Sovereign debt prices and default decisions would then be affected by the possibility of miscoordination among lenders. But is this channel quantitatively important?

A branch of the literature on quantitative sovereign default models extends the basic framework to include self-fulfilling rollover crises, along the lines of Cole and Kehoe (2000). In Chatterjee and Eyigungor (2012), debt maturities as those observed in the data render the rollover problem unimportant. Roch and Uhlig (2018) argue that a lender-of-last-resort can eliminate the risk of miscoordination. A similar effect if present in Kirsch and Ruhmkorf (2017), but in their model financial assistance raises the level of debt and the probability of default. In Bocola and Dovis (2019), rollover risk is still important with long-term bonds and the sovereign adjusts debt maturity in response to changes in the odds of miscoordination among lenders. Bianchi et al. (2018) and Hernandez (2017) argue that some of the observed accumulation of reserves is a response of sovereigns to the risk of rollover crises.

We find that the impact of rollover risk can be very small, very large, or something in between, depending on the assumption about the default cost function. With default costs as in Arellano (2008), rollover risk has either a negligible effect or a very small effect, depending on debt maturity. In contrast, with default costs as in Bocola and Dovis (2019), rollover risk is extremely important. With one-period bonds, a 5%-probability of miscoordination among lenders reduces the debt-to-output ratio from 152% to 18% and turns the correlation between GDP and debt-to-output from -0.87 to 0.59. With an average debt maturity of 2 years, removing rollover risk from the model raises the average debt to output by about 50% and takes the correlation between GDP and debt-to-output from -0.85 to -0.10.

et al. (2018) (also used in Roettger (2019)), Arellano et al. (2019) and D'Erasmo and Mendoza (2019).

These findings imply that the results in this literature are not robust to changes in the specification of the output costs of default. Chatterjee and Eyigungor (2012) argues that rollover risk justifies the existence of long-term debt, but that does not hold in their models if default costs are as in Arellano (2008). In Bocola and Dovis (2019), the maturity structure of debt responds to shocks to the probability of miscoordination among lenders, which is crucial for their analysis, but this effect would be irrelevant if default costs were as in Arellano (2008) or Chatterjee and Eyigungor (2012). Last, the frequency of default in Arellano (2008) would fall to virtually zero with default costs as in Aguiar and Amador (2014) or Bocola and Dovis (2019).

The output costs in these models are supposed to reflect the losses implied by a default episode. There is no reason for them to vary according to the question a paper asks. Since some economies seem more prone to default than others, default costs might be different across countries, but why should the relationship between default costs and (detrended) output differ between Argentina and Greece? Which function should be more inclined?

The output losses from default are likely to result from a combination of factors, and it makes sense to include them in the model. Owing to the difficulties in estimating these costs, choosing one or two parameters to match relevant targets seems to be a sensible strategy. The literature has gone one step further, choosing default cost functions that allow the model to fit moments from the data. This practice may be criticized, but it is arguably in line with the prescription in Kydland and Prescott (1996) that the model economy should "mimic the world as closely as possible along a limited, but clearly specified, number of dimensions".

This paper shows, however, that the set of admissible choices for the default cost function allows for drastically different results. The discipline needed for a sound computational experiment, as envisioned by Kydland and Prescott (1996), seems to be missing. A better understanding of how to incorporate default costs in quantitative models of sovereign default is a crucial bottleneck in this literature.

A few models in this literature obtain endogenous costs of default. Examples include Mendoza and Yue (2012), Niemann and Pichler (2017), Park (2017), Sosa-Padilla (2018), and Thaler (2019). While there is much to learn from these papers, it is not clear this would be the best approach to the problem in general, since each model typically considers only one source of default cost.

A growing literature attempts to estimate default costs (see, e.g., Zymek (2012), Hebert and Schreger (2017) and Andrade and Chhaochharia (2018)). From the findings of this literature to a function linking costs of default to pre-default output, there is still a long way to go. Nevertheless, moments from the data might hold the key to the problem. Sovereign defaults are relatively rare events, but research using large historical databases has uncovered many important findings in this literature (see, e.g.,Tomz and Wright (2007), Cruces and Trebesch (2013), and Meyer et al. (2019)). Empirical estimates about output falls in the aftermath of sovereign default would provide much-needed targets for quantitative models.

The paper is organized as follows. Section 2 presents the model, Section 3 shows the results, and Section 4 concludes.

2 Model

The model follows Chatterjee and Eyigungor (2012). They build on Aguiar and Gopinath (2006) and Arellano (2008), borrowing the self-fulfilling debt crises features from Cole and Kehoe (2000) and including a maturity structure to sovereign debt.

Time is discrete and runs forever. The decision maker is a benevolent government of a small open economy that borrows from risk-neutral foreign creditors. The economy is endowed with an exogenous stochastic output stream Y_t, where $Y_t = \exp\{y_t\}$ and:

$$y_{t+1} = \rho y_t + \eta \varepsilon_{y,t+1}, \qquad \varepsilon_{y,t+1} \sim \mathcal{N}(0,1).$$

Households within the country are identical and maximize:

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t u\left(c_t\right), \quad \text{with} \quad u(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma}$$

The only credit instrument available to the government is a multi-period non state contingent bond B with exogenous maturity decay λ traded in international credit markets. In equilibrium, risk neutral lenders must be compensated by the endogenously determined default risk.

Every period, a fraction λ of the outstanding debt B matures. We denote by q the current price of its whole structure of debt, so that if the government issues qB' in the current period, it will have to pay $\lambda B'$ in the following period. The budget constraint is:

$$c = Y - \lambda B + q\left(B', Y\right)\left[B' - (1-\lambda)B\right].$$

To rule out Ponzi schemes we must have $B \leq Z$ for some large enough Z that is never binding in equilibrium.

At each period t, the government chooses between defaulting and meeting its current obligations. Defaults entail two costs: the country loses access to foreign credit markets;

and output falls from Y to $h(Y)$, where $0 \leq h(Y) \leq Y$. Output returns to Y only after regaining access to international markets, which happens with probability θ at every subsequent period.

The timing of events is designed to allow for the possibility of multiple equilibria:

- Output Y and a sunspot variable s_t are realized, $s_t \sim \mathcal{U}(0,1)$.
- The government chooses B'.
- Lenders post price schedules for the bond.
- The government decides whether to default ($d_t = 1$) or not ($d_t = 0$).

Table 1 illustrates the game played by lenders and the government, where Δ represents lenders' losses in case of default.

Table 1: Static Coordination Game

	Repay	Default
Lend	$0, V_c$	$-\Delta, V_d$
Don't Lend	$0, V_{\text{no roll}}$	$0, V_d$

If $V_{\text{no roll}} \geq V_d$ there is a unique equilibrium (since it must be that $V_c \geq V_{\text{no roll}}$). However, if $V_{\text{no roll}} < V_d$, multiple equilibrium arise: both {Repay, Lend} and {Default, Don't Lend} sets of strategies are Nash equilibria in pure strategies. To select among them, we follow the literature and employ a sunspot device. In every period, there is an exogenous probability π that agents coordinate in the {Default, Don't Lend} equilibrium and a probability $(1-\pi)$ of coordinating in the {Repay, Lend} equilibrium. Hence, if $s_t \leq \pi$, agents coordinate in the equilibrium {Default, Don't Lend} if that one exists – i.e., if $V_{\text{no roll}} < V_d$.

Finally we define $\mathbb{E}_t[d_{t+1}] \equiv \delta_t$ and $\xi_t \equiv \mathbb{1}_{\{s_t \leq \pi\}}$. If $\xi_t = 1$, foreign investors will only choose to lend if even if they do not buy the bonds the government intends to issue, the government will not choose to default.

The decision problem for the government can be written using three sub-problems. The value of repaying conditional on lenders rolling over the debt, $V_c(B, y)$, is defined as follows:

$$V_c(B,Y) = \max_{B' \leq Z} \left\{ u(Y - \lambda B + q\left(B', Y\right)\left[B' - (1-\lambda)B\right]) + \beta \mathbb{E}\left[V\left(B', Y'\right)\right] \right\}.$$

The value of repaying conditional on lenders not rolling over the debt is defined by:

$$V_{\text{no roll}}(B,Y) = \left\{ u(Y - \lambda B) + \beta \mathbb{E}\left[V\left(B', Y'\right)\right] \right\}$$

Finally, the value of defaulting, $V_d(Y)$, is:

$$V_d(Y) = u\left(h\left(Y\right)\right) + \beta \mathbb{E}\left[\theta V_c\left(0, Y'\right) + (1-\theta) V_d\left(Y'\right)\right].$$

Therefore, value function of the government can then be written as:

$$V(B,Y) = \begin{cases} V_c(B,Y) & \text{if } V_{\text{no roll}}(B,Y) \geq V_d(Y) \\ V_c(B,Y) & \text{if } V_{\text{no roll}}(B,Y) < V_d(Y), \xi = 0, V_c(B,Y) > V_d(Y) \\ V_d(Y) & \text{if } V_{\text{no roll}}(B,Y) < V_d(Y), \xi = 1 \\ V_d(Y) & \text{if } V_c(B,Y) < V_d(Y) \end{cases}$$

2.1 Equilibrium

The equilibrium price that guarantees that the zero-profits condition of lenders is given by the following recursion:

$$q(B', Y) = \frac{1}{1+r}(1-\delta)\mathbb{E}\left[\lambda + (1-\lambda) q(B'', Y')\right]. \tag{1}$$

Given B' the probability of default next period is:

$$\delta = (1 - \mathbb{E}[\xi'])\mathbb{E}[\mathbb{1}_{\{V_c(B,Y) < V_d(Y)\}}] + \mathbb{E}[\xi']\mathbb{E}[\mathbb{1}_{\{V_{\text{no roll}}(B,Y) < V_d(Y)\}}]$$

Finally the recursive equilibrium comprises:

1. a pricing function $q(B', Y)$,
2. a quadruple of value functions $(V_c(B,Y), V_d(Y), V_{\text{no roll}}(B,Y), V(B,Y))$,
3. a decision rule telling the government when to default and when to pay as a function of the state (B,Y),
4. an asset accumulation rule that, conditional on choosing not to default, maps (B,Y) into B',

such that:

1. the four Bellman equations for $(V_c(B,Y), V_d(Y), V_{\text{no roll}}(B,Y), V(B,Y))$ are satisfied,
2. given the price function $q(B', Y)$, the default decision rule and the asset accumulation decision rule attains the optimal value function $V(B,Y)$ and
3. the price function $q(B', Y)$ satisfies equation (1).

2.2 Default Cost Functions

The following default cost functions can be visualized in Figure 1. The top left panel exhibits default cost function in Arellano (2008) :

$$h_1(Y) = \begin{cases} \alpha\mathbb{E}[Y] & \text{if } Y > \mathbb{E}[Y] \\ Y & \text{if } Y \leq \mathbb{E}[Y]. \end{cases}$$

The default cost function in Chatterjee and Eyigungor (2012), at the top right panel, is:

$$h_2(Y) = Y - \max\{0, d_0Y + d_1Y^2\}, d_1 \geq 0 \text{ and } d_0 < 0.$$

The default cost function in Aguiar and Amador (2014), at the bottom left panel, is:

$$h_3(Y) = Y - \tau.$$

Finally, the default cost function in Bocola and Dovis (2019), portrayed in the bottom right panel, is:[5]

$$h_4(Y) = Y - \max\{0, d_0Y + d_1Y^2\},\ d_0 = \frac{(a - b\exp\left(-3\left(\frac{\eta^2}{1-\rho^2}\right)^{1/2}\right)}{(1 - \exp\left(-3\left(\frac{\eta^2}{1-\rho^2}\right)^{1/2}\right)} \text{ and } d_1 = b - d_0.$$

2.3 Model Solution and Calibration

The solution method is standard in the literature. The process for y_t is discretized into a Markov Process using Tauchen's Quadrature Method. We set the grids for Y with 200 equally spaced points. The upper and lower bounds for B are $[0, 7.5]$ and there are 1000 equally spaced points.

The calibration of the model follows closely Chatterjee and Eyigungor (2012) and is standard in the literature. The parameter values of default cost functions are equal to those in the respective original paper. Table 2 shows all calibrated parameters.

3 Results

3.1 The effect of the output loss function with no rollover risk

We first show results for the model with no rollover risk ($\pi = 0$). This is the case considered by the vast majority of papers employing this framework. The literature is

[5]Notice that $\exp\left(-3\left(\frac{\eta^2}{1-\rho^2}\right)^{1/2}\right)$ is the value of output three standard deviations below its average value. The parameter a stands for the percentage loss in output after a default when output is three standard deviations below its average value. The parameter b represents the percentage loss in output when the latter is at its average value.

Table 2: Calibration

Parameters of the model		
β	0.954	Time discount parameter
γ	2	Risk aversion
r	0.01	International interest rate
ρ	0.9485	Persistence of output shock
η	0.0271	St. dev output shock
θ	0.0385	Prob of regaining access
Default cost functions		
h_1	$\alpha = 0.969$	
h_2	$d_0 = -0.188$ and $d_1 = 0.245$	
h_3	$\tau = 0.15$	
h_4	$a = 0.1406$ and $b = 0.225$	

typically interested in understanding the implications of the model for the debt-to-output ratio, the frequency of default, and the relation between GDP, debt-to-output and the interest rate spread. For each specification of the function h, we solve the model for the case with one-period bonds only ($\lambda = 1$) and with an average debt maturity of two years ($\lambda = 0.125$). We then simulate these economies. Table 3 reports the results. The time unit is one year.

The discussion of results focuses on the case with two-year bonds, which is the empirically more relevant one, but results are somewhat similar with $\lambda = 1$ and $\lambda = 0.125$.

The amount of debt and frequency of default vary widely. The debt-to-output ratio goes from around 10% using h_1, to around 20% using h_2, and to 130% or 160% using h_3 and h_4. The default frequency with two-year debt goes from once every 20 years using h_1, to once every 200 years with h_2, once every 2,000 years using h_4 and almost never with h_3.

In order to understand these results, it is useful to first consider a simple example with constant output and no shocks. Owing to the losses from default, repaying debt is incentive compatible for the country, but only if debt is below a certain threshold. Hence the price of debt is $1/(1+r)$ if debt is below that level (because it would be risk-free), but it is equal to 0 otherwise (because default would occur with certainty). Since the time discount factor is low, the country borrows up to the incentive-compatible level of debt.

If the economy is subject to shocks, the output losses from default might vary along the business cycle. At time t, these losses are given by the difference between Y and $h(Y)$. It is easy to see from Figure 1 that $Y - h(Y)$ varies a lot with the business cycle when

	h_1	h_2	h_3	h_4
$\lambda = 1$				
debt to ouput mean	0,057	0,195	1,214	1,524
debt to output std dev	0,024	0,023	0,120	0,081
default frequency	0,71%	0,27%	0,02%	0,04%
corr(debt to ouput, gdp)	0,947	0,905	-0,980	-0,868
corr(spread, gdp)	-0,106	-0,229	-0,238	-0,172
corr(debt to ouput, spread)	-0,057	-0,099	0,234	0,190
$\lambda = 0.125$				
debt to ouput mean	0,108	0,219	1,298	1,634
debt to output std dev	0,070	0,029	0,129	0,087
default frequency	4,86%	0,56%	0,02%	0,05%
corr(debt to ouput, gdp)	0,890	0,872	-0,977	-0,851
corr(spread, gdp)	0,071	-0,115	-0,236	-0,153
corr(debt to ouput, spread)	0,134	-0,004	0,234	0,173

Table 3: Results for the model with different specifications for the default cost: h_1, as in Arellano (2008); h_2, as in Chatterjee and Eyigungor (2012); h_3, as in Aguiar and Amador (2014); h_4, as in Bocola and Dovis (2019).

function h_1 is used, but is constant or close to constant with functions similar to h_3.

If $Y - h(Y)$ is constant or close to constant, as in the case of function h_3, the situation is similar to the deterministic case. Hence the country chooses to borrow up to the point that repaying is always incentive compatible. Further borrowing from that point on would be very expensive, since a very small increase in debt would imply a large increase in the default probability and thus a large fall in the price of debt. In contrast, if the output losses from default are given by h_1, borrowing always entails a risk, since a sequence of bad shocks taking the economy to a state of very low output would imply zero default cost in the current period and very small expected default cost in the near future. Lending in this case is always risky.

In an economy with shocks to $Y - h(Y)$, the sovereign faces a trade-off: more debt implies a larger set of states where default is the optimal choice and, consequently, a lower price of debt. With a low time discount factor, the country has incentives to borrow even if the price of debt is significantly lower than $1/(1+r)$, but not if it is too low. But if $Y - h(Y)$ is always high, repaying large amounts is incentive compatible, so issuing debt is cheap. This is why the amount of debt varies so much.

Table 3 also shows that the correlation between GDP and the debt-to-output ratio ranges from -0.98 with default function h_3 to around 0.90 with default function h_1. This is because with function h_3, the default penalty $Y - h(Y)$ is nearly constant, so the level of debt is close to constant, which leads to a correlation between output and debt-to-output close to -1. In contrast, with function h_1, the default penalty at t and the expected default penalty in the near future sharply increase with output, and so does the debt-to-output ratio.

Default functions h_2 and h_4 have the same functional form but different parameters and widely different implications for the debt-to-output ratio (-0.87 and 0.85, respectively). Again, the difference stems from how $Y - h(Y)$ varies with Y. Shifting up the intercept of h_4 in 1 would raise that correlation. Different parameters could yield any number in the range $[-0.87, 0.85]$.

The interest rate spread varies vary little with default functions h_3 and h_4 (and no rollover risk) because the odds of default are always very low. Hence the correlations between the spread and other variables are less interesting in these cases. With default functions h_1 and h_2, the default penalty $Y - h(Y)$ varies a lot along the business cycle, leading to large changes in the amount a country can borrow without exposing itself to default risk. Since the time-preference factor β is relatively low, countries will choose to go beyond that level, but the correlation between default risk (given by the interest rate spread) and output (ou debt) might go either way. Table 3 shows that the shape of the default cost function affects whether this correlation is (mildly) positive (with default cost function h_1) or (mildly) negative (with default cost h_2).

3.2 Does rollover risk matter?

Table 4 shows how the inclusion of rollover risk affects the model for each specification of the function h. As before, we consider an economy with one-period bonds only ($\lambda = 1$) and with an average debt maturity of two years ($\lambda = 0.125$), and the time unit is one year. The table shows results for $\pi = 0$ and $\pi = 5\%$, but Table 5 in the appendix shows that larger values of π and $\pi = 5\%$ yield very similar results. A five-percent increase in the probability of default would lead to an increase in interest rates of 5-percent per quarter, hence with a five-percent probability of miscoordination, the sovereign will avoid exposing itself to rollover risk.[6] In consequence, debt crises owing to miscoordination among lenders will be rare. However, this high probability of pessimistic beliefs might affect the level of debt governments can sustain in order to avoid this risk.

[6] With smaller values of π, rollover crises would occur more often in equilibrium, but the comparison across specifications of the h function would yield similar insights.

	$\lambda = 1$		$\lambda = 0.125$	
h_1	$\pi = 0$	$\pi = 0.05$	$\pi = 0$	$\pi = 0.05$
debt to ouput mean	0,057	0,052	0,108	0,107
debt to output std dev	0,024	0,022	0,070	0,070
default frequency	0,71%	0,75%	4,86%	4,89%
corr(debt to ouput, gdp)	0,947	0,960	0,890	0,891
corr(spread, gdp)	-0,106	-0,201	0,071	0,070
corr(debt to ouput, spread)	-0,057	-0,141	0,134	0,134
h_2	$\pi = 0$	$\pi = 0.05$	$\pi = 0$	$\pi = 0.05$
debt to ouput mean	0,195	0,096	0,219	0,217
debt to output std dev	0,023	0,011	0,029	0,030
default frequency	0,27%	0,23%	0,56%	0,60%
corr(debt to ouput, gdp)	0,905	0,929	0,872	0,875
corr(spread, gdp)	-0,229	-0,452	-0,115	-0,133
corr(debt to ouput, spread)	-0,099	-0,356	-0,004	-0,022
h_3	$\pi = 0$	$\pi = 0.05$	$\pi = 0$	$\pi = 0.05$
debt to ouput mean	1,214	0,163	1,298	0,961
debt to output std dev	0,120	0,004	0,129	0,063
default frequency	0,02%	0,02%	0,02%	0,03%
corr(debt to ouput, gdp)	-0,980	-0,767	-0,977	-0,959
corr(spread, gdp)	-0,238	-0,277	-0,236	-0,361
corr(debt to ouput, spread)	0,234	0,329	0,234	0,366
h_4	$\pi = 0$	$\pi = 0.05$	$\pi = 0$	$\pi = 0.05$
debt to ouput mean	1,524	0,176	1,634	1,114
debt to output std dev	0,081	0,004	0,087	0,053
default frequency	0,04%	0,04%	0,05%	0,16%
corr(debt to ouput, gdp)	-0,868	0,586	-0,851	-0,105
corr(spread, gdp)	-0,172	-0,356	-0,153	-0,251
corr(debt to ouput, spread)	0,190	-0,025	0,173	0,178

Table 4: Results for the model with and without rollover risk, and with different specifications for the default cost: h_1, as in Arellano (2008); h_2, as in Chatterjee and Eyigungor (2012); h_3, as in Aguiar and Amador (2014); h_4, as in Bocola and Dovis (2019).

Figures 2-9 in Appendix A show how each of these economies behave following a particular sequence of shocks. Visualizing the path of the economy might help to build intuition. Each chart compares the case with no rollover risk ($\pi = 0$) and the case with $\pi = 0.05$. Figures 2 and 3 refer to an economy with output costs of default given by function h_1. The difference between them is the average debt maturity ($\lambda = 1$ in Figure 2 and $\lambda = 0.125$ in Figure 3). Figures 4 and 5 repeat the same exercise using output costs of default given by function h_2. Figures 6 and 7 repeat the exercise using function h_3. And Figures 8 and 9 repeat this exercise using function h_4. The time unit is a quarter.

As Table 4 makes clear, the effect of rollover risk can be anything between very small and very large depending on the default cost function.

Miscoordination among lenders forces the sovereign to repay all its maturing debt. If this is a large part of a country's quarterly GDP, the marginal utility in case of debt repayment might be very high and default might be the best option. Hence, rollover risk is a particularly big problem in case of one-period debt. That's why for any default cost function, raising π to 5% has a larger effect on the economy in case of one-period bonds ($\lambda = 1$).

With default cost functions given by h_1 and 2-year bonds, debt is on average around 10% of GDP, so the amount of debt maturing at a given period is not much more than 1% of GDP. Hence repaying all maturing debt is worse than rolling it over and paying interests for a longer period, but the difference is very small. Hence the effect of rollover risk is negligible. Table 4 and Figure 3 show that the economies with $\pi = 0$ and $\pi = 0.05$ behave in the same way. Incentives for default owing to changes in the default penalty (the expected values of $Y - h(Y)$) dwarf the effects of not being able to roll over maturing bonds.

Table 4 and Figure 2 show that even with one-period debt, the differences between the economies with $\pi = 0$ and $\pi = 0.05$ are small when default costs are given by h_1. If debt-to-output is around 5% of yearly GDP, repaying debt in full in a quarter is a bit costly, but changes in $Y - h(Y)$ are far more important in the decision about defaulting on debt.

The first paper to incorporate rollover risk in a quantitative model of sovereign debt and default was Chatterjee and Eyigungor (2012). In their model, without rollover risk, the government would only issue one-period debt. Once we consider the possibility of miscoordination among agents, longer maturities become optimal because they nullify rollover risk. As before, the intuition is that the amount of maturing debt is small and changes in $Y - h(Y)$ are relatively large.

Indeed, using their default cost function h_2, rollover risk is important in an economy with one-period debt only (Figure 4), but not when the average debt maturity is two years (Figure 5). Since the debt-output ratio is around 20%, the amount maturing in a given period is not large enough to affect default decisions.

Bocola and Dovis (2019) extends Chatterjee and Eyigungor (2012) framework to quantify the importance of rollover risk on Italian interest rate spreads. In their model, the government chooses not only the amount of debt it should issue, but also the optimal debt maturity. Crucially, the optimal debt maturity responds to shocks to the probability of miscoordination among lenders. This is because rollover risk is important even with long term debt.

Figure 8 shows that with one-period debt, rollover risk has a huge impact on the level of debt. As shown in Table 4, with default cost function h_4, rollover risk brings the debt-GDP ratio to about one ninth of its value when pessimistic beliefs are ruled out. The effect is still large with $\lambda = 0.125$ because with $\pi = 0$, average debt would be 163% of GDP. Hence, miscoordination among lenders would imply paying around 20% of GDP to foreigners, which would have a large effect on agents' utility and make default the optimal decision. The sovereign prefers to borrow less to avoid this problem.

Bocola and Dovis (2019) use the maturity structure of the Italian debt to assess whether default risk is driven by fundamental shocks or by lenders' pessimistic beliefs about debt repayment. If output losses are like others assume, the question has already been answered from the start, rollover risk is irrelevant with long-term debt. The answer to their question crucially depends on the default cost function employed in the quantitative analysis.

When the output losses from default are given by h_3, as in Aguiar and Amador (2014), default almost never occurs in equilibrium and debt is close to risk free. The model would require other ingredients to generate the observed fluctuations in debt prices. If the default cost is large enough, a lot of debt can be sustained in equilibrium and rollover risk matters.

One important difference between the models with default cost functions given by h_3 and h_4 is that in the latter case, with rollover risk, the correlation between debt-to-output and GDP gets close to zero with two-year bonds (and positive and large with one-period debt). For rollover risk to be important, debt maturing at a given period must be large. The default cost function h_4 guarantees that ($Y - h(Y)$ is always large), but also implies incentives for default conditional on a rollover crisis vary with the business cycle ($Y - h(Y)$ is much larger when Y is high). This leads to an increase in debt-to-output at times of high GDP. But again, this effect relies on a particular default cost function.

4 Concluding remarks

The relation between output costs of default and the business cycle is crucial for the predictions of sovereign default models. The set of admissible functions portraying this relationship allows for drastically different results. A better understanding of how large are the output losses from default and how they vary across the business cycle would enable us to discard some of the existing explanations in the literature and shape a research agenda for moving forward.

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A Additional tables and figures

	$\lambda = 1$			$\lambda = 0.125$		
h_1	$\pi = 5\%$	$\pi = 20\%$	$\pi = 90\%$	$\pi = 5\%$	$\pi = 20\%$	$\pi = 90\%$
debt to ouput mean	0,052	0,047	0,042	0,107	0,107	0,104
debt to output std dev	0,022	0,020	0,018	0,070	0,070	0,069
default frequency	0,75%	0,70%	0,60%	4,89%	4,89%	4,80%
corr(debt to ouput, gdp)	0,960	0,962	0,960	0,891	0,891	0,892
corr(spread, gdp)	-0,201	-0,226	-0,178	0,070	0,072	0,072
corr(debt to ouput, spread)	-0,141	-0,169	-0,136	0,134	0,136	0,135
h_2	$\pi = 5\%$	$\pi = 20\%$	$\pi = 90\%$	$\pi = 5\%$	$\pi = 20\%$	$\pi = 90\%$
debt to ouput mean	0,096	0,091	0,088	0,217	0,217	0,215
debt to output std dev	0,011	0,011	0,011	0,030	0,031	0,031
default frequency	0,23%	0,21%	0,18%	0,60%	0,61%	0,63%
corr(debt to ouput, gdp)	0,929	0,937	0,945	0,875	0,874	0,876
corr(spread, gdp)	-0,452	-0,410	-0,365	-0,133	-0,136	-0,141
corr(debt to ouput, spread)	-0,356	-0,318	-0,279	-0,022	-0,024	-0,030
h_3	$\pi = 5\%$	$\pi = 20\%$	$\pi = 90\%$	$\pi = 5\%$	$\pi = 20\%$	$\pi = 90\%$
debt to ouput mean	0,163	0,161	0,160	0,961	0,956	0,952
debt to output std dev	0,004	0,004	0,004	0,063	0,063	0,062
default frequency	0,02%	0,02%	0,02%	0,03%	0,03%	0,02%
corr(debt to ouput, gdp)	-0,767	-0,759	-0,785	-0,959	-0,955	-0,966
corr(spread, gdp)	-0,277	-0,227	-0,198	-0,361	-0,323	-0,289
corr(debt to ouput, spread)	0,329	0,280	0,250	0,366	0,326	0,292
h_4	$\pi = 5\%$	$\pi = 20\%$	$\pi = 90\%$	$\pi = 5\%$	$\pi = 20\%$	$\pi = 90\%$
debt to ouput mean	0,176	0,173	0,170	1,114	1,085	1,060
debt to output std dev	0,004	0,004	0,004	0,053	0,048	0,043
default frequency	0,04%	0,03%	0,03%	0,16%	0,13%	0,11%
corr(debt to ouput, gdp)	0,586	0,596	0,620	-0,105	-0,126	-0,168
corr(spread, gdp)	-0,356	-0,314	-0,277	-0,251	-0,214	-0,195
corr(debt to ouput, spread)	-0,025	-0,010	-0,002	0,178	0,174	0,180

Table 5: Results for the model with $\pi \in \{0.05, 0.20, 0.90\}$, and with different specifications for the default cost: h_1, as in Arellano (2008); h_2, as in Chatterjee and Eyigungor (2012); h_3, as in Aguiar and Amador (2014); h_4, as in Bocola and Dovis (2019).

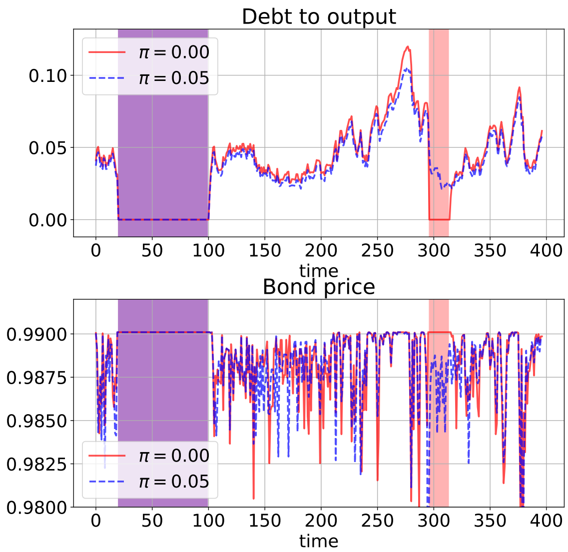

Figure 2: Simulated Data, default cost given by h_1, $\lambda = 1$. Red lines show results from a case with no rollover risk ($\pi = 0$). Dashed blue lines show results with $\pi = 0.05$. Colored shaded areas represent time excluded from financial markets after a sovereign default in each case.

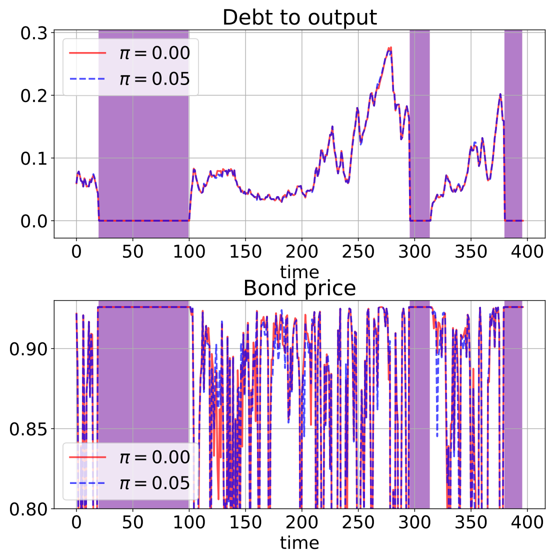

Figure 3: Simulated Data, default cost given by h_1, $\lambda = 0.125$. Red lines show results from a case with no rollover risk ($\pi = 0$). Dashed blue lines show results with $\pi = 0.05$. Colored shaded areas represent time excluded from financial markets after a sovereign default in each case.

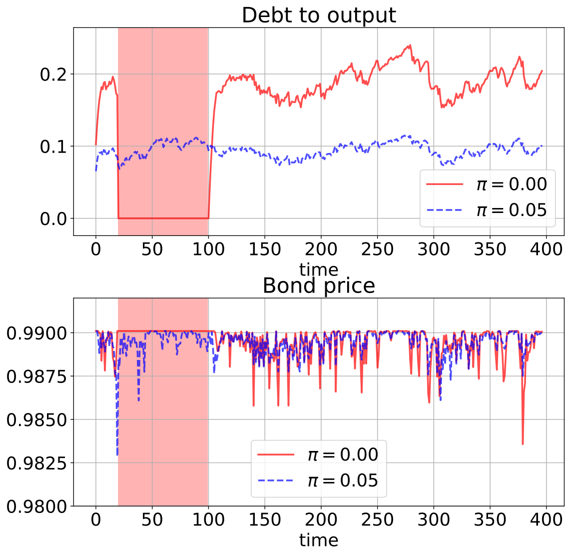

Figure 4: Simulated Data, default cost given by h_2, $\lambda = 1$. Red lines show results from a case with no rollover risk ($\pi = 0$). Dashed blue lines show results with $\pi = 0.05$. Colored shaded areas represent time excluded from financial markets after a sovereign default in each case.

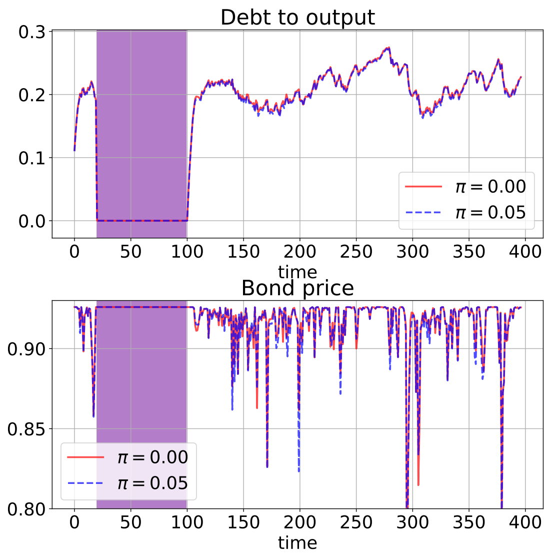

Figure 5: Simulated Data, default cost given by h_2, $\lambda = 0.125$. Red lines show results from a case with no rollover risk ($\pi = 0$). Dashed blue lines show results with $\pi = 0.05$. Colored shaded areas represent time excluded from financial markets after a sovereign default in each case.

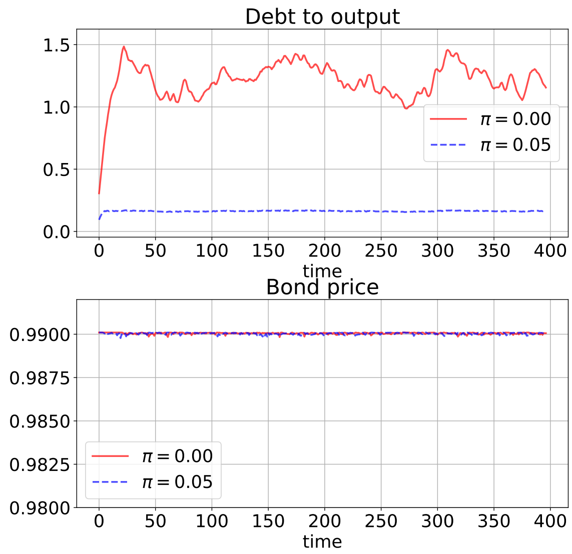

Figure 6: Simulated Data, default cost given by h_3, $\lambda = 1$. Red lines show results from a case with no rollover risk ($\pi = 0$). Dashed blue lines show results with $\pi = 0.05$. Colored shaded areas represent time excluded from financial markets after a sovereign default in each case.

Figure 7: Simulated Data, default cost given by h_3, $\lambda = 0.125$. Red lines show results from a case with no rollover risk ($\pi = 0$). Dashed blue lines show results with $\pi = 0.05$. Colored shaded areas represent time excluded from financial markets after a sovereign default in each case.

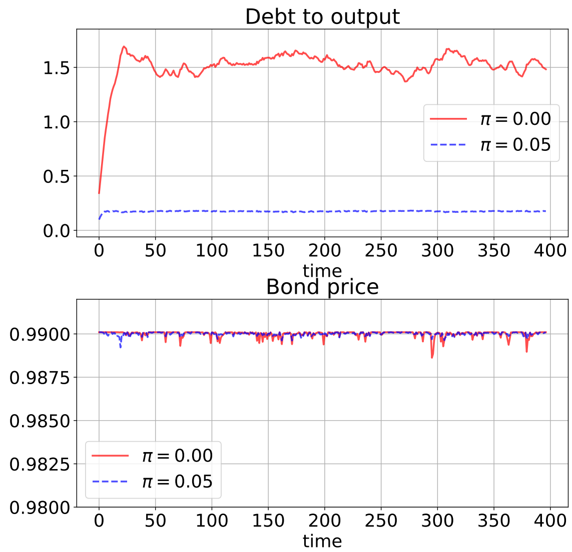

Figure 8: Simulated Data, default cost given by h_4, $\lambda = 1$. Red lines show results from a case with no rollover risk ($\pi = 0$). Dashed blue lines show results with $\pi = 0.05$. Colored shaded areas represent time excluded from financial markets after a sovereign default in each case.

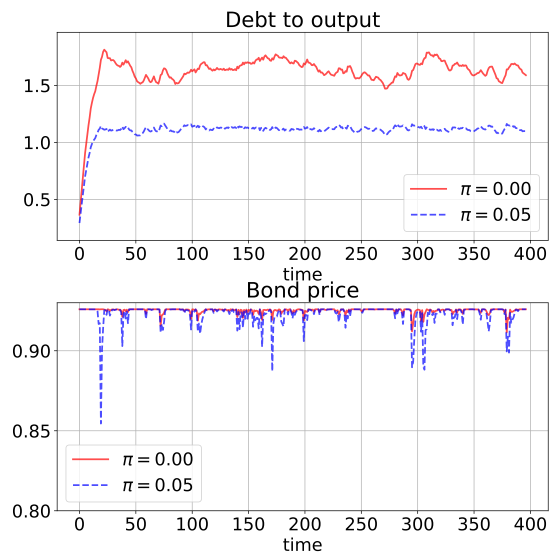

Figure 9: Simulated Data, default cost given by h_4, $\lambda = 0.125$. Red lines show results from a case with no rollover risk ($\pi = 0$). Dashed blue lines show results with $\pi = 0.05$. Colored shaded areas represent time excluded from financial markets after a sovereign default in each case.